Rational Research in Forest Production and Forest Management with Consideration of Mixed Species Forest Management and Risk

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- In order to solve the forest management optimization problems, it is necessary that the most relevant and important management options are well investigated and formulated. Models of biological growth should be developed that describe the production options available over time and space. The latest decades clearly show that detailed deterministic long term planning is irrelevant. Energy prices, prices of industrial products and environmental problems rapidly change in ways that cannot be perfectly predicted. Research in forest production planning should focus on the development of growth models that are useful when stochastic optimal control theory is applied. Since biological production takes considerable time, it is very important to create options to sequentially adjust the production to new relative prices, growth conditions, ecological problems and possible damages caused by parasites, fire or storms. In particular, valuable options can be obtained via mixed species stands. When several species are available in the young stands, the species mix can sequentially be adapted to changing product prices, costs and growth conditions. The analysis concerns the structure of relevant stochastic optimal control problems and consistent development of research in forest production. General mathematical models are analyzed. Comparative statics analysis and numerical methods are used to derive optimal results.

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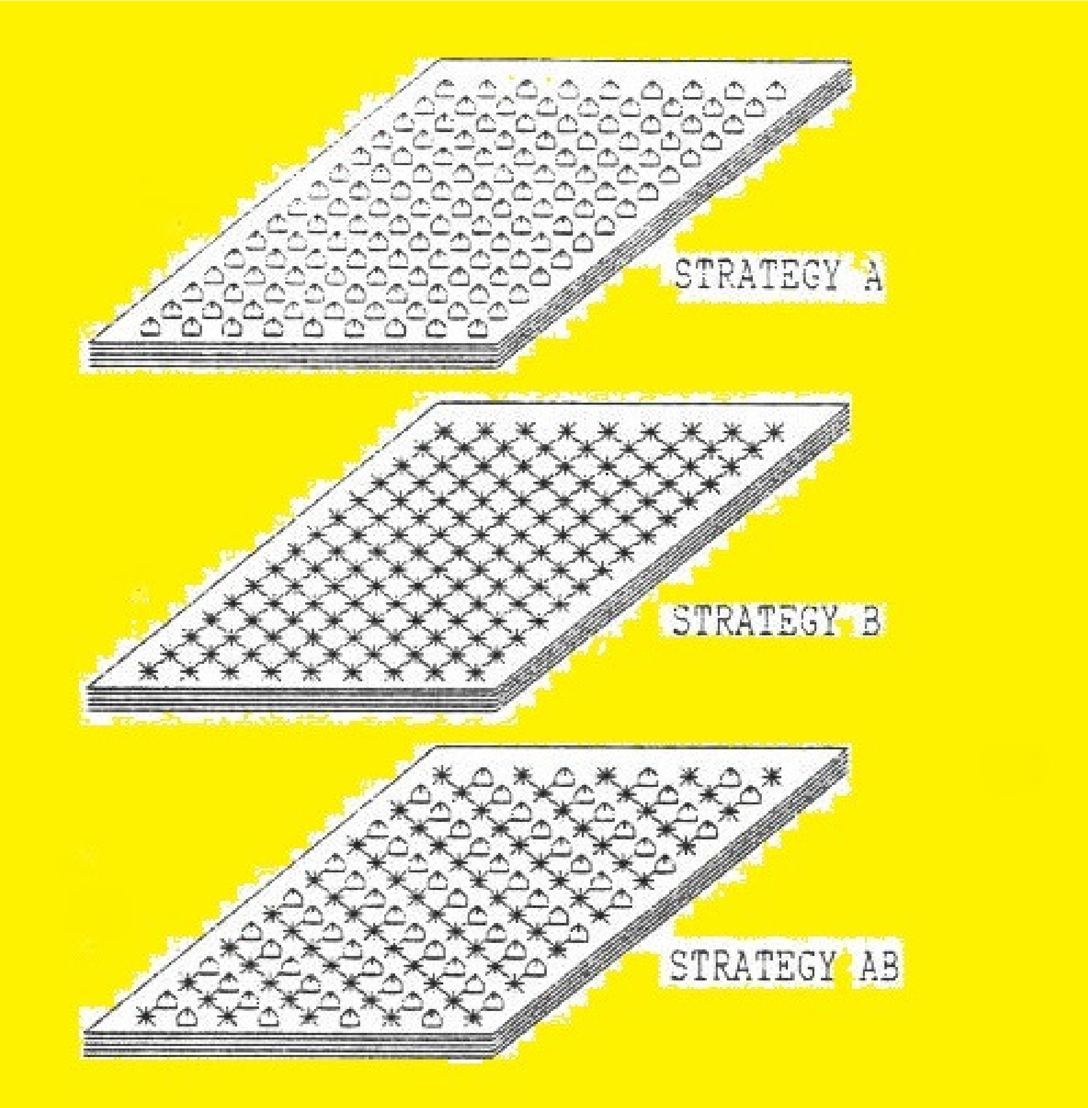
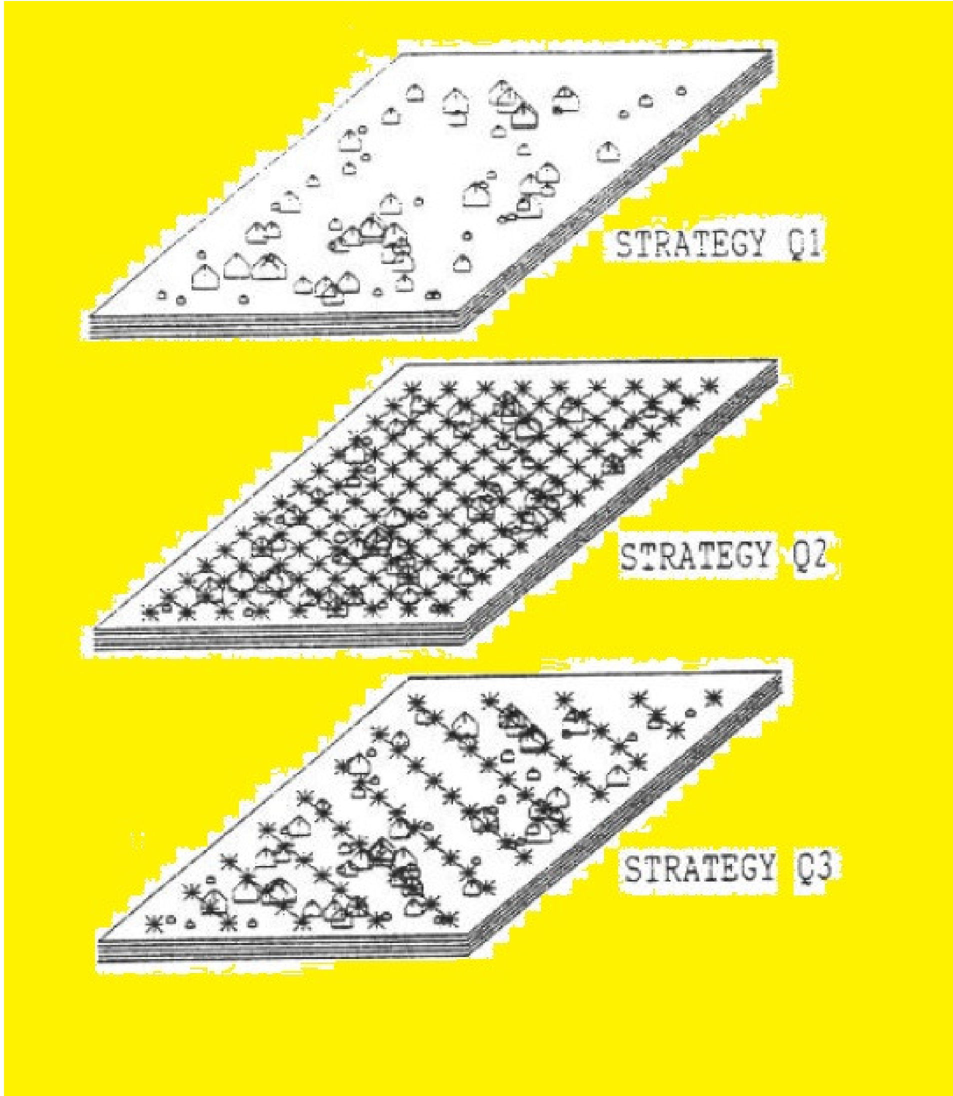

STRATEGY A
STRATEGY B
STRATEGY AB


STRATEGY Q1
STRATEGY Q2
STRATEGY Q3

Lohmander, P., The multi species forest stand, stochastic prices and adaptive selective thinning, SYSTEMS ANALYSIS - MODELLING - SIMULATION, Vol. 9, 229-250, 1992
http://www.Lohmander.com/PL_SAMS_9_1992.pdf

Lohmander, P., Economic two stage multi period species management in a stochastic environment: The value of selective thinning options and stochastic growth parameters,SYSTEMS ANALYSIS - MODELLING -SIMULATION, Vol. 11, 287-302, 1993
http://www.Lohmander.com/PL_SAMS_11_1993.pdf

Lohmander, P., Optimal sequential forestry decisions under risk, ANNALS OF OPERATIONS RESEARCH, Vol. 95, pp. 217-228, 2000
http://www.Lohmander.com/PL_AOR_95_2000.pdf

Lohmander, P., Optimala beslut inför osäker framtid, FAKTA SKOG, SUAS, Nr 10, 2001
http://www.Lohmander.com/FS01-10.pdf

Lu, F., Lohmander, P., Optimal mixed stand management under risk, in Lu, F., Optimization of forest management decision making under conditions of risk, Doctoral Thesis, Acta Universitatis Agriculturae Sueciae, Swedish University of Agricultural Sciences, No. 333, 2004

Lohmander, P., Adaptive Optimization of Forest Management in a Stochastic World, in Weintraub A. et al (Editors), Handbook of Operations Research in Natural Resources, Springer, Springer Science, International Series in Operations Research and Management Science, New York, USA, pp 525-544, 2007
http://www.amazon.ca/gp/reader/0387718141/ref=sib_dp_pt/701-0734992-1741115#reader-link
http://www.Lohmander.com/PL_Handbook2007.pdf

Lu, F., Lohmander, P., Optimal Decisions for Mixed Forests under Risk,
Scientia Silvae Sinicae, Vol. 45, No. 11, Nov. 2009
http://www.Lohmander.com/Lu_Lohmander_2009.pdf

Optimal Management Decisions for Mixed Forests under Risk

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Abstract: Different tree species have different sensitivities to damages from different kinds of fungi, insects, and vertebrates. Prices of forest products from different tree species also change over time. Mixed forests provide valuable options for sequential adaptive management. An adaptive optimization model under the risk of moose damage and prices variation has been developed to determine the initial proportion of Norway Spruce and Scots Pine in a mixed-species stand that would maximize the expected net present value. The results showed that the mixed stand was superior to the pure pine stand even no risk was considered, due to the biological mixture effect. However, when the risk of moose damage was considered, the superiority of the mixed stand was increased by 5% and 24% with or without incorporating the minimum stem number requirement of the Forest Act, respectively. The superiority of the mixed stand over a pure pine stand could be further increased by 6% when the price risk and selective thinning were included, compared to that the price was fixed

Key words: forest management decisions; uncertainty; risk, adaptive optimization

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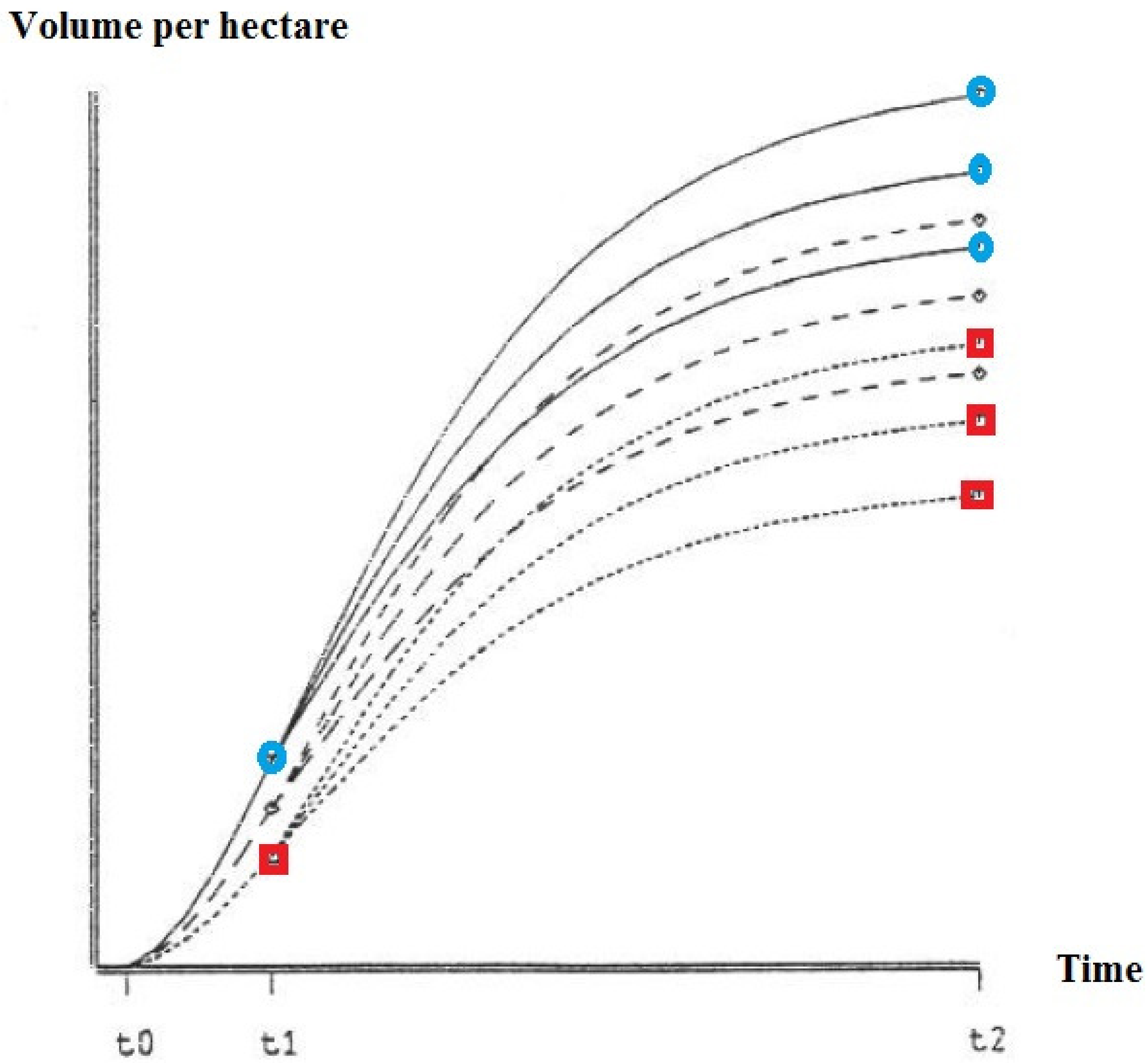
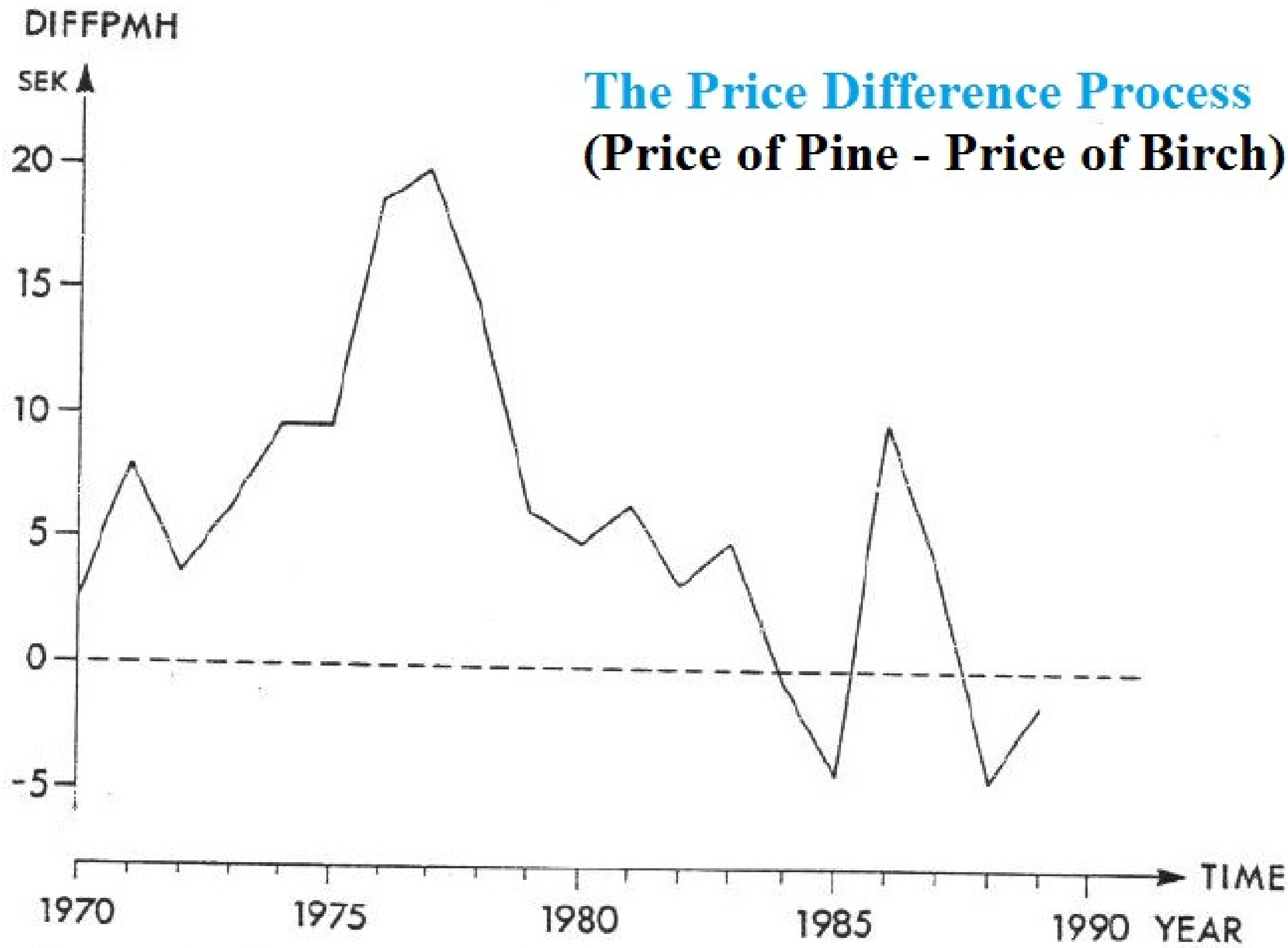
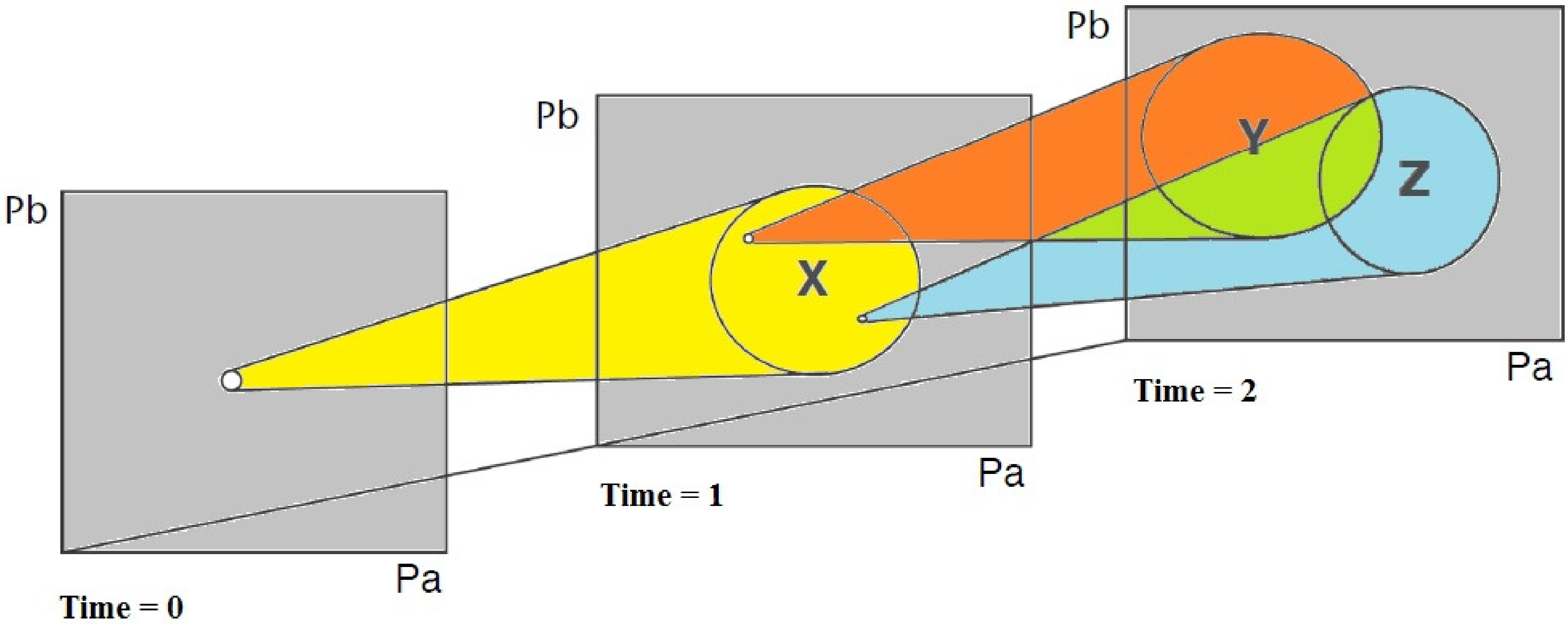
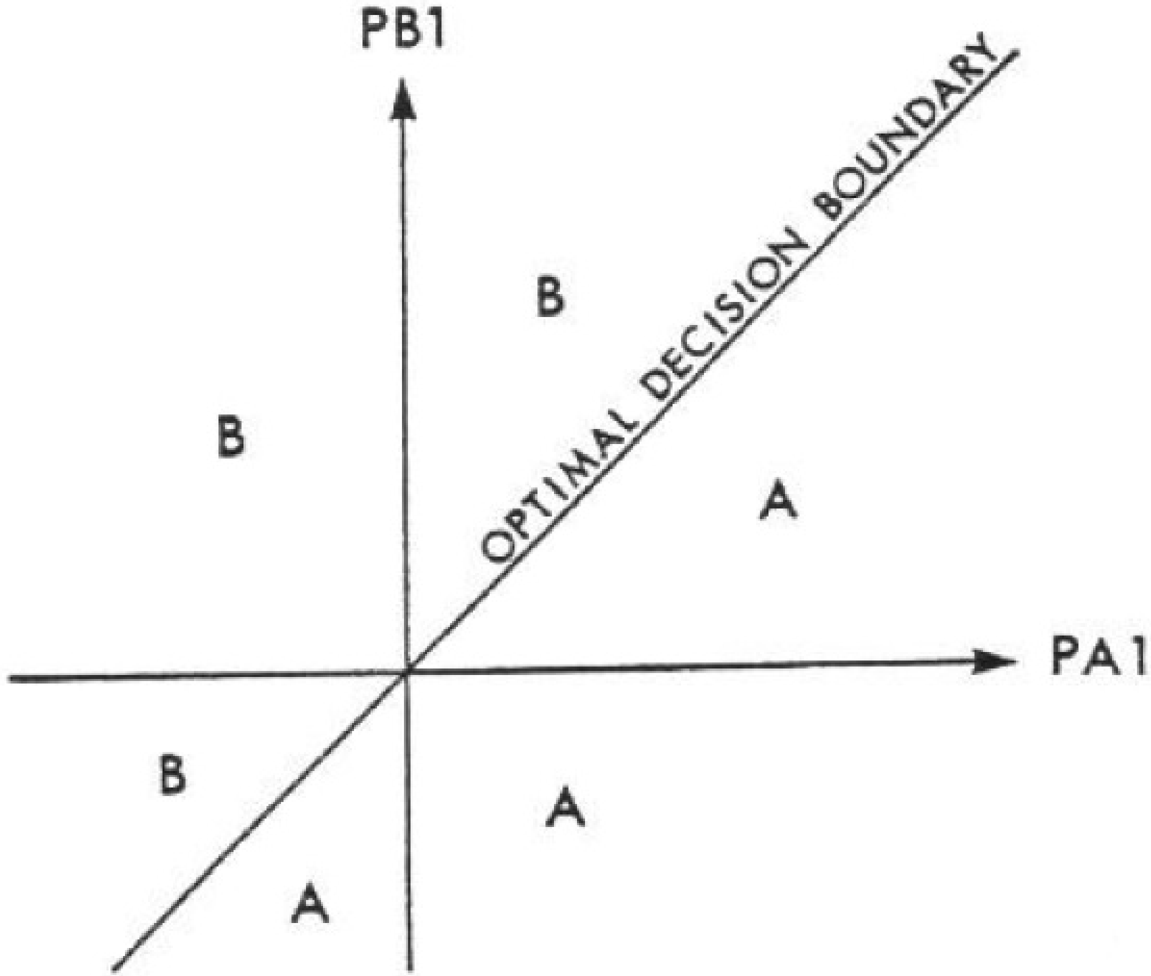


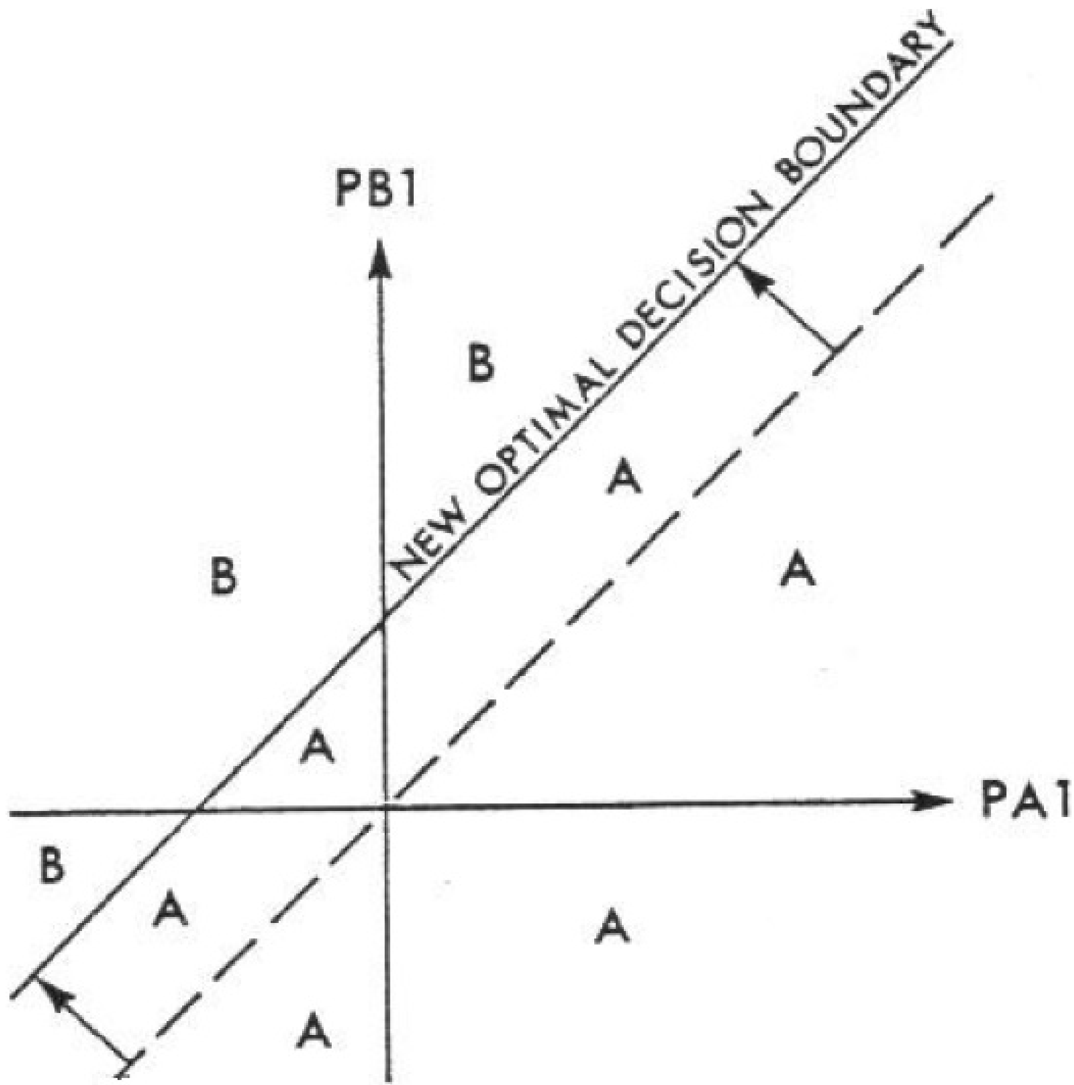
Volume per hectare
Time
t0
t1
t2

The Price Difference Process
(Price of Pine - Price of Birch)



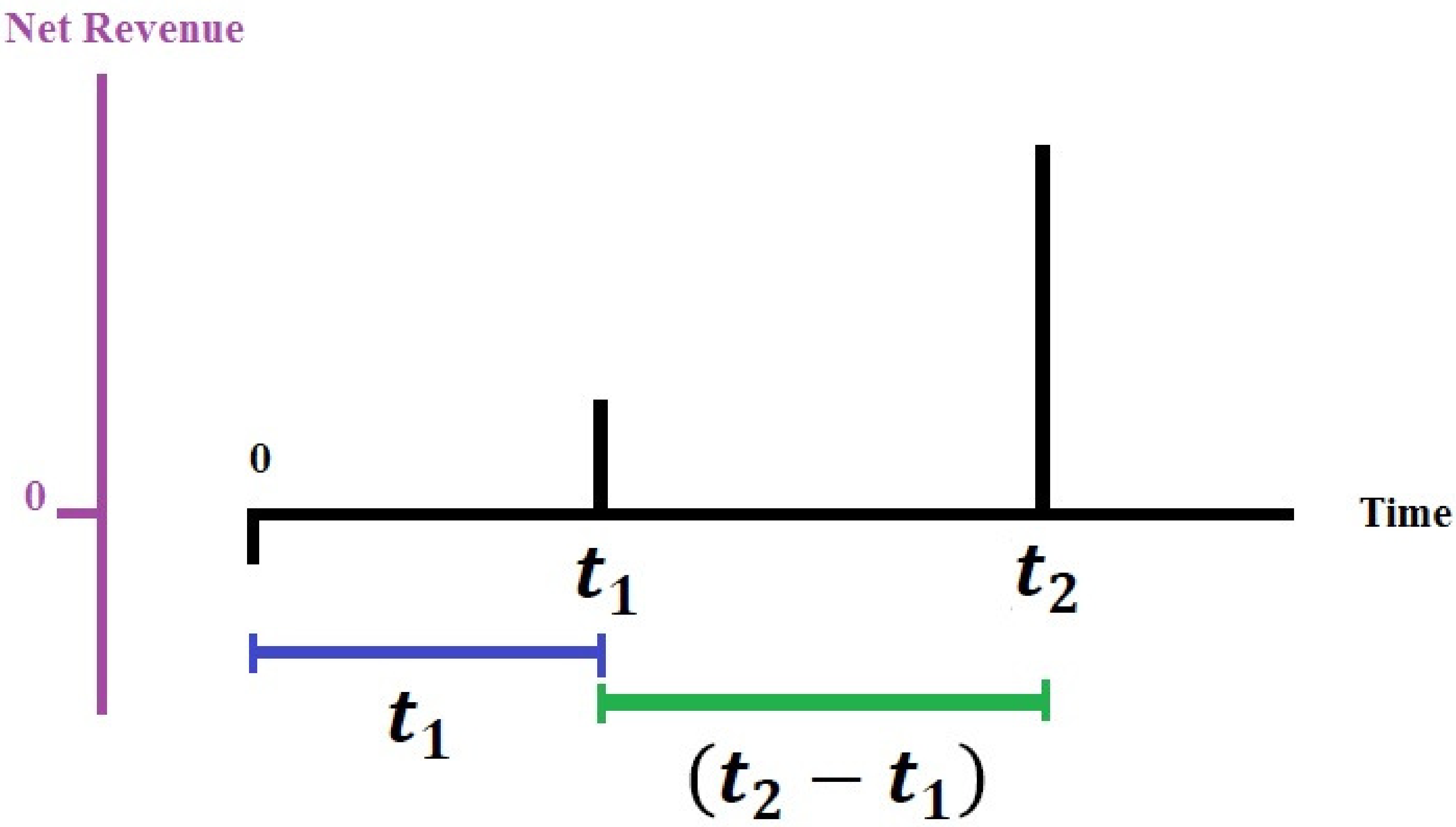

Pb
Pa
Time = 0
Pb
Pa
Time = 1
X
Pb
Pa
Time = 2
Y
Z


PB1
OPTIMAL DECISION BOUNDARY
B
B
A
PA1
B
A
A


PB1
NEW OPTIMAL DECISION BOUNDARY
B
A
B
A
A
PA1
B
A
A
A

The stochastic prices and mixed species problem

A new explorative investigation of the fundamental problem


Net Revenue
0
0
Time
t_1
t_2
t_1
$(t_2 - t_1)$

Initial price process assumptions:

Prices are real Martingale processes.

$\boldsymbol{\sigma_{AB} = 0}$.

$$P_{A_1} = P_{A_0} + \Delta P_{A_1}$$

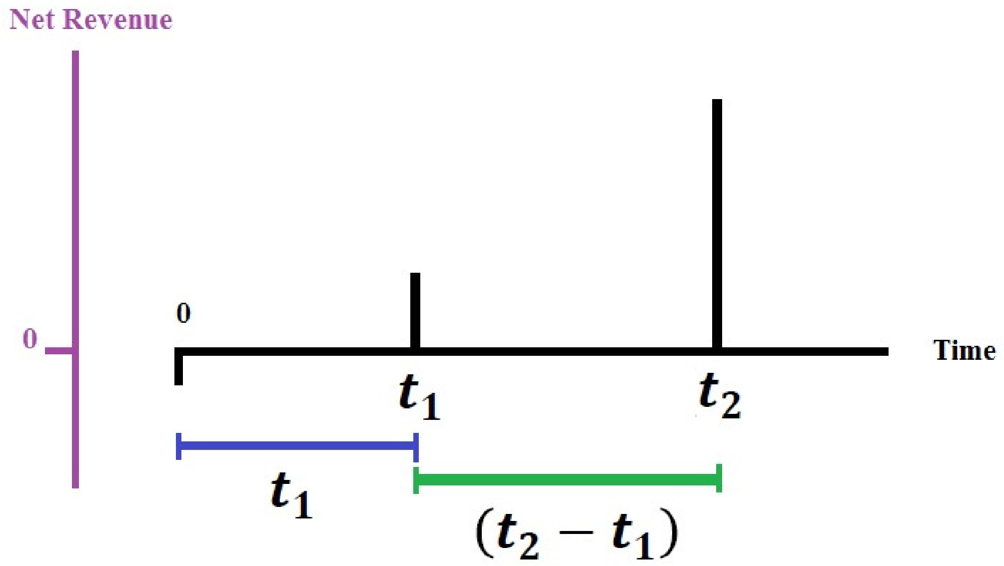
$$\Delta P_{A_1} \in N\left(0, t_1 \sigma_A^2\right)$$

$$E\left(P_{A_1} \middle| P_{A_0}\right) = P_{A_0}$$

$$\Delta P_{A_1} = \Delta_1 + \Delta_2 + \cdots + \Delta_{t_1-1} + \Delta_{t_1}$$

$$\Delta_i \in N\left(0, \sigma_A^2\right) \ \forall i$$

$$\Delta P_{A_1} \in N\left(0, t_1 \sigma_A^2\right)$$



$$P_{A_2} = P_{A_1} + \Delta P_{A_2}$$

$$\Delta P_{A_2} \in N\left(0, (t_2 - t_1)\sigma_A^2\right)$$

$$E\left(P_{A_2} \middle| P_{A_1}\right) = P_{A_1}$$

$$E\left(P_{A_2} \middle| P_{A_0}\right) = P_{A_0}$$

In most cases, $P_{A_1} \neq P_{A_0}$

Observation:

$$E(P_{A_2}|P_{A_1}) - E(P_{A_2}|P_{A_0}) \begin{cases} > 0 \; if \; (P_{A_1} > P_{A_0}) \\ = 0 \; if \; (P_{A_1} = P_{A_0}) \\ < 0 \; if \; (P_{A_1} < P_{A_0}) \end{cases}$$

$$P_{B_1} = P_{B_0} + \Delta P_{B_1}$$

$$\Delta P_{B_1} \in N\left(0, t_1 \sigma_B^2\right)$$

$$E(P_{B_1} | P_{B_0}) = P_{B_0}$$

$$P_{B_2} = P_{B_1} + \Delta P_{B_2}$$

$$\Delta P_{B_2} \in N\left(0, (t_2 - t_1) \sigma_B^2\right)$$

$$E(P_{B_2} | P_{B_1}) = P_{B_1}$$

$$E(P_{B_2} | P_{B_0}) = P_{B_0}$$

In most cases, $P_{B_1} \neq P_{B_0}$

Observation:

$$E(P_{B_2}|P_{B_1}) - E(P_{B_2}|P_{B_0}) \begin{cases} > 0 \; if \; (P_{B_1} > P_{B_0}) \\ = 0 \; if \; (P_{B_1} = P_{B_0}) \\ < 0 \; if \; (P_{B_1} < P_{B_0}) \end{cases}$$

$$d_1 = e^{-rt_1}, d_2 = e^{-rt_2}$$

Expected present value of management system A without adaptive decisions:

$$\pi_A = E\left(\left(-c_A + d_1 P_{A_1} h_{A_1} + d_2 P_{A_2} h_{A_2}\right) \middle| P_{A_0}\right)$$

$$\pi_A = -c_A + d_1 E(P_{A_1} | P_{A_0}) h_{A_1} + d_2 E(P_{A_2} | P_{A_0}) h_{A_2}$$

$$E(P_{A_1} | P_{A_0}) = P_{A_0}$$

$$E(P_{A_2} | P_{A_1}) = P_{A_1}$$

$$\pi_A = -c_A + d_1 P_{A_0} h_{A_1} + d_2 P_{A_0} h_{A_2}$$

Expected present value of management system B without adaptive decisions:

$$\pi_B = E\left(\left(-c_B + d_1 P_{B_1} h_{B_1} + d_2 P_{B_2} h_{B_2}\right) \middle| P_{B_0}\right)$$

$$\pi_B = -c_B + d_1 E\left(P_{B_1} \middle| P_{B_0}\right) h_{B_1} + d_2 E\left(P_{B_2} \middle| P_{B_0}\right) h_{B_2}$$

$$E\left(P_{B_1} \middle| P_{B_0}\right) = P_{B_0}$$

$$E\left(P_{B_2} \middle| P_{B_1}\right) = P_{B_1}$$

$$\pi_B = -c_B + d_1 P_{B_0} h_{B_1} + d_2 P_{B_0} h_{B_2}$$

Expected present value of a management system AB with adaptive decisions. (50% of the stems are removed at t_1.)

$$\pi_{AB} = \begin{pmatrix} -\dfrac{(c_A + c_B)}{2} - k \\ + \\ \phi\left(\left(d_1 P_{B_1} h_{B_1} + d_2 E(P_{A_2} \middle| P_{A_1}) h_{A_2} \geq d_1 P_{A_1} h_{A_1} + d_2 E(P_{B_2} | P_{B_1}) h_{B_2}\right) \middle| (P_{A_0}, P_{B_0})\right) Z_1 \\ + \\ \phi\left(\left(d_1 P_{B_1} h_{B_1} + d_2 E(P_{A_2} \middle| P_{A_1}) h_{A_2} < d_1 P_{A_1} h_{A_1} + d_2 E(P_{B_2} | P_{B_1}) h_{B_2}\right) \middle| (P_{A_0}, P_{B_0})\right) Z_2 \end{pmatrix}$$

$$\boldsymbol{Z_1 = E\left(d_1 P_{B_1} h_{B_1} + d_2 P_{A_2} h_{A_2} \middle| \left(P_{A_0}, P_{B_0}, \left(d_1 P_{B_1} h_{B_1} + d_2 E(P_{A_2} | P_{A_1}) h_{A_2} \geq d_1 P_{A_1} h_{A_1} + d_2 E(P_{B_2} | P_{B_1}) h_{B_2}\right)\right)\right)}$$

$$\boldsymbol{Z_2 = E\left(d_1 P_{A_1} h_{A_1} + d_2 P_{B_2} h_{B_2} \middle| \left(P_{A_0}, P_{B_0}, \left(d_1 P_{B_1} h_{B_1} + d_2 E(P_{A_2} | P_{A_1}) h_{A_2} < d_1 P_{A_1} h_{A_1} + d_2 E(P_{B_2} | P_{B_1}) h_{B_2}\right)\right)\right)}$$

$$\pi_{AB} = -\frac{(c_A + c_B)}{2} - k + \phi Z_1 + (1 - \phi) Z_2$$

$$\pi_{AB} = -\frac{(c_A + c_B)}{2} - k + \phi Z_1 + (1-\phi) Z_2$$

$$\phi = \phi\left(\left(d_1 P_{B_1} h_{B_1} + d_2 E(P_{A_2} \middle| P_{A_1}) h_{A_2} \geq d_1 P_{A_1} h_{A_1} + d_2 E(P_{B_2} \middle| P_{B_1}) h_{B_2}\right) \middle| (P_{A_0}, P_{B_0})\right)$$

$$Z_1 = E\left(d_1 P_{B_1} h_{B_1} + d_2 P_{A_2} h_{A_2} \middle| \left(P_{A_0}, P_{B_0}, (d_1 P_{B_1} h_{B_1} + d_2 E(P_{A_2}|P_{A_1}) h_{A_2} \geq d_1 P_{A_1} h_{A_1} + d_2 E(P_{B_2}|P_{B_1}) h_{B_2})\right)\right)$$

$$Z_2 = E\left(d_1 P_{A_1} h_{A_1} + d_2 P_{B_2} h_{B_2} \middle| \left(P_{A_0}, P_{B_0}, (d_1 P_{B_1} h_{B_1} + d_2 E(P_{A_2}|P_{A_1}) h_{A_2} < d_1 P_{A_1} h_{A_1} + d_2 E(P_{B_2}|P_{B_1}) h_{B_2})\right)\right)$$

Case 1:

Harvests at t_1 are not affected by the timber prices P_{A_1} and P_{B_1} (since harvests at t_1 do not give timber but other assortments, such as energy assortments and pulp wood.)

We assume that all management alternatives lead to the same net present values of harvests at t_1.

In order to make the following derivations easier to follow, we exclude the present values of harvests at t_1 from π_A, π_B and π_{AB}.

We also assume that the timber harvest volumes are the same, h, for both species.

$$\pi_A = -c_A + d_2 E(P_{A_2} \middle| P_{A_0}) h_{A_2}$$

$$\pi_A = -c_A + d_2 P_{A_0} h$$

$$\pi_B = -c_B + d_2 E(P_{B_2} \middle| P_{B_0}) h_{B_2}$$

$$\pi_B = -c_B + d_2 P_{B_0} h$$

$$\pi_{AB} = -\frac{(c_A + c_B)}{2} - k + \phi Z_1 + (1-\phi) Z_2$$

$$\phi = \phi\left(\left(d_2 E(P_{A_2} | P_{A_1}) h_{A_2} \geq d_2 E(P_{B_2} | P_{B_1}) h_{B_2}\right) \middle| (P_{A_0}, P_{B_0})\right)$$

$$\phi = \phi\left(\left(d_2\, P_{A_1} h_{A_2} \geq d_2 P_{B_1} h_{B_2}\right) \middle| (P_{A_0}, P_{B_0})\right)$$

$$\phi = \phi\left(\left(P_{A_1} \geq P_{B_1}\right) \middle| (P_{A_0}, P_{B_0})\right)$$

$$Z_1 = E\left(d_2 P_{A_2} h_{A_2} \middle| \left(P_{A_0}, P_{B_0}, \left(E(P_{A_2}|P_{A_1}) h_{A_2} \geq E(P_{B_2}|P_{B_1}) h_{B_2}\right)\right)\right)$$

$$Z_1 = d_2 E\left(P_{A_2} \middle| \left(P_{A_0}, P_{B_0}, \left(P_{A_1} \geq P_{B_1}\right)\right)\right) h$$

$$Z_2 = E\left(d_2 P_{B_2} h_{B_2} \middle| \left(P_{A_0}, P_{B_0}, \left(E(P_{A_2} | P_{A_1}) h_{A_2} < E(P_{B_2} | P_{B_1}) h_{B_2}\right)\right)\right)$$

$$Z_2 = d_2 E\left(P_{B_2} \middle| \left(P_{A_0}, P_{B_0}, \left(P_{A_1} < P_{B_1}\right)\right)\right) h$$

Summary of Case 1:

$$\pi_A = -c_A + d_2 P_{A_0} h$$

$$\pi_B = -c_B + d_2 P_{B_0} h$$

$$\pi_{AB} = -\frac{(c_A + c_B)}{2} - k + \phi Z_1 + (1 - \phi) Z_2$$

$$\pi_{AB} = -\frac{(c_A + c_B)}{2} - k + \phi Z_1 + (1 - \phi) Z_2$$

$$\phi = \phi\left(\left(P_{A_1} \geq P_{B_1}\right) \middle| \left(P_{A_0}, P_{B_0}\right)\right)$$

$$Z_1 = d_2 E\left(P_{A_2} \middle| \left(P_{A_0}, P_{B_0}, \left(P_{A_1} \geq P_{B_1}\right)\right)\right) h$$

$$Z_2 = d_2 E\left(P_{B_2} \middle| \left(P_{A_0}, P_{B_0}, \left(P_{A_1} < P_{B_1}\right)\right)\right) h$$

Case 2:

As Case 1 with the following constraints:

$$\boldsymbol{\sigma_B = 0}$$

$$\boldsymbol{(\sigma_B = 0) \Longrightarrow \left(\Delta P_{B_1} = 0\,;\ \Delta P_{B_2} = 0\right)}$$

$$\boldsymbol{P_{A_0} = P_{B_0}}$$

$$p = P_{A_1} - P_{B_1}$$

$$f(p) = \begin{cases} 0 \quad , & p \leq -g \\ g^{-1} + g^{-2}p \quad , & -g < p < 0 \\ g^{-1} \quad , & p = 0 \\ g^{-1} - g^{-2}p \quad , & 0 < p < g \\ 0 \quad , & p \geq g \end{cases}$$

$$\pi_{AB} = -\frac{(c_A + c_B)}{2} - k + \phi Z_1 + (1 - \phi) Z_2$$

$$\phi = \frac{1}{2}$$

$$Z_1 = \left(2d_2 \int_0^g (\alpha + \beta p) f(p) dp \right) h$$

$$Z_1 = \left(2d_2 \int_0^g (\alpha + \beta p) f(p) dp \right) h$$

$$Z_3 = \frac{Z_1}{2d_2 h}$$

$$Z_3 = \int_0^g (\alpha + \beta p) f(p) dp$$

$$Z_3 = \int_0^g (\alpha + \beta p) f(p) dp$$

$$Z_3 = \int_0^g (\alpha + \beta p)\left(g^{-1} - g^{-2}p\right) dp$$

$$Z_3 = \int_0^g \left(\alpha g^{-1} - \alpha g^{-2}p + \beta g^{-1}p - \beta g^{-2}p^2\right) dp$$

$$Z_3 = \int_0^g \left(\alpha g^{-1} - \alpha g^{-2} p + \beta g^{-1} p - \beta g^{-2} p^2\right) dp$$

$$Z_3 = \left[\alpha g^{-1} p - \frac{\alpha g^{-2} p^2}{2} + \frac{\beta g^{-1} p^2}{2} - \frac{\beta g^{-2} p^3}{3}\right]_0^g$$

$$Z_3 = \alpha g^{-1} g - \frac{\alpha g^{-2} g^2}{2} + \frac{\beta g^{-1} g^2}{2} - \frac{\beta g^{-2} g^3}{3}$$

$$Z_3 = \alpha g^{-1} g - \frac{\alpha g^{-2} g^2}{2} + \frac{\beta g^{-1} g^2}{2} - \frac{\beta g^{-2} g^3}{3}$$

$$Z_3 = \alpha - \frac{\alpha}{2} + \frac{\beta g}{2} - \frac{\beta g}{3}$$

$$Z_3 = \frac{\alpha}{2} + \left(\frac{1}{2} - \frac{1}{3}\right) \beta g$$

$$Z_3 = \frac{\alpha}{2} + \left(\frac{1}{2} - \frac{1}{3}\right)\beta g$$

$$Z_3 = \frac{\alpha}{2} + \left(\frac{3}{6} - \frac{2}{6}\right)\beta g$$

$$Z_3 = \frac{\alpha}{2} + \frac{\beta}{6} g$$

$$Z_1 = 2d_2h\left(\frac{\alpha}{2} + \frac{\beta}{6} g\right)$$

$$Z_1 = d_2h\left(\alpha + \frac{\beta}{3} g\right)$$

$$Z_2 = d_2 E\left(P_{B_2} \middle| \left(P_{A_0}, P_{B_0}, (P_{A_1} < P_{B_1})\right)\right) h$$

$$Z_2 = d_2 P_{B_0} h$$

Observation: $Z_1 = d_2 h\left(\alpha + \frac{\beta}{3} g\right)$

For g=0:

$$Z_1 = d_2 h \alpha$$

$$Z_2 = d_2 P_{B_0} h$$

$$d_2 h \alpha = d_2 P_{B_0} h$$

$$\alpha = P_{B_0}$$

Comparison of investments for g=0:

$$\pi_A = -c_A + d_2 P_{A_0} h$$

$$\pi_B = -c_B + d_2 P_{B_0} h$$

$$\pi_{AB} = -\frac{(c_A + c_B)}{2} - k + \phi Z_1 + (1 - \phi) Z_2$$

$$\pi_{AB} = -\frac{(c_A + c_B)}{2} - k + \phi Z_1 + (1 - \phi) Z_2$$

$$\pi_{AB} = -\frac{(c_A + c_B)}{2} - k + \frac{1}{2} d_2 h \left(\alpha + \frac{\beta}{3} g\right) + \frac{1}{2} d_2 P_{B_0} h$$

$$\pi_{AB} = -\frac{(c_A + c_B)}{2} - k + \frac{1}{2} d_2 h \left(\alpha + \frac{\beta}{3} 0\right) + \frac{1}{2} d_2 P_{B_0} h$$

$$\pi_{AB} = -\frac{(c_A + c_B)}{2} - k + \frac{1}{2} d_2 h \alpha + \frac{1}{2} d_2 P_{B_0} h$$

$$\alpha = P_{B_0}$$

$$\pi_{AB} = -\frac{(c_A + c_B)}{2} - k + \frac{1}{2} d_2 h P_{B_0} + \frac{1}{2} d_2 P_{B_0} h$$

$$\pi_{AB} = -\frac{(c_A + c_B)}{2} - k + d_2 P_{B_0} h$$

$$P_{A_0} = P_{B_0} = P_0$$

$\text{Let } c_A = c_B = c$

$$\pi_A = -c_A + d_2 P_{A_0} h$$

$$\pi_A = -c + d_2 P_0 h$$

$$\pi_B = -c_B + d_2 P_{B_0} h$$

$$\pi_B = -c + d_2 P_0 h$$

$$\boldsymbol{\pi_{AB} = -\frac{(c_A + c_B)}{2} - k + d_2 P_{B_0} h}$$

$$\boldsymbol{\pi_{AB} = -c - k + d_2 P_0 h}$$

Then,

$$\boldsymbol{\pi_A = \pi_B = \pi_1}$$

Standard assumption: $\mathbf{k > 0}$.

$$(\mathbf{g = 0} \wedge \boldsymbol{k} > \mathbf{0}) \Rightarrow (\boldsymbol{\pi_{AB} < \pi_A = \pi_B})$$

Comparative statics for g > 0:

$$\pi_{AB} = -\frac{(c_A + c_B)}{2} - k + \phi Z_1 + (1-\phi) Z_2$$

$$\pi_{AB} = -\frac{(c_A + c_B)}{2} - k + \frac{1}{2} d_2 h \left(\alpha + \frac{\beta}{3} g\right) + \frac{1}{2} d_2 P_{B_0} h$$

$$\frac{d\pi_{AB}}{dg} = \frac{d_2 h \beta}{6} > 0$$

Observation:

$$\Delta\pi = \pi_{AB} - \pi_1 = -k + \frac{d_2 h\beta}{6} g$$

$$\Delta\pi = \pi_{AB} - \pi_1 \begin{cases} < 0, & k > \dfrac{d_2 h\beta}{6} g \\ = 0, & k = \dfrac{d_2 h\beta}{6} g \\ > 0, & k < \dfrac{d_2 h\beta}{6} g \end{cases}$$

General comparative statics analysis:

$$\pi_{AB} = -\frac{(c_A + c_B)}{2} - k + \phi Z_1 + (1 - \phi) Z_2$$

$$\phi = \phi(\sigma_A, \dots)$$
$$Z_1 = Z_1(\sigma_A, \dots)$$
$$Z_2 = Z_2(\sigma_A, \dots)$$

$$\pi_{AB} = -\frac{(c_A + c_B)}{2} - k + \phi(\sigma_A)Z_1(\sigma_A) + (1 - \phi(\sigma_A))Z_2(\sigma_A)$$

$$\pi_{AB} = -\frac{(c_A + c_B)}{2} - k + \phi(\sigma_A)Z_1(\sigma_A) + (1 - \phi(\sigma_A))Z_2(\sigma_A)$$

$$\frac{d\pi_{AB}}{d\sigma_A} = \frac{d\phi(\sigma_A)}{d\sigma_A}Z_1(\sigma_A) + \phi(\sigma_A)\frac{dZ_1(\sigma_A)}{d\sigma_A} - \frac{d\phi(\sigma_A)}{d\sigma_A}Z_2(\sigma_A) + (1 - \phi(\sigma_A))\frac{dZ_2(\sigma_A)}{d\sigma_A}$$

$$\frac{d\pi_{AB}}{d\sigma_A} = \frac{d\phi(\sigma_A)}{d\sigma_A}(Z_1 - Z_2) + \phi(\sigma_A)\frac{dZ_1(\sigma_A)}{d\sigma_A} + (1 - \phi(\sigma_A))\frac{dZ_2(\sigma_A)}{d\sigma_A}$$

Observation:

On the decision boundary, the expected present values of A and B are the same,

$$Z_1 - Z_2 = 0$$

Usually,

$$\left|\frac{dZ_2(\sigma_{A,}.)}{d\sigma_A}\right| \ll \left|\frac{dZ_1(\sigma_{A,}.)}{d\sigma_A}\right| \text{ and } \frac{dZ_1(\sigma_A)}{d\sigma_A} > 0$$

$\phi(\sigma_A,.)$ **and** $\left(1 - \phi(\sigma_A,.)\right)$ **are strictly positive and of the same order of magnitude. Then:**

$$\frac{d\pi_{AB}}{d\sigma_A} \approx \phi(\sigma_A)\frac{dZ_1(\sigma_A)}{d\sigma_A} > 0$$

Observation:

$$\Delta\pi = \pi_{AB} - \pi_1 = -k + w(\sigma_A)$$

$$\frac{dw(\sigma_A)}{d\sigma_A} > 0$$

$$\Delta\pi = \pi_{AB} - \pi_1 \begin{cases} < 0\ , & k > w(\sigma_A) \\ = 0\ , & k = w(\sigma_A) \\ > 0\ , & k < w(\sigma_A) \end{cases}$$

CONCLUSIONS:

In typical cases:

If $\sigma_A = 0$, a one species plantation gives the highest expected present value.

At some strictly positive and unique value of σ_A , investments in one or two species give the same expected present value.

At higher values of σ_A , a two species investment gives a strictly higher expected present value than a one species investment.

Rational Research in Forest Production and Forest Management with Consideration of Mixed Species Forest Management and Risk

- The latest decades clearly show that detailed deterministic long term planning is irrelevant.
- Energy prices, prices of industrial products and environmental problems rapidly change in ways that cannot be perfectly predicted.
- **Research in forest production planning should focus on the development of growth models that are useful when stochastic optimal control theory is applied.**
- **Since biological production takes considerable time, it is very important to create options to sequentially adjust the production to new relative prices, growth conditions, ecological problems and possible damages caused by parasites, fire or storms.**
- In particular, valuable options can be obtained via mixed species stands. When several species are available in the young stands, the species mix can sequentially be adapted to changing product prices, costs and growth conditions.

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