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ECONOMIC TWO STAGE MULTI SPECIES MANAGEMENT IN A STOCHASTIC ENVIRONMENT: THE VALUE OF SELECTIVE THINNING OPTIONS AND STOCHASTIC GROWTH PARAMETERS

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In a perfectly predictable world, there is often no reason to plant more than one species in a particular forest stand. It is easy to calculate the present value of each possible species alternative and to select the species which maximizes the present value of the investment. However, since we cannot predict future soil and climate conditions several decades in advance and the effects on forest growth are yet unknown, the optimal investment strategy may be quite different: In this paper it is shown that a mixed species stand may be the optimal investment. Then, there is an option to obtain the optimal species through selective thinning when more and better information concerning future conditions is available.

KEY WORDS Forest growth modelling, forest management.

1. INTRODUCTION

Flexibility is a concept which can be defined in many different contexts. The value of flexibility, particularly in nature preservation problems, has been intensively debated by Arrow and Fisher [2], Henry [7], Viscusi and Zeckhauser [32] and by Miller and Lad [24]. In the problem of forest rotation economics, flexibility may be defined as the length of the time period during which clearcutting is allowed or possible in a forest stand. It has been shown by Lohmander [10, 15] that it is possible to make considerable gains from a flexible rotation age in the presence of stochastic wood prices. The relevant approach to that problem is optimal stopping theory: The decision to stop the life of the forest stand investment is taken conditional on the latest price information available. If prices are stochastic, the optimal rotation age also becomes stochastic.

Is there a value associated with product flexibility in a production system? This paper addresses this general question in a specific context. The answer to the general question is of course dependent on the properties of the system environment. In particular, if the production system can be used to produce many different kinds of products, this is an essential quality of the system if demand is changing and unpredictable. In a constant and predictable environment, flexibility is useless. Of course, system flexibility is also useless if the decision maker has to take all decisions before the demand development has been observed. In other words, a multi stage information and decision structure is needed if the advantages of the flexibility should be discovered. Sequential optimal decisions based on the latest available information is denoted by adaptive optimization. Some recent programming approaches and results in this area with applications to the forest sector enterprises have been reported by Lohmander [13, 14, 15, 17, 22].

This paper will restrict the attention to the flexibility obtainable in forest management if several species are planted in the same stand. In the single species stand, which finds strong support in several forest acts and common practise, the species composition in the latest period of production is decided already 100 years or more before the final harvest takes place. Quite clearly, it is not possible to predict the price development of different forest species without error 100 years in advance. Quite clearly, no person living today knows the future levels of acidity deposition and pollution affecting the forests or the extent to which different species will be able to accept these possibly changing environmental conditions. A deterministic approach to the economic forest management optimization problem under changing environmental conditions is however suggested as a first step by Lohmander [20]. If we invest in a mixed species stand, we can select the proper species in the final stage of production maybe 50 years later through selective thinning. Then, we will know the true development of the environmental conditions and wood prices and can make the correct adaptive choice.

The mixed species stands have been given much attention also in the past. Andersson [1] and Hägg [8, 9] have studied the effects of hardwoods on the survival, quality development and profitability of pine in mixed stands. One of the key assumptions in this paper is that thinning really increases the profitability from forestry and should be undertaken. This means that the optimal number of stems initially should be higher than what is optimal in the end of the production period. One reason for this is that the quality of the timber increases very much as the number of stems in the young stand increases. This issue is discussed by Persson [25] who shows how the spacing and the quality development of pine stands are related. Mielikäinen [23] reports from wood production investigations in mixed pine–birch stands and Tham [26, 27] investigates the development of mixed spruce–birch stands. Valsta [28, 29, 30, 31] calculates the economically optimal mixed species management programs including thinnings and clearfellings based on the assumptions of deterministic developments of the environment and the markets via deterministic dynamic programming and other methods. Similar problems are solved by Bullard, Shearli and Klemperer [4], Bare and Opalach [3] and Carlsson [5]. The dynamic optimization results reported by Carlsson [5] are based on the forest production experiments by Tham [26, 27]. One may conclude that there may be reasons to invest in mixed species stands also when the environment is regarded as deterministic. In some of the production experiments mentioned above, there were indications that the total growth of the stand and/or the total profitability of the management of the stand would increase if the single species stand was replaced by a mixed species stand. However, these reported relative improvements generally have been small and have been relevant in situations where future conditions are assumed known and detailed long term harvest planning is possible. Hence, it is urgent that the expected economic value and the optimal management decision rules of mixed species forestry are

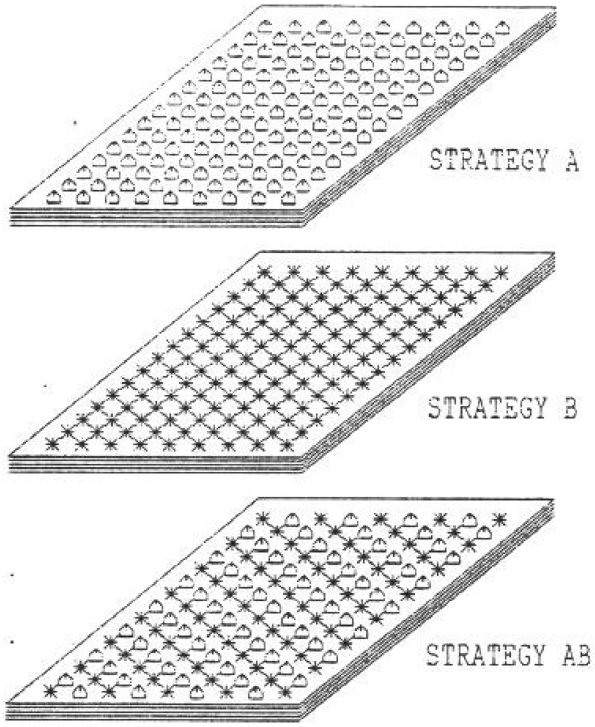

Figure 1 Strategies for forest management.

reconsidered in a relevant context. It is obvious that the expected present value difference between multi species stands and single species stands is strongly underestimated if the environment is assumed to be deterministic. The option value of late adaptive production changes in the multi species stand is not at all included in the traditional calculations.

In a perfectly predictable environment, it is frequently economically optimal to invest in a one species stand, strategy A or strategy B, with the corresponding species A and B. However, in an unpredictable environment it is valuable to have the option to select species as late as possible, when more and better information is available. Today, we do not know if and to what extent different species will be affected by different possible future levels of soil acidity and climatic changes. Furthermore, the true future levels and changes are still unknown to us. If we initially invest in a multi species stand (strategy AB), then we have the valuable option of future adaptive decision making. It is also obvious that the optimal intertemporal decisions in a mixed species stand should be dependent on the latest developments of the environment. Furthermore, the initial decisions should be affected by future unpredictability since the early decisions influence the future management options. These are two reasons why traditional deterministic multi species models will give the wrong management guidelines.

2. THE TWO STAGE DECISION PROBLEM

Of course, the true adaptive forest management optimizer should not restrict himself to a two stage decision problem. In reality, considerable changes in the environment and the forest markets may be observed on several occasions during the life of a forest stand. Every time this happens, it is possible that the optimal stand management decision is different from what was planned a moment earlier. In particular, it has been shown by Lohmander [10, 15] that the expected profitability of the final clearfelling increases very much if a new harvest decision is taken every 6 or 12 months, depending on the state of the roundwood market price. However, since the purpose of this paper is to discuss the general principles of multi species management in an unpredictable and changing environment, the degree of model complexity will be held as low as possible in order to transparently illustrate the main properties of the problem area.

At time 0, $t_0 = 0$, the first stage decision is to decide the number of plants of species A and species B. These numbers are denoted by N_A and N_B. The stochastic environmental process which represents properties such as soil acidity and temperature is denoted by X. X is defined as an exogenous Markov process. At t_0, the state of the process at t_0, X_0, is observed and known. The objective function to be optimized at t_0 is W_0, which is the expected present value of the investment. W_0 contains costs at t_0 that are known with certainty and the expected present value of profits in later periods if optimal decisions are taken then, W_1^*. The costs at t_0 are assumed to be proportional to the investment levels in the two species. The costs per plant are denoted by C_A and C_B.

$$\max_{N_A, N_B} W_0(N_A, N_B; X_0) = -C_A N_A - C_B N_B + W_1^*(N_A, N_B; X_0) \tag{2.1}$$

At time 1, t_1, the second decision, the selective thinning decision, is taken conditional on the latest information concerning the environmental state, X_1. The thinning net profit is denoted by U. Depending on the plant number decisions at t_0, different thinning options are available. The thinning decision is denoted by S and the relative thinning volumes in the two species, H_{A_1} and H_{B_1} are functions of S. The thinning net prices of the two species are P_{A_1} and P_{B_1} and the volumes at t_1 before thinning are called V_{A_1} and V_{B_1}. The volume of each species at t_1 is a function of the number of plants of the same species at t_0 and the environmental state X_1. The reason is that the environmental state changes from X_0 to X_1 directly after the initial investment decision. X_1 changes to X_2 directly after the selective thinning decision. It is possible that the volumes at t_1 in the different species also are affected by the number of plants of the other species. There are, however, biological reasons why growth is almost exponential in the initial phase of production. The interspecies competition (and competition in general) is stronger after the selective thinning decision has taken place since the availability of water and light normally place binding restrictions on the photosynthesis level in the final stage of production.

$$U(S, N_A, N_B; X_1) = P_{A_1} H_{A_1}(S) V_{A_1}(N_A(N_B); X_1) + P_{B_1} H_{B_1}(S) V_{B_1}((N_A)N_B; X_1) \tag{2.2}$$

When the expected present value of future profits, W_1^*, is determined at t_0, it is assumed that the optimal selective thinning decisions, S^*, are taken at t_1 when X_1 has been observed. The optimal thinning decisions S^* maximize the present value of the sum of the thinning profit and the expected clear felling profit at t_2, W_2. $M(X_t, X_{t+1})$ denotes the probability of transition from environmental state X_t to state X_{t+1} and M is called the transition probability matrix. n denotes the number of possible environmental states.

$$W_1^*(N_A, N_B; X_0) = \sum_{k=1}^{n} M(X_0, X_1^k)[e^{-rt_1}U(S^*, N_A, N_B; X_1^k) + W_2(N_A, N_B, S^*; X_1^k)] \quad (2.3)$$

W_2 is dependent on X_1 in two ways: X_1 and X_2 affect the growth over time of the two species and the probability distribution of X_2 is conditional on X_1. Hence, W_2 can not be determined before X_1 has been observed. P_{A_2}, P_{B_2}, V_{A_2} and V_{B_2} denote the prices and volumes at t_2, the time of clear felling.

$$W_2(N_A, N_B, S^*; X_1) = \sum_{k=1}^{n} M(X_1, X_2^k)e^{-rt_2}[P_{A_2}V_{A_2}(N_A, N_B, S^*; X_1, X_2^k) + P_{B_2}V_{B_2}(N_A, N_B, S^*; X_1, X_2^k)] \quad (2.4)$$

It should be observed that the volume functions have been given quite general forms and that a Markov representation has been avoided.

3. GENERAL RESULTS

This section is devoted to the derivation of general analytical results based on a *restricted* version of the general model specified above. For notational simplicity, we make the following assumptions and definitions: $t = 1$, $X_{t-1} = X_0 = X^k$.

$$\phi_i = M(X_{t-1}^k, X_t^i) \quad \text{for} \quad i \in (1, \ldots, n), \quad n = 3 \quad (3.1)$$

In the rest of this section, the time index of X is excluded and we implicitly assume that the time period is 1. It is time for the thinning. $\pi_y(i)$ denotes the sum of the thinning profit present value at t_1 and the expected (expectation formed at t_1) present value of the clear felling profit at t_2 when the environmental state at t_1 is X^i and the only species in the stand before (and after) thinning is y, $y \in (A, B)$.

We assume that 3 investment decisions (strategies) are available at t_0: Invest in species A only, invest in species B only or invest 50% in each of the species A and B. These strategies are denoted A, B and AB respectively. Irrespective of the selected investment strategy, the total number of seedlings and the total investment cost are assumed to be the same. When thinnings take place at t_1, 50% of the stems are taken away and we assume that it is always optimal to keep only one species in the stand during the final period of production. In Sweden, the thinning net prices are normally much lower than the final clearcutting net prices. Normally, they are very close to zero. Since thinnings affect the future profitability, they may be regarded as "investments" and are normally performed even if the instantly obtainable thinning profit is close to zero. In this section, we assume that thinning net prices are equal to zero. At t_0, the different strategies give the expected present values $\bar{W}_A$, $\bar{W}_B$ and $\bar{W}_{AB}$ respectively. (Note that the constant investment costs are not included in this section since they do not affect the strategy choice.)

$$\bar{W}_A = \sum_{i=1}^{3} \phi_i \pi_A(\mathrm{i}) \quad (3.2)$$

$$\bar{W}_B = \sum_{i=1}^{3} \phi_i \pi_B(i) \quad (3.3)$$

$\pi^*_{AB}(i)$ is defined as the sum of the thinning profit present value at t_1 and the expected clear felling present value at t_2, when the environmental state at t_1 is X^i, the investment strategy was AB and the optimal selective adaptive thinning decision is taken at t_1.

We mathematically define $\pi^*_{AB}(i)$ as:

$$\pi^*_{AB}(i) = \max[\pi_A(i), \pi_B(i)] \tag{3.4}$$

$$\bar{W}_{AB} = \sum_{i=1}^{3} \phi_i \pi^*_{AB}(i) \tag{3.5}$$

Let us define the following relations between the values of π for different values of i:

$$\pi_A(1) < \pi_B(1) \tag{3.6}$$

$$\pi_A(2) = \pi_B(2) \tag{3.7}$$

$$\pi_A(3) > \pi_B(3) \tag{3.8}$$

This means that B is the economically most favourable species to keep in the forest stand during the last period of production if the environmental state happens to be X^1. In case the environmental state happens to be X^3, then one would prefer to keep species A and if the state is X^2, the two species represent the same level of profitability in continued production. However, since it is not obvious that all three states X_1, X_2 and X_3 with the qualitative relative profitability implications defined above are representative for the conditions in every forest area, let us define four different cases. The classification is based on the environmental state probability distributions.

Case	*Properties*
1	$(\phi_1, \phi_2, \phi_3) = (0, 1, 0)$
2	$(\phi_1, \phi_2, \phi_3) = (\alpha, (1-\alpha), 0)$ such that $0 < \alpha < 1$
3	$(\phi_1, \phi_2, \phi_3) = (0, (1-\alpha), \alpha)$ such that $0 < \alpha < 1$
4	$0 < \phi_i < 1$ for $i \in (1, 2, 3)$, $\sum_{i=1}^{3} \phi_i = 1$

Now, let us investigate the expected present values of the different investment strategies in the different cases!

Case 1

$\bar{W}_A = \pi_A(2)$

$\bar{W}_B = \pi_B(2)$

$\bar{W}_{AB} = \pi_A(2) = \pi_B(2)$

Results case 1: $\bar{W}_A = \bar{W}_B = \bar{W}_{AB}$

Case 2

$\bar{W}_A = \alpha\pi_A(1) + (1-\alpha)\pi_A(2)$

$\bar{W}_B = \alpha\pi_B(1) + (1-\alpha)\pi_B(2)$

$\bar{W}_{AB} = \alpha\pi_B(1) + (1-\alpha)\pi_A(2)$

Since $\pi_A(1) < \pi_B(1)$ and $\pi_A(2) = \pi_B(2)$, we have:

Results case 2: $\bar{W}_A < \bar{W}_B = \bar{W}_{AB}$

Case 3
$\bar{W}_A = (1-\alpha)\pi_A(2) + \alpha\pi_A(3)$
$\bar{W}_B = (1-\alpha)\pi_B(2) + \alpha\pi_B(3)$
$\bar{W}_{AB} = (1-\alpha)\pi_B(2) + \alpha\pi_A(3)$

Since $\pi_A(2) = \pi_B(2)$ and $\pi_A(3) > \pi_B(3)$, we have:

Results case 3: $\bar{W}_B < \bar{W}_A = \bar{W}_{AB}$

Case 4
$\bar{W}_A = \phi_1\pi_A(1) + \phi_2\pi_A(2) + \phi_3\pi_A(3)$
$\bar{W}_B = \phi_1\pi_B(1) + \phi_2\pi_B(2) + \phi_3\pi_B(3)$
$\bar{W}_{AB} = \phi_1\pi_B(1) + \phi_2\pi_A(2) + \phi_3\pi_A(3)$

Since $\pi_A(1) < \pi_B(1)$, $\pi_A(2) = \pi_B(2)$ and $\pi_A(3) > \pi_B(3)$, we have:

Results case 4: $\bar{W}_{AB} > \bar{W}_A$, $\bar{W}_{AB} > \bar{W}_B$, $\bar{W}_A \gtreqless \bar{W}_B$

Hence, in the general case 4, where all environmental states have strictly positive probabilities, the mixed species strategy gives a strictly higher expected present value than every single species strategy thanks to the selective thinning option. The reason is that, depending on the initially unknown but later revealed environmental development, the most suitable species is selected for production in the final stage. In the general case, where the precise values of the state transition probabilities are initially not known, we also found that the best single species solution is not possible to calculate. $\bar{W}_A \gtreqless \bar{W}_B$.

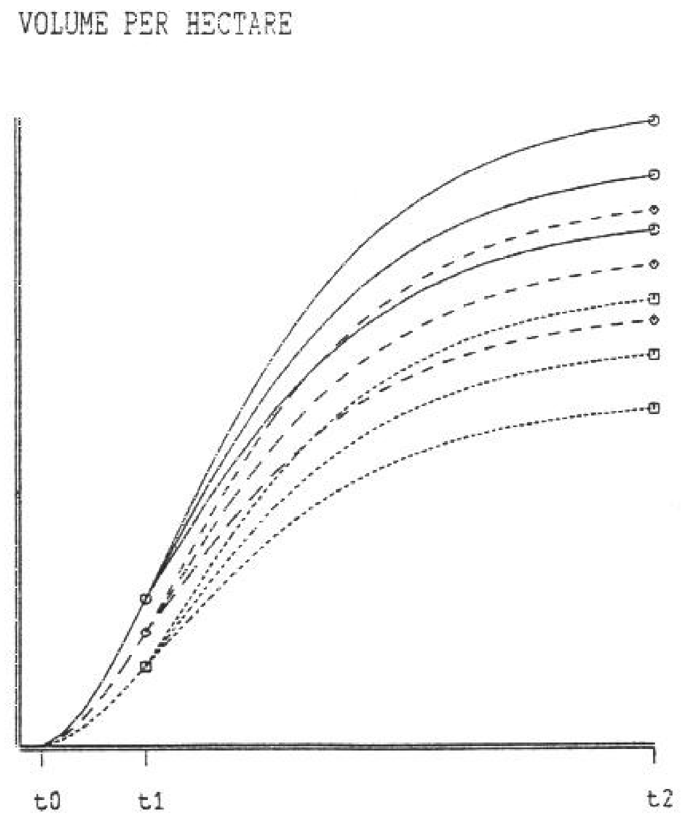

Figure 2 Volume per hectare.

If the future volume development of a species is predicted very early in the life of the stand, then the error is most likely larger than if the prediction is made later. If we want to select the species with the most favourable development, we should make the selection as late as possible. Hence, we should not invest in a one species stand based on the information available at t_0 $(=t0)$. We should invest in a mixed species stand at t_0 and make the final selective and adaptive choice of species at t_1 $(=t1)$.

4. NUMERICAL ILLUSTRATION

In order to derive quantitative solutions to the multi species management problems and to be able to investigate the properties of the optimal solutions of more complex and/or specific versions of the models in a rapid way, a simple numerical optimization model for the personal computer has been constructed and included in the *appendix*. Here, some specific model assumptions and a typical solution will be discussed. The idea is that the approach and maybe revised versions of the numerical model should be used in locally relevant contexts defined by the future investigator.

The two species A and B are possible to use in the forest stand. It is possible to plant 0, 500 or 1000 seedings of each species at t_0. Hence, there are three investment level possibilities in each species and the total number of possibilities is $3^2 = 9$. The program will investigate them all.

The costs per seedling, $C_A = C_B = 1$, the thinning net prices $P_{A_1} = P_{B_1} = 10$, the clear felling net prices $P_{A_2} = P_{B_2} = 100$, the rate of interest $R = 3\%$, $t_0 = 0$, $t_1 = 30$ and $t_2 = 60$.

The environmental state can take 3 different values in every time period: 1, 2 and 3. It is easy to change the elements of the environmental state transition probability matrix through interactive commands. In case the default definition of the environmental state transition probability matrix M is not changed, we have:

$$M = \begin{bmatrix} 0.9 & 0.1 & 0.0 \\ 0.4 & 0.5 & 0.1 \\ 0.1 & 0.4 & 0.5 \end{bmatrix} \tag{4.1}$$

Hence, if the environmental state is 1 in period t, it will take the values 1, 2 and 3 with the probabilities 0.9, 0.1 and 0 in period $t + 1$ respectively.

The growth of the two species are increasing functions of the environmental state. Different qualitative properties of the species are introduced in the following way: Species A is more affected by environmental changes than species B. If the environmental state is 2, "the normal value", then species A grows a little more than species B. Note that these simple growth functions are consistent with the assumption of exponential growth in the initial period of production!

$$V_{A_1} = 0.20 * N_A * (0.40 + 0.30 * X_1) \tag{4.2}$$

$$V_{B_1} = 0.18 * N_B * (0.80 + 0.10 * X_1) \tag{4.3}$$

There are 5 different thinning strategies, S, available at t_1 with different implications for the relative thinning volumes in the two species. These are presented below:

S (Thinning strategy)	H_{A_1} (Thinning level A)	H_{B_1} (Thinning level B)
0	100%	100%
1	100%	0%
2	0%	100%
3	0%	0%
4	50%	50%

There are two restrictions on the number of stems per hectare:

—The total number of seedlings per hectare must be at least 600 (Hence 1000 is the minimum accepted level because of the earlier restrictions). This restriction makes sure that the branches of the young trees will not grow too large and that the quality of the timber in the final clearfelling is satisfactory.

—The total number of stems per hectare after thinning in the final period of production must be below 600 (Hence 500 is the maximum accepted level because of the earlier restrictions). This makes sure that the diameter growth of the stems and hence the economic value becomes satisfactory. Because of this and the earlier restrictions, it is not feasible to have more than one species in the final period of production. Hence, the growth functions of the final period can be defined in a simple way. The volumes per hectare of the different species at t_2, instantly before the final clearfelling, are defined as functions of the volume per hectare in the species before thinning at t_1, the relative thinning volume of the species at t_1 and the environmental state during the final period of production (from directly after t_1 until t_2), X_2.

Again, as in the first period of production, species A is more sensitive to the environmental state than species B. The expected relative growth is however the same if the state happens to take the value 2.

$$V_{A_2} = (1 - H_{A_1}) * V_{A_1} * (1 + 2 * (0.40 + 0.30 * X_2)) \quad (4.4)$$

$$V_{B_2} = (1 - H_{B_1}) * V_{B_1} * (1 + 2 * (0.80 + 0.10 * X_2)) \quad (4.5)$$

The computer program, which is found in the *appendix*, solves the two stage stochastic "dynamic" optimization problem for all feasible investment strategies and initial states. The computer output shows how the optimal investment strategy and the expected present value at t_0 are dependent on the initially prevailing environmental state X_0. The optimal investment strategies (N_A^*, N_B^*) are (0, 1000), (500, 500) and (500, 500) when the initial environmental states X_0 are 1, 2 and 3 respectively.

Clearly, according to the transition probability matrix, when the initial environmental state X_0 is 1, the probabilities that the states are 1 also in the future periods are very high. Since species B grows much better than species A under these conditions, the optimal strategy is to invest in species B only. The option value of a flexible thinning decision at t_1 is low compared to the expected production loss if some B seedlings are replaced by A seedlings at t_0.

On the other hand, if the initial environmental state X_0 is equal to 2 or 3, then the future states are less predictable. The economic value of having several species and hence several adaptive thinning options available is high. Furthermore, the expected growth of the two species is not very different. Hence, the optimal strategy at t_0 is to invest in 500 seedlings of each species.

The computer output also shows the optimal adaptive thinning decisions at t_1 as functions of the states X_0 and X_1 and the different possible initial investment strategies.

5. FINAL CONCLUSIONS

In this analysis, the multi stage information and decision structure of forest management has explicitly been taken into consideration together with the fact that future environmental changes and growth responses to a large extent are unpredictable. As a consequence, qualitative results concerning the economics of mixed species forest stand management and selective thinning have been obtained that can not, and have not, been found in traditional forest management optimization models.

Traditionally, the wood markets and the environment have been assumed to be deterministic and known during the whole life cycle of the stand already at the time when the seedlings are placed in the ground. Of course, the economic values of species variation and future adaptive decisions are neglected in such contexts. Hence, even if the model defined and discussed in this paper is not very sophisticated in the sense of biometric estimations, it captures essential properties of the forest management production and decision problem that have earlier been completely neglected.

The most urgent area of future research is the following: All available empirical physiological results concerning environmental stress response levels of different species under different possible forest management regimes should be used in order to estimate the environmentally dependent multi species growth models of this paper. A general physiological approach to growth modelling is discussed by Cannell [6]. The environment may influence the development of a tree in many different ways. Acid rains may have direct effects through the air and indirect effects through the soil. If the indirect effects are the main object of analysis, the buffer capacity of the soil is of course a very important ingredient. However, soil conditions generally show considerable spatial variation in both small and large scales depending on geological history. Hence, all forest management guidelines derived from such optimization models tend to have a normative value which is only locally relevant. This does not mean that the locally relevant analysis should be avoided! Locally optimal management methods should be derived and forest economic planning should be a business where the locally relevant conditions are taken into account.

In a real world situation, one should be aware that natural regeneration from neighbour stands will affect the optimal forest management strategy. Figure 3 shows three possible strategies in the presence of naturally regenerated birch. Of course, depending on the number of plants per hectare, the plant size distribution and the spatial variation, the economically optimal decisions are different.

The area of optimal adaptive management of forest stands with stochastic regeneration from neighbour stands also deserves future attention!

Because birch seeds move far with the wind from trees in neighbour stands, we frequently obtain a forest regeneration of the species birch in Sweden without any

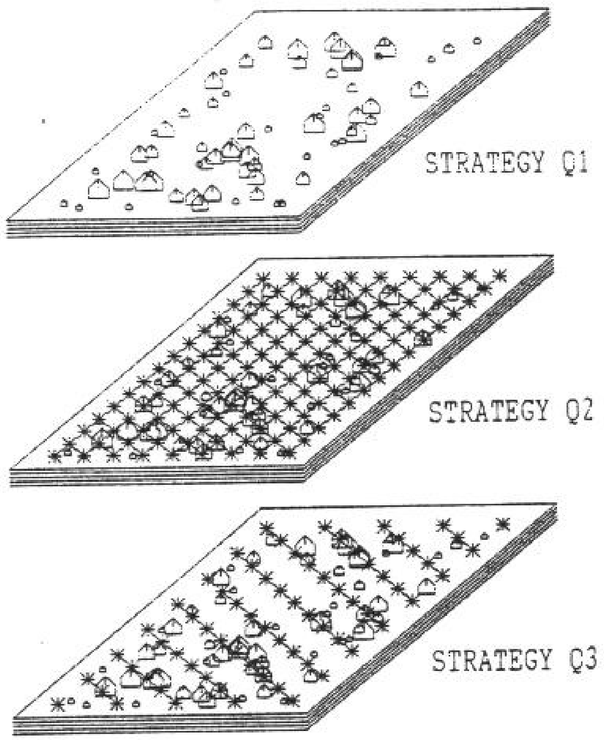

Figure 3 Investment strategies.

active management. If no further activities are undertaken, the "investment strategy" is strategy Q1. In the computer graph, and sometimes in nature, the coordinates where the seeds fall down are stochastic and have a uniform distribution over the area. Since some plants start to grow earlier than others, the plant size variation is often large. If a dense stand of spruce or pine is planted on the same land as the naturally generated birch, we have strategy Q2. The less dense spruce or pine alternative on the naturally regenerated birch land is denoted strategy Q3.

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NUMERICAL APPENDIX

In this appendix, the numerical example discussed in the paper including the relevant computer output tables is included. Furthermore, the complete adaptive optimization program for the personal computer is printed. The general principles used in the optimization program and the specific numerical assumptions are given in the main text.

The computer output contains 3 different tables: The first table is the environmental transition probability matrix. The second table gives the expected present value, WO, as a function of the initial state, XO, and the investment strategies, NA and NB. The third table contains the optimal adaptive thinning decisions as a function of the initial state, XO, the investment strategies. NA and NB and the environmental state in period 1, X1.

```
PROGRAM SELTH
LOHMANDER PETER 900404
SELECTIVE THINNING AND STOCHASTIC GROWTH PARAMETERS
+++++++++++++++++++++++++++++++++++++++++++++++++++

ENVIRONMENTAL STATE TRANSITION PROBABILITY MATRIX
(row = state in period t, column = state in period t+1)

 0.900 0.100 0.000
 0.400 0.500 0.100
 0.100 0.400 0.500

       XO        NA        NB        WO
***********************************
        1         0      1000      3169
        1       500       500      3157
        1       500      1000      2194
        1      1000         0      2325
        1      1000       500      2929
        1      1000      1000      -740

        2         0      1000      3556
        2       500       500      3874
        2       500      1000      3375
        2      1000         0      3536
        2      1000       500      3441
        2      1000      1000      -550

        3         0      1000      4028
        3       500       500      5099
        3       500      1000      4892
        3      1000         0      5086
        3      1000       500      4058
        3      1000      1000      -328
```

XO	NA	NB	OPTIMAL ADAPTIVE DECISION (X1= 1	2	3)
1	0	0	0	0	0
1	0	500	0	0	0
1	0	1000	4	4	4
1	500	0	0	0	0
1	500	500	1	2	2
1	500	1000	2	2	2
1	1000	0	4	4	4
1	1000	500	1	1	1
1	1000	1000	0	0	0
2	0	0	0	0	0
2	0	500	0	0	0
2	0	1000	4	4	4
2	500	0	0	0	0
2	500	500	1	2	2
2	500	1000	2	2	2
2	1000	0	4	4	4
2	1000	500	1	1	1
2	1000	1000	0	0	0
3	0	0	0	0	0
3	0	500	0	0	0
3	0	1000	4	4	4
3	500	0	0	0	0
3	500	500	1	2	2
3	500	1000	2	2	2
3	1000	0	4	4	4
3	1000	500	1	1	1
3	1000	1000	0	0	0

```
10 LPRINT CHR$(27):"E"
20 LPRINT CHR$(27):"G"
30 LPRINT"PROGRAM SELTH"
40 LPRINT"LOHMANDER PETER 900404"
50 LPRINT"SELECTIVE THINNING AND STOCHASTIC GROWTH PARAMETERS"
60 LPRINT"++++++++++++++++++++++++++++++++++++++++++++++++++++++"
70 LPRINT" "
80 DIM M(3,3), DECOPT(3,3,3,3)
90 DATA .9,.1,0,.4,.5,.1,.1,.4,.5
100 FOR I = 1 TO 3
110 FOR J = 1 TO 3
120 READ M(I,J)
130 NEXT J
140 NEXT I
150 INPUT"NEW TRANSITION PROBABILITIES ? (YES = 1, NO = 0)", NTP
160 IF NTP = 0 THEN 210
170 INPUT"ELEMENT COORDINATES i AND j ?",I,J
180 INPUT"PROBABILITY ?",TPR
190 M(I,J)=TPR
200 GOTO 150
210 LPRINT" "
220 LPRINT"ENVIRONMENTAL STATE TRANSITION PROBABILITY MATRIX"
230 LPRINT"(row = state in period t, column = state in period t+1)"
240 LPRINT" "
250 FOR I = 1 TO 3
260 FOR J = 1 TO 3
270 LPRINT USING"##.###";M(I,J);
280 NEXT J
290 LPRINT" "
300 NEXT I
310 LPRINT" "
320 INPUT"IS THE TRANSITION PROBABILITY MATRIX ACCEPTABLE ? (yes=1, no=0)",TA
330 IF TA = 0 THEN GOTO 150
340 R=.03
350 FOR I = 1 TO 3:FOR J = 1 TO 3:FOR K = 1 TO 3: FOR L = 1 TO 3
360 DECOPT(I,J,K,L) = 0
370 NEXT L: NEXT K: NEXT J: NEXT I
380 PA1=10
390 T1=30
400 PB1=10
410 T2=60
420 PA2=100
430 PB2=100
440 CA=1
450 CB=1
460 LPRINT"      XO       NA       NB       WO"
470 LPRINT"**********************************"
480 FOR XO=1 TO 3
490 LPRINT" "
500 FOR NA=0 TO 1000 STEP 500
510 NAS = 1 + NA/500
520 FOR NB= 0 TO 1000 STEP 500
530 NBS = 1 + NB/500
540 NSUMO = NA + NB
550 IF NSUMO < 600 THEN 970
560 W1=0
570 FOR X1 = 1 TO 3
580 VA1=(NA/250)*50*(.4+.3*X1)
590 VB1=(NB/250)*45*(.8+.1*X1)
```

```
600 API1=0
610 SOPT=100
620 FOR S=0 TO 4
630 IF S<>0 THEN 650
640 HA=1:HB=1
650 IF S<>1 THEN 670
660 HA=1:HB=0
670 IF S <>2 THEN 690
680 HA=0:HB=1
690 IF S<>3 THEN 710
700 HA=0:HB=0
710 IF S<>4 THEN 730
720 HA=.5:HB=.5
730 U=PA1*HA*VA1 + PB1*HB*VB1
740 NA2 = 0
750 IF NA>0 THEN NA2 = NA*(1-HA)
760 NB2 = 0
770 IF NB>0 THEN NB2 = NB*(1-HB)
780 NSUM2 = NA2 + NB2
790 W2=0
800 FOR X2=1 TO 3
810 VA2=VA1*(1-HA) + VA1*(1-HA)*2*(.4+.3*X2)
820 IF NSUM2>600 THEN VA2 = 0
830 VB2=VB1*(1-HB) + VB1*(1-HB)*2*(.8+.1*X2)
840 IF NSUM2>600 THEN VB2 = 0
850 W2=W2 + M(X1,X2)*(PA2*VA2 + PB2*VB2)
860 NEXT X2
870 W2 = W2*EXP(-R*T2)
880 API1EV = EXP(-R*T1)*U + W2
890 IF API1EV > API1 THEN SOPT = S
900 IF API1EV > API1 THEN API1 = API1EV
910 NEXT S
920 DECOPT(X0,X1,NAS,NBS) = SOPT
930 W1 = W1 + M(X0,X1)*API1
940 NEXT X1
950 W0 = -CA*NA - CB*NB + W1
960 LPRINT USING"########";X0;NA;NB;W0
970 NEXT NB
980 NEXT NA
990 NEXT X0
1000 LPRINT" "
1010 LPRINT" "
1020 LPRINT"      X0       NA       NB  OPTIMAL ADAPTIVE DECISION"
1030 LPRINT"                           (X1=  1          2         3)
1040 LPRINT"*************************************************************"
1050 FOR X0 = 1 TO 3
1060 FOR NAS = 1 TO 3
1070 NA = (NAS-1)*500
1080 FOR NBS = 1 TO 3
1090 NB = (NBS-1)*500
1100 LPRINT USING"########";X0;NA;NB;
1110 FOR X1 = 1 TO 3
1120 LPRINT USING"########";DECOPT(X0,X1,NAS,NBS);
1130 NEXT X1
1140 LPRINT" "
1150 NEXT NBS
1160 NEXT NAS
1170 LPRINT" "
1180 NEXT X0
1190 INPUT"A NEW OPTIMIZATION WITH NEW PROBABILITIES ? (yes=1, no=0)",TA
1200 IF TA=1 THEN 150
1210 END
```