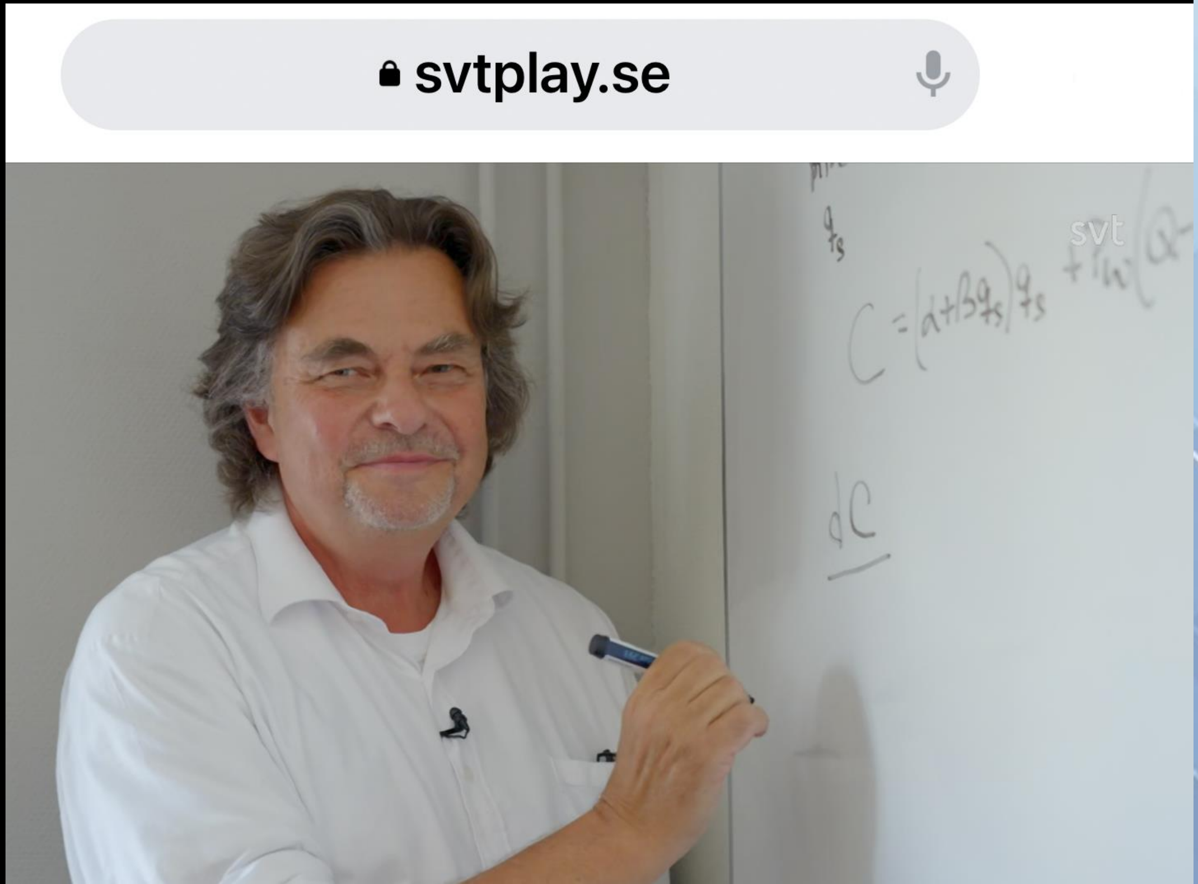
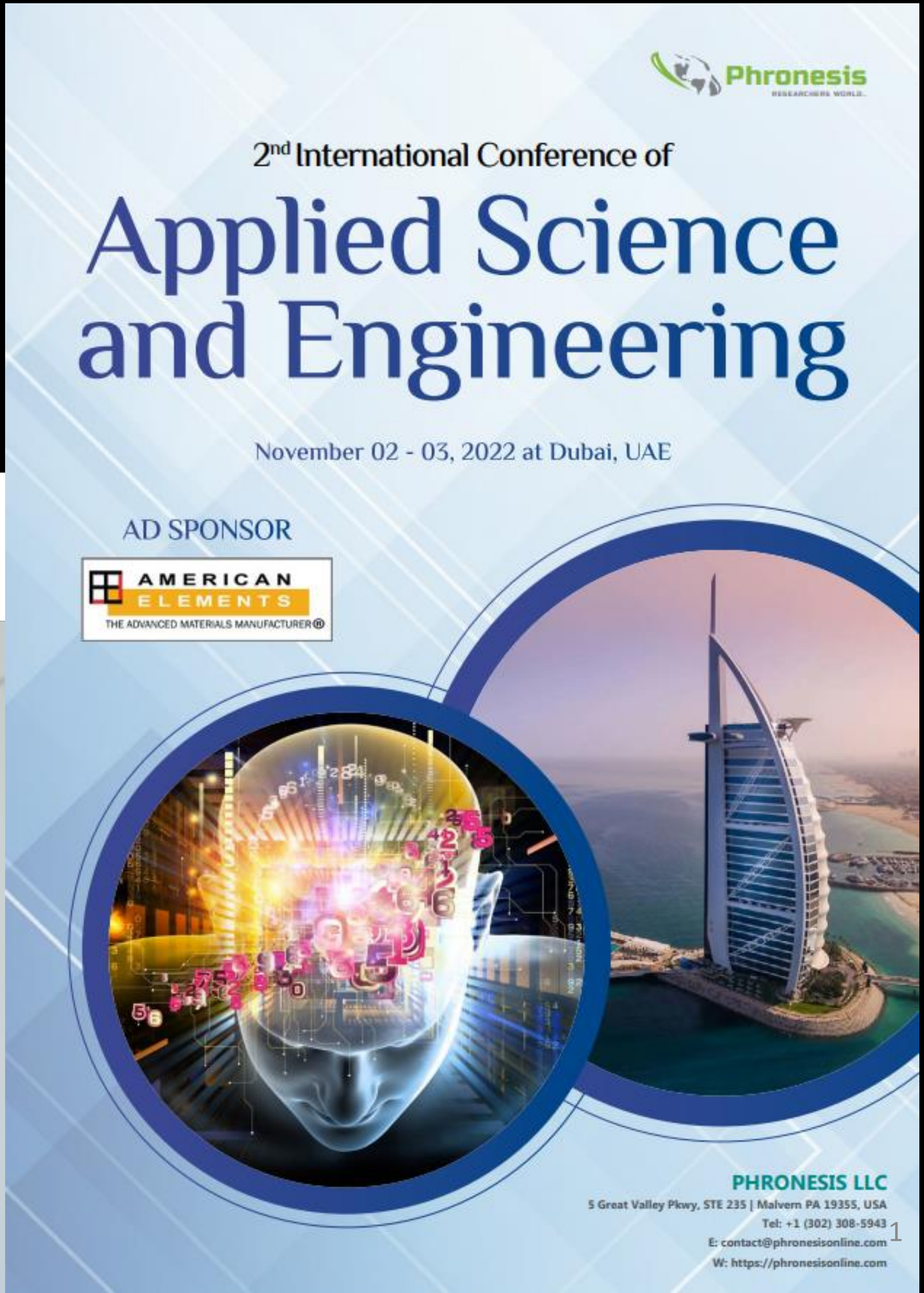
Mathematical Methods for Optimization of Adaptive Control Decisions under Risk with CCF and other Applications

by

Prof Dr **Peter Lohmander**





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Mathematical Methods for Optimization of Adaptive Control Decisions under Risk with CCF and other Applications

Our World develops rapidly, and unpredictably, in many ways. This is true in large and small scales. In order to decide what to do in order to manage some system or organization in the best way, it is necessary to apply some method that can optimize decisions that are rational under such conditions. This field is denoted adaptive optimization.

Adaptive control decisions can be determined via different methods. Most methods are more useful in some situation(s) and less useful, or more difficult to apply, in other cases.

This keynote presentation will present a number of real world problems and investigate how rational adaptive decisions can be obtained via alternative optimization methods.

These adaptive decision problems have recently been optimized with alternative mathematical methods and published in seven open access articles in different international journals. These articles are available via the included links and serve as a more detailed background to the presentation. The approaches include analytical and numerical methods, such as discrete stochastic dynamic programming, continuous stochastic optimal control theory, optimization of continuous control functions in combination with stochastic system simulation, and integer pulse control of stochastic nonlinear systems. The illustrations include continuous cover forestry, infrastructure optimization, fire risk management and wildlife management. The methods can be applied in all industrial sectors.

References:

1. Lohmander, P., 2017. Optimal Stochastic Control in Continuous Time with Wiener Processes: General Results and Applications to Optimal Wildlife Management, Iranian Journal of Operations Research, Vol. 8, No. 2, 58-67.
2. Lohmander, P. 2019. Control function optimization for stochasticcontinuous cover forest management,
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8. Lohmander, P., 2021. Dynamics, stability and sustainable optimal control in wolf-moose systems, International Robotics & Automation Journal, Volume 7, Issue 1, 2021, 24-33.
9. Lohmander, P. 2021. Optimization of Forestry, Infrastructure and Fire Management. Caspian Journal of Environmental Sciences, 19(2), 287-316. doi: 10.22124/cjes.2021.4746,
10. Lohmander, P., 2021. Optimal Adaptive Integer Pulse Control of Stochastic Nonlinear Systems: Application to the Wolf-Moose Predator Prey System, Asian Journal of Statistical Sciences, 1(1), 23-38.

Biography

Professor Dr. Peter Lohmander focuses his research on optimization, optimal control and applications to decision problems in different sectors. Since 2015, Peter Lohmander is president of his own research company, Peter Lohmander Optimal Solutions. Optimization of real dynamic and stochastic decision problems, in particular via stochastic dynamic programming and stochastic dynamic control theory, has gained considerable attention in the research projects. Application areas are economics, natural resource management, forestry, logistics, global warming, bioenergy, and the military and other areas. Peter Lohmander was full professor of forest management and economic optimization, SLU, faculty of forest sciences, Umea, Sweden, 2000 - 2015.

Updated references with links, next page.

Updated references with links:

[1] Lohmander, P., Optimal Stochastic Control in Continuous Time with Wiener Processes: General Results and Applications to Optimal Wildlife Management,
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[2] Lohmander, P., Control function optimization for stochastic continuous cover forest management, *International Robotics and Automation Journal*, 2019;5(2), pages 85-89.
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[3] Lohmander, P., Market Adaptive Control Function Optimization in Continuous Cover Forest Management, *Iranian Journal of Management Studies*, Article 1, Volume 12, Issue 3, Summer 2019, Page 335-361. DOI: 10.22059/IJMS.2019.267204.673348
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Our ***World develops rapidly, and unpredictably***, in many ways.

This is true in large and small scales.

In order to ***decide what to do*** in order to ***manage some system or organization in the best way***, it is necessary to apply some method that can optimize decisions that are rational under such conditions.

This field is denoted **adaptive optimization**.

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These adaptive decision problems have recently been optimized with alternative mathematical methods and **published in eight open access articles in different international journals**.

These articles are **available via the included links** and serve as a more detailed background to the presentation.

The approaches include analytical and numerical methods, such as

Discrete stochastic dynamic programming.

Continuous stochastic optimal control theory.

Optimization of continuous control functions in combination with stochastic system simulation.

Integer pulse control of stochastic nonlinear systems.

The illustrations include:

Continuous cover forestry.

Infrastructure optimization.

Fire risk management.

Wildlife management.

The methods can be applied in all industrial sectors

Analytical optimization of general function model	**Analytical optimization of dynamic system equilibrium**
Stochastic dynamic programming	**Stochastic optimal control theory, Stochastic differential equation etc.**
Stochastic simulation + Optimization of discrete control function parameters via enumeration	**Stochastic simulation + Nonlinear approximation of objective function + Analytical optimization of control function parameters**

Lohmander, P., Optimization of forestry, infrastructure and fire management,
***Caspian Journal of Environmental Sciences*, Vol. 19 No. 2 pp. 287-316, 2021.**
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Analytical optimization of general function model

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Illustration of some sequences of equations:

This is the form of the objective function that will be used in the following analysis:

$$Z = -I + \pi_0 + \left(A_0\pi_B - C_L - \lambda C_E\right)\left(\frac{1-e^{-rT}}{r}\right) + A_0\left(\pi_U - \pi_B\right)\left(\frac{1-e^{-(r+g)T}}{r+g}\right)$$

Expected total present value

Profit from forestry in forests that have not yet burned.

$$\pi_U = P(D)\left(m_1 S - m_2 S^2\right) \tag{32}$$

Stock level in the forests

$$\frac{d\pi_U}{dS} = P(D)\left(m_1 - 2m_2 S\right) \tag{33}$$

$$\frac{dZ}{dS} = -P(D)A_0 + A_0\left(\frac{1-e^{-(r+g)T}}{r+g}\right)P(D)\left(m_1 - 2m_2 S\right) \tag{34}$$

We want to select the stock level that maximizes the expected total present value.

The first order optimum condition is:

$$\frac{dZ}{dS} = P(D)A_0\left(-1+\left(\frac{1-e^{-(r+g)T}}{r+g}\right)\left(m_1 - 2m_2 S\right)\right) = 0 \tag{35}$$

$$\frac{d^2 Z}{dS^2} = -2P(D)A_0 m_2\left(\frac{1-e^{-(r+g)T}}{r+g}\right) \tag{36}$$

< 0

(Second order maximum condition)

We denote optimal values by stars. It is possible to derive an explicit function for the optimal stock level:

$$S^* = \frac{\left(m_1 - \left(\frac{r+g}{1-e^{-(r+g)T}}\right)\right)}{2m_2} \qquad \textbf{(43)}$$

In the same way, many other continuous decision variables can be optimized. (With several simultaneous decisions, the second order maximum conditions become more complicated.)

Analytical optimization of dynamic system equilibrium

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With the motivated differential equations, we obtain the simultaneous system (5).

$$\begin{array}{ll} \text{Moose} \longrightarrow & \begin{cases} \dot{M} = aM - bM^2 - cW \\ \dot{W} = -gW + hMW \end{cases} \\ \text{Wolf} \longrightarrow & \end{array} \qquad (5)$$

Empirical estimation of the differential equations

The two differential equations are approximated as two difference equations. The unit of time is years. The available empirical data set includes one pair of observations (M, W) each year. $\Delta t = 1$.

$$\begin{cases} \dot{M} \approx \dfrac{\Delta M}{\Delta t} = aM - bM^2 - cW \\ \dot{W} \approx \dfrac{\Delta W}{\Delta t} = -gW + hMW \end{cases} \qquad (6)$$

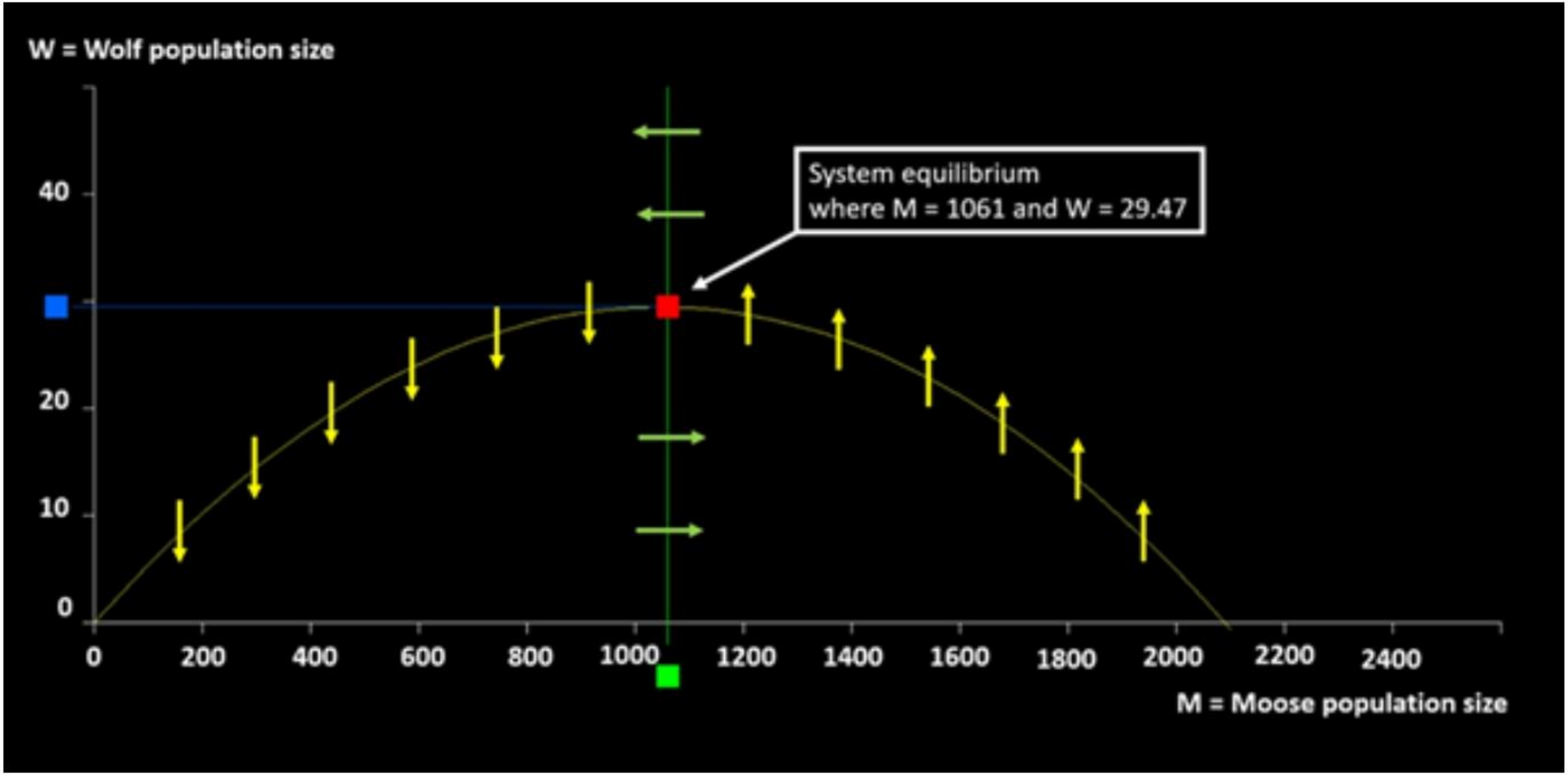

Figure 1 The phase plane of the system.

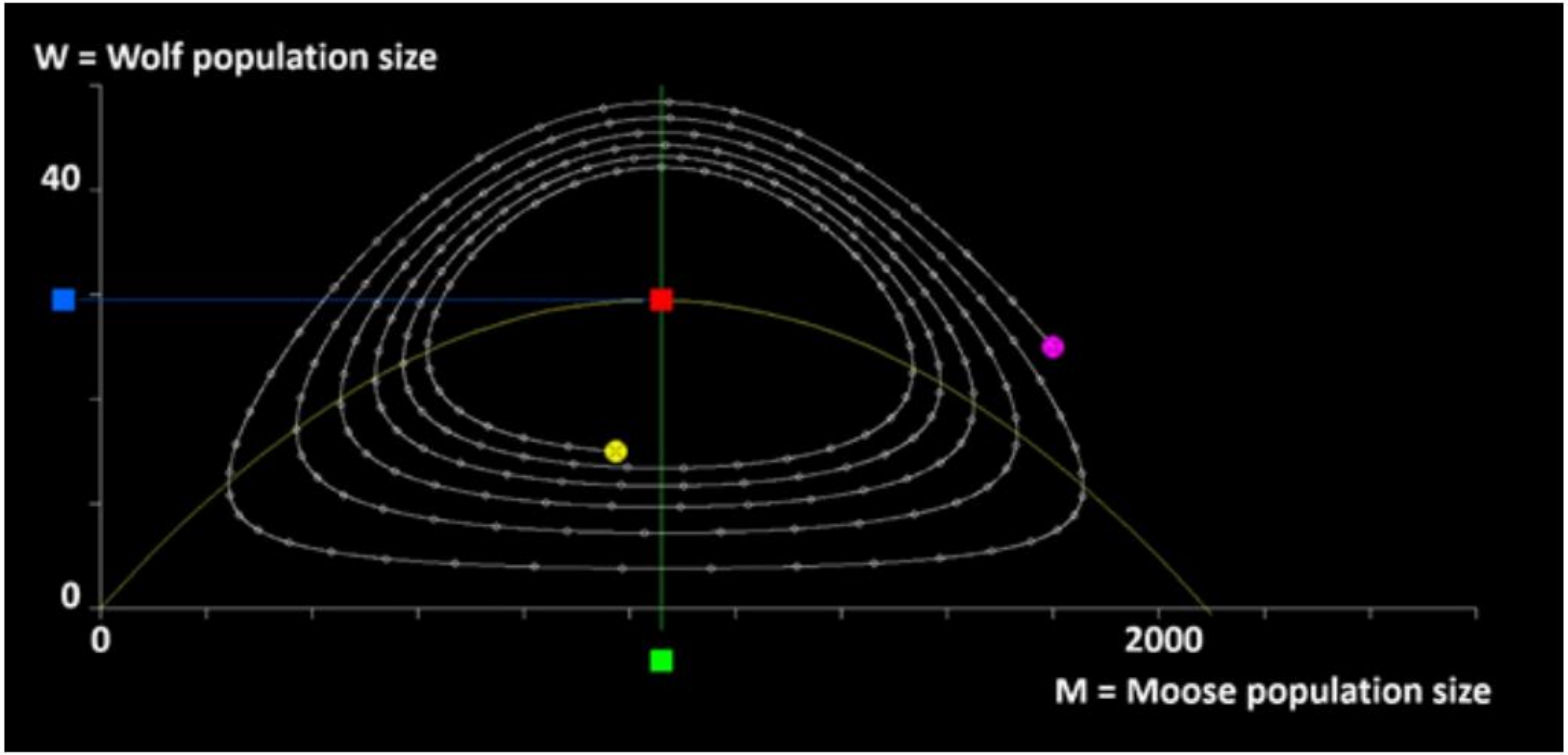

Figure 3 The time path of the populations far from the equilibrium, derived via the nonlinear differential equations in the differential equation system. The initial conditions, (M,W)=(1800,25), are marked by a purple ball and the terminal conditions are marked by a yellow ball. The total simulation time represents 200 years. Each year is marked by a small white ball.

Sustainable optimal system control

It may be rational to replace the uncontrolled development of the two species system described in the earlier part of this paper by a controlled system. Here, we assume that population level adjustments can be made rapidly and that transition periods are short. We study the optimal steady state, not the initial transition. More detailed studies of optimal transitions can sometimes be made via dynamic programming and optimal control in continuous time. We introduce R and S as control variables. R is the moose harvest level and S is the share of the wolf population that is removed from the forest. We are interested in optimal, sustainable and constant control levels. The controlled system is found in equation (98) and in (99) we find the equilibrium conditions.

$$\begin{cases} \dot{M} = aM - bM^2 - cW - R \\ \dot{W} = -gW + hMW - SW \end{cases} \tag{98}$$

Optimal hunting level of moose

hunting level (share of population) of wolf

Optimal hunting level of moose

$$\left(\dot{M}=0\right) \Rightarrow \left(R_{opt1} = aM_{opt1} - bM_{opt1}^{2} - cW_{opt1}\right) \quad (124)$$

Functions of several parameters

$$\left(\dot{W}=0 \wedge W \neq 0\right) \Rightarrow \left(S_{opt1} = -g + hM_{opt1}\right) \quad (125)$$

Optimal hunting level (share of population) of wolf

Stochastic dynamic programming

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Average burned areas per year, in different countries

Table 1 The average yearly burned areas in different countries and regions during nine years (from 2010 until 2018). The standard deviations and the relative standard deviations have also been calculated. "IFPS" denotes the region including Italy, France, Portugal and Spain. "FGLNS" denotes the region including Finland, Germany, Latvia, Norway and Sweden. Source of the list of burned areas: San-Miguel-Ayanz et al., 2019[8]

	Average Burned Area (kha)	Standard Deviation (kha)	Relative Standard Deviation
Italy	62,3	43,1	0,7
France	10,9	6,8	0,6
Portugal	144,6	156,3	1,1
Spain	95,7	66,3	0,7
Finland	0,5	0,4	0,7
Germany	0,5	0,7	1,3
Latvia	0,6	0,9	1,5
Norway	0,8	1,1	1,3
Sweden	5,1	8,5	1,7
IFPS	313,4	240,1	0,8
FGLNS	7,6	11,1	1,5

Correlations of burned areas per year, between different countries

Table 2 Correlations of burned areas in different countries during nine years (from 2010 until 2018). The original data that were used to calculate these correlations are available in the official statistics by San-Miguel-Ayanz et al., 2019[8]

	Italy	France	Portugal	Spain	Finland	Germany	Latvia	Norway	Sweden
Italy	1,000								
France	0,634	1,000							
Portugal	0,657	0,859	1,000						
Spain	0,944	0,464	0,482	1,000					
Finland	-0,492	-0,313	-0,230	-0,651	1,000				
Germany	-0,349	-0,238	-0,184	-0,369	0,666	1,000			
Latvia	-0,459	-0,291	-0,280	-0,467	0,742	0,951	1,000		
Norway	-0,427	-0,081	-0,158	-0,531	0,682	0,824	0,864	1,000	
Sweden	-0,464	-0,377	-0,356	-0,521	0,894	0,767	0,888	0,762	1,000

Stochastic dynamic programming determines how to optimally distribute water bombing air planes between countries

For $t\big|_{0 \le t < T}$, we have:

$$\phi\left(t, i_t, f_{1,t}, .., f_{n,t}\right) =$$

$$= \min_{j_t \in J_t(i_t)} \left\{ \begin{array}{l} e^{-rt} C\left(t, i_t, j_t, f_{1,t}, .., f_{n,t}\right) \\ + \sum_{f_{1,t+1}=1}^{F_{1,t+1}} \cdots \sum_{f_{n,t+1}=1}^{F_{n,t+1}} \left[\tau\left(f_{1,t+1}, .., f_{n,t+1} \middle| t, i_t, j_t, f_{1,t}, .., f_{n,t}\right) \cdot \phi\left(t+1, i_{t+1}, f_{1,t+1}, .., f_{n,t+1}\right) \right] \end{array} \right\}$$

$$\forall t\big|_{0 \le t < T}, i_t, f_{1,t}, .., f_{n,t}$$

Stochastic optimal control theory, Stochastic differential equation etc.

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2. The General Stochastic Optimal Control Problem

The general optimal stochastic control methodology in continuous time is briefly presented here. Related introductions with more details are found in Sethi and Thompson [5], Malliaris and Brock [4] and Winston [6]. Lohmander [1] presented connected methods in discrete time. We maximize the objective function,

$$E\left[\int_0^T F(X_t, U_t, t)dt + S(X_t, T)\right] \tag{1}$$

where X_t is the state variable and U_t is the closed loop control variable. Time is represented by t and T is the time horizon. $F(.)$ is the instantaneous profit rate and $S(.)$ is the salvage value. $E[.]$ denotes expected value. z_t is a standard Wiener process.

$$dX_t = f(X_t, U_t, t)dt + G(X_t, U_t, t)dz_t, X_0 = x_0 \tag{2}$$

According to the Bellman principle of optimality, we may determine the value function $V(x,t)$ as the maximum of the sum of the net reward during the first short time interval, $F(.)dt$, and the value function directly after that time interval:

$$V(x,t) = \max_u E\left[F(x,u,t)dt + V(x + dX_t, t + dt)\right]. \tag{3}$$

A Taylor approximation gives

$$V(x + dX_t, t + dt) = V(x,t) + V_x dX_t + V_t dt + \frac{V_{xx}\left(dX_t\right)^2}{2} + \frac{V_{tt}(dt)^2}{2} + V_{xt}(dX_t)(dt) + o(.) \tag{4}$$

In the Taylor approximation, we need

$$\left(dX_t\right)^2 = f^2\left(dt\right)^2 + 2fG(dt)(dz_t) + G^2(dz_t)^2 \tag{5}$$

$$
\max E\left(\int_0^\infty e^{-rt}(ku - pu^2 - fx)dt\right)
$$

$$
\begin{aligned}
&s.t. \qquad dx = (gx - u)\,dt + sx\,dz \\
&\qquad\qquad k > 0, p > 0, f > 0, s > 0.
\end{aligned} \tag{16}
$$

The net profit at a particular point in time is

$$
R(u, x) = (ku - pu^2 - fx)\,. \tag{17}
$$

$$u^* = \frac{k(r-g)+f}{2p(g+s^2)} + (2g - r + s^2)x.$$

Now, we know that the optimal control is a linear function of the population level. The intercept and the slope of this linear control function are explicit functions of the initially specified parameters.

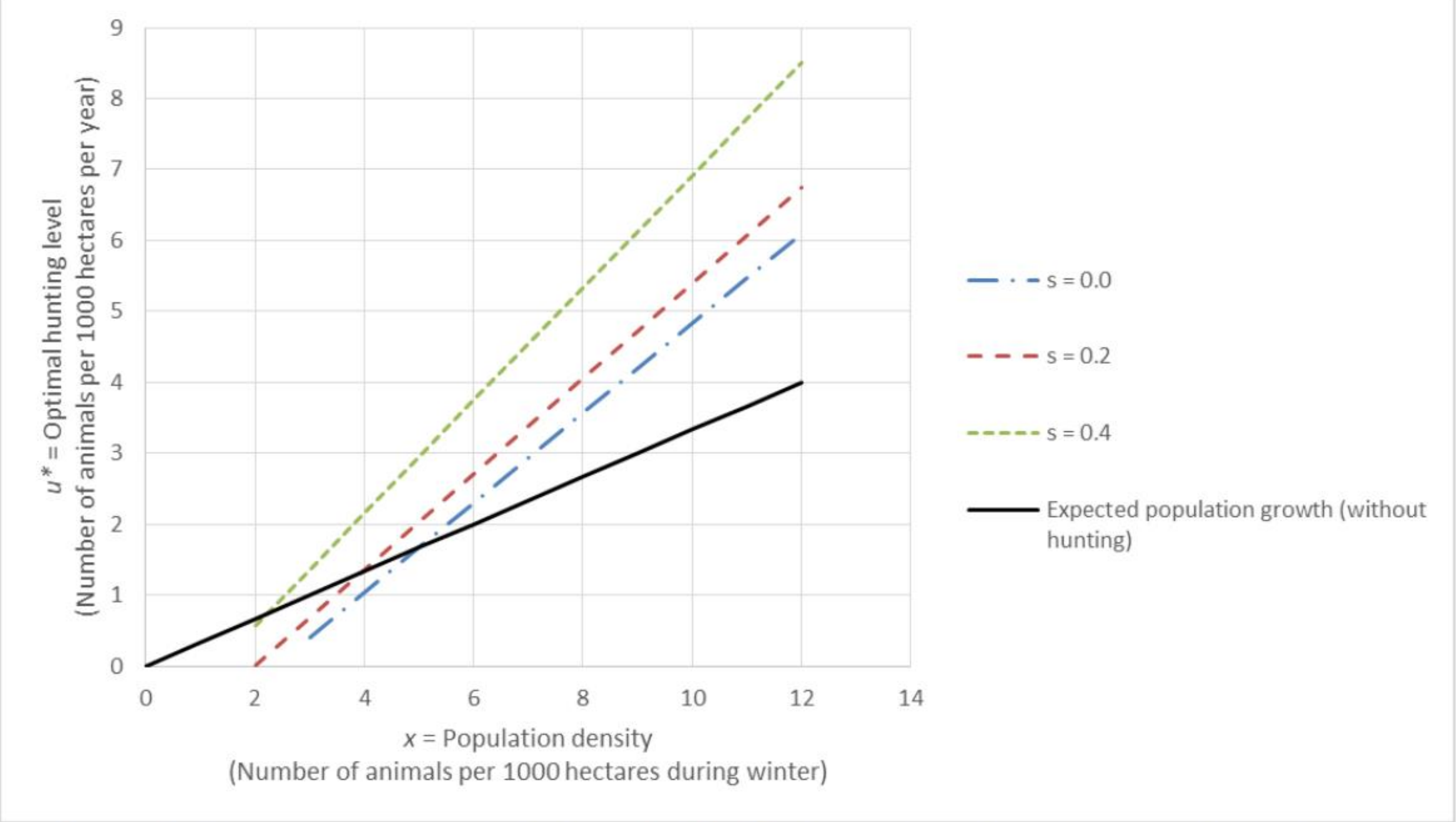

Figure 2. The optimal control, the hunting level, u^*, as a function of the population density, x, and the stochastic parameter s.

Stochastic simulation +
Optimization of discrete control function parameters via enumeration

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$$\pi = E\left(\sum_t e^{-rt}\left(P_M u(t,\bullet) + P_W v(t,\bullet) + P_{ARW}\Psi(W(t,\bullet))\right)\right) \tag{1}$$

The simultaneous difference equation system in general form, found in (2), describes how the populations of moose and wolf, develop over time. The difference equations contain stochastic components, ϵ_M and ϵ_W. These are described in more details in (7) and (9).

$$\begin{cases} \Delta M & = \phi\left(M, W, u, \varepsilon_M\right) \quad , \forall t \\ \Delta W & = \varphi\left(M, W, v, \varepsilon_W\right) \quad , \forall t \end{cases} \tag{2}$$

In equation (3) we see how the expected value of the objective function is estimated from N complete stochastic scenarios. In every stochastic scenario, n, the adaptive control functions are the same. However, the random number sequences, $\epsilon_M(t,n)$ and $\epsilon_W(t,n)$, are different for different n.

$$\pi = N^{-1}\sum_t \sum_{n=1}^{N} e^{-rt}\left(P_M u(t,n) + P_W v(t,n) + P_{ARW}\Psi(W(t,n))\right) \tag{3}$$

When we introduce the controls, u and v, that are functions of different variables and parameters, we get (11):

$$\begin{cases} \Delta M & = 0.372M - 0.178 \times 10^{-3} M^2 - 6.593W - u(\bullet) + M\varepsilon_M \\ \Delta W & = -0.244W + 0.230 \times 10^{-3} MW - v(\bullet) + W\varepsilon_W \end{cases} \tag{11}$$

In several cases, it is interesting to see how the amount of risk in the difference equations of moose and wolf, influence the optimal adaptive control rules and expected results. We introduce "RISK" to describe this influence. Compare equation (12). If RISK is 0, then we consider the system to be completely deterministic. If RISK=1, then we have the degree of risk that describes reality, according to the empirical data.

$$\begin{cases} \Delta M & = 0.372M - 0.178 \times 10^{-3} M^2 - 6.593W - u(\bullet) + M\varepsilon_M \times RISK \\ \Delta W & = -0.244W + 0.230 \times 10^{-3} MW - v(\bullet) + W\varepsilon_W \times RISK \end{cases} \tag{12}$$

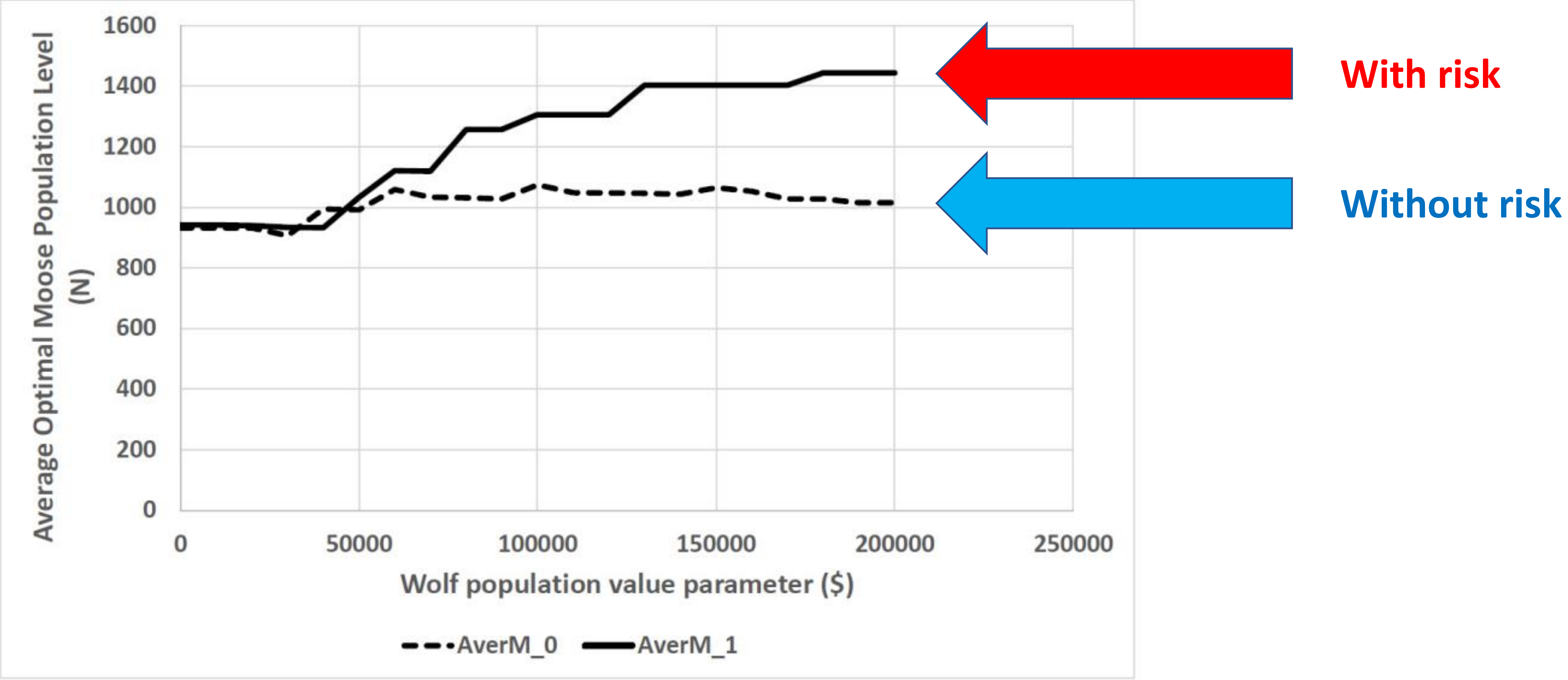
Average Optimal Moose Population Level (N)
1600
1400
1200
1000
800
600
400
200
0
0
50000
100000
150000
200000
250000
Wolf population value parameter ($)
AverM_0
AverM_1
With risk
Without risk

Figure 3.

Stochastic simulation + Nonlinear approximation of objective function + Analytical optimization of control function parameters

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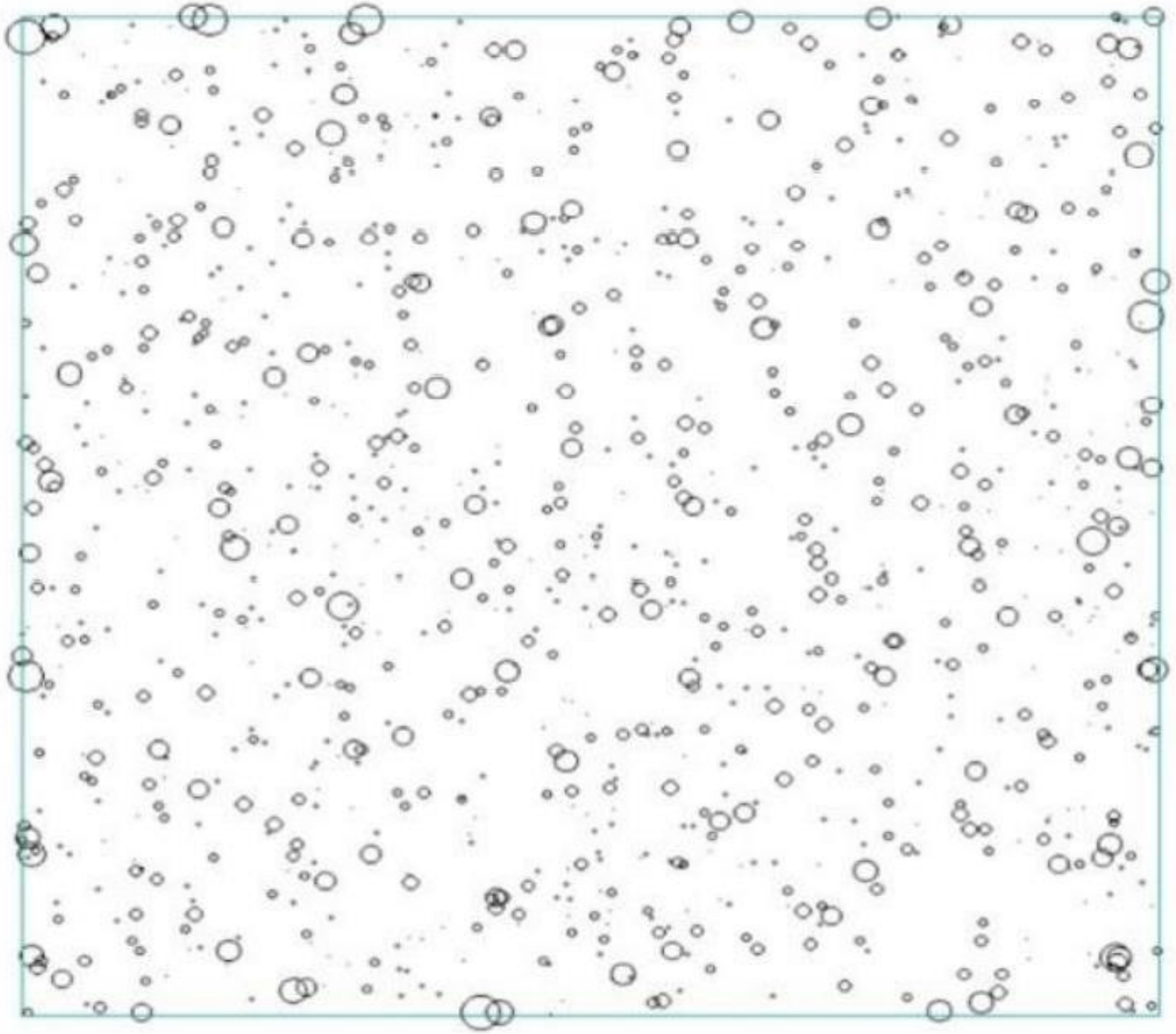


Figure 1. Spatial map of simulated trees in a CCF forest in year 35. The map represents one hectare where the four sides of the square are 100 meters each. The circles represent trees and the diameters of the circles are proportional to the diameters of the trees.

Table 1. Adaptive control function parameters in the particular numerical case.

Parameter name:	Corresponding parameter in the general model:	Constant in the limit diameter function	Derivative of limit diameter with respect to:	These transformations simplify the analysis:
dlim_0	α_1	Yes		dlim_0 = 0.1x
dlim_c	α_L	No	Local competition	dlim_c = 0.001z
dlim_q		No	Timber quality of the particular tree	
dlim_p	α_P	No	Timber price – expected timber price	dlim_p = 0.001y

The approximating function is $\Phi(.)$. Using the transformations described in Table 1. and treating dlim_q as an already known parameter, we have the approximating function $\Phi = \Phi(x, y, z)$. This function is specified to have this functional form:

$$\Phi = k_x x + k_{xx} x^2 + k_{xy} xy + k_{yyy} y^3 + k_z z + k_{zz} z^2 + k_{xz} xz \qquad (23)$$

Multiple regression analysis was used to test the function and to estimate the parameter values. To create the data set, 448 alternative parameter combinations with stochastic full system simulations were used. The approximating function fits the data well. The R2 value exceeds 99.2%. All of the p-values of the estimated parameters of the approximating function obtained values below the 5% significance limit. The highest of these values was 0,03769708l. Five of the seven p-values were far below 0,001.

In the rest of the analysis, we consider this approximation to be a correct representation of the expected present value per hectare, π. Hence,

$$\pi = 92.237x - 15.124x^2 - 3.9714xy + 0.16242y^3 - 10.873z - 1.2477z^2 + 1.2729xz \tag{25}$$

The first order optimum conditions are:

$$\begin{cases} \dfrac{d\pi}{dx} = 92.237 - 30.248x - 3.9714y + 1.2729z = 0 \\ \dfrac{d\pi}{dy} = -3.9714x + 0.48726y^2 = 0 \\ \dfrac{d\pi}{dz} = -10.873 + 1.2729x - 2.4954z = 0 \end{cases} \tag{26}$$

$$\pi^* = \pi(x^*, y^*, z^*) = 195{,}6554283 \quad (49)$$

Now, it is time to calculate the optimal parameters of the the limit diameter function:

$$\text{dlim\_0} \approx \frac{x^*}{10} \approx 0.366 \quad (50)$$

$$\text{dlim\_p} \approx \frac{y^*}{1000} \approx -0.00546 \quad (51)$$

$$\text{dlim\_c} \approx \frac{z^*}{1000} \approx -0.00249 \quad (52)$$

The optimized, numerically specified version of the limit diameter function is:

$$D_i(t) = 0.366 - 0.00546\left(P(S(i),t) - E\left(P(S(i),t)\right)\right) \quad (53)$$

$$-.00249\,L(t,i) + 0.0600\,Q(t,i)$$

CONCLUSIONS

Our ***World develops rapidly, and unpredictably***, in many ways.

This is true in large and small scales.

In order to ***decide what to do*** in order to ***manage some system or organization in the best way***, it is necessary to apply some method that can optimize decisions that are rational under such conditions.

This field is denoted **adaptive optimization**.

Adaptive control decisions can be determined via ***different methods***.

Most methods are more useful in some **situation**(s) and less useful, or more difficult to apply, in other cases.

Now, we have seen a number of ***real world problems*** and investigated how ***rational adaptive decisions could be obtained via alternative optimization methods***.

These adaptive decision problems have recently been optimized with alternative mathematical methods and **published in eight open access articles in different international journals**.

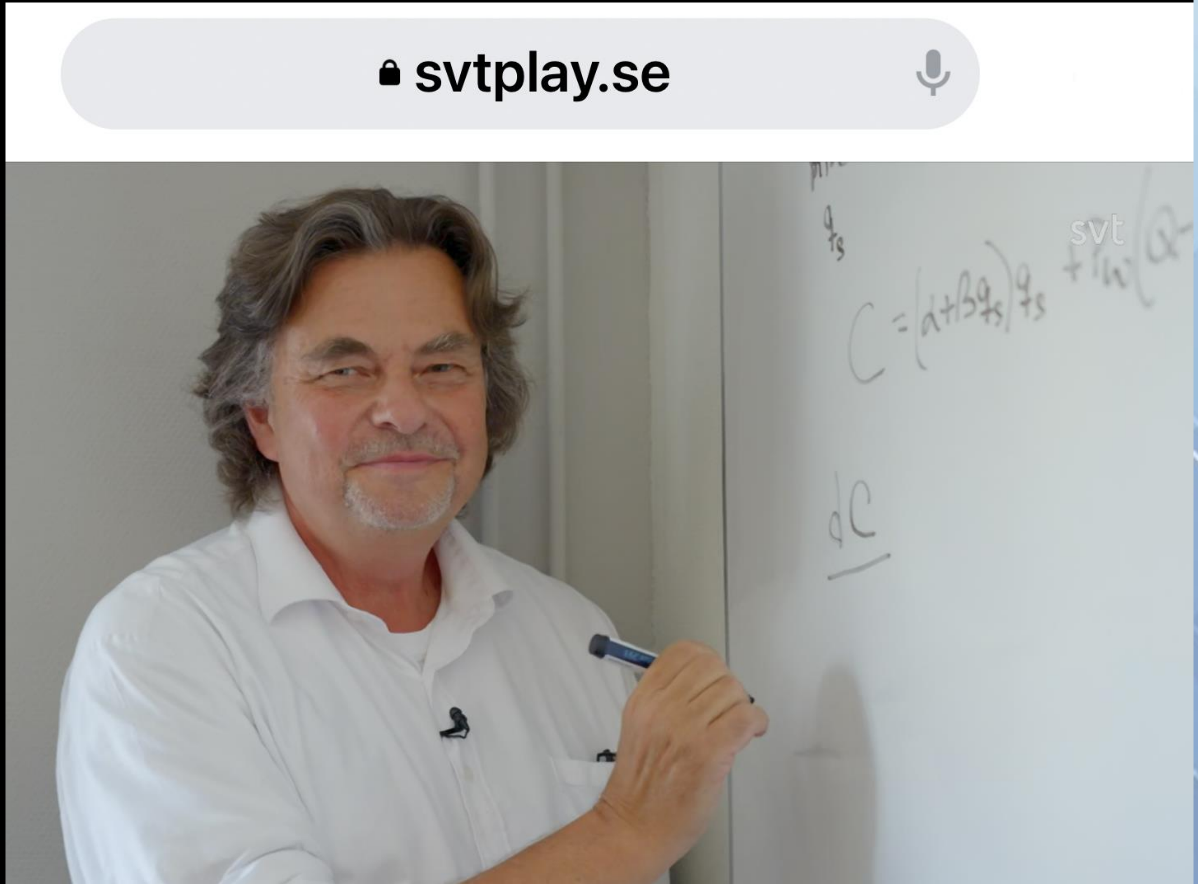
These articles are **available via the included links.**

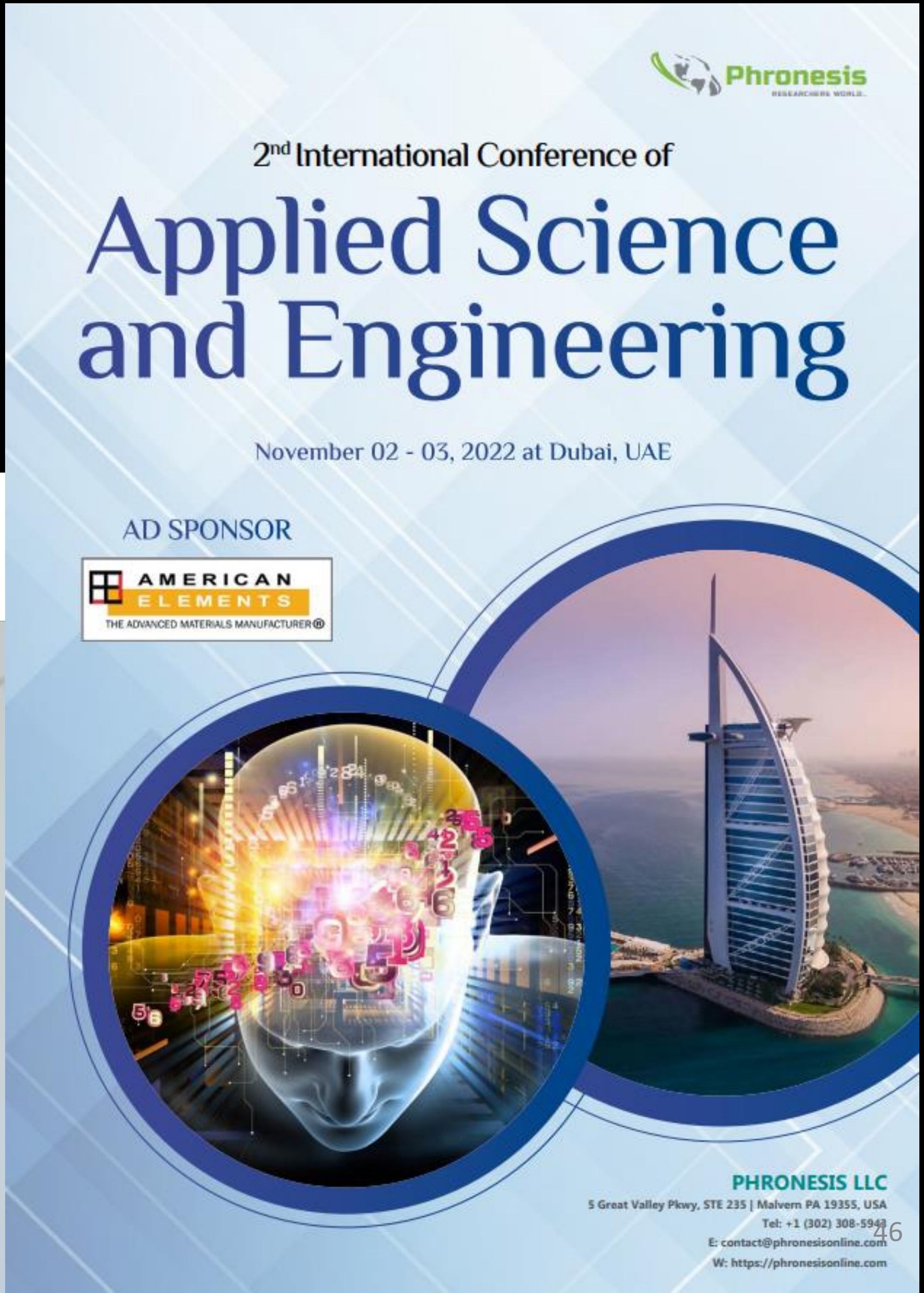
THANK YOU FOR YOUR TIME!

Mathematical Methods for Optimization of Adaptive Control Decisions under Risk with CCF and other Applications

by

Prof Dr Peter Lohmander





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