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# Handbook of Operations Research in Natural Resources

*Edited by*

Andrés Weintraub | Carlos Romero
Trond Bjørndal | Rafael Epstein

With the collaboration of Jaime Miranda

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*Volume Contributors*
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Ragnar Arnason
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Frank Asche
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Andrés Weintraub
Slim Zekri

Andres Weintraub
University of Chile
Santiago, Chile

Carlos Romero
Technical University of Madrid
Madrid, Spain

Trond Bjørndal
CEMARE, University of Portsmouth
Portsmouth, United Kingdom

Rafael Epstein
University of Chile
Santiago, Chile

*Series Editor:*
Fred Hillier
Stanford University
Stanford, CA, USA

*With the collaboration of:*
Jaime Miranda
Diego Portales University,
Santiago, Chile

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Chapter 28

# ADAPTIVE OPTIMIZATION OF FOREST MANAGEMENT IN A STOCHASTIC WORLD

Peter Lohmander
*Department of Forest Economics, SLU, Sweden*

**Abstract** Management decisions should be based on the sequentially revealed information concerning prices, growth, physical damages etc. Future flexibility is valuable in a stochastic world and should be optimized. Stochastic dynamic programming, stochastic scenario tree optimization, and optimization of adaptive control functions with stochastic simulation of the objective function are relevant alternatives.

Keywords: Stochastic dynamic optimization, forest management

## 1 INTRODUCTION

Economic optimization of forest management is a highly interesting area, which covers theories from several quantitative fields and optimization methods of all kinds. One reason for this fact is that trees grow. Trees represent interesting biological units that form stands and forests. Growth can be modelled at the tree level and at higher levels. The stock may be observed and controlled over time via activities such as thinning and fertilization. Growth, in several dimensions, is a function of the measurable state. Hence, we may view the forest as a controllable Markov process.

Future growth is affected not only by the present properties of the trees. Wind throws, insect damages, fungi damages, rodents, moose, elephants and other animals, changes in climate, air pollution, forest fires, and many other factors affect forest damages and growth and the future state of the forest. In most cases, such factors may not be perfectly predicted. Storms, fires, and different forms of other damages may usually be regarded as stochastic. It should also be clear that there are many different types of "stochastic disturbances" of this nature. Different types of disturbances may have very different effects on the forest state. Some disturbances will only influence some species and leave other species unaffected. Other disturbances will affect some parts of the trees, such as the roots or some branches. Some damages are local and affect only one tree. Other types of damages have spatial dimensions and influence the development of several square kilometres, such as forest fire damages and wind throws.

In most places, it is very expensive to go to the forest to harvest and to collect just one tree. We have to consider the different alternatives on a larger scale: Is it economically optimal to harvest one or several forest stands in the area in this period via clear felling or should we perform partial harvesting in the form of continuous cover forestry? We conclude that there are mostly considerable economies of scale in harvesting and other operations. For this reason, it is seldom optimal to perform "continuous" harvesting. During a time interval when you harvest in an area, you harvest much more than the growth during the same time interval. It would in most cases be far too expensive to keep the harvester, the forwarder, and the labour in the forest stand for ever. Hence, irrespective of whether you make a clear felling or make a thinning, it is economically optimal to wait a considerable time, usually many years, before you visit the site again in order to harvest.

When we consider stochastic events of importance to forest management and economic optimization, we must not forget the markets. The market value of a tree is a function of the general market conditions. If we want to determine the economic values of trees, we have to consider the prices of logs of different qualities and dimensions in different market places, the prices of pulpwood and fuel wood and several other alternative wood products. We also have to consider the costs of harvesting and forwarding, the capacity of the forest road network at different points in time, and the availability and prices of trucks and labour. Clearly, in most cases the timber producers do not control the market prices. Inventions and innovations in process technology will change future relative prices and can, by definition, not be perfectly predicted. If you can predict the exact properties of a future invention, it has already taken place! Furthermore, political changes at the national and international scales may change the markets in ways that are impossible to predict. As a confirmation, econometrics research has not yet been able to give us perfect price predictions covering long horizons.

Hence, if we want to address the relevant forest management problems, we have to admit that future prices cannot be perfectly predicted. This has not been possible in the past and there is no reason to hope for such options in the future. As a result, we may conclude the following:

We may view the forest management problem this way: There is a controllable Markov process. The state of the forest is affected by growth and damages that have to be regarded as stochastic. The stochastic disturbances may have many different properties and influence the forest in different scales. The market prices and other external conditions usually change over time. These changes cannot be perfectly predicted and may be regarded as stochastic processes with many different properties. There are considerable set-up costs in harvesting and other operations. Each time you undertake some operation, you have to move harvesters, forwarders and labour considerable distances between objects. Hence, you visit each stand or forest area only rarely. In the mean time, between these visits, the forest grows and may be affected by different kinds of stochastic damages. Furthermore, between these visits, considerable market changes may take place.

Now, it is time to take a close look at earlier attempts and new ways to handle the forest management decisions in an economically optimal fashion. Of course, this text cannot cover all kinds of relevant problems and applications in the area. Hence, just a small number of typical and economically important examples from forest management will be analyed in detail. The reference list contains studies that discuss many more applications of the presented methods and may be consulted by the reader at a later point in time.

## 2 GENERAL MATHEMATICAL TOOLS AND METHODS

Economic forest management decision problems have been addressed by all kinds of mathematical optimization methods during the years. Faustmann (1849) defined the present value of an infinite series of identical forest generations. It was implicitly assumed that everything was known with certainty. A number of authors have continued in the same tradition. Johansson and Löfgren (1985) give a survey of this field. They use a number of different methods in different chapters, all but one essentially based on deterministic derivations and optimization. During more than one century, the dominating forest economic decision problem has been the determination of the optimal harvest year, based on alternative deterministic assumptions. Typically, the decision problems were solved stand by stand via one-dimensional present value maximization. The options to simultaneously optimize the number of seedlings per hectare, the number of thinnings, the timing and intensity of thinnings, and the year of the final felling, were mostly neglected. As one typical example, Johansson and Löfgren (1985) present a very detailed one-dimensional analysis of the optimal harvest year decision problem, assuming that all future forest generations will be identical to the present.

Linear programming made optimization and coordination at higher levels possible. George Danzig developed an efficient method for linear programming problems many years before he published his well-known book, Danzig (1963). Linear programming rapidly became a widely used forest planning tool, partly thanks to the development of computers and easily available standard software. Mostly, the forest management model assumptions included perfect information, large numbers of forest compartments, and long horizons. Much later, it became known that linear programming also is a very useful tool when it is necessary to solve a completely different type of problem with stochastic disturbances of many different kinds. See later sections.

Bellman (1957) presented a conceptually new method: dynamic programming. In many of the typical applications, dynamic programming is used to optimize decisions over time under the assumption that future parameters are known with certainty. However, dynamic programming can also be used for many other purposes. It turns out that you may handle set-up costs, sequential information, and adaptive decisions in a very simple and consistent way. Ross (1983) concentrates on the very important and relevant field stochastic dynamic programming. Among other things, Ross (1983) shows how you can find an optimal stationary policy for stochastic dynamic programming problems based on Markov chains via linear programming. Winston (2004), Chapter 19, gives a very convenient summary of this approach and related methods.

It has often been considered more elegant and sometimes more relevant to deal with continuous time formulations and solutions. Pontryagin (1961) is often regarded as the founder of optimal control theory. Fleming and Rishel (1975) present most of the general theory and proofs in a complete fashion. Sethi and Thompson (2000) give a very good description of the area and derive the central proofs using dynamic programming in an efficient way. In fact, several authors have used continuous time optimal control theory to derive optimal solutions in forestry. Clark (1976) derives the optimal thinning policy in forestry using deterministic optimal control in continuous time. Sethi and Thompson (2000) develop the model from Clark

(1976) in a similar way. Clark (1976) and Sethi and Thompson (2000) refer to Kilkki and Vaisanen (1969), who developed the original optimization model in discrete time using dynamic programming. The later authors then transformed the model into continuous time.

The author of this paper is not convinced that the later continuous time versions of the Kilkki and Vaisanen (1969) model are more relevant or better in any way. The optimal stock-level function is smooth and elegant in the continuous time version. The discrete time model however has the very important advantage that the harvest cost function in a very simple way can include the set-up cost, the cost of moving harvesters, forwarders and labour to the site, and many other parameters that may vary in many different ways between periods. Hence, the discrete time version may come as close as you want to the true optimal solution. It is never optimal to "continuously harvest" the forest. The prices and variable harvest costs per unit are assumed constant over time in the Sethi and Thompson (2000) continuous time version. Furthermore, there are unfortunate errors in the Hamiltonian function analysis in the otherwise very well-written Sethi and Thompson (2000) book, since the discounting factor was forgotten. The original dynamic programming version is much more easily described to the reader and the dynamic programming version can also easily be extended to the really relevant case where you may have stochastic disturbances of many different kinds. Then, we simply go to stochastic dynamic programming in discrete time. A final argument is that very few, if any, real things are continuous. In the time dimension, as one example, we note that growth and harvesting conditions normally change over the year. Some seasons are warm, others are cold, some are wet, and some are dry.

Furthermore, as the discrete time intervals approach zero, we should not neglect the existence of day and night, since the light conditions usually affect biological growth in forests and elsewhere and most connected activities in the economies. As a result, it may be better to handle the real and relevant periods via discrete time dynamic programming than to assume that they do not exist, using continuous time optimal control. Continuous time optimal control can be extended to stochastic continuous time optimal control. This approach has found many applications. Sethi and Thompson (2000) give a good summary with typical applications from different fields. The reader should be aware that the underlying process assumptions often are very restrictive. If the assumptions are relevant to the application at hand, this may not be a problem. Gleit (1978) investigates a stochastic optimal control problem in continuous time and derives an optimal harvest function. The mathematical analysis is well performed and the analytical difficulties are openly demonstrated. In order to be able to derive some explicit results, several highly restrictive assumptions have to be introduced. One of these is the assumption that the expected growth is proportional to the stock level. From a biological perspective, such assumptions are seldom relevant. Even such a very special analysis with simplifying assumptions required 13 pages of advanced formulae. The author of this paper is convinced that it is far from easy to handle the types of stochastic disturbances described in the introduction via continuous time optimal control theory. Discrete time stochastic dynamic programming is mostly a more relevant approach, in particular since this makes it possible to include set-up costs, most types of functional forms (or at least discrete approximations), and periods with all kinds of different properties in a convenient way.

## 3 STOCHASTIC DYNAMIC PROGRAMMING

Here, a very general definition of the economic management problem is given along the lines found in Winston (2004) and many earlier publications. $W(i,t)$ is the expected present value at time $t$, $i$ is the state, and $r$ is the rate of interest. $h(i,t)$ denotes the control, such as the harvest, as a function of state and time. We use stars to indicate optimal values. $W^*(i,t) = W(i,t,h^*(i,t))$. $R(i,t,h)$ is the profit at time $t$. $T$ is the horizon. $W^*(i,T+1) = 0\ \forall i$.

$\tau(j|i,t,h)$ denotes the conditional probability of reaching state $j$ in period $t+1$ and $\sum_{j=1}^{J} \tau(j \mid i,t,h)$ is the expected value of the entering state in period $t+1$.

For $t \in \{1,2,\ldots T\}$ and $i \in \{1,2,\ldots I\}$, we determine the optimal harvest (and other) decisions.

$$W^*(i,t) = \max_{\substack{h \\ h \in H(i)}} \left( R(i,t,h) + e^{-r} \sum_{j=1}^{J} \tau(j \mid i,t,h) W^*(j,t+1) \right),$$

where $H(i)$ is the feasible control set and may sometimes also contain a time argument, which may be important if harvest capacity is changed over time.

If the planning horizon is infinite and functions do not change over time, we can simplify the problem by dropping the time index, using the following definition:

$W^*(i,t) = W^*(i) = W_i^*$. In the same manner, we write:

$$R(i,t,h) = R(i,h) = R_{i,h}.$$

Since each $W_i$ should be optimal and independent of $t$, the following inequalities must hold.

$$W_i \geq R_{i,h} + e^{-r} \sum_{j=1}^{J} \tau(j \mid i,h) W_j, \quad \forall i,h \mid h \in H(i).$$

Furthermore, there is an upper bound on each $W_i$. $W_i$ cannot exceed the value obtained if the best decision is selected.

$$W_i^* = R_{i,h^*} + e^{-r} \sum_{j=1}^{J} \tau(j \mid i,h^*) W_j^*.$$

Hence, the optimal values can be determined from this linear programming formulation:

$$\min Z = \sum_{i=1}^{I} W_i$$

s.t.

$$W_i - e^{-r} \sum_{j=1}^{J} \tau(j \mid i,h) W_j \geq R_{i,h} \quad \forall i,h \mid h \in H(i).$$

## 4 STOCHASTIC SCENARIO TREE OPTIMIZATION

The approach of multi-period stochastic programming used below has been well described by Birge and Louveaux (1997). The particular forest management problem was suggested by the author and first presented at the MODFOR conference in 2002. Let us denote adaptive multi period stochastic programming with scenarios, A1, and stochastic dynamic programming, A2. A1 and A2 have different advantages and disadvantages in practical applications. The author has used A2 in many applications with very good results but thinks that A1 may be an approach which is more easily used as a "default value" standard tool within some application areas. Here, we describe A1: We maximize the expected present value, $W$, of the net profits from all periods, 1, 2, …, $n$.

$$
\begin{aligned}
W = & \sum_{d}\sum_{s_1} k_1\theta(s_1)(p_{ds_1}v_{ds_1} + L_d)h_{ds_1} + \\
& \sum_{d}\sum_{s_{12}} k_2\theta(s_{12})(p_{ds_{12}}v_{ds_{12}} + L_d)h_{ds_{21}} + \\
& \ldots \\
& + \sum_{d}\sum_{s_{12\ldots n}} k_n\theta(s_{12\ldots n})(p_{ds_{12\ldots n}}v_{ds_{12\ldots n}} + L_d)h_{ds_{12\ldots n}}
\end{aligned}
$$

$d$ = Forest department

$s_{12\ldots t}$ = One scenario defining the states of the exogenous stochastic process(es) from period 1 until period $t$

$\theta(.)$ = Probability (of a scenario) estimated before the exogenous stochastic process outcomes have been observed

$r$ = Rate of interest in the capital market

$m$ = Time length of a period

$K$ = Discounting factor. We assume that all results from a period occur in the middle of that period. Time zero is the point in time where the first period starts. The discounting factors of the different periods, $t$, are $k_t$:

$$
\begin{aligned}
k_1 &= e^{-(1-\frac{1}{2})mr} \\
k_2 &= e^{-(2-\frac{1}{2})mr} \\
&\ldots \\
k_n &= e^{-(n-\frac{1}{2})mr}
\end{aligned}
$$

$p_{ds_{12\ldots t}}$ = Net price (price - harvesting costs per volume unit) in forest department $d$ in period $t$ if the scenario which has been followed until period $t$ is $s_{12\ldots t}$

$v_{ds_{12\ldots t}}$ = Volume per area unit (density) in department $d$ in period $t$ if the scenario which has been followed until period $t$ is $s_{12\ldots t}$

$L_d$ = Land value (of bare land) in department $d$

$h_{ds_{12...t}}$ = Harvest area in department $d$ in period $t$ if the scenario which has been followed until period $t$ is $s_{12...t}$

$d$ = Forest department

$s_{12...t}$ = One scenario defining the states of the exogenous stochastic process(es) from period 1 until period $t$

$\theta(.)$ = Probability (of a scenario) estimated before the exogenous stochastic process outcomes have been observed

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$$k_1 = e^{-(1-\frac{1}{2})mr}$$

$$k_2 = e^{-(2-\frac{1}{2})mr}$$

$$\ldots$$

$$k_n = e^{-(n-\frac{1}{2})mr}$$

$p_{ds_{12...t}}$ = Net price (price - harvesting costs per volume unit) in forest department $d$ in period $t$ if the scenario which has been followed until period $t$ is $s_{12...t}$

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$L_d$ = Land value (of bare land) in department $d$

$h_{ds_{12...t}}$ = Harvest area in department $d$ in period $t$ if the scenario which has been followed until period $t$ is $s_{12...t}$

**Constraints and equations:**

The probabilities of the different scenarios $s_{12\ldots t}$ should be calculated via the (possibly time dependent) state transition probabilities for each $t$ such that $2 \leq t \leq n$. The definitions of the exogenous stochastic process(es) may be different. The exogenous stochastic processes usually influence the variables in the problem, the objective function coefficients, the set of feasible decisions, or something else. All of these effects must be calculated for each time period and scenario. The parameter vectors of different scenarios may be identical in the first periods. In such cases, constraints must exist that make sure that also the decisions are the same. As long as the parameter vectors are the same, the "true" scenario cannot be identified. This has been discussed by Lohmander (1994).

If there are physical constraints, such that a particular forest department area only can be harvested once during the defined time horizon, these constraints must be defined for each scenario covering the complete time horizon. The dynamic timber volume developments in the different departments should be calculated through relevant difference equations. There may be other constraints, connecting the possible decisions in different forest departments. Perhaps the total harvest volume has to stay within some interval which is feasible because of delivery contracts? Perhaps there is a local pulp mill which needs a more or less constant flow of pulpwood? Then, constraints have to be defined which make sure that, for each scenario, the total harvest volumes stay within the feasible sets in the different periods. In some cases, there are harvest area constraints because of forest act regulations. In Sweden, such rules define dynamic sets of feasible harvest activities at forest properties. One Swedish forest property application including the forest act regulations is found here:

http://www.lohmander.com/kurser/MODFOR02/M DFOR02.htm

A forest logistics application of the stochastic scenario method is found in Lohmander and Olsson (2005).

## 5 OPTIMIZATION OF ADAPTIVE CONTROL FUNCTIONS WITH STOCHASTIC SIMULATION OF THE OBJECTIVE FUNCTION

This approach is very flexible. You can in principle handle all kinds of functions and constraints. You just define the complete model as a stochastic simulation model. You determine your adaptive strategy, the principles that give the decisions as a function of the revealed state of the system, time, etc. Then, when you simulate the system, you let the system decide your decisions according to your suggested adaptive strategy and calculate your objective function value, for instance the expected present value. You simulate the complete system a large number of times, let us say 1,000 times, and divide the sum of the objective function values by the same number, for instance 1,000. That way, you get an estimate of the expected present value. Then you may use different numerical methods in order to search a strategy that is close to the optimum. One approach which has been tested in typical forestry applications and turned out to come close to the optimum rather rapidly is the following: Let us assume that the decision, such as the harvest level, is an adaptive function of the state of the system including information about the stock level and the price. We assume that this function has two parameters defined by the point $(X,Y)$. $(X,Y)$ should be optimized. An example of such an adaptive harvest function could be $h(V, P, X, Y)$ where $h$ denotes harvest as a function of stock, $V$, and price, $P$. The parameters are $X$ and $Y$.

Initially, you guess the optimal parameter combination $(X,Y)$. You investigate, via a large number of stochastic simulations, the expected objective function value, $F$, in this position. Then, you take steps, $dX$ and $dY$, in each direction, and visit points $(X + dX,Y)$ and $(X,Y + dY)$. In these coordinates, you investigate the expected objective function values. Now, you have useful information and can determine approximations to the partial derivatives of the expected objective function value $\partial F/\partial X$ and $\partial F/\partial Y$. With this information, you determine (an approximation of) the direction of the steepest path. You take steps of equal size in that direction and investigate $F$ each time. As long as $F$ increases, you continue in the same direction. When you have passed the peak, you reduce the step size and go back. You continue to go back and forward, reducing the step size, until you are "satisfied". Seen from the "peak" in the first selected direction, it is possible that the derivatives $\partial F/\partial X$ and $\partial F/\partial Y$ are different. Hence, you check these derivatives again and go in the "locally best" direction until you find a new peak. The process is repeated until you are satisfied. This approach is conceptually simple and flexible. One problem is that we do not know if we find a global maximum. Another problem is that stochastic deviations from the locally optimal solutions are likely, because we cannot "afford" to spend too much computing time with very large numbers of system simulations in each position. The step size is another issue that deserves some attention. Still, despite all the numerical problems with this approach, it may serve as a good alternative in some cases. The studies by Lohmander (1992a, 1993) are two such examples.

## 6 ADAPTIVE STAND-LEVEL OPTIMIZATION WITH FINAL HARVESTS

Forest management has been optimized at the stand level via stochastic dynamic programming in discrete time with continuous probability density functions of stochastic prices by Lohmander (1983, 1987, 1988). Other authors have later published the same type of models and results. This approach makes it possible to derive optimal reservation prices and expected optimal values of the forest stand and land explicitly via recursion and analytical methods. A reservation price is the price which makes you indifferent between the alternatives to harvest directly and to wait longer. If the price offer is higher than the reservation price, you should harvest directly. If the price offer is lower than the reservation price, you should wait longer. Here, the main structure of the model will be presented. $W_t$ is the expected present value in period $t$ before the price $p$ in the same period has been observed. Prices are stochastic and have the probability density function $f(p)$. $f(p) > 0 \forall p$. In each period, new stochastic prices are revealed in the market. This type of price process is a special case of a stationary stochastic process. If we consider time periods of 5 years, this is a model that is not easy to reject on statistical grounds in the Swedish market and many other markets. The prices really denote "real net prices", nominal prices – harvesting costs per volume unit deflated by consumer price indices. The reservation price in a particular period is denoted by $q_t$. If $q_t > p_t$, then you should wait at least one period more for a new price offer. If $q_t = p_t$, the probability of which is almost zero, you are indifferent between the alternatives to harvest at once and to wait at least one more period. If $q_t < p_t$, then you should harvest directly. $V$ is the volume per area unit and $L$ is the value of the land released after harvest. We select to start the calculations far in the future, at the horizon, $T$. It turns out that the particular choice of $T$ is not critical to the results as long as $T$ is sufficiently large, for instance three times the age of the optimal forest rotation age in a deterministic analysis.

$$W_T^* = 0$$

$$W_t = \int_{-\infty}^{q_t} W_{t+1}^* f(p)dp + \int_{q_t}^{\infty} e^{-rt}(pV(t) + L)f(p)dp, \quad \forall t | t < T.$$

The reservation prices are optimized for each period $t$ such that $t < T$.

$$\frac{\mathrm{d}W_t}{\mathrm{d}q_t} = f(q_t)\left(W_{t+1}^* - e^{-rt}(q_t V(t) + L)\right) = 0$$

$$f(q_t) > 0 \Rightarrow \left(W_{t+1}^* - e^{-rt}(q_t V(t) + L)\right) = 0$$

$$\left[\frac{\mathrm{d}W_t}{\mathrm{d}q_t}=0\right]\Rightarrow\left[e^{-rt}(q_tV(t)+L)=W_{t+1}^*\right].$$

The present value of harvesting, given that the price is equal to the reservation price, is the same as the expected present value in case you wait at least one more period and take optimal decisions in future periods based on the revealed outcomes of the stochastic prices. In optimum,

$$\left(W_{t+1}^*-e^{-rt}(q_tV(t)+L)\right)=0\text{, which implies that}$$

$$\frac{\mathrm{d}^2W_t}{\mathrm{d}q_t^{\,2}}=-f(q_t)e^{-rt}V(t)<0.$$

We find that the solution is a unique maximum in each period. The optimal reservation prices and expected present values are determined recursively, starting from $T$ via the backward algorithm of dynamic programming.

$$q_t^*=\frac{e^{rt}W_{t+1}^*-L}{V(t)}.$$

You may state the optimization problem as:

$$W_t^*=\int_{-\infty}^{\infty}\max\left\{W_{t+1}^*,e^{-rt}\left(pV(t)+L\right)\right\}f(p)\mathrm{d}p,\quad\forall t\middle|t<T.$$

As a result, it is clear that the expected present value is a nonstrictly decreasing function of time. $W_t^*\geq W_{t+1}^*$. In most empirically relevant cases, $q_t<\infty$. $f(p)>0\forall p$ and $W_t^*>W_{t+1}^*$. As a result, the expected present value is a strictly decreasing function of time. This is not really surprising: If we move forward in the time dimension and still have not harvested, this indicates that we have not experienced prices above the optimal reservation prices yet. The longer we have to wait before we experience such a good price, the worse. Before we knew that this would happen, we had many valuable options ahead of us. That is why the expected present value is a decreasing function of time. The reader should be aware that we should not decide the harvest year in advance if the market prices are stochastic. We should wait and see what happens in the market before we decide to harvest or to wait longer. The reservation prices can however be optimized along the lines suggested in this section. Often, the optimal reservation prices are decreasing functions of time. However, this does not always have to be the case. Furthermore, if we make a more detailed analysis, we have to consider the changing dimensions of the trees, proportions of possible timber and pulpwood harvest levels, and quality-dependent price lists. If we look at the decision problems using shorter periods, such as months or years, we usually have to consider autocorrelation issues in detail. Different kinds of stochastic price and growth processes may lead to different results. Such detailed analyses have been done by, for instance, Lohmander (1987). General findings in this area have been reported by Lohmander (2000).

## 7 ADAPTIVE CONTINUOUS COVER OPTIMIZATION AT THE STAND LEVEL

Now, we move on to discrete time optimization of continuous cover forest management using dynamic programming. The analysis will start with a deterministic model that becomes transformed to a stochastic version. The analysis below was originally presented by the author at EURO/Informs, Istanbul, 2003. The volume stock levels, $S$, are defined in such a way that the stock moves up one level per 5-year period. Volume is denoted by $V$. The volume growth is assumed to follow this process $G(V) = \alpha V + \beta V^2$, which is consistent with the classical logistic growth assumption in natural resource economics. We may rewrite the function in this way: $G(V) = sV(1 - V / K)$, where $s$ is the "intrinsic growth rate" and $K$ is the "carrying capacity" of the environment. In the analysis, we assume that $s = 5\%$ and $K = 400$ cubic metres per hectare, which are typical parameters in some Swedish forests. The particular numerical values are however not important to the qualitative discussions. Of course, other numerical values will be the results if other growth parameters are used. The qualitative results are however the same. In the cases without price risk, it is assumed that the real price per cubic metre is 300 SEK, which is close to the average value in Sweden. The real variable harvesting cost is 100 SEK per cubic metre. (7 SEK≈\$US 1). When there are set-up costs, representing the costs of moving harvesters, forwarders, and labour to the objects, these are assumed to be 500 SEK per hectare and occasion in real terms, which is typical in Sweden. When stochastic prices are assumed, then the prices per cubic metre are 220, 260, 300, 340, and 380 when the "business states" are 1, 2, 3, 4, or 5 respectively. Then, all business states are assumed to have equal probability. This degree of price variation is typical in Sweden and it is hard to statistically reject the hypothesis that a more or less uniform net price probability distribution is relevant, using historical data. All of the analysis in this section concerns optimization with infinite horizon. In the examples, a 3% real rate of interest is used. The stock level is constrained to simplify the illustration: $0 < S < 13$ (Table 1).

**Table 1.** The "classical" and most simple case with a constant price of 300 SEK per cubic metre and without set-up costs: the table shows the optimal harvest volumes per 5-year period as a function of the entering stock level.

| Entering stock (cubic metres per hectare) | Optimal harvest volume (cubic metres per hectare) |
|---|---|
| 30 | 0 |
| 37 | 0 |
| 45 | 0 |
| 55 | 0 |
| 67 | 0 |
| 81 | 14 |
| 97 | 30 |
| 116 | 49 |
| 136 | 69 |
| 159 | 92 |
| 183 | 116 |
| 207 | 140 |

In the relevant case, demonstrated in Table 2, the set-up costs and stochastic prices are treated consistently and simultaneously. The optimal harvest is an increasing function of the stock level and the price level. We also note that "low volume harvesting" should be avoided when possible, which is natural since we have set-up costs.

**Table 2.** The relevant case with set-up costs and price risk: The table shows the optimal harvest volumes (cubic metres per hectare) per 5-year period as a function of the entering stock level and the price level.

| Entering stock (cubic metres per hectare) | Price (SEK per cubic metre) | | | | |
|---|---|---|---|---|---|
| | 220 | 260 | 300 | 340 | 380 |
| 30 | 0 | 0 | 0 | 0 | 0 |
| 37 | 0 | 0 | 0 | 0 | 0 |
| 45 | 0 | 0 | 0 | 0 | 0 |
| 55 | 0 | 0 | 0 | 0 | 0 |
| 67 | 0 | 0 | 0 | 0 | 37 |
| 81 | 0 | 0 | 0 | 0 | 51 |
| 97 | 0 | 0 | 0 | 0 | 67 |
| 116 | 0 | 0 | 0 | 49 | 86 |
| 136 | 0 | 0 | 0 | 69 | 106 |
| 159 | 0 | 0 | 0 | 92 | 129 |
| 183 | 0 | 0 | 46 | 116 | 153 |
| 207 | 25 | 25 | 71 | 140 | 178 |

# 8 ADAPTIVE COMPANY-LEVEL FOREST MANAGEMENT OPTIMIZATION

In forest companies, you often face global constraints, such as harvest capacity constraints, delivery contracts, etc. Optimal harvesting decisions in forestry under the influence of stochastic prices have mostly been studied under the assumption of complete stand separability. In situations when the harvester capacity is a binding constraint, the optimal stands to harvest cannot be determined without explicit consideration of this constraint, which means that stand separability assumptions are not relevant. Then, it may be optimal to harvest a particular stand if the timber price is low but not if the price is high. The profitability of harvesting may be more sensitive to the timber price in some other stand. Hence, it may be more important to use the machines in another stand if the timber price is very high. The expected shadow price of harvester capacity is an increasing function of the degree of timber price variation, indicating that the optimal harvester capacity is higher under risk than under certainty in some cases. Many other types of "global constraints" usually exist in forest sector enterprises. Many of them are expected to give "disturbances" to the classical "optimal economic stand management" decision rules. You may think that dynamic programming cannot be applied to relevant problems because of the curse of dimensionality. Maybe this is not always quite true. Optimal adaptive decisions can be determined at the forest company level. Some examples are found in the reference list: Lohmander (1992b, 1993, 1997a). Optimal infrastructure investments are also important. Compare Olsson and Lohmander (2005).

The expected present value increases if the risk of the stochastic prices increases and there are options to wait for the best harvesting occasions. The positive effects of increased price risk are reduced in case the harvesting capacity is low. If the harvesting capacity increases, you have a more flexible system and can take advantage of sudden price increases in a better way. The expected shadow price of harvest capacity is an increasing function of the degree of timber price variation. The optimal harvest capacity is higher under risk than under certainty in some cases. The traditional deterministic analysis does not give the correct shadow prices. This, in turn, leads to too low-capacity investments. The approach in this paper gives the correct expected shadow prices and can be used to optimize harvest capacity investments.

## 9 DISCUSSION

No paper is complete in the sense that all relevant issues are mentioned. The area of adaptive optimization of forest management contains many different kinds of special topics and if all of these should be discussed, you would not have been able to go into details in any particular direction. Hence, a selection of some of the most important problems had to be made. This paper contains a treatment of the final harvesting problem with adaptive optimization. We have also analysed the continuous cover forestry problems with and without price risk and set-up costs. Some other problems were mentioned and different types of global constraints were discussed in connection to the stochastic programming formulation and typical methodological tools were described. Now, the reader is advised to search for relevant applications in other directions. Optimal forest sector logistics is one area where new and relevant results can be obtained, in particular since roads and other parts of the logistics network may be disturbed by unexpected rains, snow and ice, and other problems that cannot be perfectly predicted. If you cannot deliver a sufficient flow of wood to the pulp mills, you may have to stop production, which may be very costly. Hence, there is an optimal combination of road capacity, trucks, pulpwood storage, and locations, which is not always easy to optimize. Some efforts in this direction have however been made in recent years and can be found in the reference list. Stochastic damages of many kinds have been analysed and the optimal adaptive strategies have been determined. Some of the recently investigated areas in this class concern species-selective damages caused by moose in Sweden and the optimal mix of species and selective thinnings in plantations. In large parts of Northern Sweden, moose damages to Scots pine cause severe problems and mixed species plantations are sometimes the economically best solution. Compare Lu and Lohmander (2005).

Another topic with reported optimal results is the spatial and temporal management of forest areas where stochastic winds randomly cause windthrows. In Sweden, the windthrow topic has been quite dominating during the spring of 2005 because a hurricane, named Gudrun, destroyed very large forest areas in southern Sweden completely. Research results existed much earlier, indicating that the optimal harvest ages are lower in stormy areas, that one should keep large areas together without partial harvests since the stands protect each other from the wind and that one should modify the spacing and thinning intensity. Compare Lohmander and Helles (1987) and Lohmander (1987b). However, the Swedish forest act did not take such things into account and the forest owners could not deviate from the detailed forestry regulations. With some luck, the forest act may be modified in the near future and consider these problems more carefully. So far, we have not mentioned the fact that the markets sometimes may be described as dynamic games. When the number of players is small, which is sometimes the case, at least locally, we may use deterministic or stochastic differential or difference games to study the optimal decisions. Compare Lohmander (1997b). This is a very large field that deserves much more efforts in the future. The area of stochastic difference games may be regarded as a very natural extension of adaptive optimization. Of course, this is highly relevant in the forest sector.

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