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Peter Lohmander

Swedish University of Agricultural Sciences, Faculty of Forestry, Department of Forest Economics, S-901 83 Umeå, Sweden

E-mail: peter.lohmander@sekon.slu.se

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This paper is a summary of central and typical concepts, ideas and results in the field of sequential optimization and stochastic phenomena in forestry. The sequential optimization methods can be applied to all forestry decisions. The text covers forestry decisions and forest economics issues that are based on sequential decision making. An illustration covers optimal decisions in the presence of stochastic market prices. Stochastic (and/or deterministic but for different reasons unpredictable) changes in the economic and physical environments can be considered in decision making over time as soon as they are revealed. For this reason, the information and decision processes are sequential.

1. Introduction

The traditional forest planning, management and economics theory is based on deterministic assumptions. Also today, the deterministic physical and economic environment is the standard assumption in long term and short term modeling in the forest sector. Most concepts, ideas and economic management rules are derived from models based on the assumptions of perfect information concerning future conditions. One reason for the assumption of a deterministic world is that such a world makes it possible to understand and analyze everything in a simple way. (Models may in some cases be rather complicated also if they do not explicitly cover risk and uncertainty aspects.)

2. Sources of risk

Most things change over time and perfect predictions are seldom possible. Some phenomena are very difficult to predict. Forest growth models usually give fairly good predictions of the future stand volumes. Prices are much harder to predict with the same precision. After all, the value of the wood is the net price multiplied by the volume. Hence, if we are interested in the optimization of the value of wood, we should concentrate on the risk associated with the net price. How should we optimize our decisions when we consider this risk? We usually have a large risk in the net price and a low risk in the volume. If possible, we should try to treat the different sources of risk in the same optimization. This will be discussed below. A large body of literature on uncertainty with respect to timber growth exists. Those studies have mostly concentrated on decision problems, which are not sequential. For this reason, they will not be discussed further in this paper.

3. Modeling and implementation in a risky world

If you can understand and explain concepts and ideas in a simple way to most decision makers in industry, then it is likely that the suggested solutions to decision problems really will be tested in reality.

If, on the other hand, you suggest some (optimal) strategy to a real world decision problem to a manager who is not competent enough to understand the assumptions and analyses which are the foundations of the recommendation, it is very likely that the decision maker will not test your suggestion.

One pedagogical problem with optimal decision rules in a stochastic world is the following: suppose that you define a decision optimization model based on information available at time t. The model is relevant and correct in every objective sense. You derive the optimal decision to be undertaken by some decision maker, D. The optimal decision is to harvest a forest stand directly. D undertakes the suggested action. A new market situation appears. The timber price reaches an extremely high level, much higher than ever before. There was no indication that this would happen and your objective and absolutely correct analysis of the time series of price had earlier shown that such a price level had a positive but very low probability, perhaps 0.0004. Then, D may say that it would have been better to follow the recommendations of some other consultant, C. C may have given the recommendation that D should not harvest at time t. You may know, however, that the decision suggested by C was based on a logically incorrect decision support model. C may not even have considered price levels at all when he gave his suggestion!

In other words: you may have a perfect decision optimization model, which for instance maximizes the expected present value. You consider all available information, sequential structures of information, the capital market etc. Still, you may have bad luck in the sense illustrated above. For that reason, some less qualified person with a simple decision support model with logical and numerical errors may suggest a decision, which afterwards turns out to be a more profitable decision.

Many business, management and market consultants use decision and prediction models that are not available for public inspection. Of course, they argue that their models are business secrets that must not be revealed to the competitors. That opinion may be justified. On the other hand, the buyer of a prediction or a strategy and decision suggestion does not know the quality of the product bought. (On the other hand, consultants are sometimes asked to sell a strategy suggestion already determined by the buyer. The strategy suggestion may be personally beneficial to the buyer and the buyer wants some "external" expert to motivate his choice of strategy to other interest groups. It is the firm opinion of the author that this type of "economic management consultancy" is a very large and profitable business. However, it is very hard to prove this.)

If you run a forest company and follow suggestions given by some consultant with a secret model, you will have difficulties to compare the economic results which you obtain to some optimal results. Which results would you have obtained if you had followed the suggestions of some other consultant with a well documented decision optimization model?

Should you compare the results 20 years later? Then, it will be too late to alter the already undertaken harvest programs etc. and you may already have left the company.

There is also a tendency that decision makers who afterwards turn out to have made the correct decisions (or simply have been lucky), want to convince the world that they have been very qualified decision makers and that their way of considering different options always is the best. The long list of best seller publications with business and management success stories in the market is a proof of that. Of course, you sometimes are lucky. A small number of decision makers are very lucky during some decade and they write the books. There is no proof that their way of decision making normally maximizes the expected present value. Probably, they took a lot of chances and simply were extremely lucky! They could easily have failed completely but would then not have affected the best seller book market afterwards. The winners write the history.

It is difficult to understand the methodology of relevance to optimal decision making in a stochastic world. Nowadays, several high level courses at universities all around the world include useful methodology. However, the concrete application of those methods to relevant decision problems is also a difficult task.

It usually turns out that a large body of special knowledge is needed from different disciplines if the methods are to be used in a relevant way.

Often, the optimal solution to a real world decision problem (in which the stochastic development of the world is really considered) is not to determine a long term plan concerning what to do in future periods. Often, the optimal decision is to undertake some specific action in some future period *in case* the world develops in some specific way. Hence, the optimal future decisions can not be determined in advance. *The optimal future decisions are stochastic* when we look at them from the present point in time. This may seem discouraging to those who want to know what to do in the future and who like long time planning. On the other hand, sequentially updated market information and conditional decisions are what you typically observe in reality in every business area every day.

Hence, it turns out that many forest owners without a rigorous traditional forestry education of long term planning, easily understand and accept that it can not be optimal (and reasonable) to undertake detailed long term forest planning. Why should they decide the harvest level ten or perhaps even fifty years from now without knowing the future timber price and the availability of harvest contractors?

The concept of a reservation price is very simple. It is very easy to convince any forest owner without a traditional long term planning education that there has to exist a reservation price with the following properties:

If the timber price happens to be higher than the reservation price, then it is optimal to harvest. If the price happens to be lower than the reservation price, then you should wait at least one more period for a new market price. If the price happens to be exactly as high as the reservation price, then you should be indifferent between the alternatives "harvest" and "wait". Optimal reservation prices in forestry have been calculated by Lohmander [6,7,10,23], Yin and Newman [33,34] and Carter and Newman [3]. Nevertheless, it is not easy to explain how to calculate the optimal reservation price. In order to understand the fundamental principles of such a calculation, a special course is usually needed, including some calculus, stochastic processes, time series analysis and applications of relevance to the issue in question. It is possible to obtain sufficient understanding of the fundamental principles if the student is talented and spends some weeks dealing with similar problems.

4. The market situation and reasons for stochastic prices

We may wonder why the markets are not easy to predict. The market may seem to be a rather simple, albeit large, equation system, completely determined by the parameters of the supply and demand functions. Usually, econometric studies have the goal to determine the coefficients of supply and demand equations, sometimes in the forest sector.

Often, very convincing econometric estimates of the parameters of these simultaneous equation systems are reported. The intersections of the equations of supply and demand are calculated and the solutions are compared to the historical price and quantity series. Sometimes, such comparisons look very good.

However, these market models contain exogenous variables that are assumed known in advance. The developments of these exogenous variables are of course not perfectly predictable in reality. For this reason, the market models with exogenous variables can not be used to predict future prices and quantities directly. In order to derive predictions with these market models, the exogenous variables must first be predicted. When we consider the final price and quantity predictions of the market model, we must not forget the errors in the "exogenous variable predictions" which affect the price and volume predictions.

Shifts in these exogenous variables are the reasons why the demand and supply functions "shift" over time and why we obtain changing price and quantity levels in our time series and in our solutions to the estimated market models. The reader is encouraged to investigate this by a look at econometric models and results in the forest econometrics literature.

We may conclude that the variation in the prices in the forest sector, which can not be explained or predicted many years in advance, is large. We may perhaps say that the prices in the forest sector will possibly be "explained" in the future, when we know how the "explaining" exogenous variables developed. That kind of knowledge, however, is irrelevant to our decision problems here and now.

Lacking complete information and deterministic laws which give us perfect predictions of future prices and other conditions of relevance, we may consider these future values of prices and other variables to be stochastic. Deterministic laws may exist that could be used to make perfect predictions based on perfect information concerning the present state of the world. We do not know these laws and complete information will be extremely expensive. Hence, we have to consider the world to be a realization of a stochastic process.

Hence, the reasons for the existence of stochastic (in our special sense) prices are in general not precisely known. There are reasons in some market situations, however, why actors should behave in a certain way which is not possible to predict by the competitors. Recent results in this area are found in Lohmander [22,25]. Oligopoly and oligopsony markets have such characteristics.

5. Highly important information which is not at all perfectly predictable

Prices are very important to every producer in many ways. In the case of forestry, there are many different prices that strongly affect the firms, directly and indirectly. Let us make some observations:

1. Product price changes (prices of timber, pulpwood, fuel wood etc.) directly influence the profit of the producer in a particular year also if the production (harvest etc.) decisions are held constant.
2. Product price changes indirectly influence the profit in a particular year since they may affect the production (harvest etc.) decisions in that year.
3. Product price changes indirectly influence the profit during future years since those profits are affected by future harvest levels. These future harvest levels are affected by present harvest decisions, which are functions of present prices.

Example. Future harvest levels decrease if the present harvest level increases because a volume has already been cut and can not be cut again in the near future.

4. Product price changes indirectly influence present and future decisions because: The price mean and the degree of price variation will affect the optimal response (with respect to harvest level) to price changes. Every new price observation will affect the estimates of the price mean and variation.

Example. If the timber price increases very much, you will experience three effects. First: forest owners may increase the harvest level because the price is higher than the reservation price in many stands. Second: future harvest levels (with future prices held constant) will decrease because forest owners have revised their price distribution assumptions and will in general wait for higher prices (in other words: they will increase the reservation prices). Third: future harvest levels decrease if the present harvest level increases because a volume has already been cut and can not be cut again in the near future.

Of course, the prices relevant to the forest owner, the prices of pulpwood and timber etc. are partly caused by conditions in the forest industry and in the markets of pulp, paper and timber. Several sequential decision optimization models relevant to forest industry have also been constructed. Typical decisions to optimize with such models include market state dependent optimal raw material purchases, production, storage and sales. Stochastic dynamic optimization models of that type can also be used as submodels when the production and storage capacity investment problems are under consideration. Such a capacity investment model has been used by Lohmander [17] to show that the optimal production capacity is strictly higher than what you find using a deterministic optimization model.

6. To consider relevant stochastic phenomena or details

When we realize that we want to consider the development of some important stochastic variables over time and sequentially optimize decisions based on the information, we may need to reduce the level of detail in other dimensions in order to be able to handle the problem numerically. In particular, in most multistage optimizations with stochastic variables, we need to include a "state space". We must select which variables are to be represented in the state space and the level of detail, the number of different states, or levels, in each dimension. For a given total size of the computer memory available for storing relevant information, we can not include more state dimensions or a higher state resolution without reducing the number of states in some other dimension.

$$X = M^N . \tag{1}$$

The total number of states, X, is defined above. M is the number of state levels in each dimension and N is the number of state dimensions. For instance, if there are 2 state dimensions, such as stock level and price, and 100 different state levels in each dimension (100 different possible price levels and 100 different possible stock levels), then we have 10,000 different states. With one more dimension of the same resolution, we will have 1,000,000 different states. (The third dimension could for instance be the acidity (PH level) of the soil.) With four state dimensions (and the same resolution), we end up with 100,000,000 different states. (The fourth dimension could be the number of trucks, harvesters, the snow depth or even the size of the population in some region.) We realize that we cannot easily include new stochastic variables (dimensions) in our problems without severely reducing the possibility to consider other things. We face the "curse of dimensionality" of stochastic dynamic programming.

7. Analytical and numerical approaches

Dynamic programming, DP, was invented by Bellman [1]. As we have already mentioned, one relevant approach to real world problems is stochastic dynamic programming, SDP. SDP is a special version of dynamic programming, DP, in which some information, such as the price level, P, is sequentially revealed. The decisions in a particular period, t, such as the harvest decision, $h(t)$, are not taken before $P(t)$ has been observed. In other words, the optimal harvest level is a function of time and of the price level, $h(t, P(t))$.

In some cases, the following model is relevant:

$$W_t = \int_{-\infty}^{q_t} W_{t+1}^* f(P)\,\mathrm{d}P + \int_{q_t}^{\infty} \mathrm{e}^{-rt}[PV(t) + L] f(P)\,\mathrm{d}P. \tag{2}$$

Above we find the expected present value, W, at time t (such that $t \leqslant T$) as a function of the reservation price, q, at time t. The vector W has to be determined recursively, starting from a very distant future point in time, the "horizon", T. P is the timber (net) price, V is the stand density (volume per hectare) and L is the value of the land released after harvest per hectare. (In some studies L is determined endogenously. However, there is usually no reason to assume that the land will be used in the same way in the future. In any case, L is usually very small in relation to W, and the assumption that L is exogenous does not affect the optimal decisions very much.) r is the rate of interest in the capital market. $f(P)$ is the probability density function of the timber (net) price.

The recursion is started with the following condition:

$$W_{T+1}^* = 0. \tag{3}$$

Then, the optimization problems, solved recursively, are

$$W_t^* = W_t(q_t^*; .) \quad \forall t \leqslant T. \tag{4}$$

The first order optimum condition is

$$\frac{\delta W_t}{\delta q_t} = f(q_t)[W_{t+1}^* - \mathrm{e}^{-rt}(q_t V(t) + L)] = 0, \tag{5}$$

$$\frac{\delta W_t}{\delta q_t} = f(\cdot) g(\cdot) = 0. \tag{6}$$

For most probability density functions, such as the normal distribution, $f(P) > 0$ for all P. Hence, if we have found an optimum, then $g(\cdot) = 0$. This implies that there is a unique extremum determined from

$$q_t^* = q_t = \frac{\mathrm{e}^{rt} W_{t+1}^* - L}{V(t)}. \tag{7}$$

We can show that this solution to q really is a maximum:

$$\frac{\delta^2 W_t}{\delta q_t^2} = \frac{\delta f}{\delta q} g(\cdot) + f(\cdot) \frac{\delta g}{\delta q}. \tag{8}$$

We have an extremum, $f(\cdot) > 0$ and $g(\cdot) = 0$. Hence,

$$\frac{\delta^2 W_t}{\delta q_t^2} = f(q_t)\left[-\mathrm{e}^{-rt} V(t)\right] < 0. \tag{9}$$

In other words, we have a maximum.

Recent numerical results from forestry in Sweden based on these reservation price models are found in Lohmander [26,27].

In order to determine what to do in a particular case (a particular state and stage), we must know the probabilities of different future states. The transition probability matrix contains all such information. The dynamic properties of the stochastic price process influence the optimal decisions significantly, which has been shown by Lohmander [21]. Usually, the transition probabilities are calculated from the past observations of state transitions.

However, the number of such past transitions may be low. Even if, for example, we let the price state be represented by no more than 10 different possible price levels (which may seem to be a very low resolution), we would need at least 100 price observations in order to get on average 1 transition from each price state in period t to every other price state in period $t+1$.

In fact, if we would like on average 10 transitions from each state in period t to states in period $t+1$, we would need 1000 price observations. (If we have fewer observations, the estimated probabilities of transitions will contain very large relative errors.) If we have one observation from each year, we would need a price series from the year 997 until 1996! However, we can not expect such a series to exist and we can not assume that the first sequence of such a series is very relevant to present harvest decisions.

For that reason, a more practical approach is to estimate the parameters of a stochastic Markov process from the "modern", short and available price series. Such a price process will contain an error term, usually assumed to be normally distributed. With the help of the estimated Markovian process, the elements of the corresponding transition probability matrix can be calculated for any selected level of resolution.

The tails of the normal distribution have to be truncated, however. The probability mass has to be adjusted to take this into account. The probability of going from one state to some other state (out of many possible) always has to be one.

When we use discrete state stochastic dynamic programming, we will have the following mathematical structure of the decision problem:

$$W_i^* = \max W_i(d) = R_{i,d} + \beta \sum_{j=1}^{j=N} Z(j \mid i, d) W_j^* \quad \forall i,\ d \in D(i). \tag{10}$$

W (with state index i) is the expected present value of starting in state i and sequentially selecting optimal decisions, d, in every future period. We assume an infinite horizon and stationary conditions. R denotes the instant economic reward and β is the one period discount factor. In these calculations, the transition probability matrix, Z, is needed. The elements in Z are the probabilities of coming to states j from states i if decisions d are taken.

In particular, with a strictly positive discount factor, β, and stationary processes, we can use some convenient and available linear programming software to solve the stochastic dynamic programming problem

$$\min W = W_1 + W_2 + \cdots + W_N \tag{11}$$

$$\text{subject to} \quad W_i - \beta \sum_{j=1}^{j=N} Z(j \mid i, d) W_j \geqslant R_{i,d} \quad \forall i, d|_{d \in D(i)}. \tag{12}$$

The minimization approach used here is just a "technicality". The results obtained really maximize the vector W. Compare Wagner [31], Markland [28] and Winston [32]. The constraints make sure that the state, i, dependent decisions, d, maximize the sum of the instant economic reward, R, and the expected present value of future rewards.

We should remember that the state index i (or j) used here may contain information concerning the state in several dimensions. The set of feasible decisions, D, is a function of the state, i.

Among the many discrete state stochastic dynamic programming forestry models we find Norstrom [29], Risvand [30], Lohmander [6,8,11,12,15] and Buongiorno [2].

8. Some general observations

In the presence of stochastic phenomena, it is important to have many options available. You should adapt harvesting and all other decisions to the very latest state information, that is, as late as possible.

It may turn out that it is very profitable to deliver wood rapidly. Hence, we must have stocks available for rapid delivery.

A mixed species plantation has some special advantages in comparison to single species stands in a stochastic world (Lohmander [13,18]):

- The prices of different species usually unpredictably develop differently over time (Lohmander [14]).
- The general, partly not predictable, environmental changes such as the acidity of soils, the climate etc. will usually affect different species differently (Lohmander [16]).
- Parasites such as insects and fungi may appear, causing species specific damage.

Hence, it is valuable to sequentially change the species compositions via selective thinning. This is possible only if we have several species to select from in the young stand.

Since wood used optimally has a higher value than wood used in a market insensitive way, the optimal investment intensity (plantation density) increases. (Lohmander [19], Gong [4] and Zhou [35].)

You must have some extra production capacity to utilize in the mills and some extra raw material stocks and product stocks if you rapidly want to be able to take advantage of periods with very good prices. The stocks of standing timber should of course be bought during low price periods and harvested when prices are high. Sometimes, prices are very high, and we harvest the main part of these timber stocks.

Optimal sequential forestry decisions under risk may also concern "continuous" harvesting (Lohmander [5,9]). Optimal sequential forestry logistics is another area of importance. Compare Lohmander [20,24].

We must also be aware of the special spatial considerations that are optimal and typical in the presence of stochastic wind throws. The classical approach is to say that if the probability of strong winds increases, the probability of a wind throw in that stand increases (in particular when the trees are old and tall). Hence, you should cut the stand earlier than in the case of no winds in order to maximize the expected present value. However, the different forest stands protect each other from the winds. For this reason, the harvest timing decisions can not be taken separately, stand by stand. It may even be optimal to harvest some stands later in the presence of strong winds. If they should be cut, many other stands may be wind thrown and large economic values may be lost. Generally, it is optimal to keep large forest areas covered OR clear cut. (Compare Lohmander [6,8].) When one of the stands is cut, the neighboring stands should simultaneously be harvested. Special thinning rules can also be derived.

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