

**L8** (By Peter Lohmander 2009-10-23)

Dynamic game theory. General theory, application examples, analytical solutions and numerical solutions via computer programming.

**One period game theory and applications:**

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<http://www.lohmander.com/mil/EUROInf03.doc>

<http://www.lohmander.com/mil/Tactical7.doc>

<http://www.lohmander.com/mil/NashLP2.doc>

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**The Nash equilibrium is the optimal mixed strategy solution to a two person zero sum game based on linear programming**

**By Peter Lohmander Version 2003-06-26**

(This document is a part of a presentation by Peter Lohmander at EURO/Informs, Military Operations Research III, Istanbul, 2003.)

<http://www.lohmander.com/mil/EUROInf03.doc>

$$\begin{bmatrix} \alpha_{11} & \alpha_{12} & \cdot & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \cdot & \alpha_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ \alpha_{m1} & \alpha_{m2} & \cdot & \alpha_{mn} \end{bmatrix}$$

$$\begin{bmatrix} x_1 y_1 & \cdot & x_1 y_n \\ \cdot & \cdot & \cdot \\ x_m y_1 & \cdot & x_m y_n \end{bmatrix}$$

**The linear programming solution**

**LP of A:**

$$\max E$$

*s.t.*

$$E \leq \alpha_{11}x_1 + \dots + \alpha_{m1}x_m \quad (\text{against } \beta_1)$$

$$E \leq \alpha_{12}x_1 + \dots + \alpha_{m2}x_m \quad (\text{against } \beta_2)$$

.

$$E \leq \alpha_{1n}x_1 + \dots + \alpha_{mn}x_m \quad (\text{against } \beta_n)$$

$$1 = x_1 + \dots + x_m$$

**Assumptions:**

$$0 < x_i < 1 \quad \forall i$$

$$0 < y_j < 1 \quad \forall j$$

$$\frac{\partial L}{\partial \lambda_j} \lambda_j = 0$$

These equations determine  $E$  and  $x_i \quad \forall i$

$$E - \alpha_{11}x_1 - \dots - \alpha_{m1}x_m = 0$$

$$E - \alpha_{12}x_1 - \dots - \alpha_{m2}x_m = 0$$

.

$$E - \alpha_{1n}x_1 - \dots - \alpha_{mn}x_m = 0$$

$$x_1 + \dots + x_m = 1$$

**LP of B:**

$$\min E$$

*s.t.*

$$E \geq \alpha_{11}y_1 + \dots + \alpha_{1n}y_n \quad (\textit{against } \alpha_1)$$

$$E \geq \alpha_{21}y_1 + \dots + \alpha_{2n}y_n \quad (\textit{against } \alpha_2)$$

.

$$E \geq \alpha_{m1}y_1 + \dots + \alpha_{mn}y_n \quad (\textit{against } \alpha_m)$$

$$1 = y_1 + \dots + y_n$$

These equations determine  $E$  and  $y_j \quad \forall j$

$$E - \alpha_{11}y_1 - \dots - \alpha_{1n}y_n = 0$$

$$E - \alpha_{21}y_1 - \dots - \alpha_{2n}y_n = 0$$

.

$$E - \alpha_{m1}y_1 - \dots - \alpha_{mn}y_n = 0$$

$$y_1 + \dots + y_n = 1$$

**The Nash mixed strategy equilibrium solution:**

$$\pi = \sum_{i=1}^{m-1} \sum_{j=1}^{n-1} \alpha_{ij} x_i y_j + \sum_{j=1}^{n-1} \alpha_{mj} x_m y_j + \sum_{i=1}^{m-1} \alpha_{in} x_i y_n + \alpha_{mn} x_m y_n$$

where

$$x_m = 1 - \sum_{i=1}^{m-1} x_i$$

$$y_n = 1 - \sum_{j=1}^{n-1} y_j$$

**This can be rewritten as:**

$$\begin{aligned} \pi = & \sum_{i=1}^{m-1} \sum_{j=1}^{n-1} \alpha_{ij} x_i y_j + \sum_{j=1}^{n-1} \alpha_{mj} \left( 1 - \sum_{i=1}^{m-1} x_i \right) y_j \\ & + \sum_{i=1}^{m-1} \alpha_{in} x_i \left( 1 - \sum_{j=1}^{n-1} y_j \right) + \alpha_{mn} \left( 1 - \sum_{i=1}^{m-1} x_i \right) \left( 1 - \sum_{j=1}^{n-1} y_j \right) \end{aligned}$$



**Nash conditions of A:**

$$\frac{\partial \pi}{\partial x_i | (i \neq m)} = \sum_{j=1}^{n-1} \alpha_{ij} y_j - \sum_{j=1}^{n-1} \alpha_{mj} y_j + \alpha_{in} \left( 1 - \sum_{j=1}^{n-1} y_j \right) - \alpha_{mn} \left( 1 - \sum_{j=1}^{n-1} y_j \right)$$

$$\frac{\partial \pi}{\partial x_i | (i \neq m)} = \sum_{j=1}^{n-1} \alpha_{ij} y_j - \sum_{j=1}^{n-1} \alpha_{mj} y_j + \alpha_{in} y_n - \alpha_{mn} y_n$$

$$\frac{\partial \pi}{\partial x_i | (i \neq m)} = \sum_{j=1}^n \alpha_{ij} y_j - \sum_{j=1}^n \alpha_{mj} y_j$$

$$\frac{\partial \pi}{\partial x_i | (i \neq m)} = \sum_{j=1}^n \alpha_{ij} y_j - E$$

**In Nash equilibrium:**

$$\frac{\partial \pi}{\partial x_i | (i \neq m)} = \sum_{j=1}^n \alpha_{ij} y_j - E = 0$$

**(This is also the result from the LP solution.)**

**Nash conditions of B:**

$$\frac{\partial \pi}{\partial y_j | (j \neq n)} = \sum_{i=1}^{m-1} \alpha_{ij} x_i + \alpha_{mj} \left( 1 - \sum_{i=1}^{m-1} x_i \right) - \sum_{i=1}^{m-1} \alpha_{in} x_i - \alpha_{mn} \left( 1 - \sum_{i=1}^{m-1} x_i \right)$$

$$\frac{\partial \pi}{\partial y_j | (j \neq n)} = \sum_{i=1}^{m-1} \alpha_{ij} x_i + \alpha_{mj} x_m - \sum_{i=1}^{m-1} \alpha_{in} x_i - \alpha_{mn} x_m$$

$$\frac{\partial \pi}{\partial y_j | (j \neq n)} = \sum_{i=1}^m \alpha_{ij} x_i - \sum_{i=1}^m \alpha_{in} x_i$$

$$\frac{\partial \pi}{\partial y_j | (j \neq n)} = \sum_{i=1}^m \alpha_{ij} x_i - E$$

**In Nash equilibrium:**

$$\frac{\partial \pi}{\partial y_j | (j \neq n)} = \sum_{i=1}^m \alpha_{ij} x_i - E = 0$$

**(This is also the result from the LP solution.)**

**Observations:**

**The Nash equilibrium and the LP solution are the same.**

**This is remarkable since the ideas and principles are quite different:**

**The Nash solution is derived from the condition that no player would benefit from selecting other probabilities (different from the Nash solution).**

**The LP solution is derived from the condition that no player can get a worse outcome irrespective of what the opponent does.**

**Optimal response if the opponent does not play according to Nash (or LP):**

**A does not benefit from changing the probabilities (controlled by A) in case B plays according to the Nash solution.**

**In case B selects probabilities different from the Nash solution, A benefits from deviation from the Nash solution.**

**Case 1.  $\frac{\partial \pi}{\partial x_i} > 0$  Then A should increase  $x_i$ .**

**Case 2.  $\frac{\partial \pi}{\partial x_i} < 0$  Then A should decrease  $x_i$ .**

**Of course, B benefits from moving from the Nash (or LP) solution if A does not play according to the Nash (or LP) solution.**

**Case 3.  $\frac{\partial \pi}{\partial y_j} > 0$  Then B should decrease  $y_j$ .**

**Case 4.  $\frac{\partial \pi}{\partial y_j} < 0$  Then B should increase  $y_j$ .**

```

! DG_091020_1016;
! Peter Lohmander;

model:

sets:
row/1..2/:X, Xopt;
col/1..2/:Y, Yopt;
rowcol(row,col):a;
player/1..2/:V;
endsets

data:
a = 1 0
    0 1
;
enddata

@FREE(V1);
@FREE(V2);
@FREE(V(1));
@FREE(V(2));
@FREE(DEV);

!*****;
submodel Dec1:
[Dec1obj] max = V1;
@for(col(j): V1 <= @SUM(row(i): a(i,j)*X(i)));
@SUM(row(i): X(i)) <= 1;

endsubmodel
!*****;
submodel Dec2:
[Dec2obj] min = V2;
@for(row(i): V2 >= @SUM(col(j):a(i,j)*Y(j)));
@SUM(col(j): Y(j)) >= 1;

endsubmodel
!*****;

CALC:
@SET('DEFAULT');
@SET('TERSEO', 2);

Nstrat1 = 2;
Nstrat2 = 2;

@SOLVE(Dec1);
V(1) = V1;
@for(row(i):Xopt(i) = X(i));

@SOLVE(Dec2);
V(2) = V2;
@for(col(j):Yopt(j) = Y(j));

Value = V(1);
DEV = V(1) - V(2);

@WRITE(@NEWLINE(1));

```

```

@IFC( @SQR(DEV) #GT# .001: @WRITE(' *** OPTIMIZATION ERROR *** '));
@WRITE('Value = ', @FORMAT(Value, '7.5G'));
@WRITE(@NEWLINE(1));
  @WRITE('Optimal Decision Frequences of Player 1:',@NEWLINE(1) );
  @FOR(row(i)|i #LE# Nstrat1:
  @WRITE( @Format(Xopt(i), '7.5G'), 5*' ');
  );
  @WRITE(@NEWLINE(1));

  @WRITE('Optimal Decision Frequences of Player 2:',@NEWLINE(1) );
  @FOR(col(j)|j #LE# Nstrat2:
  @WRITE( @Format(Yopt(j), '7.5G'), 5*' ');
  );
  @WRITE(@NEWLINE(1));
  @WRITE(@NEWLINE(1));

ENDCALC

end

```

```

Value =      0.5
Optimal Decision Frequences of Player 1:
    0.5      0.5
Optimal Decision Frequences of Player 2:
    0.5      0.5

```

```

! DG_091020_0957;
! Peter Lohmander;

model:

sets:
row/1..3/:X, Xopt;
col/1..4/:Y, Yopt;
rowcol(row,col):a;
player/1..2/:V;
endsets

data:
a = .3 .2 .1 0
    .1 .5 .5 .1
    0 .1 .2 .2
;
enddata

@FREE(V1);
@FREE(V2);
@FREE(V(1));
@FREE(V(2));
@FREE(DEV);

!*****;
submodel Dec1:
[Dec1obj] max = V1;
@for(col(j): V1 <= @SUM(row(i): a(i,j)*X(i)));
@SUM(row(i): X(i)) <= 1;

endsubmodel
!*****;
submodel Dec2:
[Dec2obj] min = V2;
@for(row(i): V2 >= @SUM(col(j):a(i,j)*Y(j)));
@SUM(col(j): Y(j)) >= 1;

endsubmodel
!*****;

CALC:
@SET('DEFAULT');
@SET('TERSEO', 2);

Nstrat1 = 3;
Nstrat2 = 4;

@SOLVE(Dec1);
V(1) = V1;
@for(row(i):Xopt(i) = X(i));

@SOLVE(Dec2);
V(2) = V2;
@for(col(j):Yopt(j) = Y(j));

Value = V(1);
DEV = V(1) - V(2);

```

```

@WRITE(@NEWLINE(1));
@IFC( @SQR(DEV) #GT# .001: @WRITE(' *** OPTIMIZATION ERROR *** '));
@WRITE('Value = ', @FORMAT(Value, '7.5G'));
@WRITE(@NEWLINE(1));
  @WRITE('Optimal Decision Frequences of Player 1:',@NEWLINE(1) );
  @FOR(row(i)|i #LE# Nstrat1:
  @WRITE( @Format(Xopt(i), '7.5G'), 5*' ');
  );
  @WRITE(@NEWLINE(1));

  @WRITE('Optimal Decision Frequences of Player 2:',@NEWLINE(1) );
  @FOR(col(j)|j #LE# Nstrat2:
  @WRITE( @Format(Yopt(j), '7.5G'), 5*' ');
  );
  @WRITE(@NEWLINE(1));
  @WRITE(@NEWLINE(1));

ENDCALC

end

```

```

Value =      0.12
Optimal Decision Frequences of Player 1:
  0.4      0      0.6
Optimal Decision Frequences of Player 2:
  0.4      0      0      0.6

```



```
! DG_091020_1318;
! Peter Lohmander;
```

```
model:
```

```
sets:
```

```
row/1..10/:X, Xopt;
col/1..10/:Y, Yopt;
rowcol(row,col):a;
player/1..2/:V;
endsets
```

```
@FREE(V1);
@FREE(V2);
@FREE(V(1));
@FREE(V(2));
@FREE(DEV);
```

```
!*****;
```

```
submodel Dec1:
```

```
[Dec1obj] max = V1;
```

```
@for(col(j)|j#LE#Nstrat2: V1 <= @SUM(row(i)|i#LE#Nstrat1: a(i,j)*X(i)));
```

```
@SUM(row(i)|i#LE#Nstrat1: X(i)) <= 1;
```

```
endsubmodel
```

```
!*****;
```

```
submodel Dec2:
```

```
[Dec2obj] min = V2;
```

```
@for(row(i)|i#LE#Nstrat1: V2 >= @SUM(col(j)|j#LE#Nstrat2: a(i,j)*Y(j)));
```

```
@SUM(col(j)|j#LE#Nstrat2: Y(j)) >= 1;
```

```
endsubmodel
```

```
!*****;
```

```
CALC:
```

```
@SET('DEFAULT');
```

```
@SET('TERSEO', 2);
```

```
t = 0;
```

```
@WHILE( t#LE#6:
```

```
t = t + 1;
```

```
Nstrat1 = t + 1;
```

```
Nstrat2 = t + 1;
```

```
@WRITE(@NEWLINE(1));
```

```
@WRITE('***** NEW GAME *****',@NEWLINE(1));
```

```
@WRITE('Nstrat1 = ', @FORMAT(Nstrat1, '3.0G'),' Nstrat2 = ',
@FORMAT(Nstrat2, '3.5G') );
```

```
@for(rowcol(i,j)|i#LT#j #OR# i#GT#j :a(i,j) = 0);
```

```
@for(rowcol(i,j)|i#EQ#j: a(i,j) = 1);
```

```
@WRITE(@NEWLINE(1));
```

```
@WRITE('Game matrix = ', @NEWLINE(1));
```

```
@WRITE(Nstrat2*'-----',@NEWLINE(1));
```

```
@WRITE(@NEWLINE(1));
```

```

@for(row(i)|i#LE#Nstrat1:
  @for(col(j)|j#LE#Nstrat2:
    @WRITE( @Format(a(i,j), '7.5G'), 5*' ');
  );
  @WRITE(@NEWLINE(1));
);
@WRITE(Nstrat2*'-----',@NEWLINE(1));

@SOLVE(Dec1);
V(1) = V1;
@for(row(i):Xopt(i) = X(i));

@SOLVE(Dec2);
V(2) = V2;
@for(col(j):Yopt(j) = Y(j));

Value = V(1);
DEV = V(1) - V(2);

@WRITE(@NEWLINE(1));
@IFC( @SQR(DEV) #GT# .001: @WRITE(' *** OPTIMIZATION ERROR *** '));
@WRITE('Value of the game = ', @FORMAT(Value, '7.5G'));
@WRITE(@NEWLINE(1));
@WRITE('Optimal Decision Frequences of Player 1:',@NEWLINE(1) );
@FOR(row(i)|i #LE# Nstrat1:
@WRITE( @Format(Xopt(i), '7.5G'), 5*' ');
);
@WRITE(@NEWLINE(1));

@WRITE('Optimal Decision Frequences of Player 2:',@NEWLINE(1) );
@FOR(col(j)|j #LE# Nstrat2:
@WRITE( @Format(Yopt(j), '7.5G'), 5*' ');
);
@WRITE(@NEWLINE(1));
@WRITE(@NEWLINE(1));

);

ENDCALC

end

***** NEW GAME *****
Nstrat1 = 2 Nstrat2 = 2
Game matrix =
-----
      1      0
      0      1
-----

Value of the game = 0.5
Optimal Decision Frequences of Player 1:
      0.5      0.5
Optimal Decision Frequences of Player 2:
      0.5      0.5

```

\*\*\*\*\* NEW GAME \*\*\*\*\*

Nstrat1 = 3 Nstrat2 = 3

Game matrix =

```
-----
      1      0      0
      0      1      0
      0      0      1
-----
```

Value of the game = 0.33333

Optimal Decision Frequencies of Player 1:

0.33333 0.33333 0.33333

Optimal Decision Frequencies of Player 2:

0.33333 0.33333 0.33333

\*\*\*\*\* NEW GAME \*\*\*\*\*

Nstrat1 = 4 Nstrat2 = 4

Game matrix =

```
-----
      1      0      0      0
      0      1      0      0
      0      0      1      0
      0      0      0      1
-----
```

Value of the game = 0.25

Optimal Decision Frequencies of Player 1:

0.25 0.25 0.25 0.25

Optimal Decision Frequencies of Player 2:

0.25 0.25 0.25 0.25

\*\*\*\*\* NEW GAME \*\*\*\*\*

Nstrat1 = 5 Nstrat2 = 5

Game matrix =

```
-----
      1      0      0      0      0
      0      1      0      0      0
      0      0      1      0      0
      0      0      0      1      0
      0      0      0      0      1
-----
```

Value of the game = 0.2

Optimal Decision Frequencies of Player 1:

0.2 0.2 0.2 0.2 0.2

Optimal Decision Frequencies of Player 2:

0.2 0.2 0.2 0.2 0.2

\*\*\*\*\* NEW GAME \*\*\*\*\*

Nstrat1 = 6 Nstrat2 = 6

Game matrix =

---

1	0	0	0	0	0	0
0	1	0	0	0	0	0
0	0	1	0	0	0	0
0	0	0	1	0	0	0
0	0	0	0	1	0	0
0	0	0	0	0	1	0
0	0	0	0	0	0	1

---

Value of the game = 0.16667

Optimal Decision Frequencies of Player 1:

0.16667    0.16667    0.16667    0.16667    0.16667    0.16667

Optimal Decision Frequencies of Player 2:

0.16667    0.16667    0.16667    0.16667    0.16667    0.16667

.... Cont .....

```

! TRP_091023;
! Transition program;
! Peter Lohmander 091023;
model:
sets:
res/1..10/:prob;
res2/1..10/;;
g/1..10/;;
c/1..10/;;
ResGC(res,res2,g,c):TransP;
endsets

calc:
@SET('DEFAULT');
@SET('TERSEO', 2);

@for(ResGC: TransP = 0);

@for(res(x):
RESTOT = x-1;
@for(g(i)|i#LE#x:
GS1 = i-1;
@for(c(j):
CA2 = j-1;

!           @WRITE(@NEWLINE(1),'Resources = ', RESTOT, ' GS1 = ',
@FORMAT(GS1, '6G'),
!           ' CA2 = ', @FORMAT(CA2, '6G'), @NEWLINE(1) );

Pdeath = 0;
@IFC(CA2#GE#1: Pdeath = 1 - 1/(1+CA2));

    @for(res(n): prob(n) = 0);

        @for(res2(n)|n#LE#x :
            RESTOTnext = n;
            REDUCT = RESTOTnext - RESTOT;
            prob(n) = - (@PBN(Pdeath, GS1, - REDUCT) - @PBN(Pdeath, GS1, -
(REDUCT-1))) ;
            TransP(x,n,i,j) = prob(n);
        );

!     @for(res(n):
!         @WRITE( @Format(prob(n), '7.5G'), 5*' ');
!     );
!     @WRITE(@NEWLINE(1));

);
);
);

count3 = -1;
@WHILE(count3#LE#3:
count3 = count3+1;
@WRITE(@NEWLINE(1));

count2 = -1;
@WHILE( count2#LE#8:
count2 = count2 + 1;
@WRITE(@NEWLINE(1));

```

```

count = -1;
@while( count#LE#8:
count = count + 1;
restot = count2;
res2tot = count;
GS = 2;
CA = count3;
restotcorr = restot+1;
res2totcorr = res2tot+1;
GScorr = GS+1;
CACorr = CA+1;
TransPcalc = TransP(restotcorr, res2totcorr, GScorr, CACorr);
@IFC(GS#LE#restot: @WRITE('restot = ', restot, ' res2tot = ', res2tot,
' GS = ', GS, ' CA = ', CA, ' TransP = ', TransPcalc);
@WRITE(@NEWLINE(1))
);
);
);
);
endcalc
end

```

```

restot = 2 res2tot = 0 GS = 2 CA = 0 TransP = 0
restot = 2 res2tot = 1 GS = 2 CA = 0 TransP = 0
restot = 2 res2tot = 2 GS = 2 CA = 0 TransP = 1
restot = 2 res2tot = 3 GS = 2 CA = 0 TransP = 0
restot = 2 res2tot = 4 GS = 2 CA = 0 TransP = 0
restot = 2 res2tot = 5 GS = 2 CA = 0 TransP = 0
restot = 2 res2tot = 6 GS = 2 CA = 0 TransP = 0
restot = 2 res2tot = 7 GS = 2 CA = 0 TransP = 0
restot = 2 res2tot = 8 GS = 2 CA = 0 TransP = 0
restot = 2 res2tot = 9 GS = 2 CA = 0 TransP = 0

```

```

restot = 3 res2tot = 0 GS = 2 CA = 0 TransP = 0
restot = 3 res2tot = 1 GS = 2 CA = 0 TransP = 0
restot = 3 res2tot = 2 GS = 2 CA = 0 TransP = 0
restot = 3 res2tot = 3 GS = 2 CA = 0 TransP = 1
restot = 3 res2tot = 4 GS = 2 CA = 0 TransP = 0
restot = 3 res2tot = 5 GS = 2 CA = 0 TransP = 0
restot = 3 res2tot = 6 GS = 2 CA = 0 TransP = 0
restot = 3 res2tot = 7 GS = 2 CA = 0 TransP = 0
restot = 3 res2tot = 8 GS = 2 CA = 0 TransP = 0
restot = 3 res2tot = 9 GS = 2 CA = 0 TransP = 0

```

```

restot = 4 res2tot = 0 GS = 2 CA = 0 TransP = 0
restot = 4 res2tot = 1 GS = 2 CA = 0 TransP = 0
restot = 4 res2tot = 2 GS = 2 CA = 0 TransP = 0
restot = 4 res2tot = 3 GS = 2 CA = 0 TransP = 0
restot = 4 res2tot = 4 GS = 2 CA = 0 TransP = 1
restot = 4 res2tot = 5 GS = 2 CA = 0 TransP = 0
restot = 4 res2tot = 6 GS = 2 CA = 0 TransP = 0
restot = 4 res2tot = 7 GS = 2 CA = 0 TransP = 0
restot = 4 res2tot = 8 GS = 2 CA = 0 TransP = 0
restot = 4 res2tot = 9 GS = 2 CA = 0 TransP = 0

```













```

restot = 6 res2tot = 0 GS = 2 CA = 3 TransP = 0
restot = 6 res2tot = 1 GS = 2 CA = 3 TransP = 0
restot = 6 res2tot = 2 GS = 2 CA = 3 TransP = 0
restot = 6 res2tot = 3 GS = 2 CA = 3 TransP = 0
restot = 6 res2tot = 4 GS = 2 CA = 3 TransP = 0.5625
restot = 6 res2tot = 5 GS = 2 CA = 3 TransP = 0.375
restot = 6 res2tot = 6 GS = 2 CA = 3 TransP = 0.06249999999999999
restot = 6 res2tot = 7 GS = 2 CA = 3 TransP = 0
restot = 6 res2tot = 8 GS = 2 CA = 3 TransP = 0
restot = 6 res2tot = 9 GS = 2 CA = 3 TransP = 0

restot = 7 res2tot = 0 GS = 2 CA = 3 TransP = 0
restot = 7 res2tot = 1 GS = 2 CA = 3 TransP = 0
restot = 7 res2tot = 2 GS = 2 CA = 3 TransP = 0
restot = 7 res2tot = 3 GS = 2 CA = 3 TransP = 0
restot = 7 res2tot = 4 GS = 2 CA = 3 TransP = 0
restot = 7 res2tot = 5 GS = 2 CA = 3 TransP = 0.5625
restot = 7 res2tot = 6 GS = 2 CA = 3 TransP = 0.375
restot = 7 res2tot = 7 GS = 2 CA = 3 TransP = 0.06249999999999999
restot = 7 res2tot = 8 GS = 2 CA = 3 TransP = 0
restot = 7 res2tot = 9 GS = 2 CA = 3 TransP = 0

restot = 8 res2tot = 0 GS = 2 CA = 3 TransP = 0
restot = 8 res2tot = 1 GS = 2 CA = 3 TransP = 0
restot = 8 res2tot = 2 GS = 2 CA = 3 TransP = 0
restot = 8 res2tot = 3 GS = 2 CA = 3 TransP = 0
restot = 8 res2tot = 4 GS = 2 CA = 3 TransP = 0
restot = 8 res2tot = 5 GS = 2 CA = 3 TransP = 0
restot = 8 res2tot = 6 GS = 2 CA = 3 TransP = 0.5625
restot = 8 res2tot = 7 GS = 2 CA = 3 TransP = 0.375
restot = 8 res2tot = 8 GS = 2 CA = 3 TransP = 0.06249999999999999
restot = 8 res2tot = 9 GS = 2 CA = 3 TransP = 0

restot = 9 res2tot = 0 GS = 2 CA = 3 TransP = 0
restot = 9 res2tot = 1 GS = 2 CA = 3 TransP = 0
restot = 9 res2tot = 2 GS = 2 CA = 3 TransP = 0
restot = 9 res2tot = 3 GS = 2 CA = 3 TransP = 0
restot = 9 res2tot = 4 GS = 2 CA = 3 TransP = 0
restot = 9 res2tot = 5 GS = 2 CA = 3 TransP = 0
restot = 9 res2tot = 6 GS = 2 CA = 3 TransP = 0
restot = 9 res2tot = 7 GS = 2 CA = 3 TransP = 0.5625
restot = 9 res2tot = 8 GS = 2 CA = 3 TransP = 0.375
restot = 9 res2tot = 9 GS = 2 CA = 3 TransP = 0.06249999999999999

restot = 2 res2tot = 0 GS = 2 CA = 4 TransP = 0.6400000000000001
restot = 2 res2tot = 1 GS = 2 CA = 4 TransP = 0.32
restot = 2 res2tot = 2 GS = 2 CA = 4 TransP = 0.03999999999999999
restot = 2 res2tot = 3 GS = 2 CA = 4 TransP = 0
restot = 2 res2tot = 4 GS = 2 CA = 4 TransP = 0
restot = 2 res2tot = 5 GS = 2 CA = 4 TransP = 0
restot = 2 res2tot = 6 GS = 2 CA = 4 TransP = 0
restot = 2 res2tot = 7 GS = 2 CA = 4 TransP = 0
restot = 2 res2tot = 8 GS = 2 CA = 4 TransP = 0
restot = 2 res2tot = 9 GS = 2 CA = 4 TransP = 0

```



```
restot = 8 res2tot = 0 GS = 2 CA = 4 TransP = 0
restot = 8 res2tot = 1 GS = 2 CA = 4 TransP = 0
restot = 8 res2tot = 2 GS = 2 CA = 4 TransP = 0
restot = 8 res2tot = 3 GS = 2 CA = 4 TransP = 0
restot = 8 res2tot = 4 GS = 2 CA = 4 TransP = 0
restot = 8 res2tot = 5 GS = 2 CA = 4 TransP = 0
restot = 8 res2tot = 6 GS = 2 CA = 4 TransP = 0.6400000000000001
restot = 8 res2tot = 7 GS = 2 CA = 4 TransP = 0.32
restot = 8 res2tot = 8 GS = 2 CA = 4 TransP = 0.03999999999999999
restot = 8 res2tot = 9 GS = 2 CA = 4 TransP = 0

restot = 9 res2tot = 0 GS = 2 CA = 4 TransP = 0
restot = 9 res2tot = 1 GS = 2 CA = 4 TransP = 0
restot = 9 res2tot = 2 GS = 2 CA = 4 TransP = 0
restot = 9 res2tot = 3 GS = 2 CA = 4 TransP = 0
restot = 9 res2tot = 4 GS = 2 CA = 4 TransP = 0
restot = 9 res2tot = 5 GS = 2 CA = 4 TransP = 0
restot = 9 res2tot = 6 GS = 2 CA = 4 TransP = 0
restot = 9 res2tot = 7 GS = 2 CA = 4 TransP = 0.6400000000000001
restot = 9 res2tot = 8 GS = 2 CA = 4 TransP = 0.32
restot = 9 res2tot = 9 GS = 2 CA = 4 TransP = 0.03999999999999999
```

```

! DG_091023;
! Peter Lohmänder;
model:
sets:
row/1..4/:X, Xopt;
col/1..4/:Y, Yopt;
rowcol(row,col):a;
player/1..2/:V;
time/1..7/;;
TXY(time,row,col):VALOPT;

res/1..4/:prob;
res2/1..4/;;
g/1..4/;;
c/1..4/;;
ResGC(res,res2,g,c):TransP;

endsets
@FREE(V1);
@FREE(V2);
@FREE(V(1));
@FREE(V(2));
@FREE(DEV);
@for(TXY(t,i,j): @FREE(VALOPT(t,i,j)));

!*****;
submodel Dec1:
[Dec1obj] max = V1;
@for(col(j)|j#LE#Nstrat2: V1 <= @SUM(row(i)|i#LE#Nstrat1: a(i,j)*X(i)));
@SUM(row(i)|i#LE#Nstrat1: X(i)) = 1;

endsubmodel
!*****;
submodel Dec2:
[Dec2obj] min = V2;
@for(row(i)|i#LE#Nstrat1: V2 >= @SUM(col(j)|j#LE#Nstrat2: a(i,j)*Y(j)));
@SUM(col(j)|j#LE#Nstrat2: Y(j)) = 1;

endsubmodel
!*****;

CALC:
@SET('DEFAULT');
@SET('TERSEO', 2);

```

```

@for(ResGC: TransP = 0);
@for(res(xxx):
RESTOT = xxx-1;
@for(g(i)|i#LE#xxx:
GS1 = i-1;
@for(c(j):
CA2 = j-1;
Pdeath = 0;
@IFC(CA2#GE#1: Pdeath = 1 - 1/(1+CA2));
  @for(res(n): prob(n) = 0);
    @for(res2(n)|n#LE#xxx :
      RESTOTnext = n;
      REDUCT = RESTOTnext - RESTOT;
      prob(n) = - (@PBN(Pdeath, GS1, - REDUCT)
                  - @PBN(Pdeath, GS1, -(REDUCT-1))) ;
      TransP(xxx,n,i,j) = prob(n);
    );
  );
);
);

EXTRA = 100;

TMAX = 5;
TMAXP1 = TMAX+1;

@for(TXY(t,i,j)|t#EQ#TMAXP1: VALOPT(t,i,j) = 0);

t = TMAXP1;
@WHILE( t#GT#1:
t = t - 1;
@WRITE(@NEWLINE(1),'t = ', @FORMAT(t, '2G'),@NEWLINE(1) );

GS1max = -1;
@while(GS1max#LT#3:
GS1max = GS1max+1;

GS2max = -1;
@while(GS2max#LT#3:
GS2max = GS2max+1;

Resource_1 = GS1max;
Resource_2 = GS2max;
@WRITE(@NEWLINE(1));
@WRITE('*****',@NEWLINE(1));
@WRITE('t = ',@FORMAT(t, '2G'), ' Resource_1 = ',
@FORMAT(Resource_1, '3G'), ' Resource_2 = ',
@FORMAT(Resource_2, '3G'),@NEWLINE(1) );

```



```

Nstrat1 = GS1max + 1;
Nstrat2 = GS2max + 1;

@for(rowcol(i,j)|i#LE#Nstrat1 #AND# j#LE#Nstrat2 :a(i,j) = 0);

@for(rowcol(i,j)|i#LE#Nstrat1 #AND# j#LE#Nstrat2 :
GS1 = i-1;
CA1 = GS1max - GS1;
GS2 = j-1;
CA2 = GS2max - GS2;

R = ( (GS1)/(1+CA2/4) - (GS2)/(1 + CA1/4) ) ;
disc = .9;

tnext = t+1;

a(i,j) = R + EXTRA;

restot1now = Nstrat1-1;
restot1nowcorr = restot1now+1;
GS1corr = GS1+1;
CA1corr = CA1+1;

restot2now = Nstrat2-1;
restot2nowcorr = restot2now+1;
GS2corr = GS2+1;
CA2corr = CA2+1;

summan = 0;
res1index = 0;
@WHILE(res1index#LT#4:
res1index = res1index+1;
res2index = 0;
@WHILE(res2index#LT#4:
res2index = res2index+1;
summan = summan +
TransP(restot1nowcorr,res1index,GS1corr,CA2corr)
* TransP(restot2nowcorr,res2index,GS2corr, CA1corr)*
VALOPT(tnext,res1index, res2index)
);
);

a(i,j) = a(i,j) + disc*summan;

);

@WRITE('Nstrat1 = ', @FORMAT(Nstrat1, '3.0G'),' Nstrat2 = ',
@FORMAT(Nstrat2, '3.5G') );

@WRITE(@NEWLINE(1));
@WRITE(Nstrat2*'-----',@NEWLINE(1));
@for(row(i)|i#LE#Nstrat1:
@for(col(j)|j#LE#Nstrat2:
@WRITE( @Format(a(i,j)-EXTRA, '7.5G'), 5*' ');
);
@WRITE(@NEWLINE(1));
);
@WRITE(Nstrat2*'-----',@NEWLINE(1));

```

```

@SOLVE(Dec1);
V(1) = V1;
@for(row(i):Xopt(i) = X(i));

@SOLVE(Dec2);
V(2) = V2;
@for(col(j):Yopt(j) = Y(j));

Value = V(1)-EXTRA;
DEV = V(1) - V(2);
VALOPT(t,1+Resource_1,1+Resource_2) = Value;

@IFC( @SQR(DEV) #GT# .1: @WRITE(' *** OPTIMIZATION ERROR *** '));
@WRITE('Value = ', @FORMAT(Value, '7.5G'));
@WRITE(@NEWLINE(1));
@WRITE('Optimal Decision Frequences of Player 1:',@NEWLINE(1) );
@FOR(row(i)|i #LE# Nstrat1:
@WRITE( @Format(Xopt(i), '7.5G'), 5* ' ');
);
@WRITE(@NEWLINE(1));

@WRITE('Optimal Decision Frequences of Player 2:',@NEWLINE(1) );
@FOR(col(j)|j #LE# Nstrat2:
@WRITE( @Format(Yopt(j), '7.5G'), 5* ' ');
);
@WRITE(@NEWLINE(1));
@WRITE(@NEWLINE(1));

);
);
);

@for(TXY(tt,xx,yy)|tt#LE#5 #AND# XX#LE#4 #AND# YY#LE#4:
@WRITE(@NEWLINE(1),'t = ', tt, ' x = ', xx, ' y = ', yy,
' VALOPT() = ',VALOPT(tt,xx,yy) );

ENDCALC

end

```

t = 5

\*\*\*\*\*  
t = 5 Resource\_1 = 0 Resource\_2 = 0  
Nstrat1 = 1 Nstrat2 = 1

-----  
0  
-----

Value = 0  
Optimal Decision Frequences of Player 1:  
1  
Optimal Decision Frequences of Player 2:  
1

\*\*\*\*\*  
t = 5 Resource\_1 = 0 Resource\_2 = 1  
Nstrat1 = 1 Nstrat2 = 2

-----  
0 -1  
-----

Value = -1  
Optimal Decision Frequences of Player 1:  
1  
Optimal Decision Frequences of Player 2:  
0 1

\*\*\*\*\*  
t = 5 Resource\_1 = 0 Resource\_2 = 2  
Nstrat1 = 1 Nstrat2 = 3

-----  
0 -1 -2  
-----

Value = -2  
Optimal Decision Frequences of Player 1:  
1  
Optimal Decision Frequences of Player 2:  
0 0 1

\*\*\*\*\*  
t = 5 Resource\_1 = 0 Resource\_2 = 3  
Nstrat1 = 1 Nstrat2 = 4

-----  
0 -1 -2 -3  
-----

Value = -3  
Optimal Decision Frequences of Player 1:  
1  
Optimal Decision Frequences of Player 2:  
0 0 0 1

```
*****
t = 5 Resource_1 = 1 Resource_2 = 0
Nstrat1 = 2 Nstrat2 = 1
```

```
-----
0
1
-----
```

```
Value = 1
Optimal Decision Frequences of Player 1:
0 1
Optimal Decision Frequences of Player 2:
1
```

```
*****
t = 5 Resource_1 = 1 Resource_2 = 1
Nstrat1 = 2 Nstrat2 = 2
```

```
-----
0 -0.8
0.8 0
-----
```

```
Value = 0
Optimal Decision Frequences of Player 1:
0 1
Optimal Decision Frequences of Player 2:
0 1
```

```
*****
t = 5 Resource_1 = 1 Resource_2 = 2
Nstrat1 = 2 Nstrat2 = 3
```

```
-----
0 -0.8 -1.6
0.66667 -0.2 -1
-----
```

```
Value = -1
Optimal Decision Frequences of Player 1:
0 1
Optimal Decision Frequences of Player 2:
0 0 1
```

```
*****
t = 5 Resource_1 = 1 Resource_2 = 3
Nstrat1 = 2 Nstrat2 = 4
```

```
-----
0 -0.8 -1.6 -2.4
0.57143 -0.33333 -1.2 -2
-----
```

```
Value = -2
Optimal Decision Frequences of Player 1:
0 1
Optimal Decision Frequences of Player 2:
0 0 0 1
```

```
*****
t = 5 Resource_1 = 2 Resource_2 = 0
Nstrat1 = 3 Nstrat2 = 1
```

```
-----
0
1
2
```

```
-----
Value = 2
Optimal Decision Frequences of Player 1:
0 0 1
Optimal Decision Frequences of Player 2:
1
```

```
*****
t = 5 Resource_1 = 2 Resource_2 = 1
Nstrat1 = 3 Nstrat2 = 2
```

```
-----
0 -0.66667
0.8 0.2
1.6 1
```

```
-----
Value = 1
Optimal Decision Frequences of Player 1:
0 0 1
Optimal Decision Frequences of Player 2:
0 1
```

```
*****
t = 5 Resource_1 = 2 Resource_2 = 2
Nstrat1 = 3 Nstrat2 = 3
```

```
-----
0 -0.66667 -1.3333
0.66667 0 -0.6
1.3333 0.6 0
```

```
-----
Value = 0
Optimal Decision Frequences of Player 1:
0 0 1
Optimal Decision Frequences of Player 2:
0 0 1
```

```
*****
t = 5 Resource_1 = 2 Resource_2 = 3
Nstrat1 = 3 Nstrat2 = 4
```

```
-----
0 -0.66667 -1.3333 -2
0.57143 -0.13333 -0.8 -1.4
1.1429 0.33333 -0.4 -1
```

```
-----
Value = -1
Optimal Decision Frequences of Player 1:
0 0 1
Optimal Decision Frequences of Player 2:
0 0 0 1
```

```
*****
t = 5 Resource_1 = 3 Resource_2 = 0
Nstrat1 = 4 Nstrat2 = 1
```

```
-----
0
1
2
3
```

```
-----
Value = 3
Optimal Decision Frequences of Player 1:
0 0 0 1
Optimal Decision Frequences of Player 2:
1
```

```
*****
t = 5 Resource_1 = 3 Resource_2 = 1
Nstrat1 = 4 Nstrat2 = 2
```

```
-----
0 -0.57143
0.8 0.33333
1.6 1.2
2.4 2
```

```
-----
Value = 2
Optimal Decision Frequences of Player 1:
0 0 0 1
Optimal Decision Frequences of Player 2:
0 1
```

```
*****
t = 5 Resource_1 = 3 Resource_2 = 2
Nstrat1 = 4 Nstrat2 = 3
```

```
-----
0 -0.57143 -1.1429
0.66667 0.13333 -0.33333
1.3333 0.8 0.4
2 1.4 1
```

```
-----
Value = 1
Optimal Decision Frequences of Player 1:
0 0 0 1
Optimal Decision Frequences of Player 2:
0 0 1
```

```
*****
t = 5 Resource_1 = 3 Resource_2 = 3
Nstrat1 = 4 Nstrat2 = 4
```

```
-----
      0      -0.57143      -1.1429      -1.7143
0.57143      0      -0.53333      -1
  1.1429      0.53333      0      -0.4
  1.7143      1      0.4      0
-----
```

```
Value = 0
Optimal Decision Frequences of Player 1:
      0      0      0      1
Optimal Decision Frequences of Player 2:
      0      0      0      1
```

```
t = 4
```

```
*****
t = 4 Resource_1 = 0 Resource_2 = 0
Nstrat1 = 1 Nstrat2 = 1
```

```
-----
      0
-----
Value = 0
Optimal Decision Frequences of Player 1:
      1
Optimal Decision Frequences of Player 2:
      1
```

```
*****
t = 4 Resource_1 = 0 Resource_2 = 1
Nstrat1 = 1 Nstrat2 = 2
```

```
-----
      -0.9      -1.9
-----
Value = -1.9
Optimal Decision Frequences of Player 1:
      1
Optimal Decision Frequences of Player 2:
      0      1
```

```
*****
t = 4 Resource_1 = 0 Resource_2 = 2
Nstrat1 = 1 Nstrat2 = 3
```

```
-----
      -1.8      -2.8      -3.8
-----
Value = -3.8
Optimal Decision Frequences of Player 1:
      1
Optimal Decision Frequences of Player 2:
      0      0      1
```

```
*****
t = 4 Resource_1 = 0 Resource_2 = 3
Nstrat1 = 1 Nstrat2 = 4
```

```
-----
-2.7      -3.7      -4.7      -5.7
-----
```

Value = -5.7

Optimal Decision Frequences of Player 1:

1

Optimal Decision Frequences of Player 2:

0 0 0 1

```
*****
t = 4 Resource_1 = 1 Resource_2 = 0
Nstrat1 = 2 Nstrat2 = 1
```

```
-----
0.9
```

```
1.9
-----
```

Value = 1.9

Optimal Decision Frequences of Player 1:

0 1

Optimal Decision Frequences of Player 2:

1

```
*****
t = 4 Resource_1 = 1 Resource_2 = 1
Nstrat1 = 2 Nstrat2 = 2
```

```
-----
0      -0.35
```

```
0.35      0
-----
```

Value = 0

Optimal Decision Frequences of Player 1:

0 1

Optimal Decision Frequences of Player 2:

0 1

```
*****
t = 4 Resource_1 = 1 Resource_2 = 2
Nstrat1 = 2 Nstrat2 = 3
```

```
-----
-0.9      -1.25      -1.6
-0.83333  -1.55      -1.9
-----
```

Value = -1.6

Optimal Decision Frequences of Player 1:

1 0

Optimal Decision Frequences of Player 2:

0 0 1



```
*****
t = 4 Resource_1 = 1 Resource_2 = 3
Nstrat1 = 2 Nstrat2 = 4
```

```
-----
-1.8      -2.15      -2.5      -2.85
-1.9036   -2.7333   -3.45     -3.8
-----
```

Value = -2.85

Optimal Decision Frequences of Player 1:

1            0

Optimal Decision Frequences of Player 2:

0            0            0            1

```
*****
t = 4 Resource_1 = 2 Resource_2 = 0
Nstrat1 = 3 Nstrat2 = 1
```

```
-----
1.8
2.8
3.8
-----
```

Value = 3.8

Optimal Decision Frequences of Player 1:

0            0            1

Optimal Decision Frequences of Player 2:

1

```
*****
t = 4 Resource_1 = 2 Resource_2 = 1
Nstrat1 = 3 Nstrat2 = 2
```

```
-----
0.9      0.83333
1.25     1.55
1.6      1.9
-----
```

Value = 1.6

Optimal Decision Frequences of Player 1:

0            0            1

Optimal Decision Frequences of Player 2:

1            0

```
*****
t = 4 Resource_1 = 2 Resource_2 = 2
Nstrat1 = 3 Nstrat2 = 3
```

```
-----
0      -0.066667   -0.13333
0.066667  0      0.3
0.13333   -0.3      0
-----
```

Value = 0

Optimal Decision Frequences of Player 1:

0            1            0

Optimal Decision Frequences of Player 2:

0            1            0

```
*****
t = 4 Resource_1 = 2 Resource_2 = 3
Nstrat1 = 3 Nstrat2 = 4
```

```
-----
-0.9      -0.96667    -1.0333    -1.1
-1.0036   -1.1833     -1.25     -0.95
-1.1071   -1.7667     -2.2      -1.9
-----
```

Value = -1.0727

Optimal Decision Frequences of Player 1:

0.81818      0.18182      0

Optimal Decision Frequences of Player 2:

0              0      0.40909      0.59091

```
*****
t = 4 Resource_1 = 3 Resource_2 = 0
Nstrat1 = 4 Nstrat2 = 1
```

```
-----
2.7
3.7
4.7
5.7
-----
```

Value = 5.7

Optimal Decision Frequences of Player 1:

0              0              0              1

Optimal Decision Frequences of Player 2:

1

```
*****
t = 4 Resource_1 = 3 Resource_2 = 1
Nstrat1 = 4 Nstrat2 = 2
```

```
-----
1.8      1.9036
2.15     2.7333
2.5      3.45
2.85     3.8
-----
```

Value = 2.85

Optimal Decision Frequences of Player 1:

0              0              0              1

Optimal Decision Frequences of Player 2:

1              0

```
*****
t = 4 Resource_1 = 3 Resource_2 = 2
Nstrat1 = 4 Nstrat2 = 3
```

```
-----
0.9      1.0036   1.1071
0.96667  1.1833   1.7667
1.0333   1.25     2.2
1.1      0.95     1.9
-----
```

```
Value = 1.0727
Optimal Decision Frequences of Player 1:
0      0      0.40909  0.59091
Optimal Decision Frequences of Player 2:
0.81818  0.18182  0
```

```
*****
t = 4 Resource_1 = 3 Resource_2 = 3
Nstrat1 = 4 Nstrat2 = 4
```

```
-----
0      0.10357  0.20714  0.31071
-0.10357  0      0.21667  0.8
-0.20714  -0.21667  0      0.95
-0.31071  -0.8     -0.95   0
-----
```

```
Value = 0
Optimal Decision Frequences of Player 1:
1      0      0      0
Optimal Decision Frequences of Player 2:
1      0      0      0
```

t = 3

```
*****
t = 3 Resource_1 = 0 Resource_2 = 0
Nstrat1 = 1 Nstrat2 = 1
```

```
-----
0
-----
```

```
Value = 0
Optimal Decision Frequences of Player 1:
1
Optimal Decision Frequences of Player 2:
1
```

```
*****
t = 3 Resource_1 = 0 Resource_2 = 1
Nstrat1 = 1 Nstrat2 = 2
```

```
-----
-1.71   -2.71
-----
```

```
Value = -2.71
Optimal Decision Frequences of Player 1:
1
Optimal Decision Frequences of Player 2:
0      1
```

```
*****
t = 3 Resource_1 = 0 Resource_2 = 2
Nstrat1 = 1 Nstrat2 = 3
```

```
-----
-3.42      -4.42      -5.42
-----
```

```
Value = -5.42
Optimal Decision Frequences of Player 1:
1
Optimal Decision Frequences of Player 2:
0      0      1
```

```
*****
t = 3 Resource_1 = 0 Resource_2 = 3
Nstrat1 = 1 Nstrat2 = 4
```

```
-----
-5.13      -6.13      -7.13      -8.13
-----
```

```
Value = -8.13
Optimal Decision Frequences of Player 1:
1
Optimal Decision Frequences of Player 2:
0      0      0      1
```

```
*****
t = 3 Resource_1 = 1 Resource_2 = 0
Nstrat1 = 2 Nstrat2 = 1
```

```
-----
1.71
2.71
-----
```

```
Value = 2.71
Optimal Decision Frequences of Player 1:
0      1
Optimal Decision Frequences of Player 2:
1
```

```
*****
t = 3 Resource_1 = 1 Resource_2 = 1
Nstrat1 = 2 Nstrat2 = 2
```

```
-----
0      0.055
-0.055      0
-----
```

```
Value = 0
Optimal Decision Frequences of Player 1:
1      0
Optimal Decision Frequences of Player 2:
1      0
```

```
*****
t = 3 Resource_1 = 1 Resource_2 = 2
Nstrat1 = 2 Nstrat2 = 3
```

```
-----
-1.44      -1.52      -1.5325
-2.0933    -2.63      -2.44
-----
```

Value = -1.5325

Optimal Decision Frequences of Player 1:

1            0

Optimal Decision Frequences of Player 2:

0            0            1

```
*****
t = 3 Resource_1 = 1 Resource_2 = 3
Nstrat1 = 2 Nstrat2 = 4
```

```
-----
-2.565     -2.8025     -2.9612     -3.0469
-3.9173    -4.6083     -5.0475     -4.565
-----
```

Value = -3.0469

Optimal Decision Frequences of Player 1:

1            0

Optimal Decision Frequences of Player 2:

0            0            0            1

```
*****
t = 3 Resource_1 = 2 Resource_2 = 0
Nstrat1 = 3 Nstrat2 = 1
```

```
-----
3.42
4.42
5.42
-----
```

Value = 5.42

Optimal Decision Frequences of Player 1:

0            0            1

Optimal Decision Frequences of Player 2:

1

```
*****
t = 3 Resource_1 = 2 Resource_2 = 1
Nstrat1 = 3 Nstrat2 = 2
```

```
-----
1.44      2.0933
1.52      2.63
1.5325    2.44
-----
```

Value = 1.5325

Optimal Decision Frequences of Player 1:

0            0            1

Optimal Decision Frequences of Player 2:

1            0

```
*****
t = 3 Resource_1 = 2 Resource_2 = 2
Nstrat1 = 3 Nstrat2 = 3
```

```
-----
      0      0.29333      0.82667
-0.29333      0      0.975
-0.82667     -0.975      0
```

```
-----
Value = 0
Optimal Decision Frequences of Player 1:
      1      0      0
Optimal Decision Frequences of Player 2:
      1      0      0
```

```
*****
t = 3 Resource_1 = 2 Resource_2 = 3
Nstrat1 = 3 Nstrat2 = 4
```

```
-----
-0.96545     -0.98848     -0.80061     -0.38242
-1.5937     -1.6292     -1.4213     -0.55318
-2.765      -3.1939     -3.2064     -1.9655
```

```
-----
Value = -0.98848
Optimal Decision Frequences of Player 1:
      1      0      0
Optimal Decision Frequences of Player 2:
      0      1      0      0
```

```
*****
t = 3 Resource_1 = 3 Resource_2 = 0
Nstrat1 = 4 Nstrat2 = 1
```

```
-----
      5.13
      6.13
      7.13
      8.13
```

```
-----
Value = 8.13
Optimal Decision Frequences of Player 1:
      0      0      0      1
Optimal Decision Frequences of Player 2:
      1
```

```
*****
t = 3 Resource_1 = 3 Resource_2 = 1
Nstrat1 = 4 Nstrat2 = 2
```

```
-----
      2.565      3.9173
      2.8025      4.6083
      2.9612      5.0475
      3.0469      4.565
```

```
-----
Value = 3.0469
Optimal Decision Frequences of Player 1:
      0      0      0      1
Optimal Decision Frequences of Player 2:
      1      0
```

```
*****
t = 3 Resource_1 = 3 Resource_2 = 2
Nstrat1 = 4 Nstrat2 = 3
```

```
-----
0.96545    1.5937    2.765
0.98848    1.6292    3.1939
0.80061    1.4213    3.2064
0.38242    0.55318   1.9655
-----
```

Value = 0.98848

Optimal Decision Frequences of Player 1:

0 1 0 0

Optimal Decision Frequences of Player 2:

1 0 0

```
*****
t = 3 Resource_1 = 3 Resource_2 = 3
Nstrat1 = 4 Nstrat2 = 4
```

```
-----
0    0.15266    0.662    1.6678
-0.15266    0    0.51758    1.8745
-0.662    -0.51758    0    1.5652
-1.6678    -1.8745    -1.5652    0
-----
```

Value = 0

Optimal Decision Frequences of Player 1:

1 0 0 0

Optimal Decision Frequences of Player 2:

1 0 0 0

t = 2

```
*****
t = 2 Resource_1 = 0 Resource_2 = 0
Nstrat1 = 1 Nstrat2 = 1
```

0

Value = 0

Optimal Decision Frequences of Player 1:

1

Optimal Decision Frequences of Player 2:

1

```
*****
t = 2 Resource_1 = 0 Resource_2 = 1
Nstrat1 = 1 Nstrat2 = 2
```

```
-----
-2.439    -3.439
-----
```

Value = -3.439

Optimal Decision Frequences of Player 1:

1

Optimal Decision Frequences of Player 2:

0 1

```
*****
t = 2 Resource_1 = 0 Resource_2 = 2
Nstrat1 = 1 Nstrat2 = 3
```

```
-----
-4.878      -5.878      -6.878
-----
```

Value = -6.878

Optimal Decision Frequences of Player 1:

1

Optimal Decision Frequences of Player 2:

0                    0                    1

```
*****
t = 2 Resource_1 = 0 Resource_2 = 3
Nstrat1 = 1 Nstrat2 = 4
```

```
-----
-7.317      -8.317      -9.317      -10.317
-----
```

Value = -10.317

Optimal Decision Frequences of Player 1:

1

Optimal Decision Frequences of Player 2:

0                    0                    0                    1

```
*****
t = 2 Resource_1 = 1 Resource_2 = 0
Nstrat1 = 2 Nstrat2 = 1
```

```
-----
2.439
```

```
3.439
-----
```

Value = 3.439

Optimal Decision Frequences of Player 1:

0                    1

Optimal Decision Frequences of Player 2:

1

```
*****
t = 2 Resource_1 = 1 Resource_2 = 1
Nstrat1 = 2 Nstrat2 = 2
```

```
-----
0            0.4195
-0.4195       0
-----
```

Value = 0

Optimal Decision Frequences of Player 1:

1                    0

Optimal Decision Frequences of Player 2:

1                    0



```
*****
t = 2 Resource_1 = 1 Resource_2 = 2
Nstrat1 = 2 Nstrat2 = 3
```

```
-----
-1.3792    -1.4896    -1.3351
-3.0451    -3.3286    -2.3792
-----
```

Value = -1.4896

Optimal Decision Frequences of Player 1:

1 0

Optimal Decision Frequences of Player 2:

0 1 0

```
*****
t = 2 Resource_1 = 1 Resource_2 = 3
Nstrat1 = 2 Nstrat2 = 4
```

```
-----
-2.7422    -2.8607    -2.9752    -2.9551
-5.6019    -6.1254    -6.2296    -4.7422
-----
```

Value = -2.9752

Optimal Decision Frequences of Player 1:

1 0

Optimal Decision Frequences of Player 2:

0 0 1 0

```
*****
t = 2 Resource_1 = 2 Resource_2 = 0
Nstrat1 = 3 Nstrat2 = 1
```

```
-----
4.878
5.878
6.878
-----
```

Value = 6.878

Optimal Decision Frequences of Player 1:

0 0 1

Optimal Decision Frequences of Player 2:

1

```
*****
t = 2 Resource_1 = 2 Resource_2 = 1
Nstrat1 = 3 Nstrat2 = 2
```

```
-----
1.3792    3.0451
1.4896    3.3286
1.3351    2.3792
-----
```

Value = 1.4896

Optimal Decision Frequences of Player 1:

0 1 0

Optimal Decision Frequences of Player 2:

1 0

```
*****
t = 2 Resource_1 = 2 Resource_2 = 2
Nstrat1 = 3 Nstrat2 = 3
```

```
-----
      0      0.25283      1.4477
-0.25283      0      1.3091
-1.4477      -1.3091      0
-----
```

```
Value = 0
Optimal Decision Frequences of Player 1:
      1      0      0
Optimal Decision Frequences of Player 2:
      1      0      0
```

```
*****
t = 2 Resource_1 = 2 Resource_2 = 3
Nstrat1 = 3 Nstrat2 = 4
```

```
-----
-0.88964      -0.96321      -0.81918      0.025384
-1.7076      -1.6554      -1.4264      -0.38424
-4.0569      -4.2363      -3.8228      -1.8896
-----
```

```
Value = -0.96321
Optimal Decision Frequences of Player 1:
      1      0      0
Optimal Decision Frequences of Player 2:
      0      1      0      0
```

```
*****
t = 2 Resource_1 = 3 Resource_2 = 0
Nstrat1 = 4 Nstrat2 = 1
```

```
-----
      7.317
      8.317
      9.317
     10.317
-----
```

```
Value = 10.317
Optimal Decision Frequences of Player 1:
      0      0      0      1
Optimal Decision Frequences of Player 2:
      1
```

```
*****
t = 2 Resource_1 = 3 Resource_2 = 1
Nstrat1 = 4 Nstrat2 = 2
```

```
-----
      2.7422      5.6019
      2.8607      6.1254
      2.9752      6.2296
      2.9551      4.7422
-----
```

```
Value = 2.9752
Optimal Decision Frequences of Player 1:
      0      0      1      0
Optimal Decision Frequences of Player 2:
      1      0
```

```
*****
t = 2 Resource_1 = 3 Resource_2 = 2
Nstrat1 = 4 Nstrat2 = 3
```

```
-----
0.88964      1.7076      4.0569
0.96321      1.6554      4.2363
0.81918      1.4264      3.8228
-0.025384    0.38424      1.8896
-----
```

Value = 0.96321

Optimal Decision Frequences of Player 1:

0 1 0 0

Optimal Decision Frequences of Player 2:

1 0 0

```
*****
t = 2 Resource_1 = 3 Resource_2 = 3
Nstrat1 = 4 Nstrat2 = 4
```

```
-----
0      0.095799    0.73324    2.6545
-0.095799    0      0.53081    2.5844
-0.73324    -0.53081    0      1.8766
-2.6545    -2.5844    -1.8766    0
-----
```

Value = 0

Optimal Decision Frequences of Player 1:

1 0 0 0

Optimal Decision Frequences of Player 2:

1 0 0 0

t = 1

```
*****
t = 1 Resource_1 = 0 Resource_2 = 0
Nstrat1 = 1 Nstrat2 = 1
```

0

Value = 0

Optimal Decision Frequences of Player 1:

1

Optimal Decision Frequences of Player 2:

1

```
*****
t = 1 Resource_1 = 0 Resource_2 = 1
Nstrat1 = 1 Nstrat2 = 2
```

```
-----
-3.0951    -4.0951
-----
```

Value = -4.0951

Optimal Decision Frequences of Player 1:

1

Optimal Decision Frequences of Player 2:

0 1

```
*****
t = 1 Resource_1 = 0 Resource_2 = 2
Nstrat1 = 1 Nstrat2 = 3
```

```
-----
-6.1902      -7.1902      -8.1902
-----
```

Value = -8.1902

Optimal Decision Frequences of Player 1:

1

Optimal Decision Frequences of Player 2:

0                    0                    1

```
*****
t = 1 Resource_1 = 0 Resource_2 = 3
Nstrat1 = 1 Nstrat2 = 4
```

```
-----
-9.2853      -10.285      -11.285      -12.285
-----
```

Value = -12.285

Optimal Decision Frequences of Player 1:

1

Optimal Decision Frequences of Player 2:

0                    0                    0                    1

```
*****
t = 1 Resource_1 = 1 Resource_2 = 0
Nstrat1 = 2 Nstrat2 = 1
```

```
-----
3.0951
```

```
4.0951
-----
```

Value = 4.0951

Optimal Decision Frequences of Player 1:

0                    1

Optimal Decision Frequences of Player 2:

1

```
*****
t = 1 Resource_1 = 1 Resource_2 = 1
Nstrat1 = 2 Nstrat2 = 2
```

```
-----
0            0.74755
-0.74755            0
-----
```

Value = 0

Optimal Decision Frequences of Player 1:

1                    0

Optimal Decision Frequences of Player 2:

1                    0

```
*****
t = 1 Resource_1 = 1 Resource_2 = 2
Nstrat1 = 2 Nstrat2 = 3
```

```
-----
-1.3407    -1.4703    -1.1614
-3.907     -3.9654     -2.3407
-----
```

Value = -1.4703

Optimal Decision Frequences of Player 1:

1 0

Optimal Decision Frequences of Player 2:

0 1 0

```
*****
t = 1 Resource_1 = 1 Resource_2 = 3
Nstrat1 = 2 Nstrat2 = 4
```

```
-----
-2.6777    -2.8092    -2.9397    -2.8506
-7.062     -7.4161    -7.1815    -4.6777
-----
```

Value = -2.9397

Optimal Decision Frequences of Player 1:

1 0

Optimal Decision Frequences of Player 2:

0 0 1 0

```
*****
t = 1 Resource_1 = 2 Resource_2 = 0
Nstrat1 = 3 Nstrat2 = 1
```

```
-----
6.1902
7.1902
8.1902
-----
```

Value = 8.1902

Optimal Decision Frequences of Player 1:

0 0 1

Optimal Decision Frequences of Player 2:

1

```
*****
t = 1 Resource_1 = 2 Resource_2 = 1
Nstrat1 = 3 Nstrat2 = 2
```

```
-----
1.3407    3.907
1.4703    3.9654
1.1614    2.3407
-----
```

Value = 1.4703

Optimal Decision Frequences of Player 1:

0 1 0

Optimal Decision Frequences of Player 2:

1 0

```
*****
t = 1 Resource_1 = 2 Resource_2 = 2
Nstrat1 = 3 Nstrat2 = 3
```

```
-----
      0      0.22711      2.0137
-0.22711      0      1.6179
-2.0137      -1.6179      0
```

```
-----
Value = 0
Optimal Decision Frequences of Player 1:
      1      0      0
Optimal Decision Frequences of Player 2:
      1      0      0
```

```
*****
t = 1 Resource_1 = 2 Resource_2 = 3
Nstrat1 = 3 Nstrat2 = 4
```

```
-----
-0.86689      -0.95563      -0.8338      0.39788
-1.6535      -1.6173      -1.4107      -0.23184
-5.1384      -5.0799      -4.2769      -1.8669
```

```
-----
Value = -0.95563
Optimal Decision Frequences of Player 1:
      1      0      0
Optimal Decision Frequences of Player 2:
      0      1      0      0
```

```
*****
t = 1 Resource_1 = 3 Resource_2 = 0
Nstrat1 = 4 Nstrat2 = 1
```

```
-----
9.2853
10.285
11.285
12.285
```

```
-----
Value = 12.285
Optimal Decision Frequences of Player 1:
      0      0      0      1
Optimal Decision Frequences of Player 2:
      1
```

```
*****
t = 1 Resource_1 = 3 Resource_2 = 1
Nstrat1 = 4 Nstrat2 = 2
```

```
-----
2.6777      7.062
2.8092      7.4161
2.9397      7.1815
2.8506      4.6777
```

```
-----
Value = 2.9397
Optimal Decision Frequences of Player 1:
      0      0      1      0
Optimal Decision Frequences of Player 2:
      1      0
```

```
*****
t = 1 Resource_1 = 3 Resource_2 = 2
Nstrat1 = 4 Nstrat2 = 3
```

```
-----
0.86689      1.6535      5.1384
0.95563      1.6173      5.0799
 0.8338      1.4107      4.2769
-0.39788     0.23184      1.8669
-----
```

Value = 0.95563

Optimal Decision Frequences of Player 1:

```
0      1      0      0
```

Optimal Decision Frequences of Player 2:

```
1      0      0
```

```
*****
t = 1 Resource_1 = 3 Resource_2 = 3
Nstrat1 = 4 Nstrat2 = 4
```

```
-----
0      0.07874     0.68841     3.4545
-0.07874     0      0.50411     3.1339
-0.68841     -0.50411     0      2.0899
-3.4545     -3.1339     -2.0899     0
-----
```

Value = 0

Optimal Decision Frequences of Player 1:

```
1      0      0      0
```

Optimal Decision Frequences of Player 2:

```
1      0      0      0
```

```
t = 1 x = 1 y = 1 VALOPT() = 0
t = 1 x = 1 y = 2 VALOPT() = -4.095100000000002
t = 1 x = 1 y = 3 VALOPT() = -8.190200000000004
t = 1 x = 1 y = 4 VALOPT() = -12.285299999999999
t = 1 x = 2 y = 1 VALOPT() = 4.095100000000002
t = 1 x = 2 y = 2 VALOPT() = 0
t = 1 x = 2 y = 3 VALOPT() = -1.470331249999987
t = 1 x = 2 y = 4 VALOPT() = -2.93974492187499
t = 1 x = 3 y = 1 VALOPT() = 8.190200000000004
t = 1 x = 3 y = 2 VALOPT() = 1.470331249999987
t = 1 x = 3 y = 3 VALOPT() = 0
t = 1 x = 3 y = 4 VALOPT() = -0.9556303030303042
t = 1 x = 4 y = 1 VALOPT() = 12.285299999999999
t = 1 x = 4 y = 2 VALOPT() = 2.93974492187499
t = 1 x = 4 y = 3 VALOPT() = 0.9556303030303042
t = 1 x = 4 y = 4 VALOPT() = 0
t = 2 x = 1 y = 1 VALOPT() = 0
t = 2 x = 1 y = 2 VALOPT() = -3.439000000000007
t = 2 x = 1 y = 3 VALOPT() = -6.878
t = 2 x = 1 y = 4 VALOPT() = -10.316999999999999
t = 2 x = 2 y = 1 VALOPT() = 3.439000000000007
t = 2 x = 2 y = 2 VALOPT() = 0
t = 2 x = 2 y = 3 VALOPT() = -1.489624999999999
t = 2 x = 2 y = 4 VALOPT() = -2.975171875000001
t = 2 x = 3 y = 1 VALOPT() = 6.878
t = 2 x = 3 y = 2 VALOPT() = 1.489624999999999
t = 2 x = 3 y = 3 VALOPT() = 0
t = 2 x = 3 y = 4 VALOPT() = -0.9632121212121234
```

```

t = 2 x = 4 y = 1 VALOPT() = 10.316999999999999
t = 2 x = 4 y = 2 VALOPT() = 2.975171875000001
t = 2 x = 4 y = 3 VALOPT() = 0.963212121212121234
t = 2 x = 4 y = 4 VALOPT() = 0
t = 3 x = 1 y = 1 VALOPT() = 0
t = 3 x = 1 y = 2 VALOPT() = -2.7100000000000008
t = 3 x = 1 y = 3 VALOPT() = -5.4200000000000002
t = 3 x = 1 y = 4 VALOPT() = -8.1299999999999996
t = 3 x = 2 y = 1 VALOPT() = 2.7100000000000008
t = 3 x = 2 y = 2 VALOPT() = 0
t = 3 x = 2 y = 3 VALOPT() = -1.5324999999999999
t = 3 x = 2 y = 4 VALOPT() = -3.046875
t = 3 x = 3 y = 1 VALOPT() = 5.4200000000000002
t = 3 x = 3 y = 2 VALOPT() = 1.5324999999999999
t = 3 x = 3 y = 3 VALOPT() = 0
t = 3 x = 3 y = 4 VALOPT() = -0.98848484848484848
t = 3 x = 4 y = 1 VALOPT() = 8.1299999999999996
t = 3 x = 4 y = 2 VALOPT() = 3.046875
t = 3 x = 4 y = 3 VALOPT() = 0.98848484848484859
t = 3 x = 4 y = 4 VALOPT() = 0
t = 4 x = 1 y = 1 VALOPT() = 0
t = 4 x = 1 y = 2 VALOPT() = -1.9000000000000006
t = 4 x = 1 y = 3 VALOPT() = -3.7999999999999997
t = 4 x = 1 y = 4 VALOPT() = -5.7000000000000003
t = 4 x = 2 y = 1 VALOPT() = 1.9000000000000006
t = 4 x = 2 y = 2 VALOPT() = 0
t = 4 x = 2 y = 3 VALOPT() = -1.5999999999999994
t = 4 x = 2 y = 4 VALOPT() = -2.8500000000000009
t = 4 x = 3 y = 1 VALOPT() = 3.7999999999999997
t = 4 x = 3 y = 2 VALOPT() = 1.5999999999999994
t = 4 x = 3 y = 3 VALOPT() = 0
t = 4 x = 3 y = 4 VALOPT() = -1.07272727272727263
t = 4 x = 4 y = 1 VALOPT() = 5.7000000000000003
t = 4 x = 4 y = 2 VALOPT() = 2.8500000000000009
t = 4 x = 4 y = 3 VALOPT() = 1.0727272727272728
t = 4 x = 4 y = 4 VALOPT() = 0
t = 5 x = 1 y = 1 VALOPT() = 0
t = 5 x = 1 y = 2 VALOPT() = -1
t = 5 x = 1 y = 3 VALOPT() = -2
t = 5 x = 1 y = 4 VALOPT() = -3
t = 5 x = 2 y = 1 VALOPT() = 1
t = 5 x = 2 y = 2 VALOPT() = 0
t = 5 x = 2 y = 3 VALOPT() = -1
t = 5 x = 2 y = 4 VALOPT() = -2
t = 5 x = 3 y = 1 VALOPT() = 2
t = 5 x = 3 y = 2 VALOPT() = 1
t = 5 x = 3 y = 3 VALOPT() = 0
t = 5 x = 3 y = 4 VALOPT() = -1
t = 5 x = 4 y = 1 VALOPT() = 3
t = 5 x = 4 y = 2 VALOPT() = 2
t = 5 x = 4 y = 3 VALOPT() = 1
t = 5 x = 4 y = 4 VALOPT() = 0

```