

Derivation of the principles of optimal expansion of bioenergy based on forest resources in combination with fossil fuels, CCS and combined heat and power production with consideration of economics and global warming

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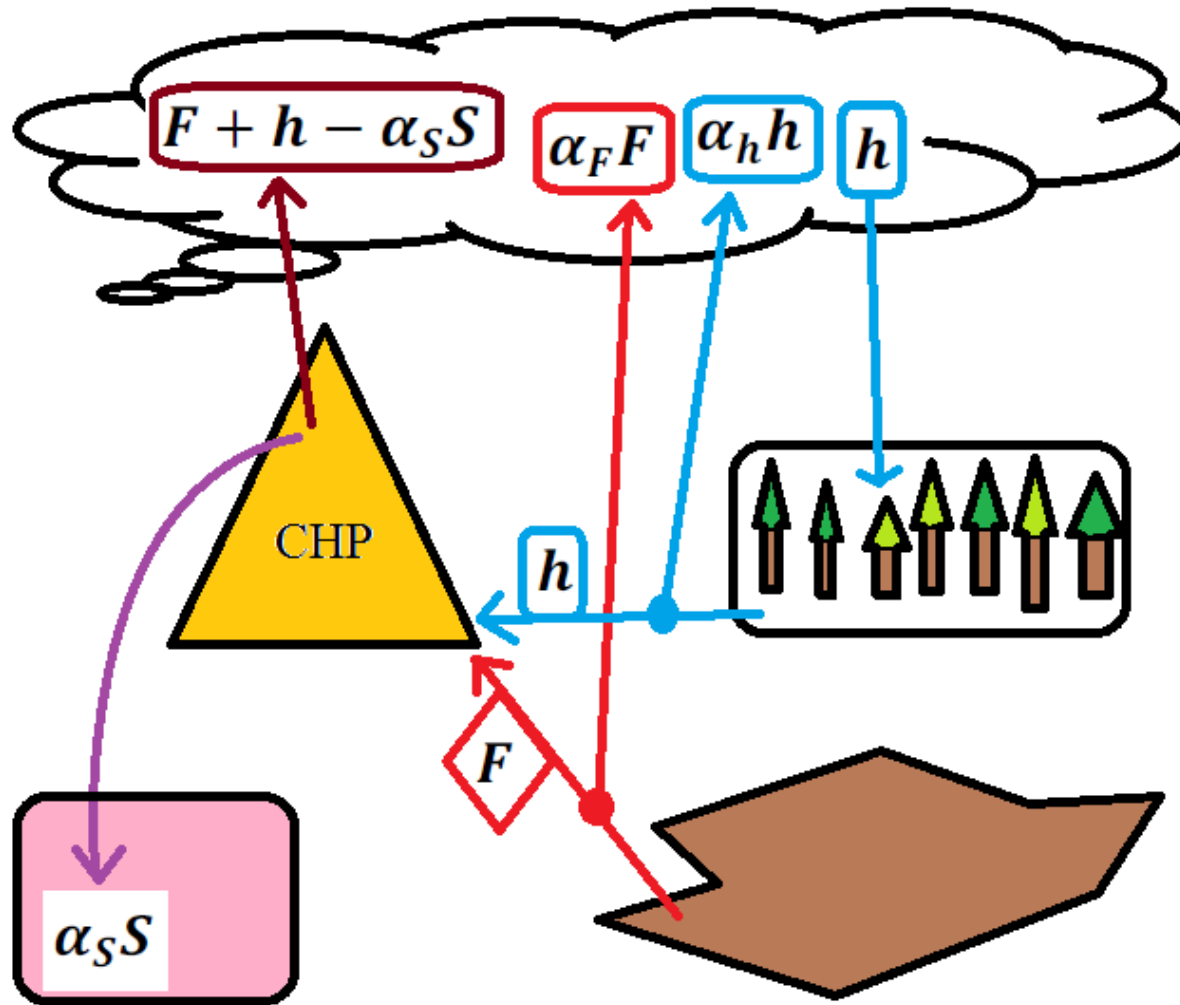
Operations Research for the Future Forest Management,
University of Guilan, Iran, March 8-12, 2014

Saturday March 8, 10.00-12.00



Derivation of the principles of optimal expansion of bioenergy based on forest resources in combination with fossil fuels, CCS and combined heat and power production with consideration of economics and global warming

- In the production of district heating, electricity and many other energy outputs, we may use fossil fuels and renewable inputs, in different combinations. The optimal input mix is a function of technological options, prices of different inputs and costs of alternative levels of environmental consequences. Even with presently existing technology in combined heat and power plants, it is usually possible to reduce the amount of fossil fuels such as coal and strongly increase the level of forest based energy inputs. In large parts of the world, such as Russian Federation, forest stocks are close to dynamic equilibria, in the sense that the net growth (and net carbon uptake) is close to zero. If these forests will be partly harvested, the net growth can increase and a larger part of the CO₂ emitted from the CHP plants will be captured by the forests. Furthermore, with CCS, Carbon Capture and Storage, the rest of the emitted CO₂ can be permanently stored. The lecture includes the definition of a general optimization model that can handle these problems in a consistent way. The model is used to derive general principles of optimal decisions in this problem area via comparative statics analysis with general functions.
- The lecture also includes case studies from different parts of the world. In several countries, it is presently profitable to replace coal by forest inputs in CHP plants. In Norway and UK, CCS has been applied and a commercially attractive option during many years and the physical potential is large. Carbon taxes on fossil fuels explain this development. With increasing carbon taxes in all parts of the world, such developments could be expected everywhere. With increasing levels of forest inputs in combination with CCS, it is possible to reduce the CO₂ in the atmosphere and the global warming problem can be managed. Furthermore, international trade in forest based energy can improve international relations, regional development and environmental conditions.
- Professor Dr. Peter Lohmander, Swedish University of Agricultural Sciences, SLU, Sweden, <http://www.Lohmander.com> Peter@Lohmander.com



The Lohmander Energy, Forest, Fossil Fuels, CCS and Climate System Optimization Model





e.on

Händelö CHP

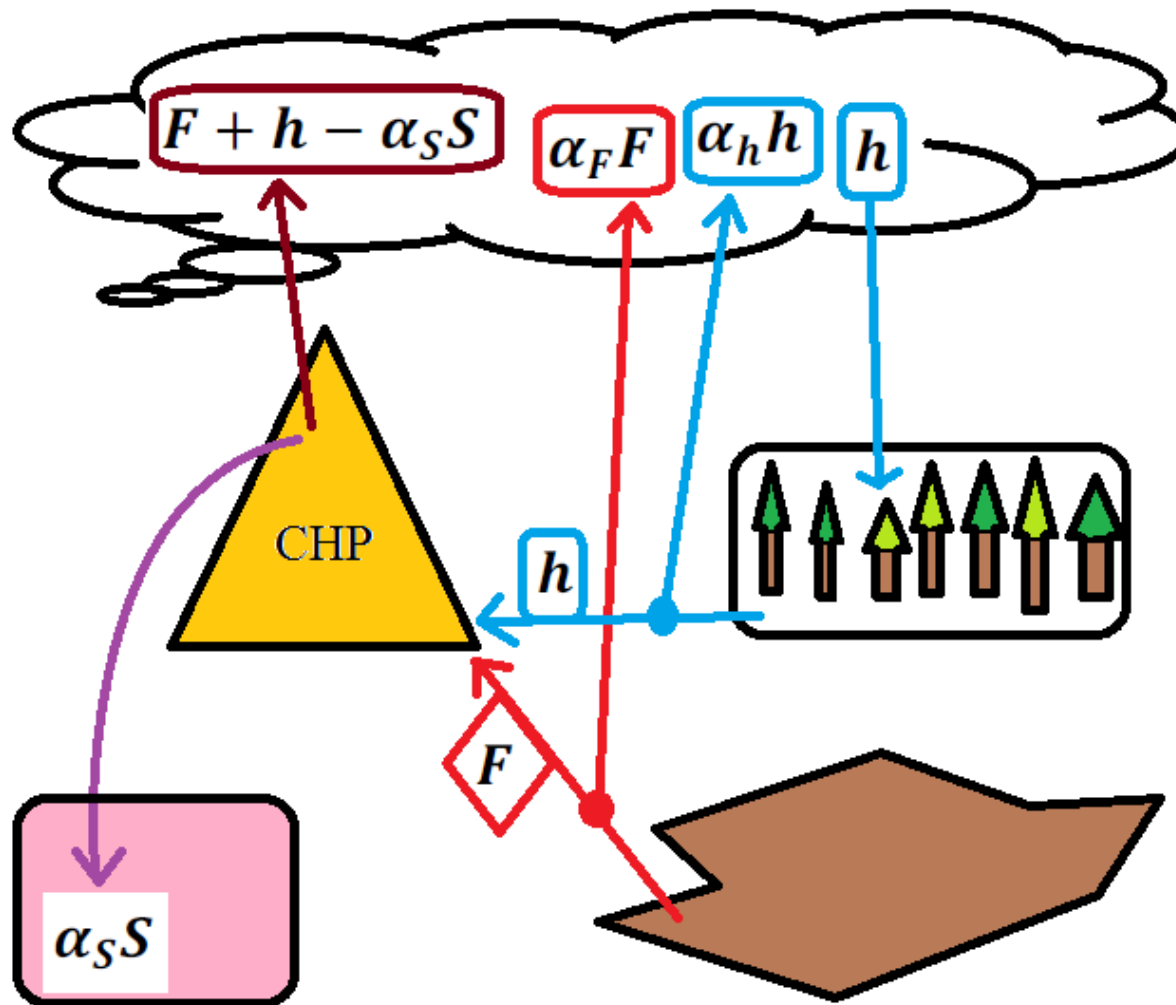
Norrköping, Sweden

Pictures by

Peter Lohmander

2008-12-11





The Lohmander Energy, Forest, Fossil Fuels, CCS and Climate System Optimization Model

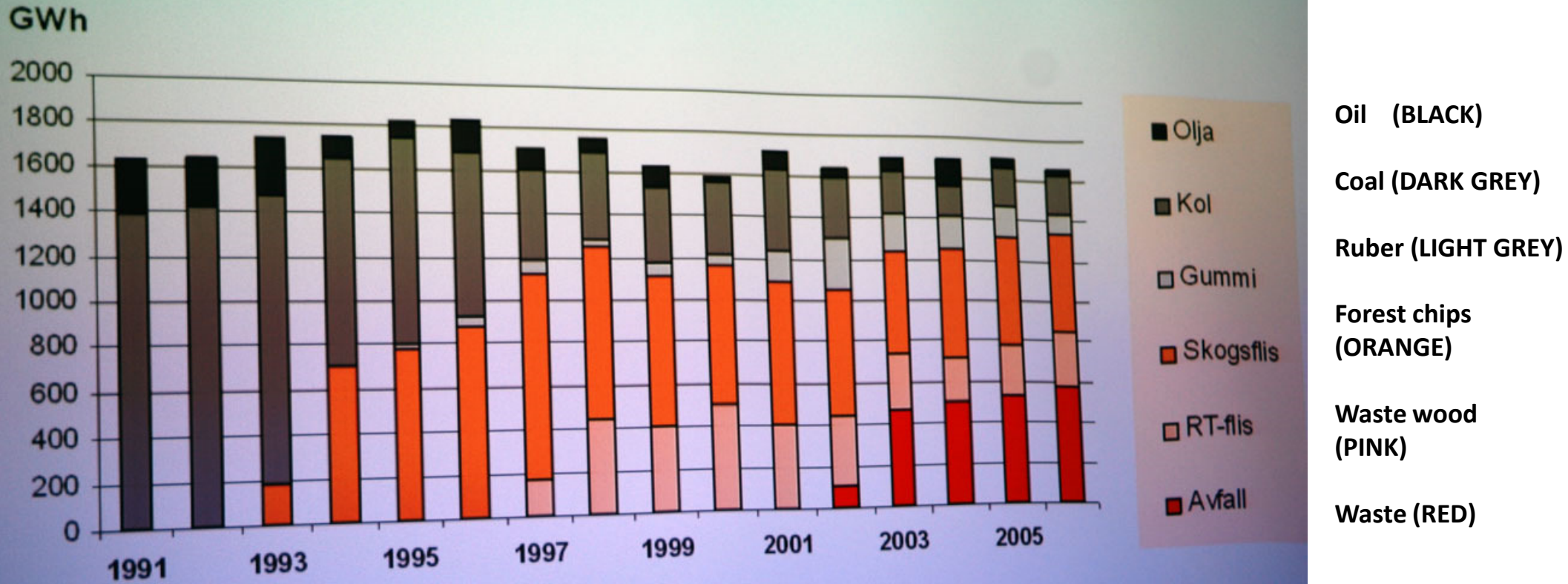


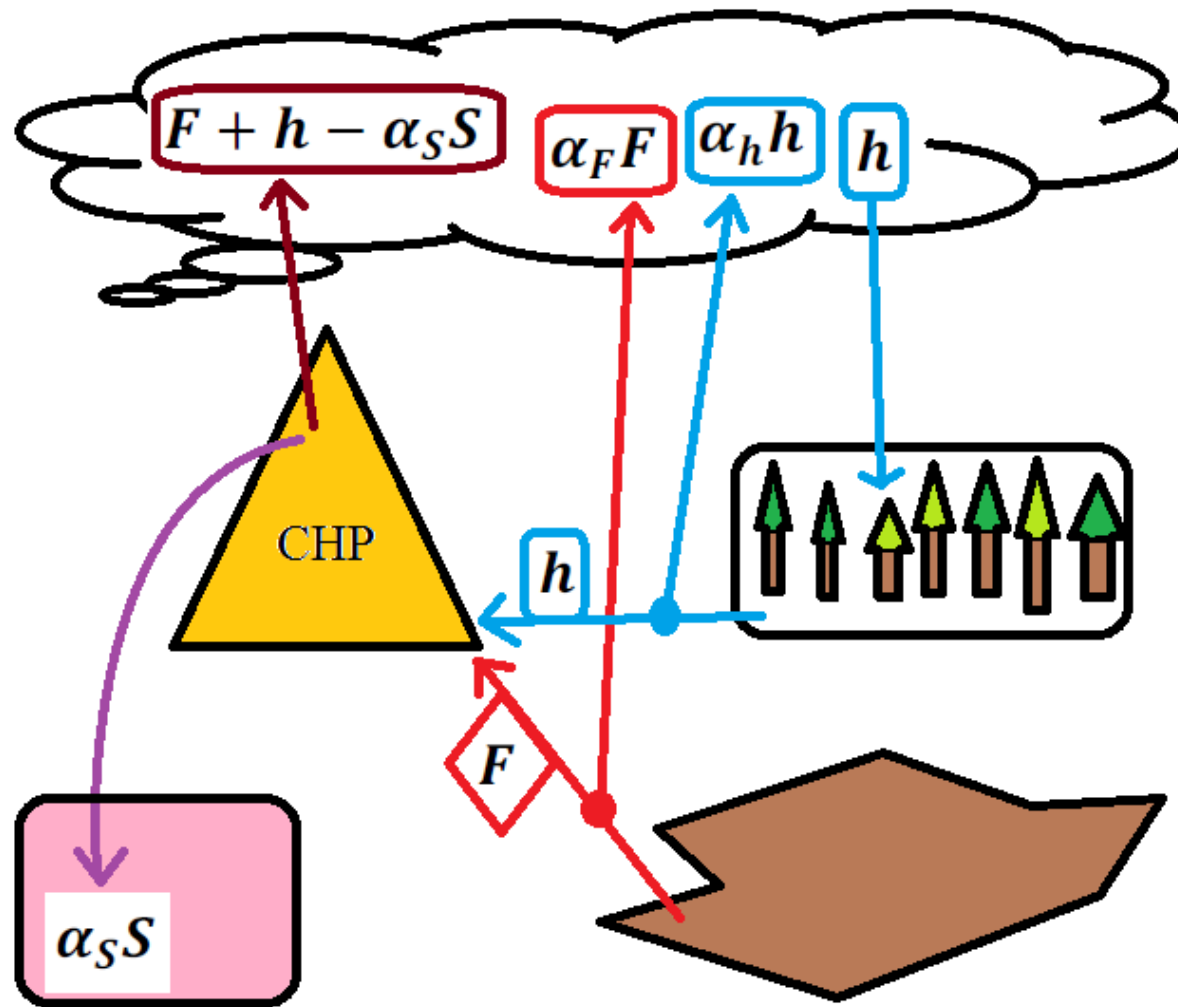




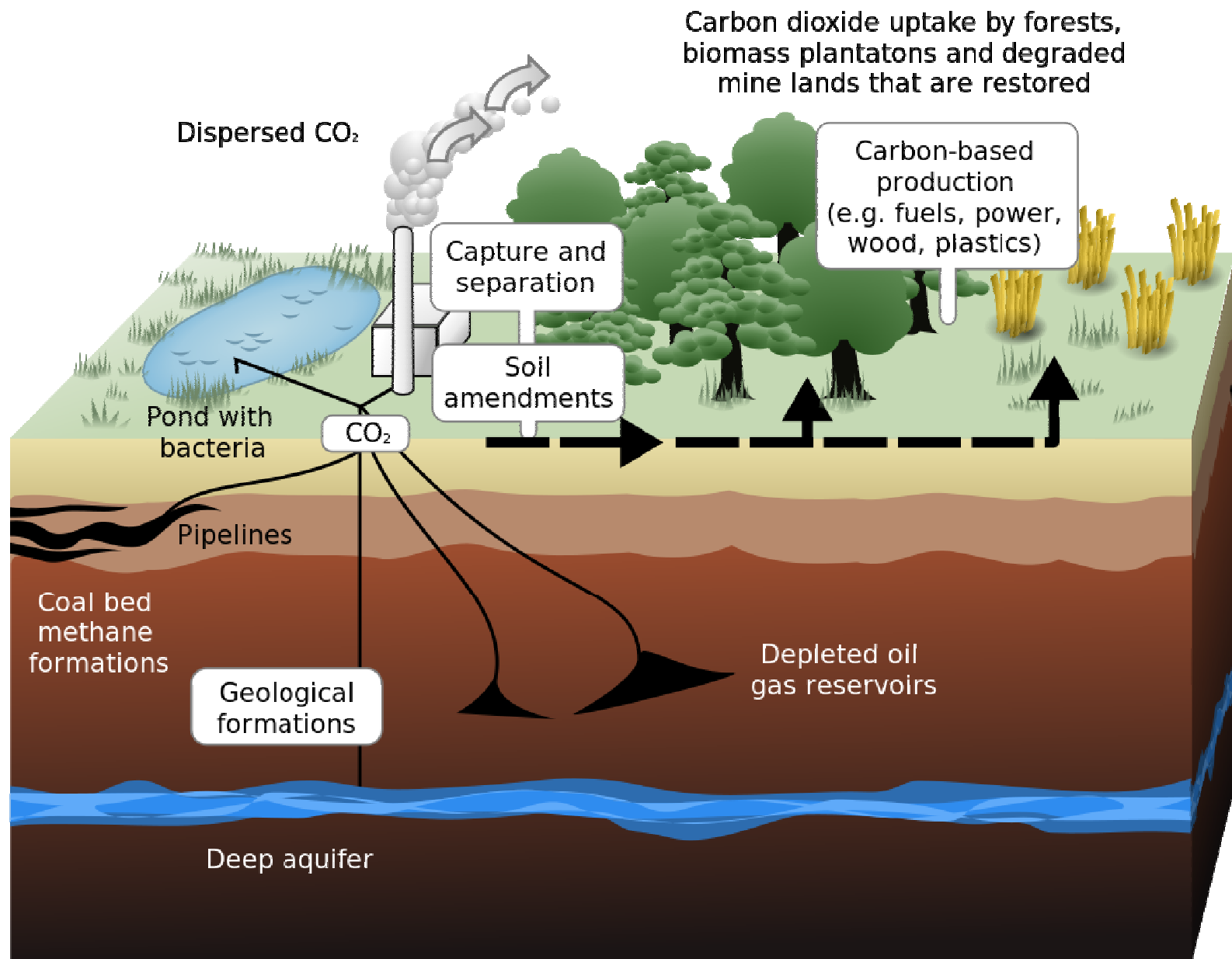
Bränslemix 1991 - 2006

Fuel mix 1991 - 2006





The Lohmander Energy, Forest, Fossil Fuels, CCS and Climate System Optimization Model



TECHNOLOGY



- Capture of CO2
- Geological storage of CO2
- Transport of CO2



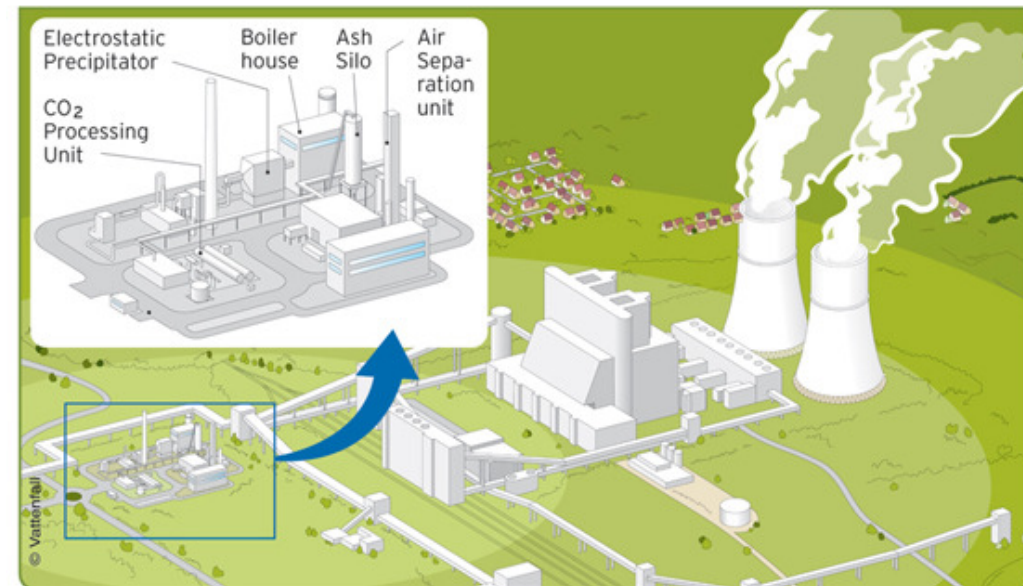
CO2 capture and storage (CCS)

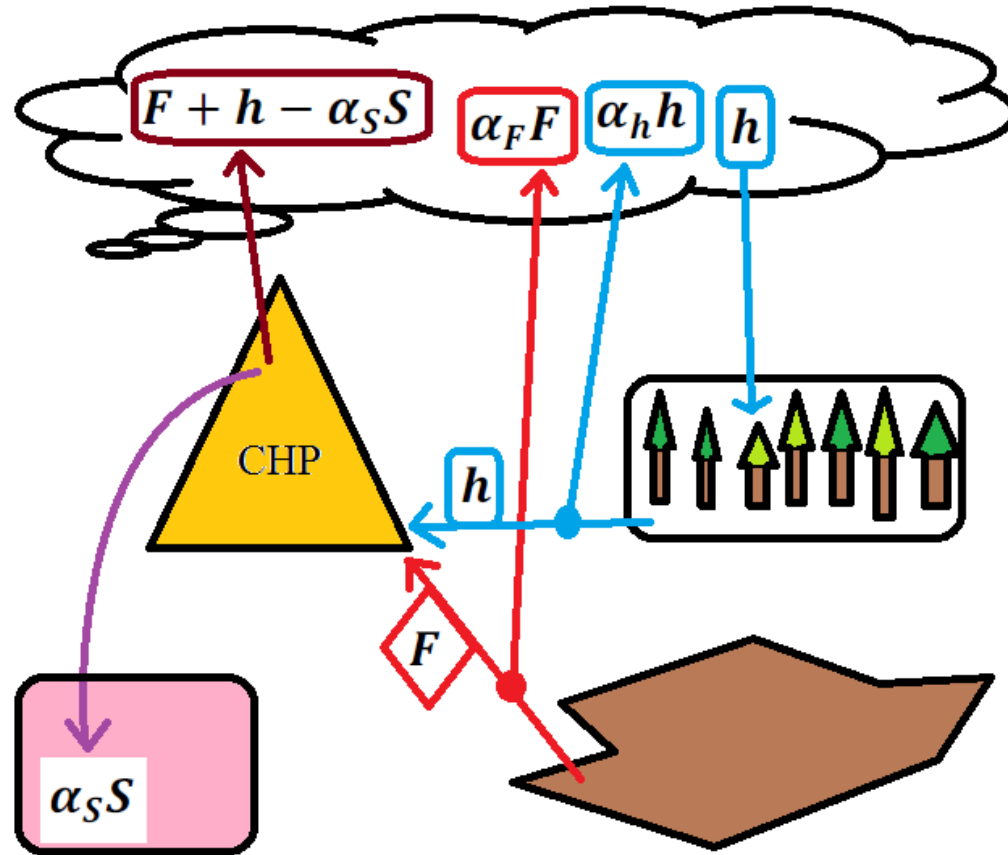


Picture of the Oxyfuel pilot plant in Schwarze Pumpe - May 2008

Vattenfall and Carbon Capture and Storage

Carbon Capture and Storage - CCS - is the method of capturing carbon dioxide compressing it into liquid form and storing it deep underground in suitable geological formations.





$$\begin{aligned}
 \text{Max } Z &= -k_\pi (C_F(F) + C_h(h) + C_S(S)) - k_G((1 + \alpha_F)F + \alpha_h h - \alpha_S S) \\
 \text{s.t. } &F + h \geq M
 \end{aligned}$$

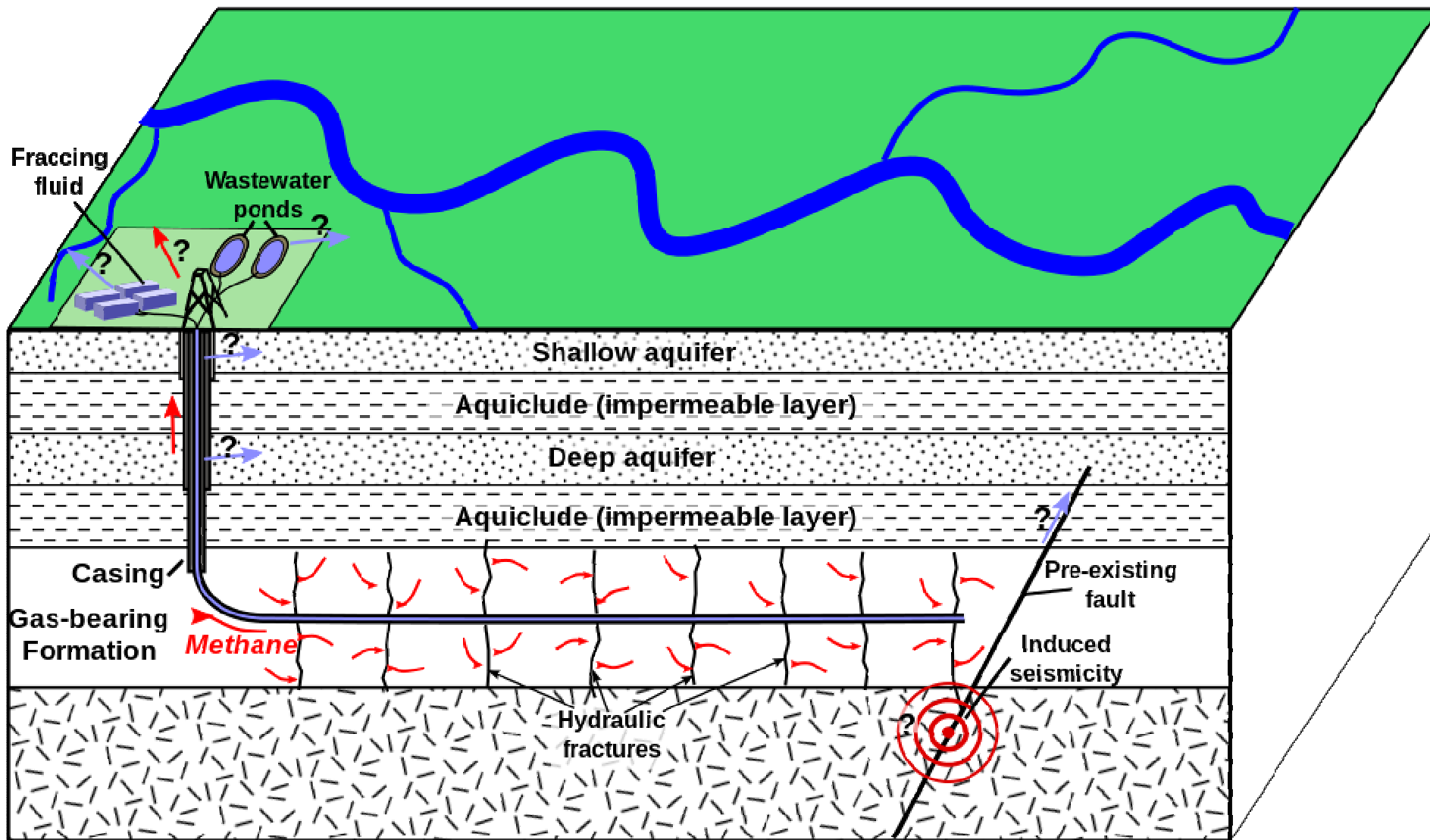


The model does not include absolute constraints on the availability of fossil fuels.

Motive:

Such constraints are assumed not to be binding in the optimal total solution.

Hydraulic fracturing also called fracturing and fracking has recently shown that there are very large quantities of fossil fuels available in the world.



Analysis:

$$\text{Max } Z = -k_{\pi}(C_F(F) + C_h(h) + C_S(S)) - k_G((1 + \alpha_F)F + \alpha_h h - \alpha_S S)$$

s.t.

$$F + h \geq M$$

Parameter assumptions:

$$k_{\pi} > 0$$

$$k_G > 0$$

$$\alpha_F > 0$$

$$\alpha_h > 0$$

$$0 < \alpha_S < 1$$

$$\alpha_h < 1 + \alpha_F$$

This constraint is not binding:

$$S < F + h$$

Cost function assumptions:

$$C'_F > 0$$

$$C''_F > 0$$

$$C'_h > 0$$

$$C''_h > 0$$

$$C'_S > 0$$

$$C''_S > 0$$







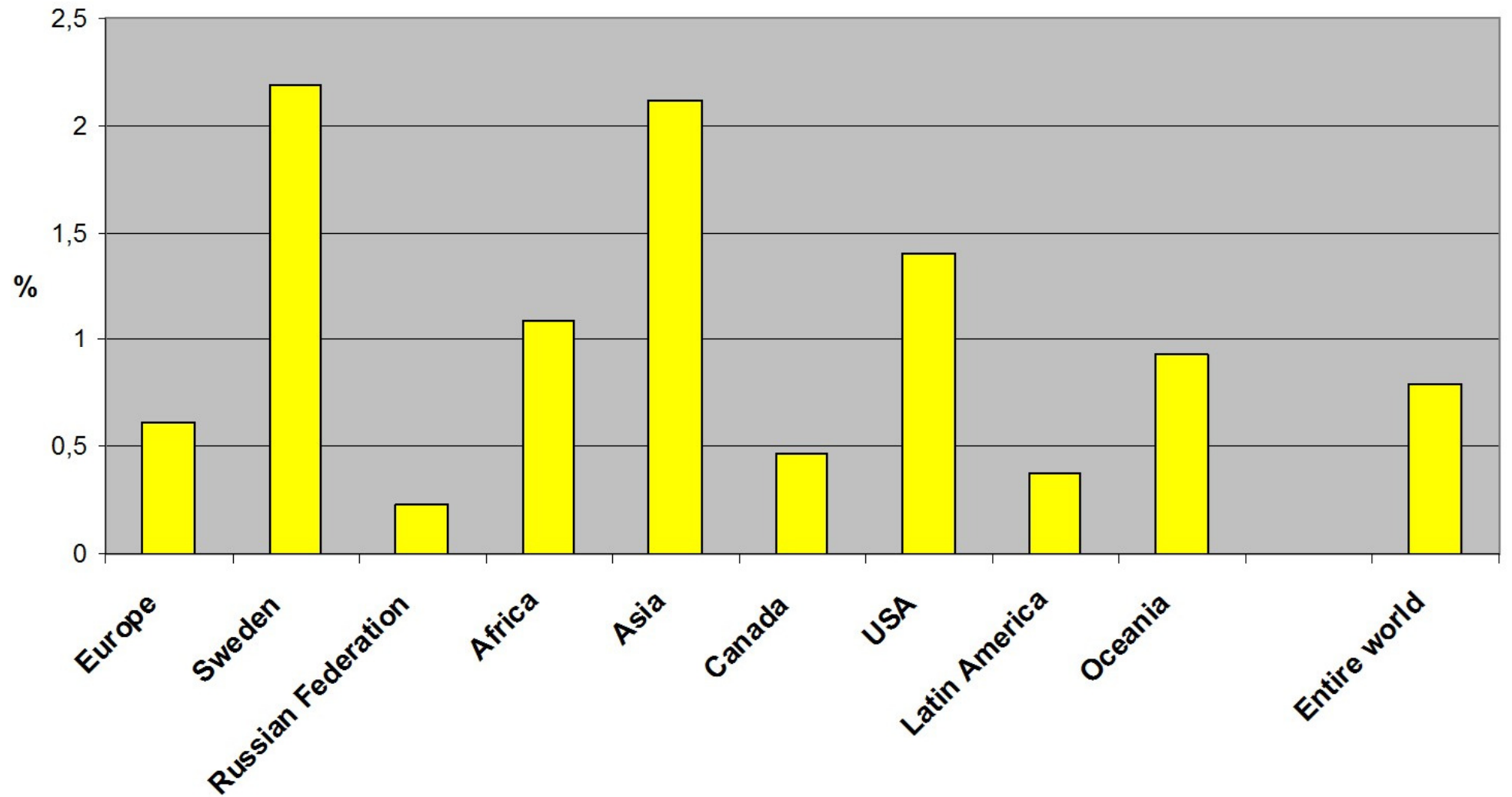




Annual Allowable Cut utilization rate, %



Harvest (2008) under bark / Stock (2005 or 2008) over bark



The classical dynamic natural resource model:

$$\frac{dx}{dt} = sx \left(1 - \frac{x}{K} \right)$$

$$x(t) = \frac{K}{1 + Ce^{-st}}$$

$$\lim_{t \rightarrow \infty} x(t) = K$$

$(s > 0)$

$$\frac{dx}{dt} = \frac{KsCe^{-st}}{(1 + Ce^{-st})^2}$$

$$\lim_{\substack{t \rightarrow \infty \\ (s > 0)}} \left(\frac{dx}{dt} \right) = 0$$

Such forests do not change the carbon levels of the atmosphere.

The Lagrange function, L , is:

$$\begin{aligned} &= -k_{\pi}(C_F(F) + C_h(h) + C_S(S)) \\ &- k_G((1 + \alpha_F)F + \alpha_h h - \alpha_S S) \\ &\quad + \lambda(F + h - M) \end{aligned}$$

The Kuhn Tucker conditions are:

$$F \geq 0; h \geq 0; S \geq 0$$

$$\frac{dL}{dF} \leq 0; \frac{dL}{dh} \leq 0; \frac{dL}{dS} \leq 0$$

$$F \frac{dL}{dF} = 0; h \frac{dL}{dh} = 0; S \frac{dL}{dS} = 0$$

$$\lambda \geq 0$$

$$\frac{dL}{d\lambda} \geq 0$$

$$\lambda \frac{dL}{d\lambda} = 0$$

Here, we have four of the constraints:

$$\frac{dL}{d\lambda} = F + h - M \geq 0$$

$$\frac{dL}{dF} = -k_{\pi} C'_F(F) - k_G(1 + \alpha_F) + \lambda \leq 0$$

$$\frac{dL}{dh} = -k_{\pi} C'_h(h) - k_G \alpha_h + \lambda \leq 0$$

$$\frac{dL}{dS} = -k_{\pi} C'_S(S) + k_G \alpha_S \leq 0$$

Assumption:

The optimal solution is an interior solution.

$$F > 0; h > 0; S > 0; \lambda > 0$$

As a consequence, we know that:

$$\frac{dL}{d\lambda} = 0; \frac{dL}{dF} = 0; \frac{dL}{dh} = 0; \frac{dL}{dS} = 0$$

This means that the following equation system should be solved:

$$\frac{dL}{d\lambda} = F + h - M = 0$$

$$\frac{dL}{dF} = -k_{\pi} C'_F(F) - k_G(1 + \alpha_F) + \lambda = 0$$

$$\frac{dL}{dh} = -k_{\pi} C'_h(h) - k_G \alpha_h + \lambda = 0$$

$$\frac{dL}{dS} = -k_{\pi} C'_S(S) + k_G \alpha_S = 0$$

The system with four equations and four endogenous variables is partly separable.

It may be split into one system with three equations and three endogenous variables (F , h and λ) and a separate equation with only one endogenous variable, S .

Stars denote optimal values.

First, we investigate S^* .

$$\frac{dL}{dS} = -k_{\pi} C'_S(S) + k_G \alpha_S \leq 0$$

$$\left(\frac{dL}{dS} = 0 \right) \Rightarrow \left(C'_S(S) = \frac{k_G \alpha_S}{k_{\pi}} \right)$$

We differentiate the first order optimum condition:

$$-k_{\pi}C''_S(S)dS^* - C'_S(S)dk_{\pi} + \alpha_S dk_G + k_G d\alpha_S = 0$$

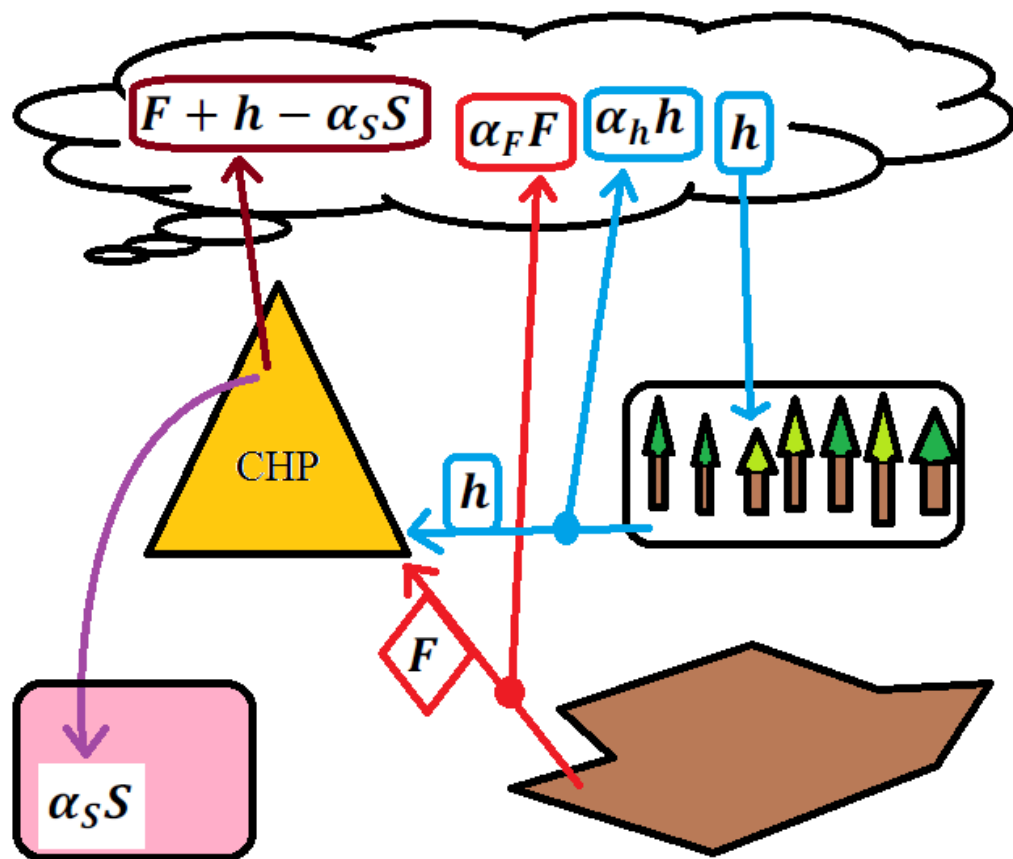
$$dS^* = \frac{1}{k_{\pi}C''_S(S)} (-C'_S(S)dk_{\pi} + \alpha_S dk_G + k_G d\alpha_S)$$

The derivatives of S^* with respect to the parameters are the following:

$$\frac{dS^*}{dk_\pi} = \frac{-C'_S(S)}{k_\pi C''_S(S)} < 0$$

$$\frac{dS^*}{dk_G} = \frac{\alpha_S}{k_\pi C''_S(S)} > 0$$

$$\frac{dS^*}{d\alpha_S} = \frac{k_G}{k_\pi C''_S(S)} > 0$$



$$\frac{dS^*}{dk_\pi} = \frac{-C'_S(S)}{k_\pi C''_S(S)} < 0$$

$$\frac{dS^*}{dk_G} = \frac{\alpha_S}{k_\pi C''_S(S)} > 0$$

$$\frac{dS^*}{d\alpha_S} = \frac{k_G}{k_\pi C''_S(S)} > 0$$

$$\begin{aligned} \text{Max } Z &= -k_\pi(C_F(F) + C_h(h) + C_S(S)) - k_G((1 + \alpha_F)F + \alpha_h h - \alpha_S S) \\ \text{s.t. } F + h &\geq M \end{aligned}$$

Next, we investigate F^* , h^* and λ^* .

The first order optimum conditions are found from this three dimensional equation system:

$$\frac{dL}{d\lambda} = F + h - M = 0$$

$$\frac{dL}{dF} = -k_{\pi} C'_F(F) - k_G(1 + \alpha_F) + \lambda = 0$$

$$\frac{dL}{dh} = -k_{\pi} C'_h(h) - k_G \alpha_h + \lambda = 0$$

We differentiate the equations:

$$\begin{bmatrix} 1 & 1 & 0 \\ -k_{\pi} C'_F & 0 & 1 \\ 0 & -k_{\pi} C'_h & 1 \end{bmatrix} \begin{bmatrix} dF^* \\ dh^* \\ d\lambda^* \end{bmatrix} = \begin{bmatrix} dM \\ C'_F dk_{\pi} + (1 + \alpha_F) dk_G + k_G d\alpha_F \\ C'_h dk_{\pi} + \alpha_h dk_G + k_G d\alpha_h \end{bmatrix}$$

When we apply Cramer's rule, we need to know $|D|$.

$$|D| = \begin{vmatrix} 1 & 1 & 0 \\ -k_{\pi} C_F'' & 0 & 1 \\ 0 & -k_{\pi} C_h'' & 1 \end{vmatrix}$$

$$|D| = k_{\pi} (C_h'' + C_F'') > 0$$

The derivatives of F^* , h^* and λ^* with respect to M are determined via Cramer's rule:

$$\frac{dF^*}{dM} = \frac{\begin{vmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -k_{\pi} C''_h & 1 \end{vmatrix}}{|D|}$$

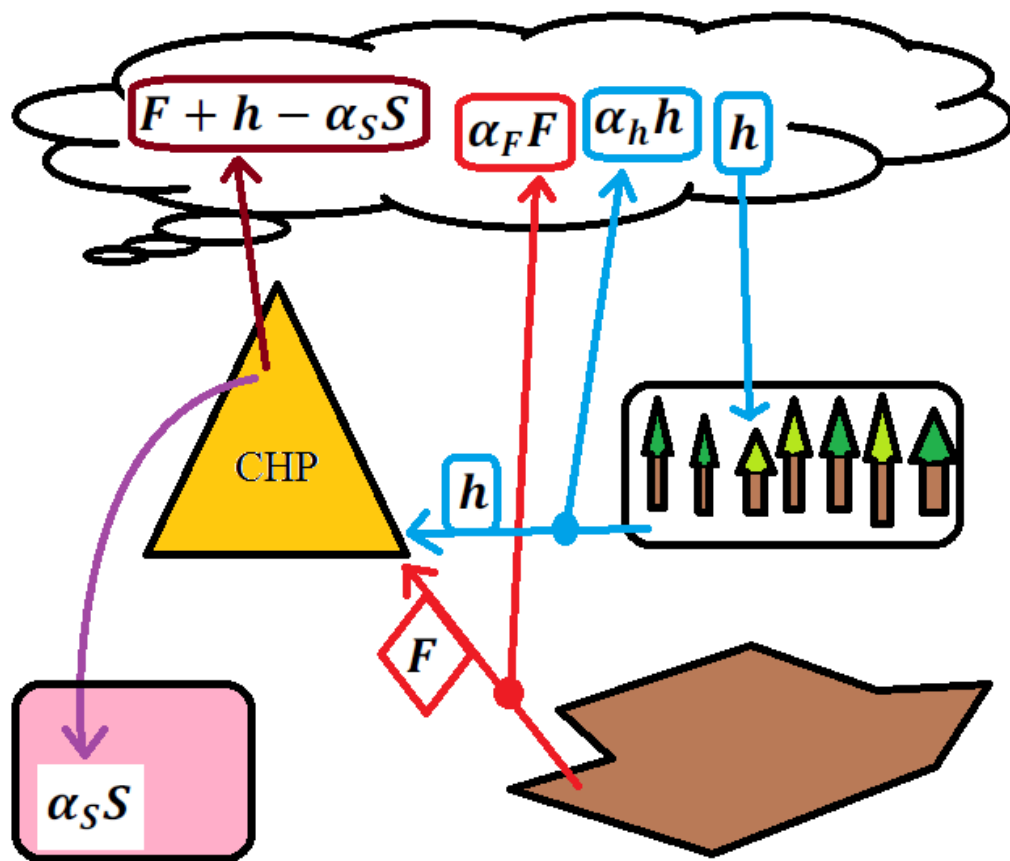
$$\frac{dF^*}{dM} = \frac{C''_h}{C''_h + C''_F} > 0$$

$$\frac{dh^*}{dM} = \frac{\begin{vmatrix} 1 & 1 & 0 \\ -k_{\pi} C_F'' & 0 & 1 \\ 0 & 0 & 1 \end{vmatrix}}{|D|}$$

$$\frac{dh^*}{dM} = \frac{C_F''}{C_h'' + C_F''} > 0$$

$$\frac{d\lambda^*}{dM} = \frac{\begin{vmatrix} 1 & 1 & 1 \\ -k_\pi C_F'' & 0 & 0 \\ 0 & -k_\pi C_h'' & 0 \end{vmatrix}}{|D|}$$

$$\frac{d\lambda^*}{dM} = \frac{k_\pi C_F'' C_h''}{C_h'' + C_F''} > 0$$



$$\frac{dF^*}{dM} = \frac{C''_h}{C''_h + C''_F} > 0$$

$$\frac{dh^*}{dM} = \frac{C''_F}{C''_h + C''_F} > 0$$

$$\frac{d\lambda^*}{dM} = \frac{k_\pi C''_F C''_h}{C''_h + C''_F} > 0$$

$$\begin{aligned} \text{Max } Z &= -k_\pi (C_F(F) + C_h(h) + C_S(S)) - k_G((1 + \alpha_F)F + \alpha_h h - \alpha_S S) \\ \text{s.t. } F + h &\geq M \end{aligned}$$

The derivatives of F^* , h^* and λ^* with respect to k_G are:

$$\frac{dF^*}{dk_G} = \frac{\begin{vmatrix} 0 & 1 & 0 \\ (1 + \alpha_F) & 0 & 1 \\ \alpha_h & -k_\pi C''_h & 1 \end{vmatrix}}{|D|}$$

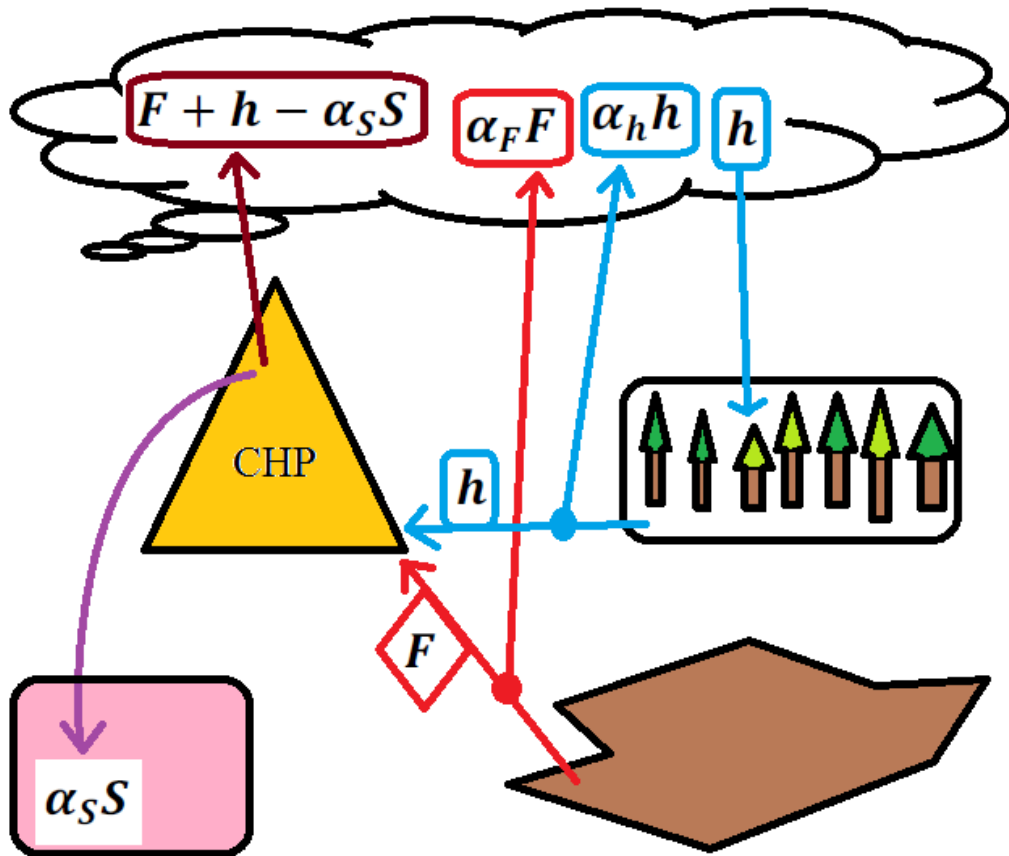
$$\frac{dF^*}{dk_G} = \frac{\alpha_h - (1 + \alpha_F)}{k_\pi (C''_h + C''_F)} < 0$$

$$\frac{dh^*}{dk_G} = \frac{\begin{vmatrix} 1 & 0 & 0 \\ -k_\pi C_F'' & (1 + \alpha_F) & 1 \\ 0 & \alpha_h & 1 \end{vmatrix}}{|D|}$$

$$\frac{dh^*}{dk_G} = \frac{(1 + \alpha_F) - \alpha_h}{k_\pi (C_h'' + C_F'')} > 0$$

$$\frac{d\lambda^*}{dk_G} = \frac{\begin{vmatrix} 1 & 1 & 0 \\ -k_\pi C_F'' & 0 & (1 + \alpha_F) \\ 0 & -k_\pi C_h'' & \alpha_h \end{vmatrix}}{|D|}$$

$$\frac{d\lambda^*}{dk_G} = \frac{C_h''(1 + \alpha_F) + C_F''\alpha_h}{C_h'' + C_F''} > 0$$



$$\frac{dF^*}{dk_G} = \frac{\alpha_h - (1 + \alpha_F)}{k_\pi(C_h'' + C_F'')} < 0$$

$$\frac{dh^*}{dk_G} = \frac{(1 + \alpha_F) - \alpha_h}{k_\pi(C_h'' + C_F'')} > 0$$

$$\frac{d\lambda^*}{dk_G} = \frac{C_h''(1 + \alpha_F) + C_F''\alpha_h}{C_h'' + C_F''} > 0$$

$$\begin{aligned} \text{Max } Z &= -k_\pi(C_F(F) + C_h(h) + C_S(S)) - k_G((1 + \alpha_F)F + \alpha_h h - \alpha_S S) \\ \text{s.t. } F + h &\geq M \end{aligned}$$

The derivatives of F^* , h^* and λ^* with respect to k_π :

$$\frac{dF^*}{dk_\pi} = \frac{\begin{vmatrix} 0 & 1 & 0 \\ C'_F & 0 & 1 \\ C'_h & -k_\pi C''_h & 1 \end{vmatrix}}{|D|}$$

$$\frac{dF^*}{dk_\pi} = \frac{C'_h - C'_F}{k_\pi (C''_h + C''_F)} \begin{cases} > \\ = \\ < \end{cases} 0$$

Observation:

The sign of $\frac{dF^*}{dk_\pi}$ is expected to change over time, since fossil fuels in the long run become more scarce and more costly to extract.

In most cases, it is assumed that $\frac{dF^*}{dk_\pi} > 0$ in the year 2014. At some future point in time, $\frac{dF^*}{dk_\pi} = 0$ and at later points in time $\frac{dF^*}{dk_\pi} < 0$.

$$\frac{dh^*}{dk_\pi} = \frac{\begin{vmatrix} 1 & 0 & 0 \\ -k_\pi C_F'' & C_F' & 1 \\ 0 & C_h' & 1 \end{vmatrix}}{|D|}$$

$$\frac{dh^*}{dk_\pi} = \frac{C_F' - C_h'}{k_\pi (C_h'' + C_F'')} \begin{cases} > \\ = \\ < \end{cases} \mathbf{0}$$

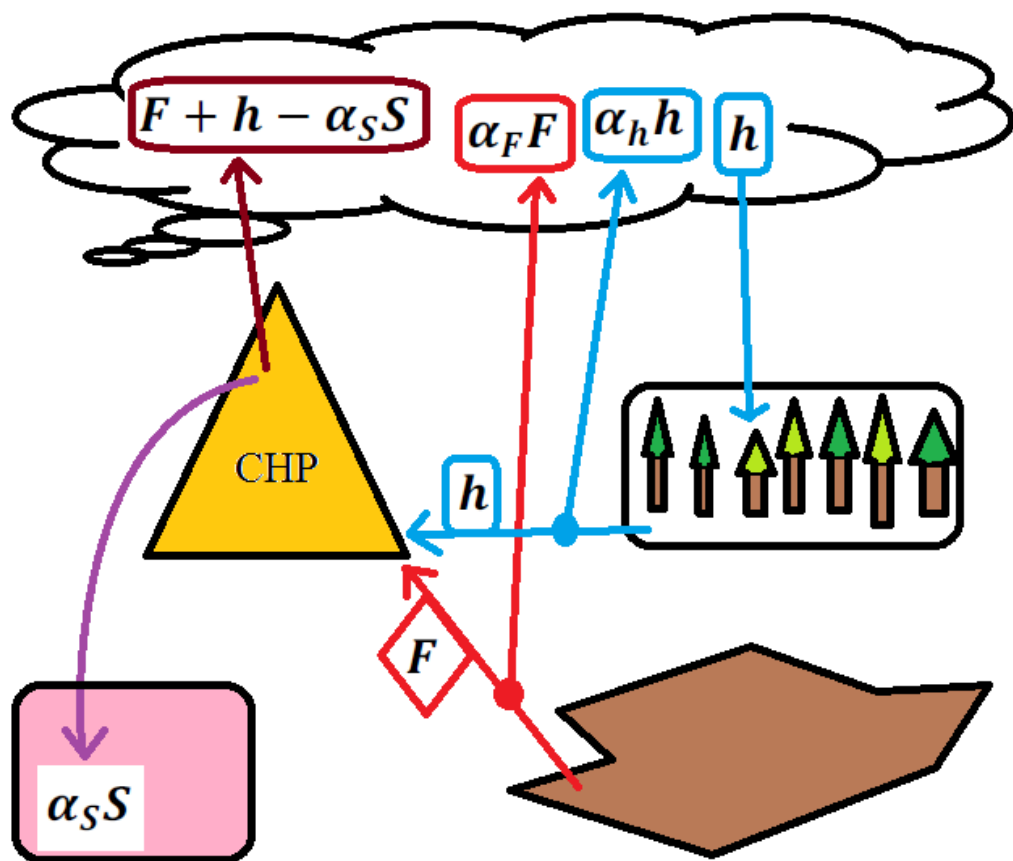
Observation:

The sign of $\frac{dh^*}{dk_\pi}$ is expected to change over time, since fossil fuels in the long run become more scarce and more costly to extract.

In most cases, it is assumed that $\frac{dh^*}{dk_\pi} < 0$ in the year 2014. At some future point in time, $\frac{dh^*}{dk_\pi} = 0$ and at later points in time $\frac{dh^*}{dk_\pi} > 0$.

$$\frac{d\lambda^*}{dk_\pi} = \frac{\begin{vmatrix} 1 & 1 & 0 \\ -k_\pi C_F'' & 0 & C_F' \\ 0 & -k_\pi C_h'' & C_h' \end{vmatrix}}{|D|}$$

$$\frac{d\lambda^*}{dk_\pi} = \frac{C_h'' C_F' + C_F'' C_h'}{C_h'' + C_F''} > 0$$



$$\frac{dF^*}{dk_\pi} = \frac{C'_h - C'_F}{k_\pi (C''_h + C''_F)} \begin{cases} > \\ = \\ < \end{cases} \mathbf{0}$$

$$\frac{dh^*}{dk_\pi} = \frac{C'_F - C'_h}{k_\pi (C''_h + C''_F)} \begin{cases} > \\ = \\ < \end{cases} \mathbf{0}$$

$$\frac{d\lambda^*}{dk_\pi} = \frac{C''_h C'_F + C''_F C'_h}{C''_h + C''_F} > \mathbf{0}$$

$$\begin{aligned} \text{Max } Z &= -k_\pi (C_F(F) + C_h(h) + C_S(S)) - k_G((1 + \alpha_F)F + \alpha_h h - \alpha_S S) \\ \text{s.t. } F + h &\geq M \end{aligned}$$

The derivatives of F^* , h^* and λ^* with respect to α_F are:

$$\frac{dF^*}{d\alpha_F} = \frac{\begin{vmatrix} 0 & 1 & 0 \\ k_G & 0 & 1 \\ 0 & -k_\pi C''_h & 1 \end{vmatrix}}{|D|}$$

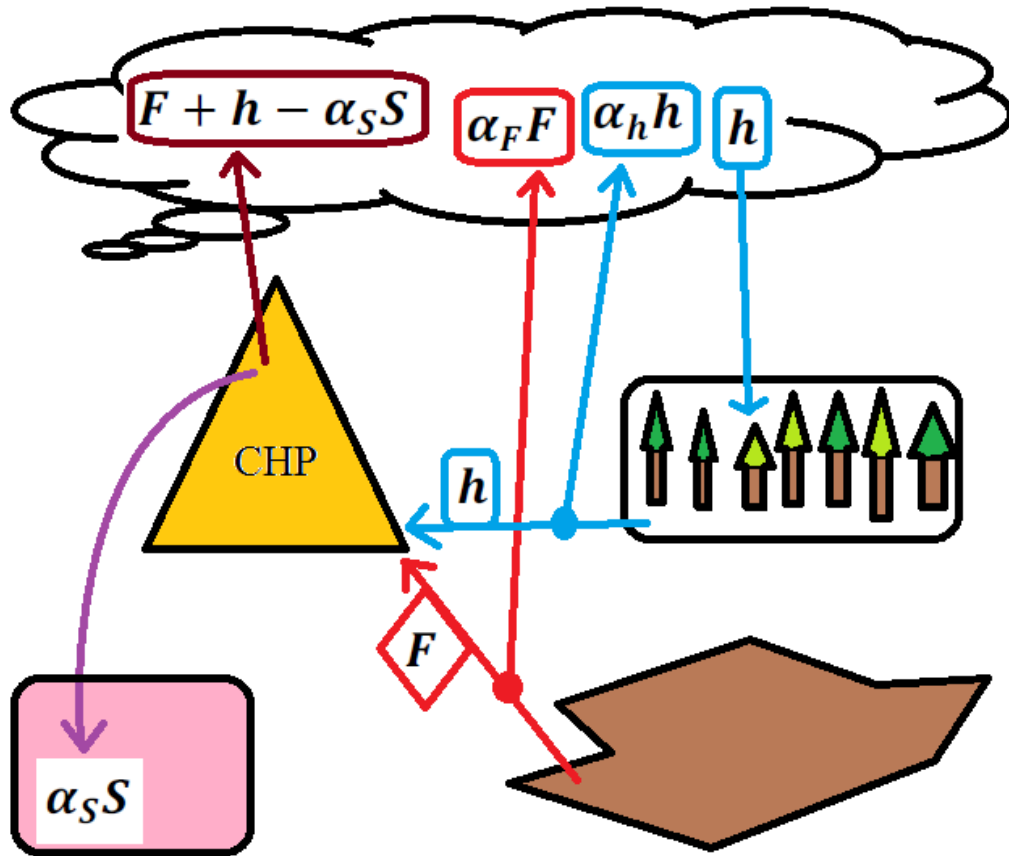
$$\frac{dF^*}{d\alpha_F} = \frac{-k_G}{k_\pi (C''_h + C''_F)} < 0$$

$$\frac{dh^*}{d\alpha_F} = \frac{\begin{vmatrix} 1 & 0 & 0 \\ -k_\pi C_F'' & k_G & 1 \\ 0 & 0 & 1 \end{vmatrix}}{|D|}$$

$$\frac{dh^*}{d\alpha_F} = \frac{k_G}{k_\pi (C_h'' + C_F'')} > 0$$

$$\frac{d\lambda^*}{d\alpha_F} = \frac{\begin{vmatrix} 1 & 1 & 0 \\ -k_\pi C_F'' & 0 & k_G \\ 0 & -k_\pi C_h'' & 0 \end{vmatrix}}{|D|}$$

$$\frac{d\lambda^*}{d\alpha_F} = \frac{C_h'' k_G}{C_h'' + C_F''} > 0$$



$$\frac{dF^*}{d\alpha_F} = \frac{-k_G}{k_\pi(C_h'' + C_F'')} < 0$$

$$\frac{dh^*}{d\alpha_F} = \frac{k_G}{k_\pi(C_h'' + C_F'')} > 0$$

$$\frac{d\lambda^*}{d\alpha_F} = \frac{C_h'' k_G}{C_h'' + C_F''} > 0$$

$$\begin{aligned} \text{Max } Z &= -k_\pi(C_F(F) + C_h(h) + C_S(S)) - k_G((1 + \alpha_F)F + \alpha_h h - \alpha_S S) \\ \text{s.t. } &F + h \geq M \end{aligned}$$

The derivatives of F^* , h^* and λ^* with respect to α_h are:

$$\frac{dF^*}{d\alpha_h} = \frac{\begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ k_G & -k_\pi C_h'' & 1 \end{vmatrix}}{|D|}$$

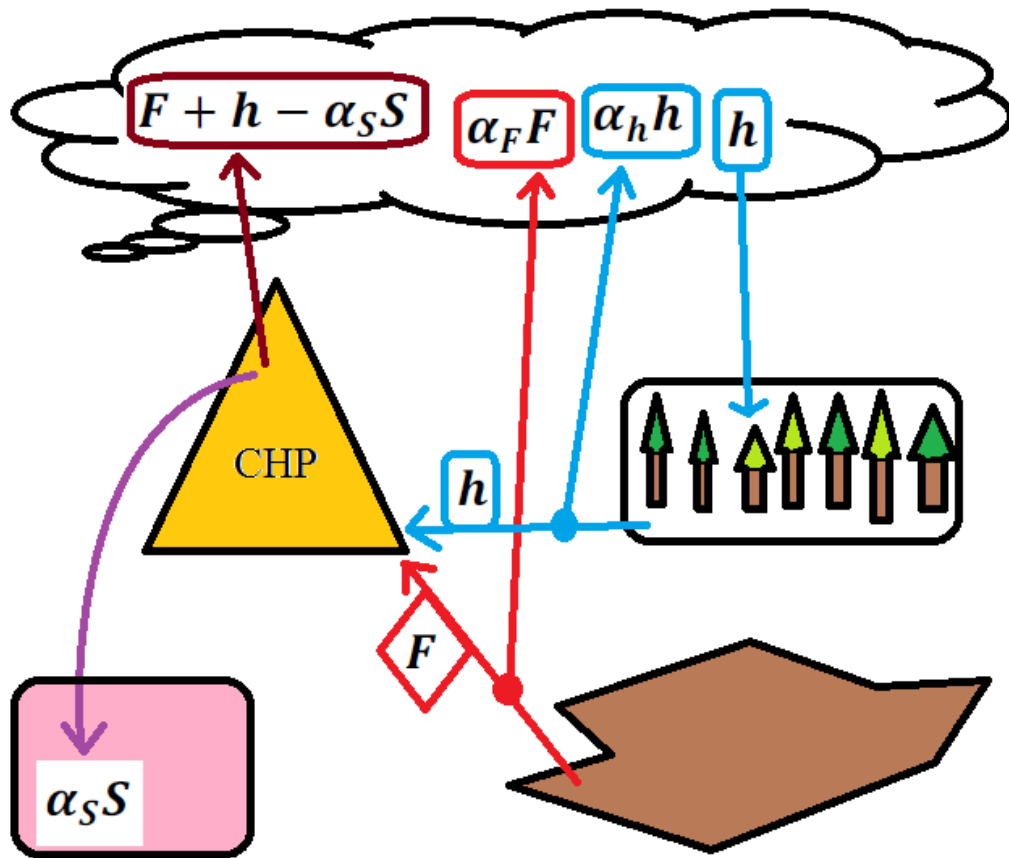
$$\frac{dF^*}{d\alpha_h} = \frac{k_G}{k_\pi (C_h'' + C_F'')} > 0$$

$$\frac{dh^*}{d\alpha_h} = \frac{\begin{vmatrix} 1 & 0 & 0 \\ -k_\pi C_F'' & 0 & 1 \\ 0 & k_G & 1 \end{vmatrix}}{|D|}$$

$$\frac{dh^*}{d\alpha_h} = \frac{-k_G}{k_\pi (C_h'' + C_F'')} < 0$$

$$\frac{d\lambda^*}{d\alpha_h} = \frac{\begin{vmatrix} 1 & 1 & 0 \\ -k_\pi C_F'' & 0 & 0 \\ 0 & -k_\pi C_h'' & k_G \end{vmatrix}}{|D|}$$

$$\frac{d\lambda^*}{d\alpha_h} = \frac{C_F'' k_G}{C_h'' + C_F''} > 0$$



$$\frac{dF^*}{d\alpha_h} = \frac{k_G}{k_\pi(C_h'' + C_F'')} > 0$$

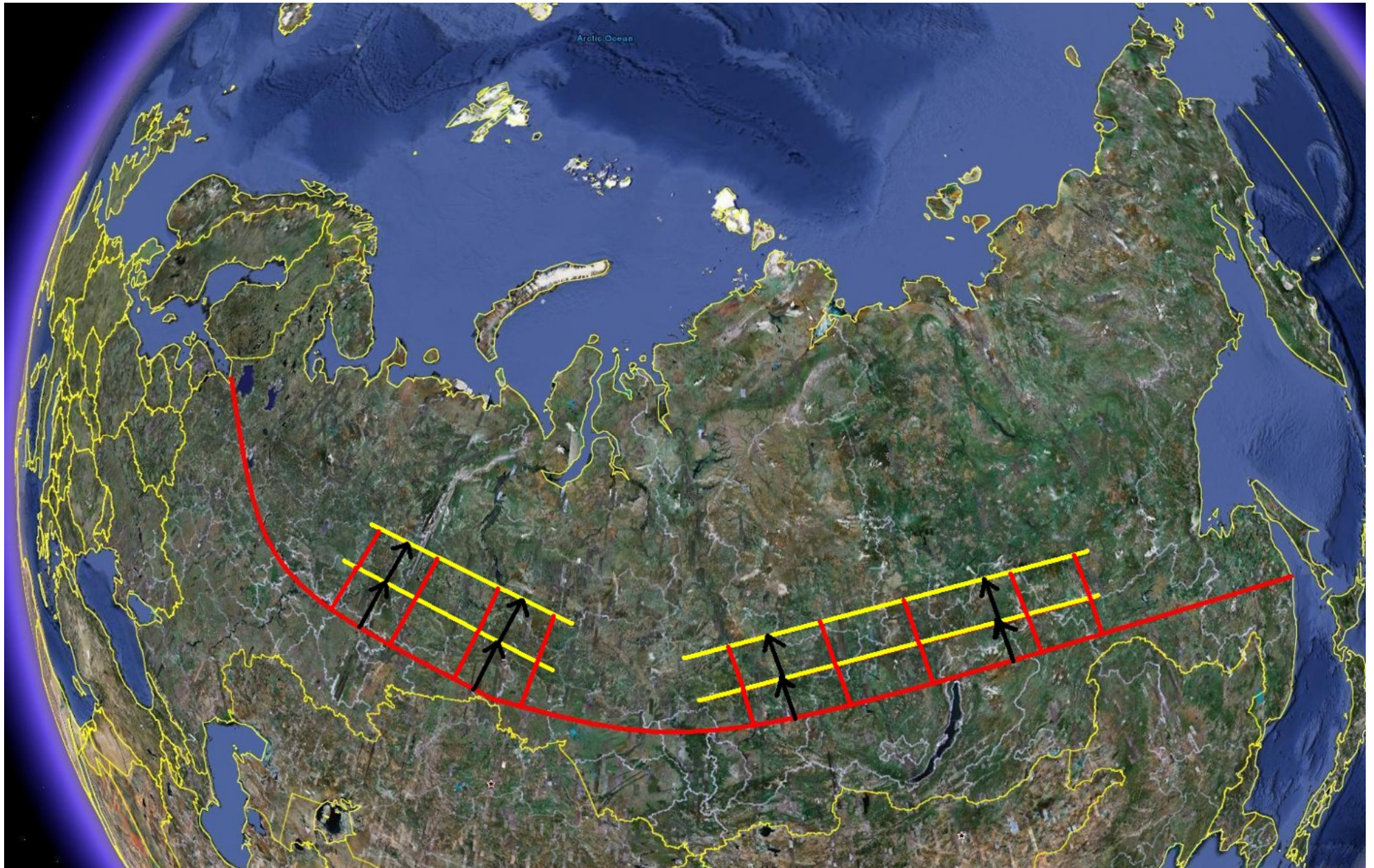
$$\frac{dh^*}{d\alpha_h} = \frac{-k_G}{k_\pi(C_h'' + C_F'')} < 0$$

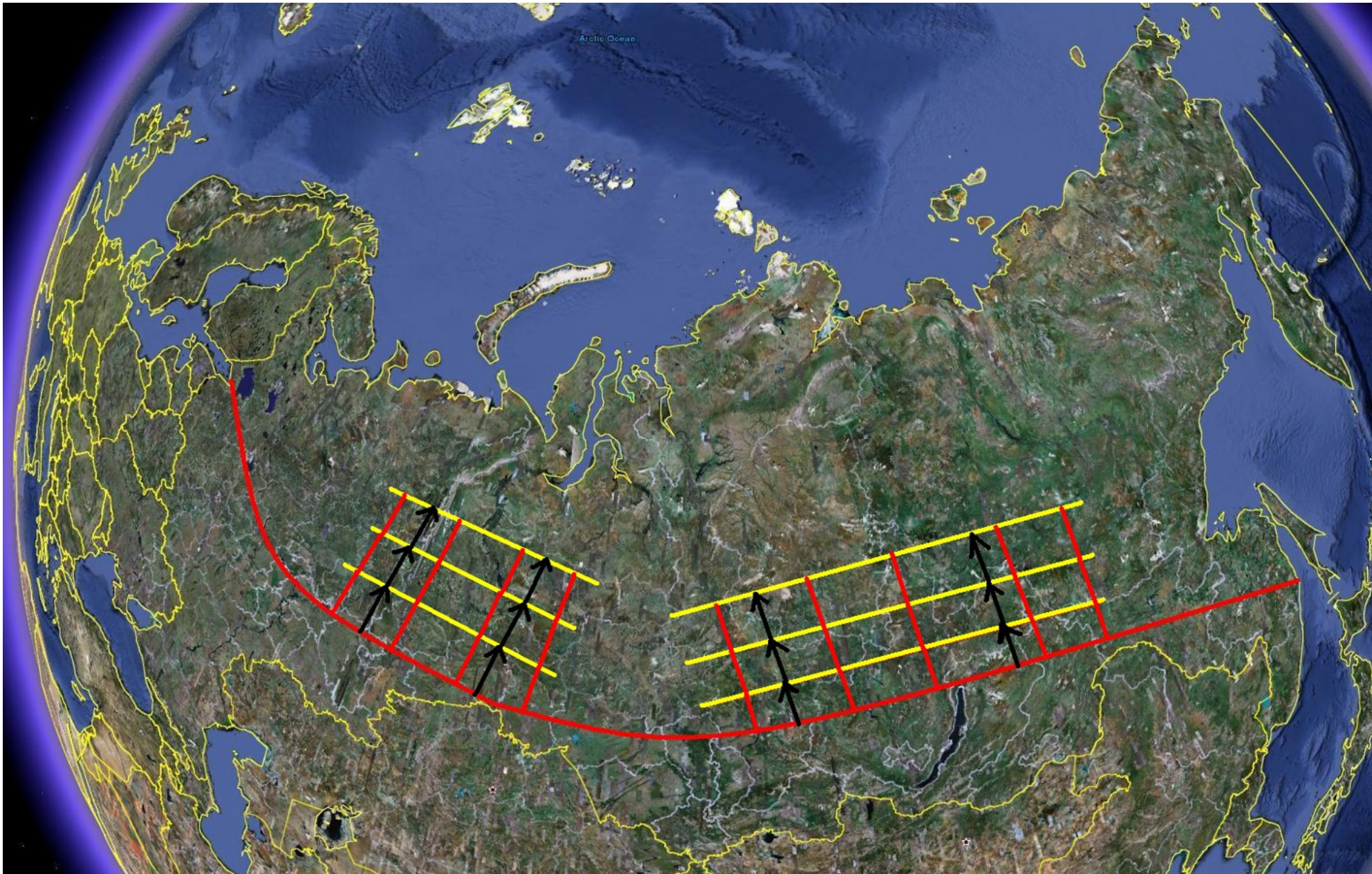
$$\frac{d\lambda^*}{d\alpha_h} = \frac{C_F'' k_G}{C_h'' + C_F''} > 0$$

$$\begin{aligned} \text{Max } Z &= -k_\pi(C_F(F) + C_h(h) + C_S(S)) - k_G((1 + \alpha_F)F + \alpha_h h - \alpha_S S) \\ \text{s.t. } F + h &\geq M \end{aligned}$$

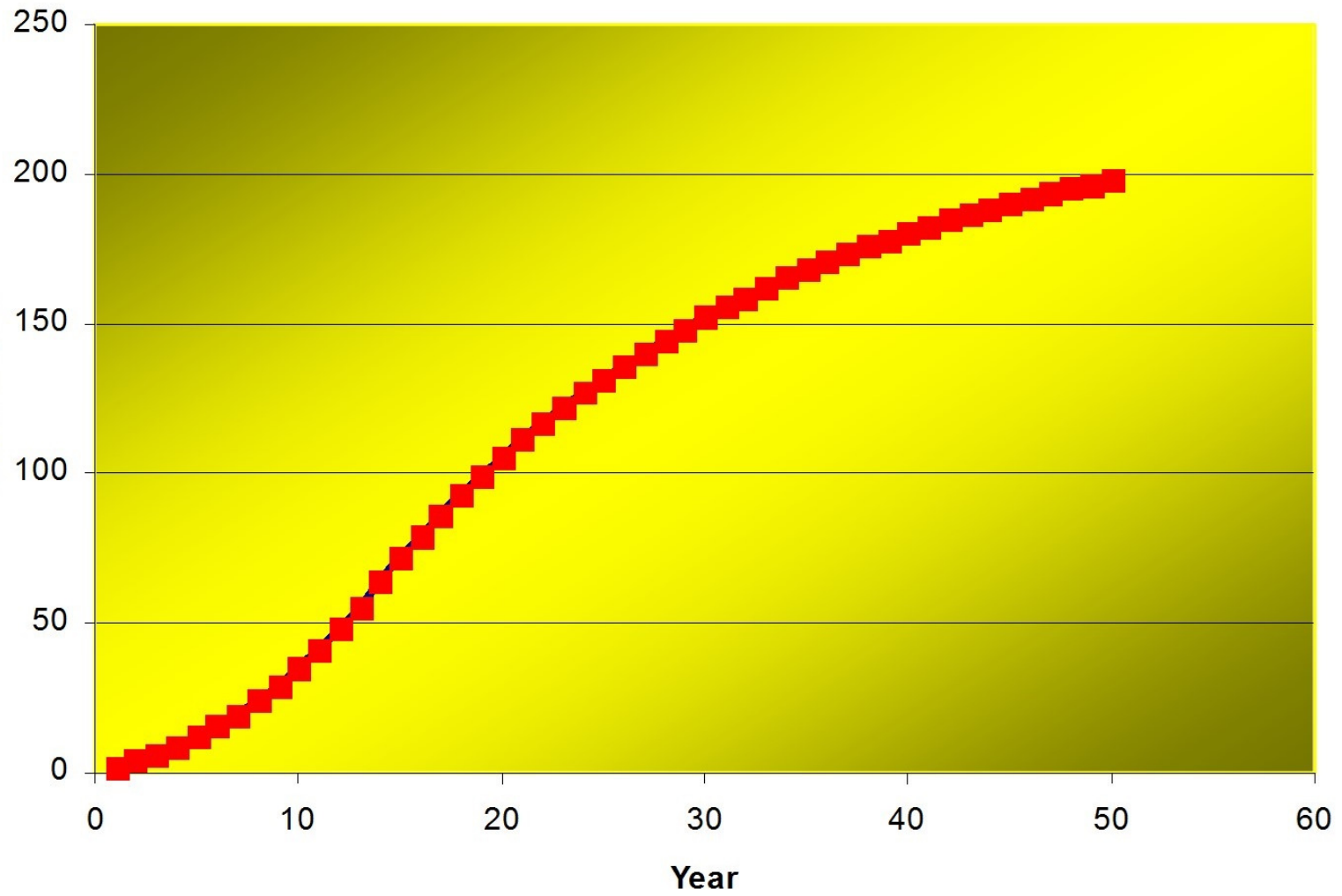








Present Value (1000 Million Euro)



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CONCLUSIONS:

With increasing levels of forest inputs in combination with CCS, it is possible to reduce the CO₂ in the atmosphere and the global warming problem can be managed.

Furthermore, international trade in forest based energy can improve international relations, regional development and environmental conditions.

All suggestions concerning future cooperation projects are welcome!

Thank you!

Peter Lohmander

APPENDIX

The classical dynamic natural resource model:

$$\frac{dx}{dt} = sx \left(1 - \frac{x}{K}\right)$$

$$\frac{dx}{dt} = sx - \frac{s}{K}x^2$$

$$\frac{dx}{dt} = sx(1 + \gamma x), \quad \gamma = -K^{-1}$$

$$\frac{1}{x(1 + \gamma x)} dx = s dt$$

$$\frac{1}{x(1 + \gamma x)} = \frac{m}{x} + \frac{n}{1 + \gamma x}$$

$$\frac{m}{x} + \frac{n}{1 + \gamma x} = \frac{(1 + \gamma x)m + nx}{x(1 + \gamma x)}$$

$$\frac{(1 + \gamma x)m + nx}{x(1 + \gamma x)} = \frac{1}{x(1 + \gamma x)}$$

$$(1 + \gamma x)m + nx = 1$$

$$m + m\gamma x + nx = 1$$

$$m + (m\gamma + n)x = 1$$

Observation:

$$m + (m\gamma + n)x = 1 \forall x$$

$$\begin{cases} m = 1 \\ m\gamma + n = 0 \end{cases}$$

Solution:

$$(m, n) = (1, -\gamma)$$

$$\frac{1}{x(1 + \gamma x)} dx = s dt$$

$$\left(\frac{m}{x} + \frac{n}{1 + \gamma x}\right) dx = s dt$$

$$\left(\frac{1}{x} - \frac{\gamma}{1 + \gamma x}\right) dx = s dt$$

$$\frac{dLN(1 + \gamma x)}{dx} = \frac{\gamma}{1 + \gamma x}$$

$$\frac{1}{x} dx - \frac{\gamma}{1 + \gamma x} dx = s dt$$

$$\int \frac{1}{x} dx - \int \frac{\gamma}{1 + \gamma x} dx = \int s dt$$

$$LN(x) - LN(1 + \gamma x) = st + C_1$$

$$LN\left(\frac{x}{1 + \gamma x}\right) = st + C_1$$

$$e^{LN\left(\frac{x}{1 + \gamma x}\right)} = e^{st + C_1}$$

$$\frac{x}{1 + \gamma x} = C_2 e^{st}$$

$$x = (1 + \gamma x) C_2 e^{st}$$

$$x(1 - C_2 \gamma e^{st}) = C_2 e^{st}$$

$$x = \frac{C_2 e^{st}}{1 - C_2 \gamma e^{st}}$$

$$x = \frac{1}{\frac{1}{C_2} e^{-st} - \gamma}$$

$$\gamma = -K^{-1}$$

$$x = \frac{1}{\frac{1}{C_2} e^{-st} + \frac{1}{K}}$$

$$x(t) = \frac{K}{1 + C e^{-st}}$$

$$\lim_{\substack{t \rightarrow \infty \\ (s > 0)}} x(t) = K$$

$$\frac{dx}{dt} = \frac{K s C e^{-st}}{(1 + C e^{-st})^2}$$

$$\lim_{\substack{t \rightarrow \infty \\ (s > 0)}} \left(\frac{dx}{dt}\right) = 0$$