**Problem definition:**















*OBS: Maybe a continuous time definition of h is not necessary.*













Particular values:







Generalized results:























**Maximization problem:**



**First order optimum conditions:**





**Reformulations of the first order optimum conditions:**









**Final version of the first order optimum conditions:**





OBS: Here, it is possible to introduce ”MC = MR interpretations” and ”Faustmann modification interpretations”.

**Observations concerning some second order derivatives and second order maximum conditions:**



The sign of is more complicated to determine. However, a unique maximum will be assumed.

, 

**Comparative statics analysis in the time interval dimension:**











Observation: Let   is strictly positive for low values of , zero for the MSY maximizing value of , , and strictly negative for higher values of .









Hence, if  increases, ceteres paribus, the optimal harvest interval increases if .

The optimal harvest interval is not changed if  and the optimal harvest decreases if .

It is NOT optimal to increase the stock level above .

**Comparative statics analysis in the volume dimension:**













***Let us assume that growth, at least locally, can be approximated by the logistic equation:***



The following equation is derived and explained in detail in

Lohmander, P., Zazykina, L., Methodology for optimization of continuous cover forestry with consideration of recreation and the forest and energy industries, Report and Abstract, Forests of Eurasia, Publishing House of Moscow State Forest University, September 19 - 25, 2010

<http://www.lohmander.com/Moscow10/Moscow10_PL_LZ.pdf>

<http://www.lohmander.com/Moscow10/Moscow10_PL_LZ.doc>

<http://www.lohmander.com/Moscow_PL_2010.pdf>

<http://www.lohmander.com/Moscow_2010/Lohmander_Zazykina_Moscow_2010.ppt>

<http://www.lohmander.com/Moscow_2010/Programma-LE_10_01.doc>









**Assumptions:**













**Calculations with very short time intervals:**







**Assumptions:**





**Observations:**



**Calculations with longer time intervals:**



Hence, if  increases from zero, then becomes more negative.

As a result, becomes more negative.

and may become strictly negative even if .

**Conclusions in the volume dimension:**

*If the time interval is very short (approximately zero):*

 and  are strictly positive for low values of , zero for  that maximizes MSY and strictly negative for larger values of .

Hence, if  and/or increase(s), ceteres paribus, the optimal stock level converges to .

It is NOT optimal to increase the stock level above .

*If the time interval is not very short:*

 and  are strictly positive for low values of  (if the time interval is sufficiently short), equal to zero for a particular value of , and strictly negative for larger values of .

Hence, if  and/or increase(s), ceteres paribus, the optimal stock level converges to a value below .

It is NOT optimal to increase the stock level above .

**Comparative statics analysis in two dimensions:**

In the derivations below, we investigate the qualitative effects (directions of changes) of the decision variables in case the imaginary parameter increases. Since  and , and since complementary assumptions are made (below), the results derived and reported below with respect to the imaginary parameter  , are relevant to both of the the real parametersand .











*Special assumptions of relevance to the two dimensional comparative statics analysis:*

 . is very small. We have a unique maximum. .

As a consequence,









***Conclusions from the one and two dimensional comparative statics analyses:***

If the initial value of is lower than  and the time interval  is sufficiently short, then and  increase if  ( and/or) increase(s).

If the initial value of is lower than  and the time interval  is sufficiently long, then  and  may be unchanged or decrease if  ( and/or) increase(s).

If the initial value of is equal to and the time interval  is very short, then and  are not changed if  ( and/or) increase(s).

If the initial value of is higher than  and/or the time interval  is sufficiently long, then and  decrease if  ( and/or) increase(s).