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PREFACE

The Society of American Forester's Systems Analysis Working Group sponsored its first major Symposium in Athens, Georgia in 1975 and followed it up with a second Symposium in the same location in 1985. The third symposium was held at the Asilomar Conference Center in Pacific Grove, California in 1988. The fourth Symposium was held in Charleston, South Carolina in 1991. The fifth was held in Valdivia, Chile in 1993. These proceedings contain the papers of the sixth symposium held in Pacific Grove in 1994.

There were 53 papers presented by participants from 13 countries. These included: USA, Canada, Sweden, Denmark, Finland, Chile, Mexico, Brazil, Australia, New Zealand, Japan, China, and Indonesia.

Larry S. Davis made preliminary arrangements and social arrangements for the meeting and J. Keith Gilless handled registration and onsite facilities supported by the Department of Environmental Policy and Management, University of California, Berkeley.

J. Douglas Brodie handled mailing and program organization and John Sessions and Doug Brodie compiled and organized the proceedings supported by the Department of Forest Engineering and the Department of Forest Resources, Oregon State University. Publishing of the proceedings was made possible through the generous support of the E4 Working Group and the USDA Forest Service. Peter Dress, Brian Turner, Peter Lohmander and Andre LaRoze provided discussion leadership in the open forums.

The participants are always the deciding factor in the success of these meetings and the level of domestic and international participation is an increasingly satisfying aspect of attendance. The fine fall weather and unequalled maritime scenery of the Monterey area contributed to the meeting success. We look forward to the next meeting scheduled internationally or in the Southern USA in 1997.

RESERVATION PRICE MODELS IN FOREST MANAGEMENT: ERRORS IN THE ESTIMATION OF PROBABILITY DENSITY FUNCTION PARAMETERS AND OPTIMAL ADJUSTMENT OF THE BIAS FREE POINT ESTIMATES

by

Peter Lohmander

ABSTRACT: Optimal reservation price strategies applied to dynamic forest harvesting problems are discussed. The sensitivity of the expected objective function value in such models to timber price probability density function parameter estimation errors, is investigated. This is important since the number of relevant price observations generally is low. The analysis explicitly treats normally distributed prices. Mean and variance estimation errors are studied. The expected objective function is approximated as a third order polynomial function of the two dimensional estimation error. It is found that it is better to underestimate than to overestimate the mean (and/or the variance) of the stochastic price when the optimal reservation price strategy is determined (if the absolute value(s) of the estimation error(s) is (are) the same in both cases.). Hence, we should not use the bias free estimate(s) of the parameter(s) but adjusted value(s). First, the optimal adjustment is determined as a function of the number of relevant price observations via the Newton-Raphson method. Then, the optimal objective function value (calculated with optimal estimate adjustment) is determined as a function of the number of relevant price observations. Prices and costs from Sweden and a typical stand growth function are used to illustrate the general principles and to derive numerical results.

KEYWORDS--Adaptive forest harvesting, estimate adjustment, stochastic dynamic programming, reservation price.

INTRODUCTION

During the latest decades a large number of applications of reservation price models have been reported from forestry. A number of those are found in the reference list. It has been found that considerable economic gains can be obtained in case those decision models based on sequential information and decisions are used instead of deterministic long term harvest planning models. The usual assumption has been that the parameters of the price probability density function are known when the optimal reservation price strategy is calculated. Usually, old price observations have been used in the estimation of the probability density functions.

Clearly, in most cases, the relevant historical price series are short and the numbers of "relevant" observations are low. Hence, at least two important estimates of properties of the price probability density function should be expected to contain errors: the mean and the variance. In particular the price variance is important to the reservation price strategy. Not even the functional form is known with certainty. In lack of an obviously relevant theory with respect to this, we may assume that the probability density function is Normal and try to reject that hypothesis with some test such as the Chi-quare test. This has sometimes been done.

However, we must be aware that the number of observations usually is low. Is the true density function Normal or perhaps triangular? It usually takes lots of observations to decide which of these possible candidates is the best. The triangular and the Normal distributions differ manily in the shape of the tails. In some problems, this may not matter very much to the results. In adaptive problems, however, the tails are very important.

The general properties of the tails of probability density functions have been investigated by Fisher and Tippett (1928), Berman (1964, 1982), Haan (1976), Davis (1979) and by Leadbetter, Lindgren and Rootze'n (1983). An example of a practically interesting paper for ocean engineering purposes is Lindgren and Rychlik (1982). Timber prices and sea waves have similarities. Recent papers on reservation prices in forestry, adaptive harvesting and related issues, have been written by Risvand (1976), Lohmander (1985, 1987, 1988a, 1988b, 1994a, 1994b, 1994c), Kaya and Buongiorno (1987, 1989), Brazee and Mendelsohn (1988), Haight (1990, 1991), Gong (1994), Mörling (1994), Petersson (1994), Ståhl (1994) and Ståhl, Carlsson and Bondesson (1994).

In adaptive forest harvesting, we wait for good prices and are very interested in the probabilities of the very highest prices, the prices in one of the tails. In a similar way, in "extreme value theory" and engineering applications, the tails of distributions are the most interesting parts. If you should construct an oil platform or a bridge, you want to know the probability that the sea waves are higher than let us say 28 meters. Such waves are extreme in most areas and most wave observations may be below five meters. Perhaps you have never even observed one wave above 25 meters. Still, the probability of such a wave will determine how you should build your construction.

Lindgren and Rootzen (1986) write:

" - For example, an important design parameter in offshore construction, is the height u100 of the "hundred year wave".

" A more detailed analysis concerns the interplay between a stochastic load process and the dynamical properties of a structure ..."

In a similar way, it is obvious that the probability density function of timber price, and the frequency of prices above a particular level, are not the only things of importance to the forest owner. It is necessary to consider the price probability density function, the growth of the forest stand and other technical questions simultaneously.

The reservation price is mostly an increasing function of the price variance, ceteres paribus. (Formal proofs are complicated and may be found in Lohmander (1987).) Hence, if the estimated variance is higher than the true variance, the reservation prices derived from the estimated variance will be higher than what they should be. Then, we may never get prices above the reservation price. Obviously, we would in that case wait for good prices for ever and never get any harvest revenues. If, on the other hand, the estimated price variance is lower than the true variance, the reservation prices are lower than optimal and we tend to harvest even if the price is too low to motivate this. We do get harvest revenues but the expected present value is not maximized.

In a similar way, the reservation prices are increasing functions of the mean of the distribution. Hence, if we over- or underestimate the mean, we get problems of the same nature as when we over- or underestimate the variance. Clearly, it seems more dangerous to overestimate than to underestimate the price variance and the mean. Furthermore, the lowest possible variance estimate is zero. With a zero variance estimate we should harvest as soon as the present value is at least as high as the present value in a deterministic problem of a similar nature. Usually, the expected objective function value using this criterion is higher than the deterministic present value. This, in turn is usually significantly greater than zero.

QUESTIONS AND STRUCTURE OF THE PAPER

In this paper, we will investigate the following: - How sensitive is the expected present value in a typical forest management harvest optimization problem, based on a reservation price strategy, to price mean and variance estimation errors?

First, the reservation prices are optimized based on the estimated (with error) price mean and variance. Then, the expected present value is determined from the true price mean and variance and the reservation prices which are not optimal in the true mean and variance case (calculated from the estimated price mean and variance).

The expected present value in the harvest optimization problem is described as a function of the price mean and variance estimation errors. If the errors are zero, the expected present value is maximized. If the price mean OR variance error increases or decreases, the expected present value decreases. However, if one error increases and the other decreases (from zero), the errors affect the derived reservation prices in different directions. The reservation prices may in that case still be optimal in the true problem. The expected present value is approximated as a two dimensional third order polynomial

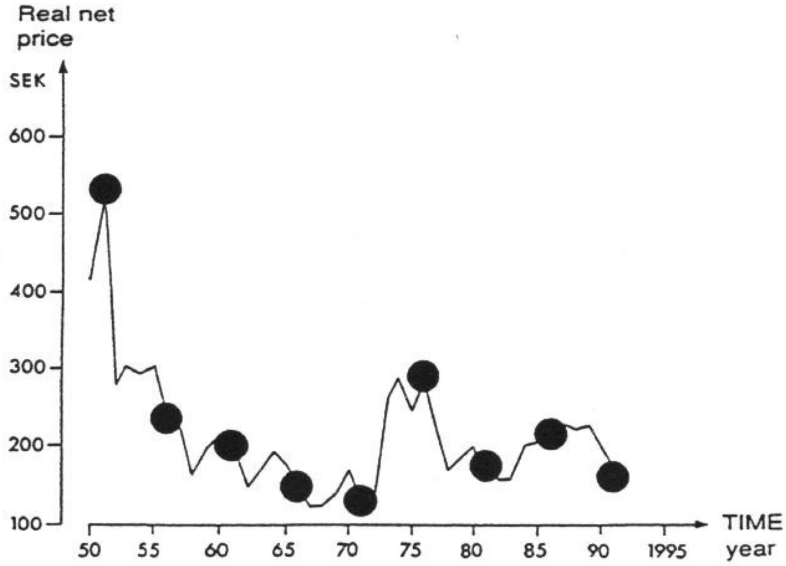

Figure 1. Real stumpage price (price - harvest cost) per cubic metre, Sweden. Source: National Board of Forestry (1993).

function of the estimation errors. This function is highly signifficant.

Next, we assume that, in different problems, we make estimates of the true price mean and variance, based on available price series. We will almost never get the true means and variances, but estimates with errors. The estimates are free from bias: The expected estimates are the true values. We determine the probability density function of the estimated mean and variance.

Then, the question is: What is the expected present value in the forest harvesting problem considering the fact that we have errors in the parameter estimates? Is it optimal to adjust the estimated price mean and/or variance? The expected present value is more negatively affected if the price mean OR variance is overestimated than if it is underestimated, if the absolute errors are the same in both cases. (This is found by inspection of the third order polynomial which is estimated.) Hence, it turns out that it is optimal to decrease the price mean OR variance from the unbiased estimate. We will find that the optimal adjustment is a function of the number of observations. The reason is that the probability of large errors in the estimates decreases with the number of observations.

If the number of observations is very low, the mean should be adjusted with about -4%. As the number of observations increases, the optimal adjustment monotonically converges to zero from below. Compare Figure 14.

EMPIRICAL BACKGROUND

The net prices, in the rest of this paper called "prices" have been very far from constant during the latest decades. This is shown in Figure 1. This is the main reason why it is interesting to consider adaptive optimization in forestry.

Clearly, in case prices are stationary and they sometimes are much higher than the mean value, we should wait for the high prices. The question is then how selective we should be or which "reservation price"

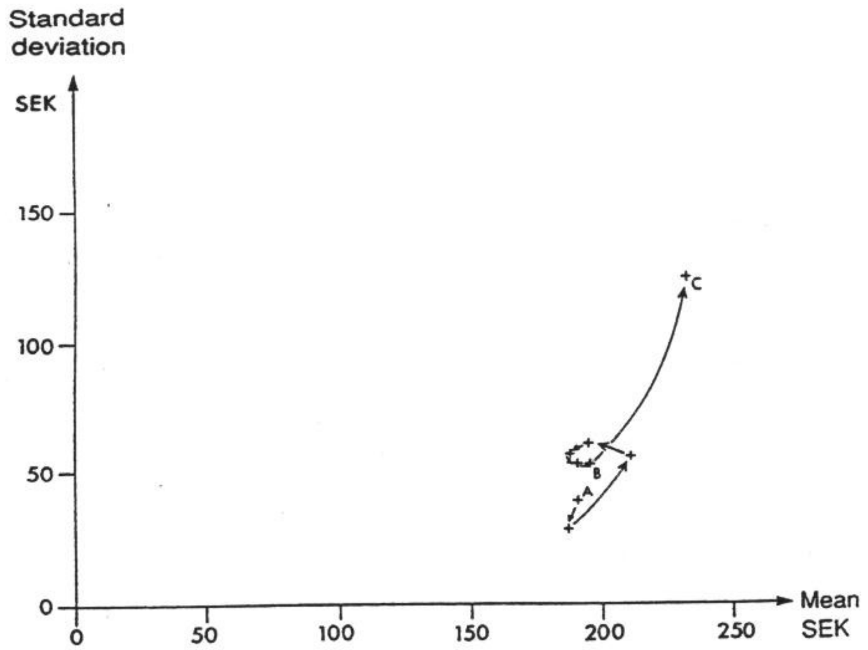

Figure 2. Estimated mean and variance for different horizons. If the horizon contains 2 (= A) to 8 (= B) periods (of five years each), the estimated mean is close to 200 SEK and the estimated standard deviation is close to 50 SEK (= 25%).

we should select. Usually, one can show that the optimal reservation price is a function of time and all economic and biological parameters in the problem. It is important to know the probability density function of price. It is interesting to know the mean and the standard deviation of price and certainly the shape of the probability density function.

In this paper, we will assume that we have normally distributed prices and hence that the only interesting parameters are the mean and the standard deviation. In Figure 2., we see how the estimates of the mean and the variance are affected if we use different numbers of observations from the time series of price. In every case, we use the latest observations in the series. There are five years between each used observation (marked in Figure 1.). The reason is that the sample autocorrelation function shows that prices that are close in the time dimension are rather similar. (The autocorrelation function is strictly positive.) For lags of more than five years, the autocorrelation is almost zero. Hence, we may regard prices in the five year period time scale to be independent and identically distributed random variables. Furthermore, if we consider five year periods, we have lots of time to adjust the harvest capacity when needed in order to change the harvest level considerably. If the periods are very short, on the other hand, capacity constraints may make it difficult or impossible to adapt harvesting to price changes.

Note in particular that the very high price level in the first period illustrated in Figure 1. affects the mean and the variance considerably if we chose to include it in the data (C in Figure 2.). Should we include it? We may argue that the very high price was an effect of the post war boom in the construction sector. Then the question is: Is it likely that we will have other post war booms in the future? Should we include them in our

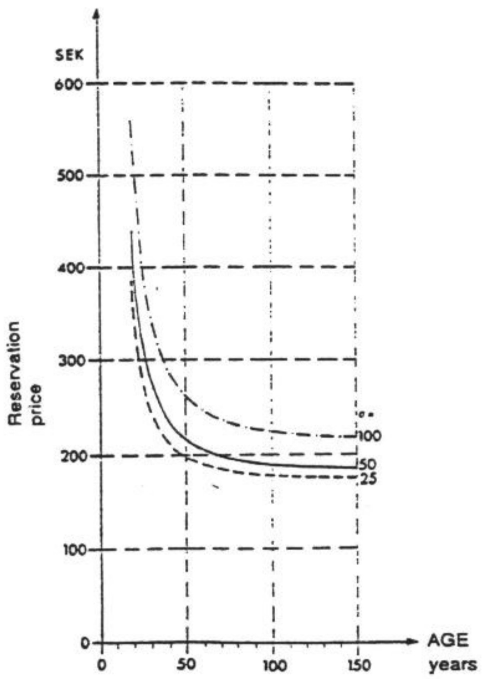

Figure 3. Optimal reservation (net) prices (net prices = price - harvest cost per cubic metre). The reservation prices are increasing functions of the price standard deviation. The different curves represent standard deviations of 25, 50 and 100 SEK. The investment cost is 10 000 SEK and the rate of interest is 3%. The mean net price is 200 SEK.

calculations? The question has no obviously correct answer. In the rest of this paper, we exclude the single observation of a very high price. However, we should be aware that if we include it, the parameters of the estimated price probability density function change considerably and the optimal reservation prices, the expected profitability and the harvesting decisions over time are strongly affected.

Some of the numerical assumptions used in the analysis are the following:

- Volume per hectare, V(t) = 630.3744(1-6.3582^(-t/60))^2.8967, where t denotes stand age. This volume function is empirically motivated, explained in higher detail and used in Lohmander (1988b). It describes the development of pinus Contorta on a typical Swedish site.

- The reported numerical results found in Figures 6, 7, 14, and 15 should be relevant in cases where (i), (ii) and (iii) hold:

  (i) The true standard deviation of the net price is close to 25% of the mean net price (compare Figure 2).

  (ii) The reforestration (= investment) cost per hectare is approximately 50 times higher than the mean net price per cubic metre. (This is a reasonable assumption in the light of the empirical observations reported in Lohmander (1994b)).

  (iii) The real rate of interest is close to 3%.

Presented ideas and results of a more qualitative nature should be relevant also in cases where the presented numerical assumptions do not hold. The absolute magnitudes of effects may however differ from the results reported in the Figures in such cases.

Optimal reservation price strategy and a typical forest harvesting problem

The optimal reservation prices, q, are determined from the expected present value, w, optimization problem.

$$w_t = \max_{q_t} \left[\int_{-\infty}^{q_t} w_{t+1} f(p)dp + \int_{q_t}^{\infty} e^{-rt}(pV(t)+L)f(p)dp \right] \qquad (1)$$

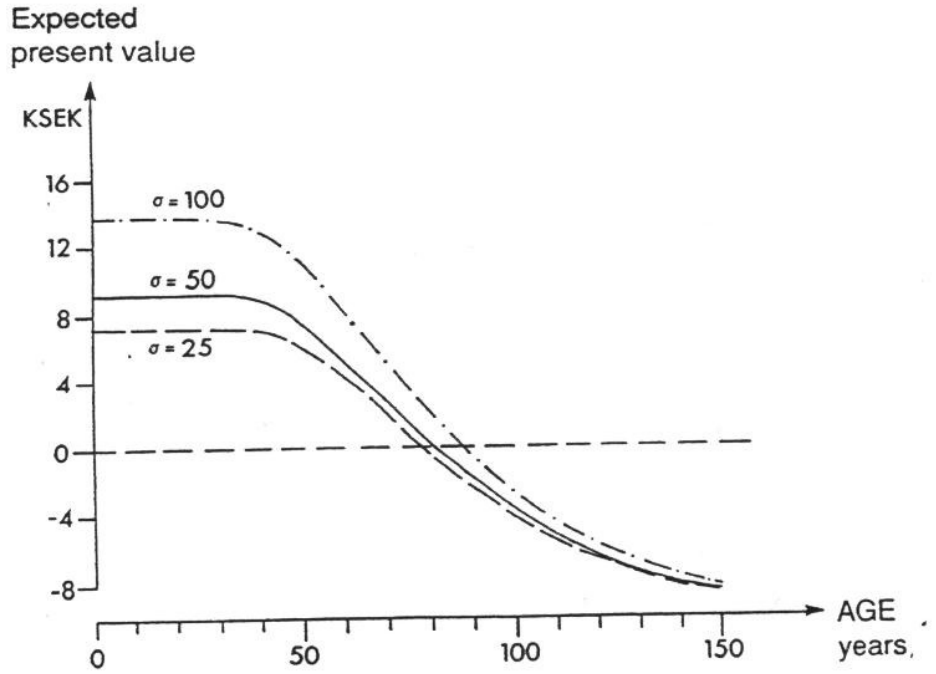

Figure 4. Optimal expected present value per hectare at different stand ages if a harvest has not yet taken place. In the deterministic case, the price standard deviation is zero, and the present value is 5 727 SEK. The expected present values are increasing functions of the price standard deviation. The different curves represent standard deviations of 25, 50 and 100 SEK. The other assumptions are the same as in Figure 3.

f(p) is the probability density function of net price, p. L is the value of the land released after a harvest. (In this analysis, L is the present value of an infinite number of identical forest generations under certainty. Other possible ways to determine L exist.) The first order optimum condition is given in (2).

$$\frac{\delta w_t}{\delta q_t} = f(q_t)\,[\,w_{t+1} - e^{-rt}(q_t V(t) + L)] = 0 \qquad (2)$$

In (2) we define g as g(.) = [...]. One can easily show that the second order maximum condition is satisfied:

$$\frac{\delta^2 w_t}{\delta q_t^2} = \underset{=0}{\frac{\delta f}{\delta q_t}} g + \underset{>0}{f \frac{\delta g}{\delta q_t}} \qquad (3)$$

$$\frac{\delta^2 w_t}{\delta q_t^2} = - \underset{>0}{e^{-rt}} \underset{>0}{f(q_t)} \underset{>0}{V(t)} < 0 \qquad (4)$$

The first order optimum condition gives (5).

$$q_t^* = \frac{e^{rt} w_{t+1} - L}{V(t)} \qquad (5)$$

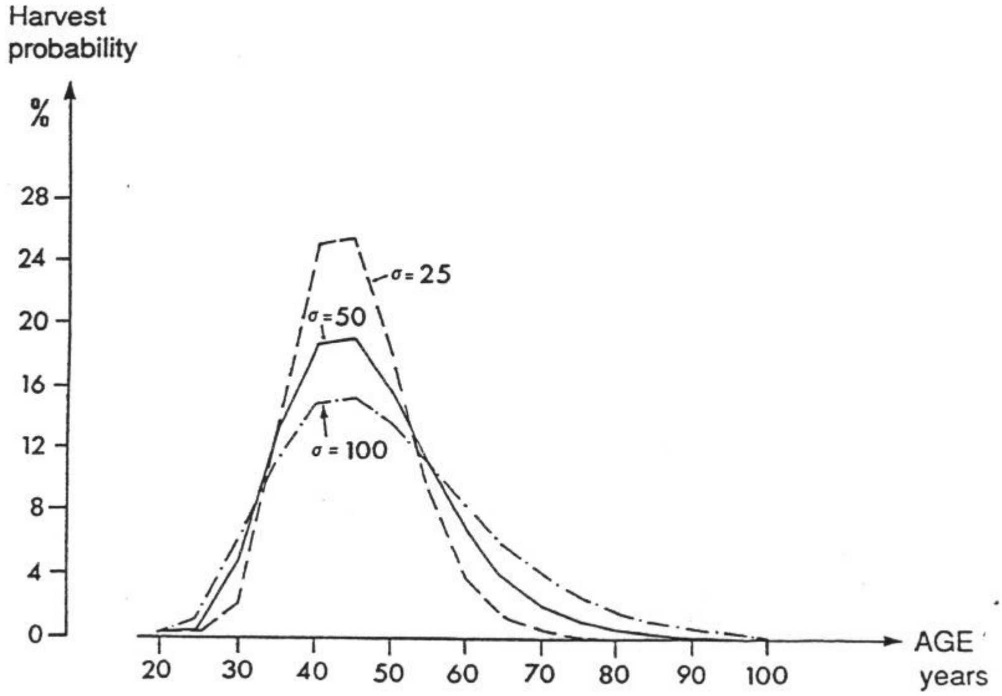

Figure 5. Harvest probabilities at different stand ages.

In (5) we find the optimal reservation price. In Figure 3, the time path of the reservation price is presented. The reservation prices and the expected present values are calculated backwards from the final period T. Note that the optimal reservation prices are increasing functions of the price standard deviation. It turns out that the precise value of T does not affect the obtained results in the early periods very much if T is sufficiently large. If T represents a stand age of 200 years, we have lots of options to harvest much earlier. The probability that it is optimal to wait until period T is almost zero (compare Figure 5). In this model, we assume that T represents the age of 200 years.

In Figure 4. we note that the expected present value is an increasing function of the price standard deviation and a decreasing function of time. Observe that in the Figures 3, 4 and 5 it is assumed that all decisions are based on the correct information about the price mean and variance. In this paper, the central observation is that we do not know the true mean and variance exactly. This will affect the results shown in the mentioned figures.

In a deterministic world, the optimal rotation age is always 45 years in this example. As the price standard deviation increases, the expected deviation from that particular age increases, which is shown in Figure 5. It is more important to adapt to the rapidly changing market conditions than to the slowly changing forest stand density.

If the price standard deviation is zero (the "deterministic case"), then the probability is 1 that harvest takes place at the age of 45 years. As the price standard deviation

Estimated mean / True mean \ Estimated standard deviation / True standard deviation	0.4	0.6	0.8	1.0	1.2	1.4	1.6
0.5	0.428	0.586	0.746	0.904	1.054	1.191	1.313
0.6	0.688	0.837	0.984	1.124	1.253	1.366	1.462
0.7	0.941	1.074	1.202	1.319	1.422	1.505	1.566
0.8	1.174	1.286	1.390	1.478	1.547	1.591	1.609
0.9	1.379	1.463	1.533	1.582	1.607	1.603	1.568
1.0	1.534	1.580	1.606	1.606	1.577	1.515	1.419
1.1	1.605	1.602	1.574	1.516	1.425	1.300	1.141
1.2	1.542	1.485	1.398	1.278	1.126	0.941	0.726
1.3	1.288	1.184	1.044	0.872	0.671	0.444	0.199
1.4	0.814	0.683	0.514	0.316	0.096	-0.137	-0.374
1.5	0.164	0.038	-0.124	-0.310	-0.507	-0.704	-0.893

Figure 6. Expected present value ratios (*) for different estimation errors: True values. (*) = The expected present value of the forest with adaptive decisions divided by the present value of an optimal deterministic program.

increases, the probability of that particular harvest age decreases. The standard deviation of the harvest year increases. The different curves represent the price standard deviations 25, 50 and 100 SEK. The other assumptions are the same as in Figure 3.

Known mean and variance errors: Expected objective function values

Now, assume that we make errors (which we always do with a limited number of observations) when we estimate the price mean and variance. The wrong price distribution parameters are used when we determine the reservation prices. Hence, we sometimes make decisions that are not optimal. What is the expected present value of a plantation if we consider this and assume that we know the estimation errors? The answers are found in Figure 6.

Note in Figure 6. that if the ratio is greater than 1, the adaptive approach is better than the deterministic approach. It is possible to make rather large errors in the mean and variance estimations and still get a better economic result with adaptive decision making than if a deterministic model is used. It is important to observe that if one of the parameters is overestimated and the other is underestimated, the decisions and the economic result may be almost the same as if the correct values were known.

If both parameters are strongly overerstimated however, the reservation prices will be much too high and the economic result is worse than if the deterministic method is used. Results of the kind found in Figure 6 were used to estimate a two dimensional third order polynomial with ordinary regression analysis. All estimated coefficients were found to be strongly signifficant. The polynomial approximation is found in Figure 7, evaluated in a number of points corresponding to the coordinates shown in Figure 6.

Estimated standard deviation / True standard deviation

Estimated mean / True mean	0.4	0.6	0.8	1.0	1.2	1.4	1.6	
0.5	0.391	0.549	0.726	0.912	1.095	1.267	1.418	
0.6	0.640	0.769	0.913	1.063	1.207	1.338	1.444	
0.7	0.904	1.004	1.116	1.230	1.336	1.424	1.486	1.5
0.8	1.157	1.227	1.306	1.384	1.452	1.498	1.515	
0.9	1.369	1.409	1.455	1.498	1.527	1.532	1.503	1.5
1.0	1.511	1.522	1.535	1.542	1.532	1.495	1.422	
1.1	1.555	1.536	1.517	1.488	1.440	1.361	1.243	1.0
1.2	1.473	1.424	1.373	1.308	1.221	1.101	0.938	
1.3	1.236	1.158	1.074	0.973	0.847	0.685	0.478	
1.4	0.815	0.708	0.591	0.455	0.290	0.086	-0.166	
1.5	0.183	0.046	-0.104	-0.275	-0.479	-0.724	-1.021	

Figure 7. Expected present value ratios for different estimation errors: Values according to estimated 2 dimensional 3 order polynomial.

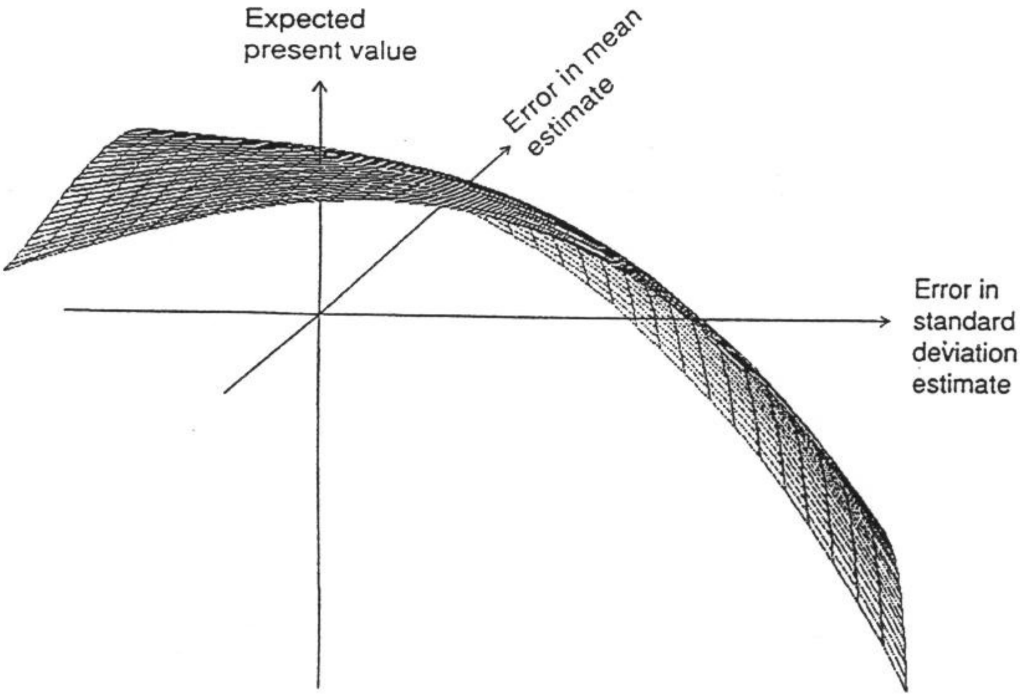

Figure 8. The expected present value surface.

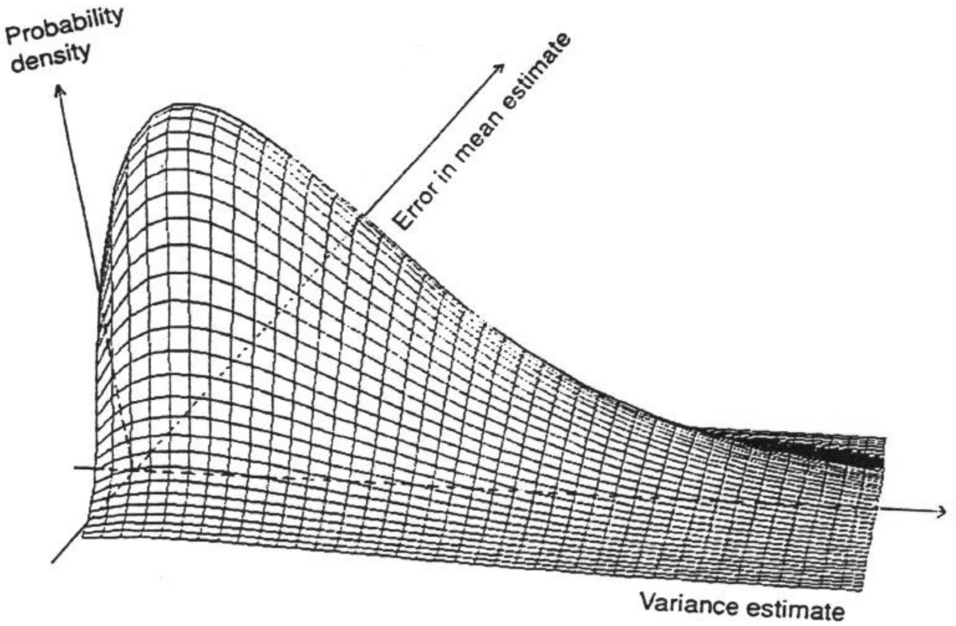

Figure 9. The mean error and variance probability density function based on 4 observations.

In equation (6), the approximating polynomial illustrated in Figures 7. and 8. is given. X = (k-1) where k = (estimated mean)/(true mean). Y = (z-1) where z = (estimated standard deviation)/(true standard deviation).

The polynomial is derived from 110 "observations" of the expected present value. The following combinations of k and z are used: k takes the values 0.5, 0.6, ...,1.5 and z takes the values 0.2, 0.3, ..., 2.0. The R2 value of the regression was 0.973. All coefficients found in (6) were signifficantly different from 0 at the 99% probability level.

$$W_o \approx 1.54 - 4.90X^2 - 1.86XY - .210YY - .205YYY - 4.75XXX - .380XYY \qquad (6)$$

A three dimensional graph of the approximating polynomial is found in Figure 8. Obviously, it is dangerous to strongly overestimate the mean and the standard deviation simultaneously.

The price mean and standard deviation errors

We should be aware that we do not know which mean and variance estimation errors we make. According to Fisher (1925) (see also for instance Rudemo (1979) and Grimmett and Stirzaker (1985)), the mean and the variance errors are independent. Furthermore, the mean error is normally distributed and the variance error is Chi-square distributed. The two dimensional probability density function of the mean error and variance is found in Figures 9 and 10. The mentioned Figures differ since the numbers of observations differ. In Figure 9, 4 observations are used and in Figure 10, 21 observations are used. Note in particular that, when the number of observations is low, the

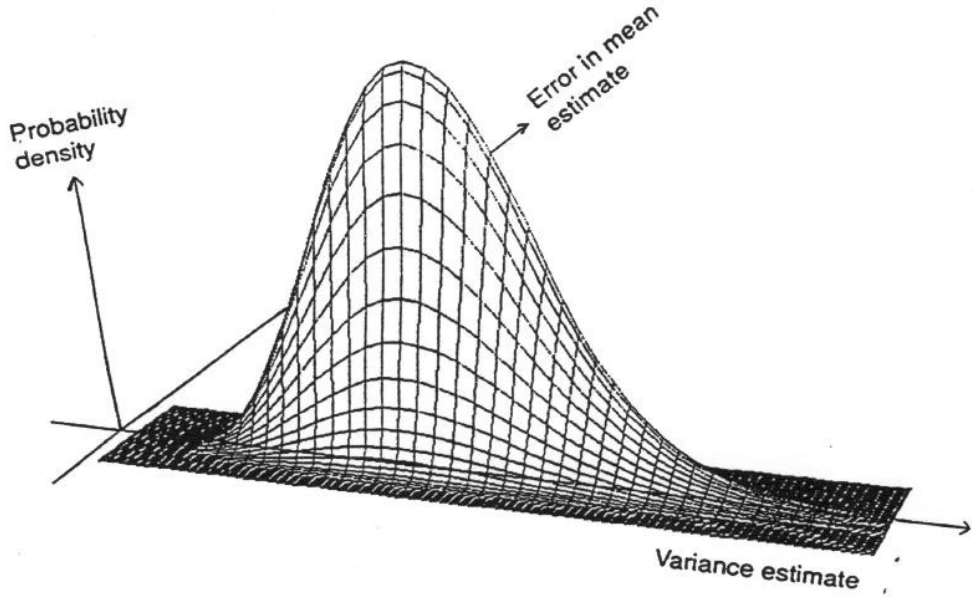

Figure 10. The mean error and variance probability density function based on 21 observations.

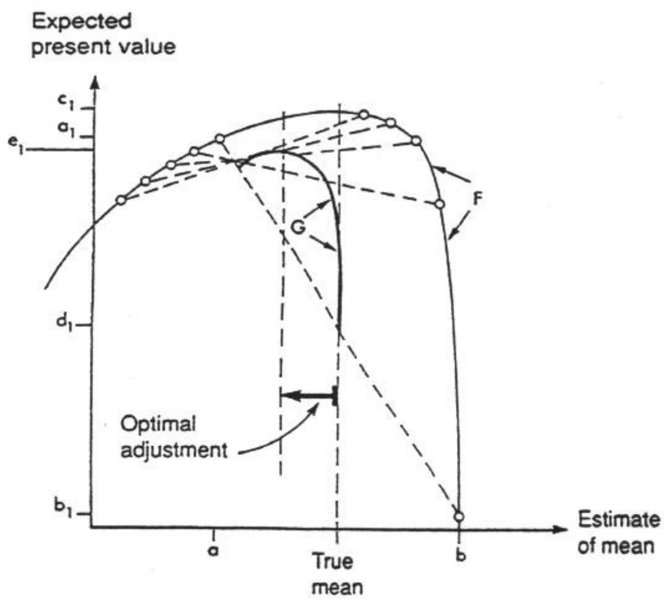

Figure 11. Optimal adjustment in the one dimensional case. F = expected present value as a function of the estimate of the mean. d1 = expected value of F when the estimate of the mean is too low (=a) in 50% of the cases and too high (=b) in the other 50% of the cases. G corresponds to d1. G is however a function of the estimate adjustment level. With optimal adjustment, G takes the value e1.

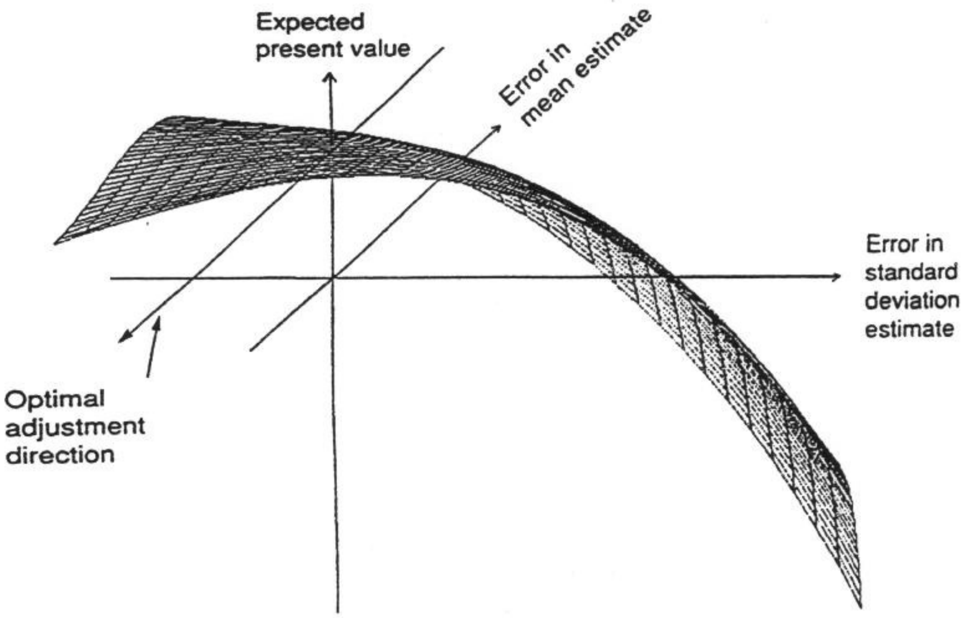

Figure 12. Optimal mean adjustment direction.

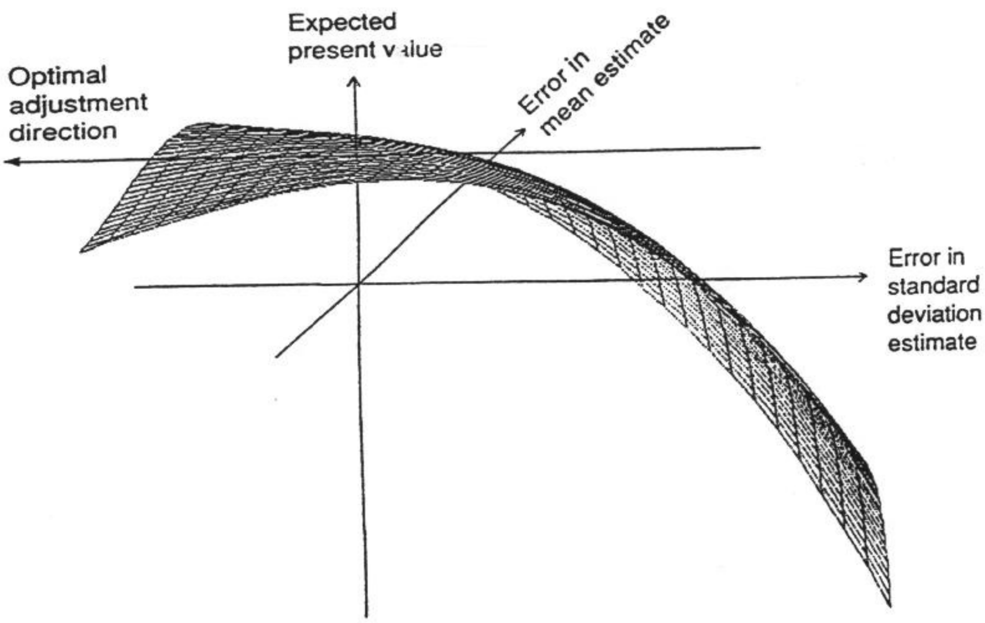

Figure 13. Optimal standard deviation adjustment direction.

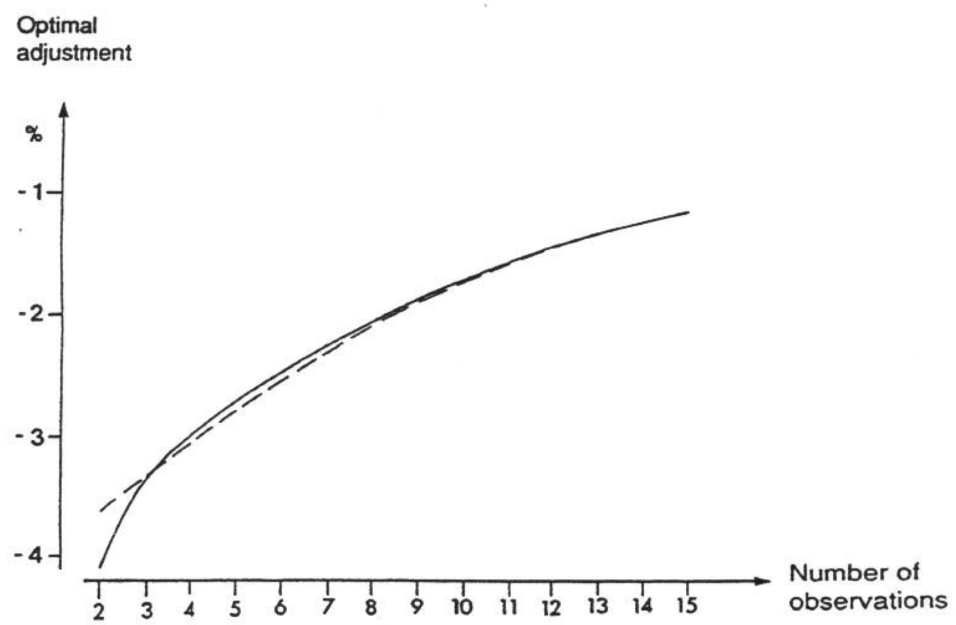

Figure 14. The optimal adjustment of the estimated mean as a function of the number of observations (= solid line). The dotted line is an approximation: - EXP (1.477214 - 0.09147 N), where N denotes number of observations.

variance estimation error probability density function is strongly assymetrical.

The optimal adjustment of the unbiased estimate

Let us first discuss the principles of optimal parameter adjustment in the one dimensional case. Assume initially that the expected value of our estimate is correct (the estimate is free from bias), but that we always make an error. The estimation error is negative in 50% of the cases and positive in the other cases. In every case, the absolute error is the same. The objective function, the expected present value, is an assymetric function of the error. A positive error makes the objective function decrease more than a negative error of the same absolute value. Then, it should be clear from Figure 11, that it is optimal to negatively adjust the estimated (bias free) parameter value. The optimal adjustment level is also found in the graph.

In the problem of this paper, we have two possible errors and adjustment options (the price mean and variance). However, which is obvious from Figure 8, we can by adjustments in one dimension reach a high objective function value even if there are errors in the other dimension. Possibly, we should get even better results by adjustments in both dimensions. From the Le Chatellier principle (compare Henderson and Quandt (1980 p. 82), we know that a constrained optimum can not be better than a free optimum.

In this case, if we restrict ourselves to a constrained adjustment (zero) in one dimension, this can not be better than if we could make optimal adjustments in both dimensions. However, Figure 8. indicates that adjustments in one dimension could improve the solution considerably. In this paper, from now on, we restrict the attention to one dimensional adjustments.

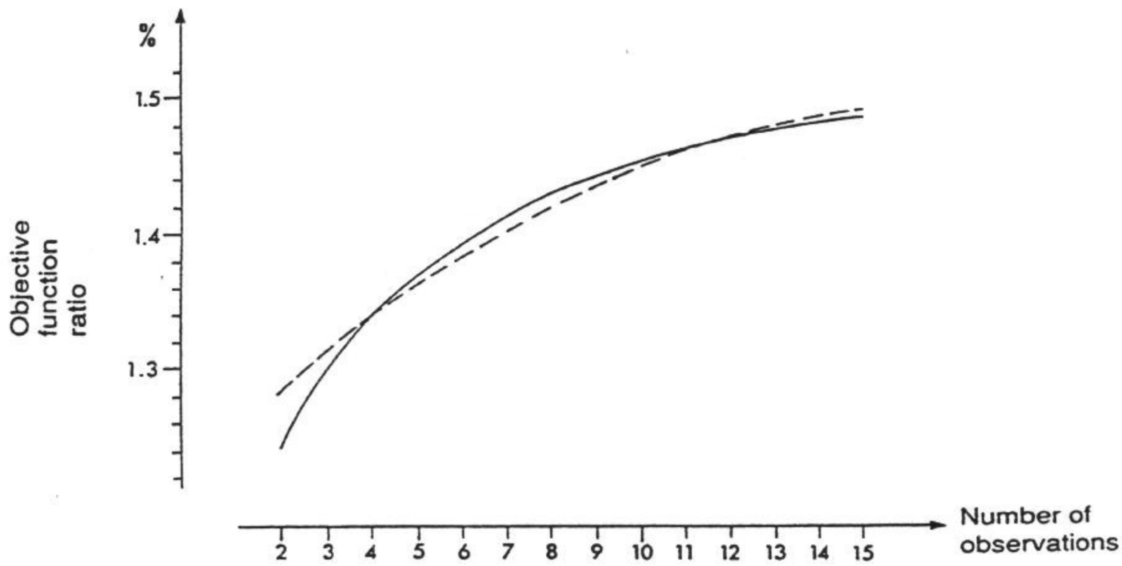

Figure 15. The objective function ratio (= The expected present value of the forest with adaptive decisions divided by the present value of an optimal deterministic program) with optimal adjustment as a function of the number of observations (= solid line). The dotted line is an approximation: 1.54204 - EXP (-1.09764 -0.12442 N), where N denotes number of observations.

Determination of the optimal adjustment

It is obvious that, since the expected absolute estimation errors are decreasing functions of the number of observations, the optimal absolute adjustments must be decreasing functions of the number of observations. Figure 14. shows that this is the case. The problem is to optimize the expected present value of the forest (w with time index 0) as a function of the adjustment level, a, (of the estimated mean) for each possible number of observations.

$$\max_{a} w_{0_a} = \iint w_0((m+a),s)\ h(m,s)\ dm\ ds \qquad (7)$$

h(m,s) is the joint probability density function of the mean, m, and of the standard deviation, s, estimates. The problem of finding the optimal adjustment, a, is solved numerically. The first and second order derivatives of the objective function are approximated numerically and the Newton-Raphson method is used to find the value of "a" which makes the first order derivative of (7) with respect to"a" equal to zero. The results are found in Figure 14. There, you also find an exponential function which approximates the different results and gives the optimal correction as an explicit function of the number of observations.

Determination of the expected present value

Now, the question is which expected present value we get when we use the optimized adjustment. This is shown in Figure 15.

Clearly, it is important to have many observations as long as we consider them relevant in the estimation of the future price probability density function. The expected present value is strongly improved if the number of price observations increases.

DISCUSSION

In this paper, the problems of market adaptive harvesting associated with the fact that we have low numbers of relevant historical price observations, have been analyzed. It was found that it is optimal to negatively adjust the bias free estimate of the mean when the reservation prices are determined. Of course, the decisions will be better if more observations are available. The sensitivity of the objective function (with optimal adjustment) to the number of observations was determined.

The author is convinced that the problem discussed in this paper is typical to most real world problems. Note that in most real world problems:

- The future state of the world is not known with certainty.
- Several decisions may be taken over time.
- Early decisions affect later options.
- The amount of relevant historical data is limited.

Hence, the author knows that the method of optimal adjustment in adaptive problems suggested in this paper will find many applications also in very different areas of decision making. Hopefully, different future general developments and applications will give new valuable insights.

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