

# Dynamic Economical Optimization of Sustainable Forest Harvesting in Russia with Consideration of Energy, other Forest Products and Recreation

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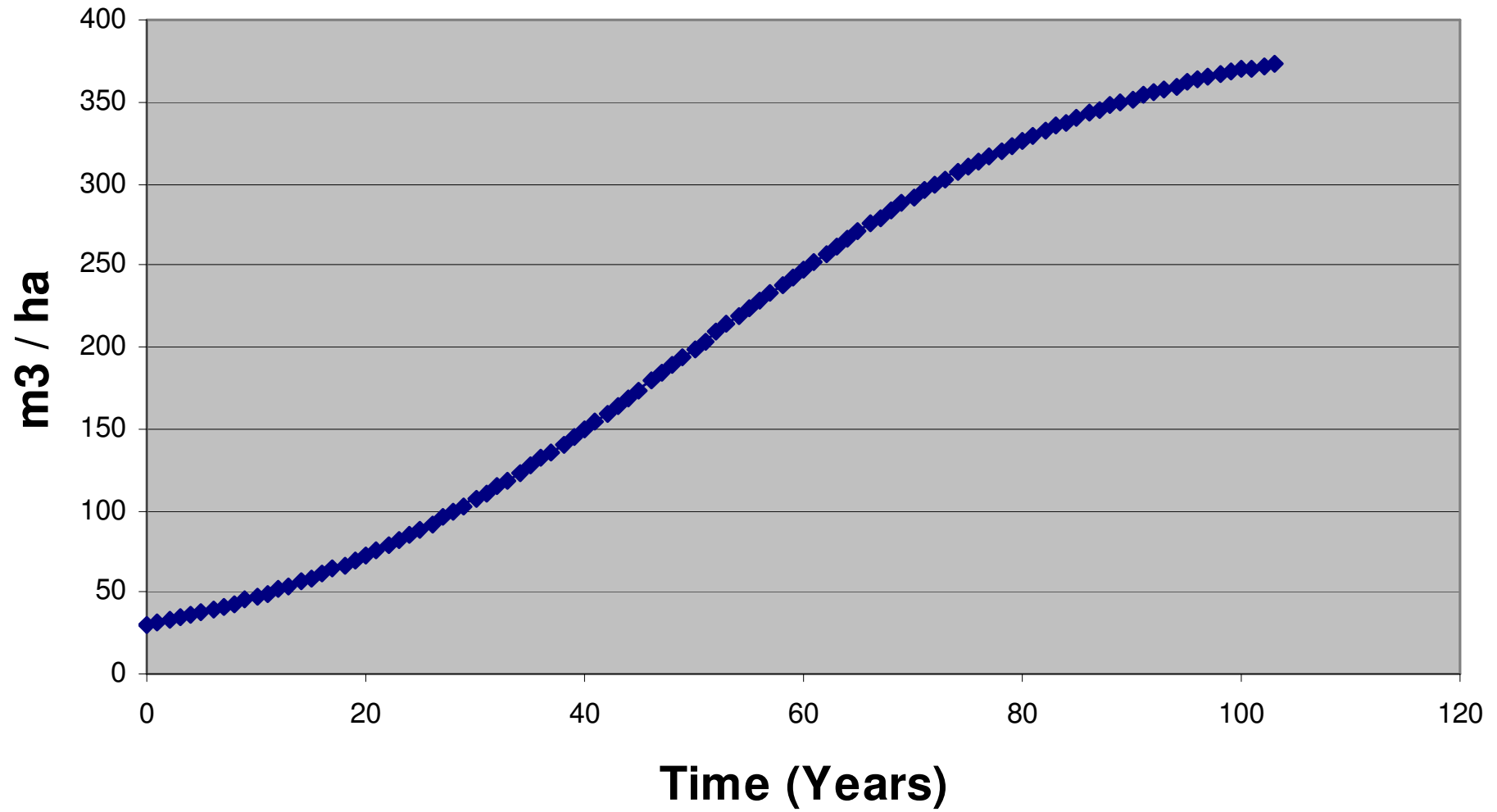
***14th SSAFR***

***Systems Analysis in Forestry,***

***Reñaca, Chile,***

***March 8-11, 2011***

# Typical Forest Stock Development Without Harvesting



# *Growth*

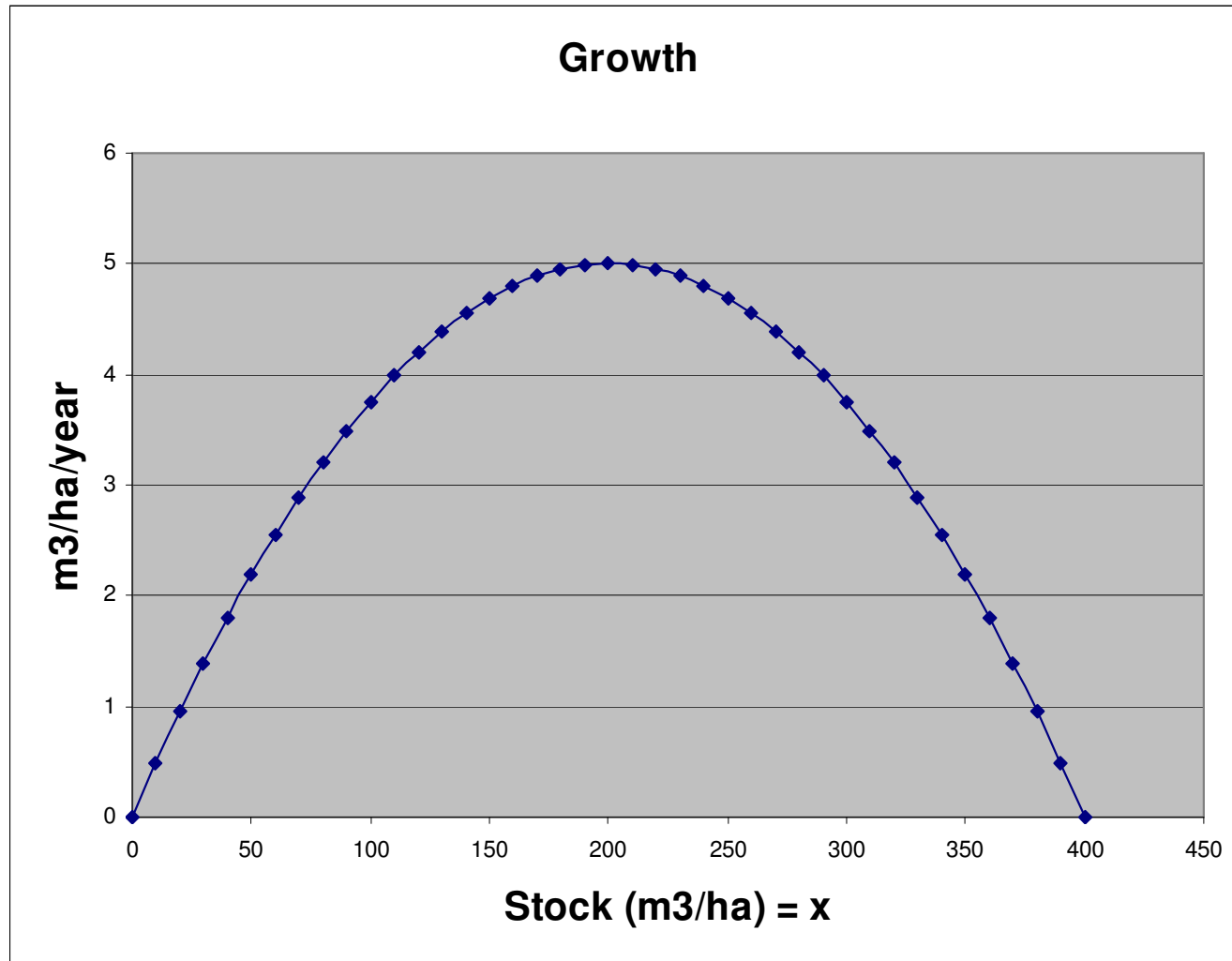
$$\bullet \quad \dot{x} = \frac{dx}{dt} = ax - bx^2$$

$$x = \textit{Stock} \quad (m^3 / ha)$$

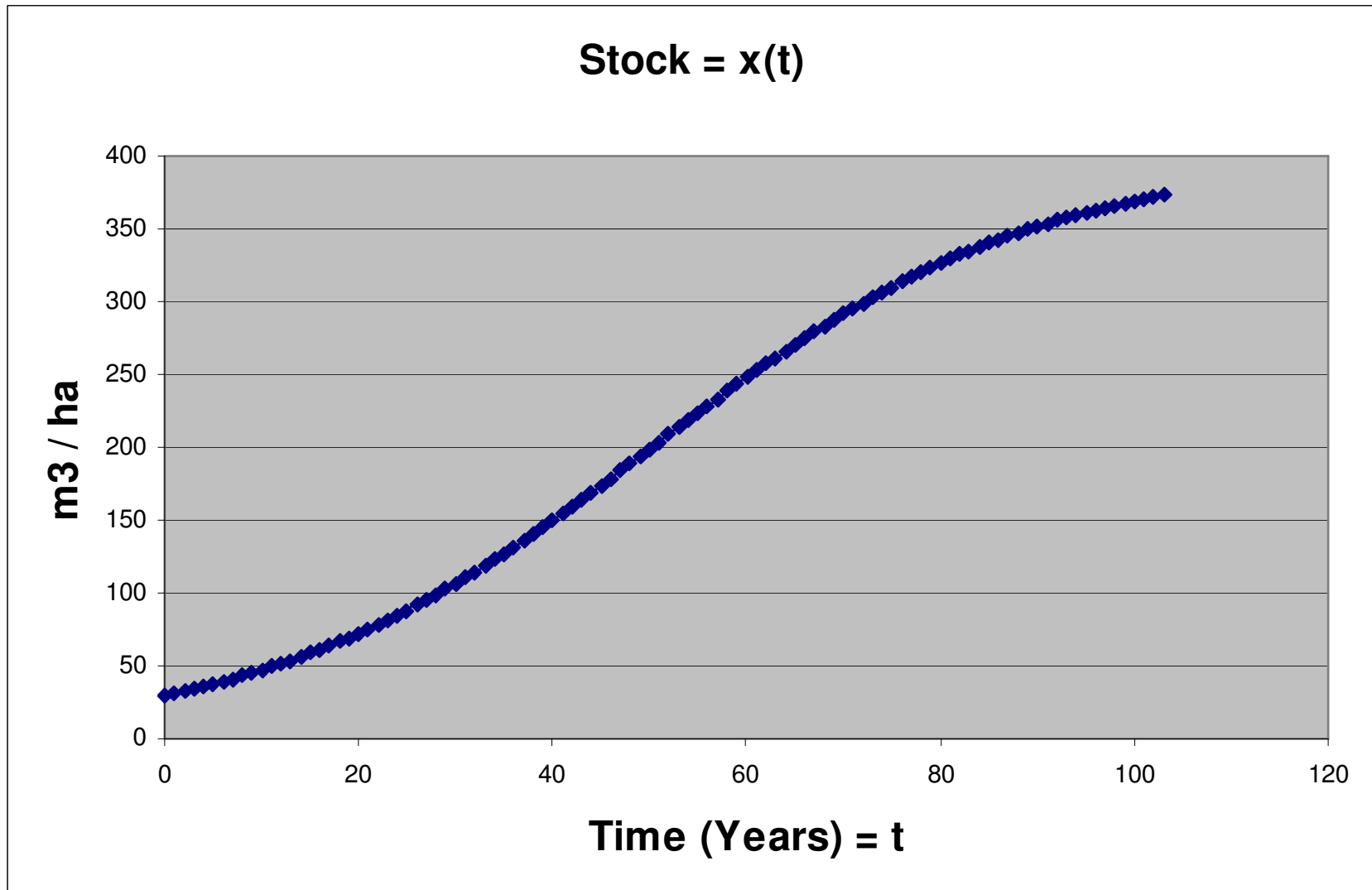
**Compare:**

Verhulst, Pierre-François (1838). ["Notice sur la loi que la population poursuit dans son accroissement"](#). *Correspondance mathématique et physique* **10**: pp. 113–121.

$$\frac{dN}{dt} = rN \left(1 - \frac{N}{K}\right)$$



$$Growth = \dot{x} = \frac{dx}{dt} = \frac{1}{20}x - \frac{1}{8000}x^2$$



$$\bullet \quad x = \frac{dx}{dt} = \frac{1}{20}x - \frac{1}{8000}x^2$$

## *A separable differential equation*

$$\bullet \quad \dot{x} = \frac{dx}{dt} = ax - bx^2$$

$$\frac{1}{ax - bx^2} dx = dt$$

$$\frac{1}{ax - bx^2} dx = dt$$

$$\frac{c}{x} + \frac{D}{(a - bx)} = \frac{1}{x(a - bx)} = \frac{1}{ax - bx^2}$$

$$\frac{c(a - bx) + Dx}{x(a - bx)} = \frac{1}{ax - bx^2}$$

$$\frac{c(a - bx) + Dx}{x(a - bx)} = \frac{1}{ax - bx^2}$$

$$ca - cbx + Dx = 1$$

$$ca + (-cb + D)x = 1$$

$$c = \frac{1}{a}$$

$$D = cb = \frac{b}{a}$$



$$\frac{1}{ax - bx^2} dx = dt$$

$$\left( \frac{c}{x} + \frac{D}{(a - bx)} \right) dx = dt$$

$$\left( \frac{\left( \frac{1}{a} \right)}{x} + \frac{\left( \frac{b}{a} \right)}{(a - bx)} \right) dx = dt$$

$$\left( \frac{1}{x} + \frac{b}{(a - bx)} \right) dx = a dt$$

$$\left( \frac{1}{x} + \frac{b}{(a-bx)} \right) dx = a dt$$

$$\int_{x_0}^{x_1} \left( \frac{1}{x} + \frac{b}{(a-bx)} \right) dx = \int_{t_0}^{t_1} a dt$$

$$\int_{x_0}^{x_1} \left( \frac{1}{x} + \frac{1}{\left(\frac{a}{b} - x\right)} \right) dx = \int_{t_0}^{t_1} a dt \quad h = \frac{a}{b}$$

$$\int_{x_0}^{x_1} \left( \frac{1}{x} + \frac{1}{(h - x)} \right) dx = \int_{t_0}^{t_1} a dt \quad h = \frac{a}{b}$$

$$\int_{x_0}^{x_1} \left( \frac{1}{x} + \frac{1}{(h-x)} \right) dx = \int_{t_0}^{t_1} a dt \quad h = \frac{a}{b}$$

$$\left[ LN(x) - LN(h-x) \right]_{x_0}^{x_1} = \left[ at \right]_{t_0}^{t_1}$$

$$LN(x_1) - LN(h-x_1) - LN(x_0) + LN(h-x_0) = a(t_1 - t_0)$$

$$LN \left( \frac{x_1 (h-x_0)}{x_0 (h-x_1)} \right) = aT \quad T = t_1 - t_0$$

$$LN\left(\frac{x_1 (h - x_0)}{x_0 (h - x_1)}\right) = aT \quad T = t_1 - t_0$$

$$\frac{x_1 (h - x_0)}{x_0 (h - x_1)} = e^{aT}$$

$$\frac{x_1}{(h - x_1)} = \frac{x_0 e^{aT}}{(h - x_0)}$$

$$x_1 = \frac{x_0 e^{aT}}{(h - x_0)} (h - x_1)$$

$$x_1 = \frac{x_0 e^{aT}}{(h - x_0)} (h - x_1)$$

$$x_1 \left( 1 + \frac{x_0 e^{aT}}{(h - x_0)} \right) = h \frac{x_0 e^{aT}}{(h - x_0)}$$

$$x_1 = \frac{h \frac{x_0 e^{aT}}{(h - x_0)}}{1 + \frac{x_0 e^{aT}}{(h - x_0)}}$$

$$x_1 = \frac{h \frac{x_0 e^{aT}}{(h-x_0)}}{1 + \frac{x_0 e^{aT}}{(h-x_0)}}$$

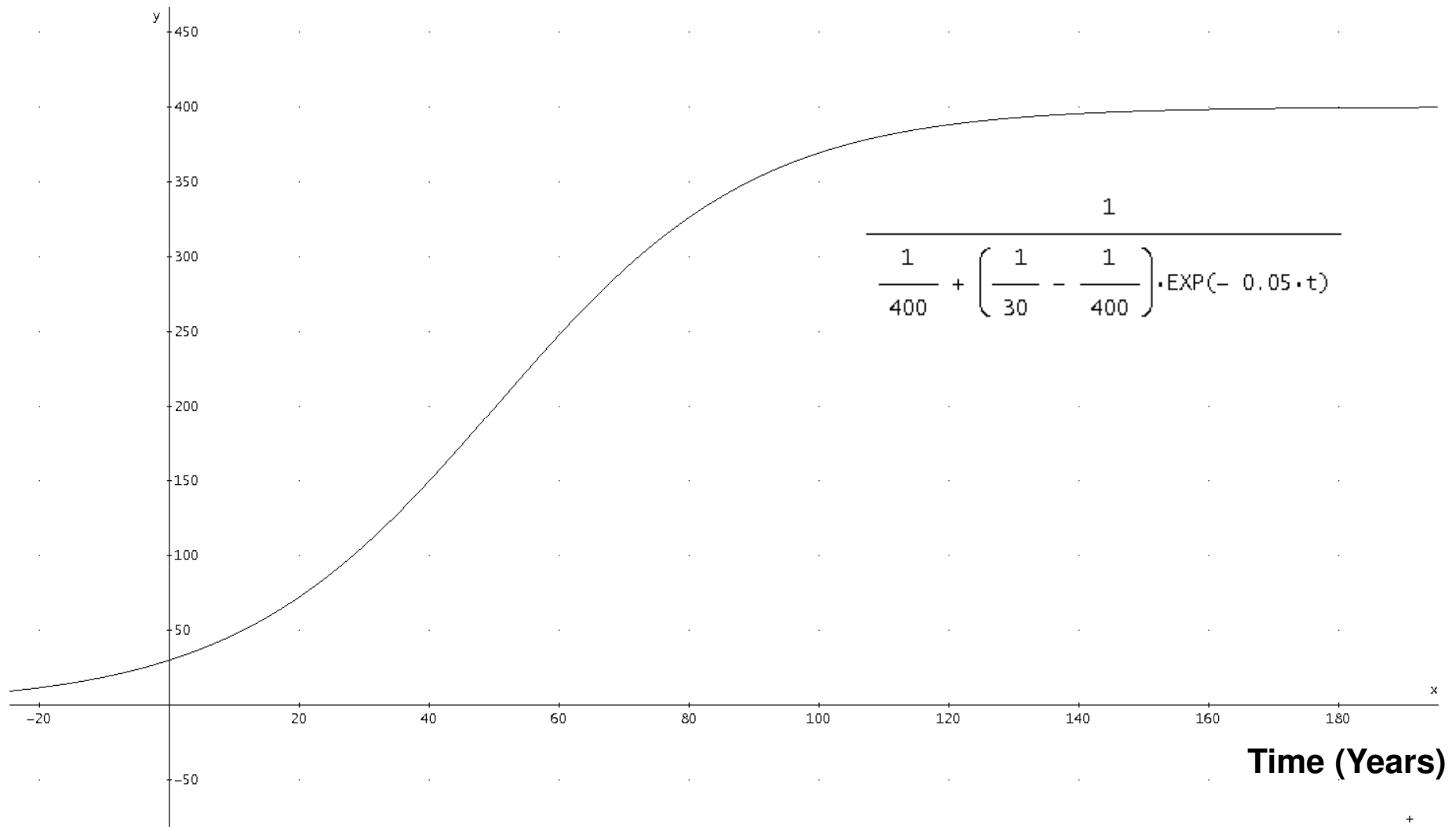
$$x_1 = \frac{1}{\frac{(h-x_0)}{hx_0} e^{-aT} + \frac{1}{h}}$$

$$x_1 = \frac{1}{\left( \frac{1}{x_0} - \frac{1}{h} \right) e^{-aT} + \frac{1}{h}}$$

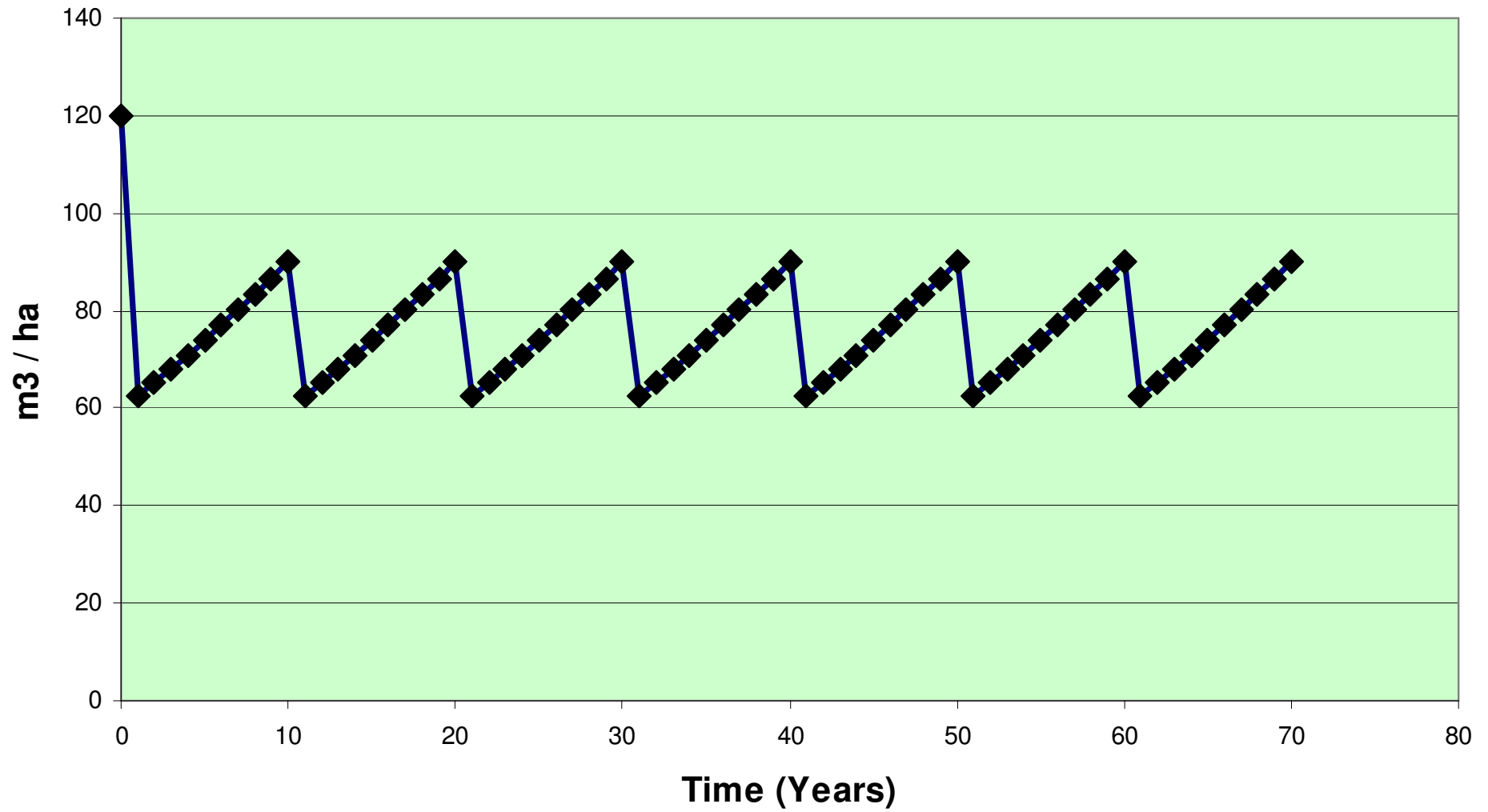
$$x_1 = \frac{1}{\frac{1}{h} + \left( \frac{1}{x_0} - \frac{1}{h} \right) e^{-aT}}$$



## Stock (m3/ha)



# Stock Development

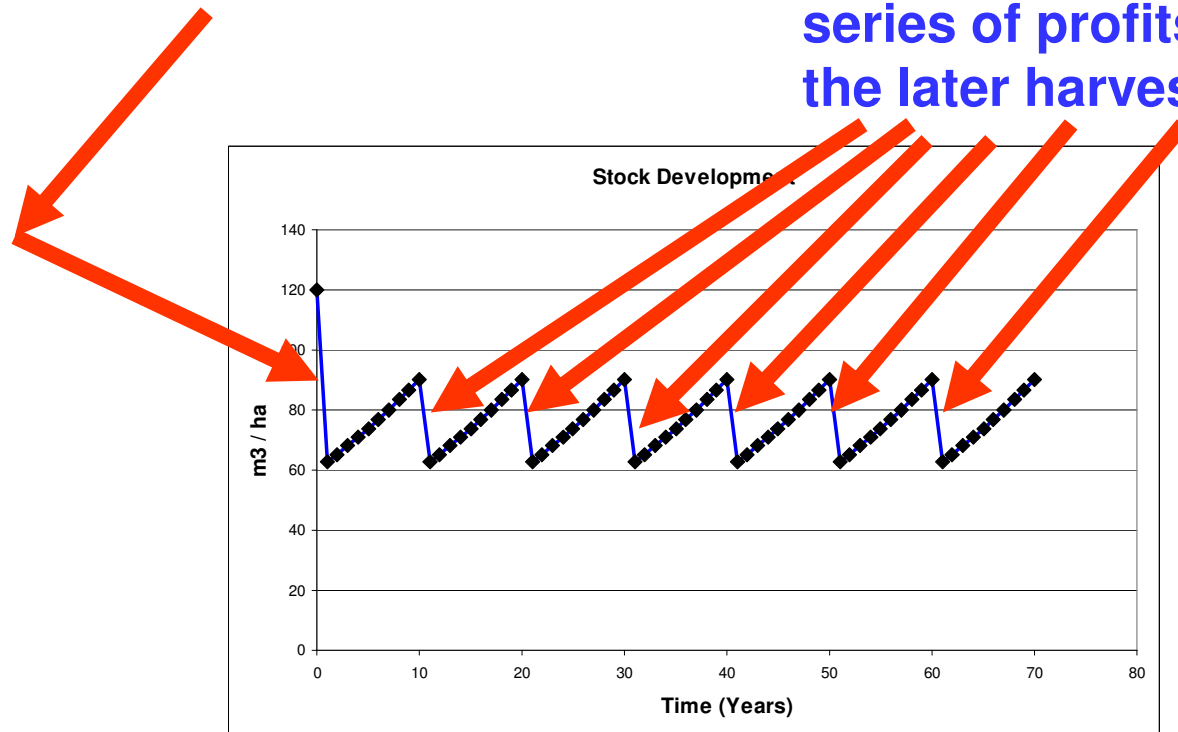


# The Present Value Function

$$\Pi(.) = (-c + ph_0) + \frac{(-c + ph_1(t, h_0))}{e^{rt} - 1}$$

Profit from the first harvest

Present value of an infinite series of profits from the later harvest



$$(-c + ph_0) + \frac{(-c + ph_1(t, h_0))}{e^{rt} - 1}$$

Initial stock level

$$-500 + 200 \cdot y + \frac{-500 + 200 \cdot \left[ \frac{1}{400} + \left( \frac{1}{300 - y} - \frac{1}{400} \right) \cdot \text{EXP}(-0.05 \cdot x) \right] - (300 - y)}{\text{EXP}(0.03 \cdot x) - 1}$$

Rate of interest

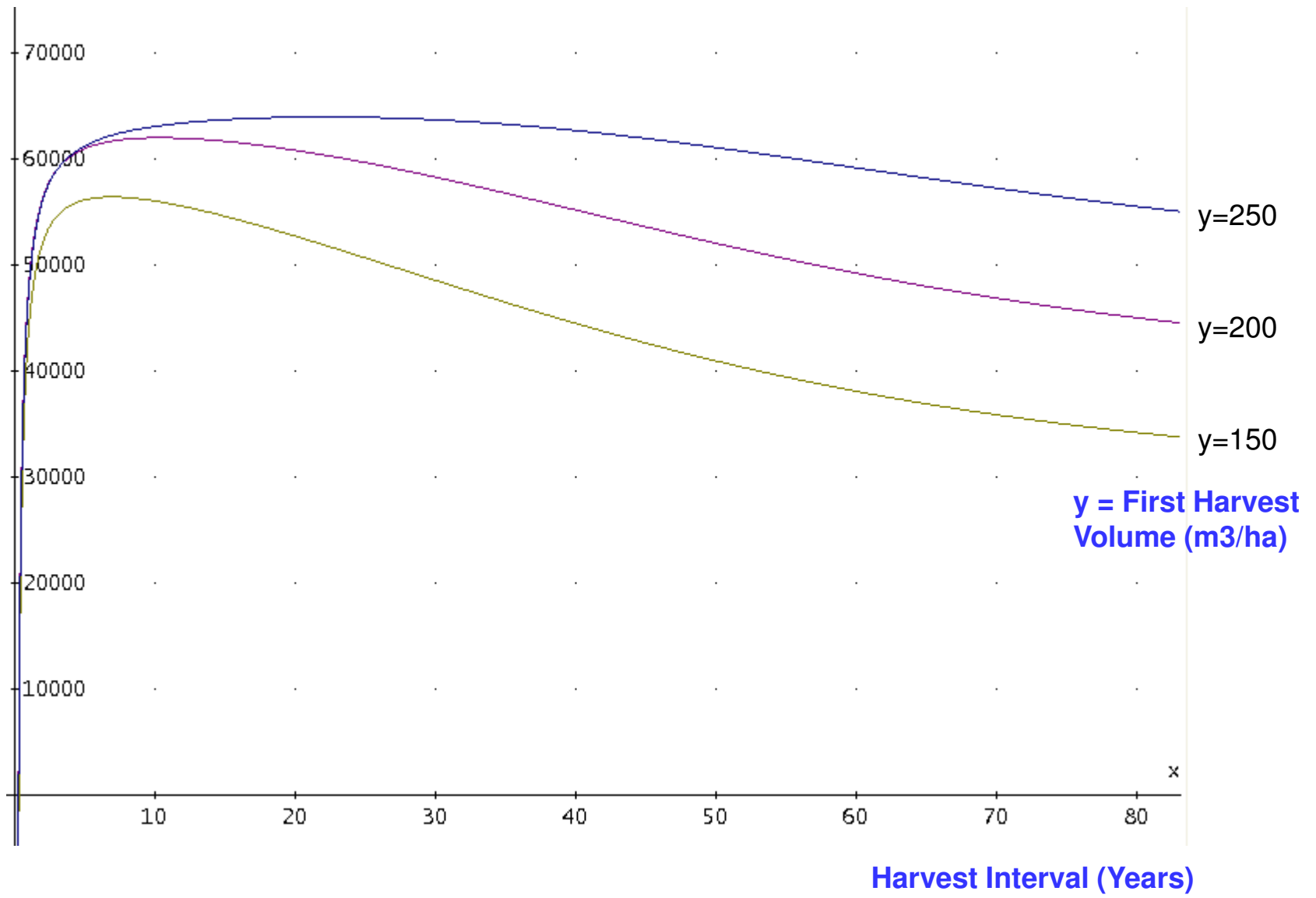
Harvest interval

$$-500 + 200 \cdot 200 + \frac{-500 + 200 \cdot \left[ \frac{1}{400} + \left( \frac{1}{300 - 200} - \frac{1}{400} \right) \cdot \text{EXP}(-0.05 \cdot x) \right] - (300 - 200)}{\text{EXP}(0.03 \cdot x) - 1}$$

$$-500 + 200 \cdot 150 + \frac{-500 + 200 \cdot \left[ \frac{1}{400} + \left( \frac{1}{300 - 150} - \frac{1}{400} \right) \cdot \text{EXP}(-0.05 \cdot x) \right] - (300 - 150)}{\text{EXP}(0.03 \cdot x) - 1}$$

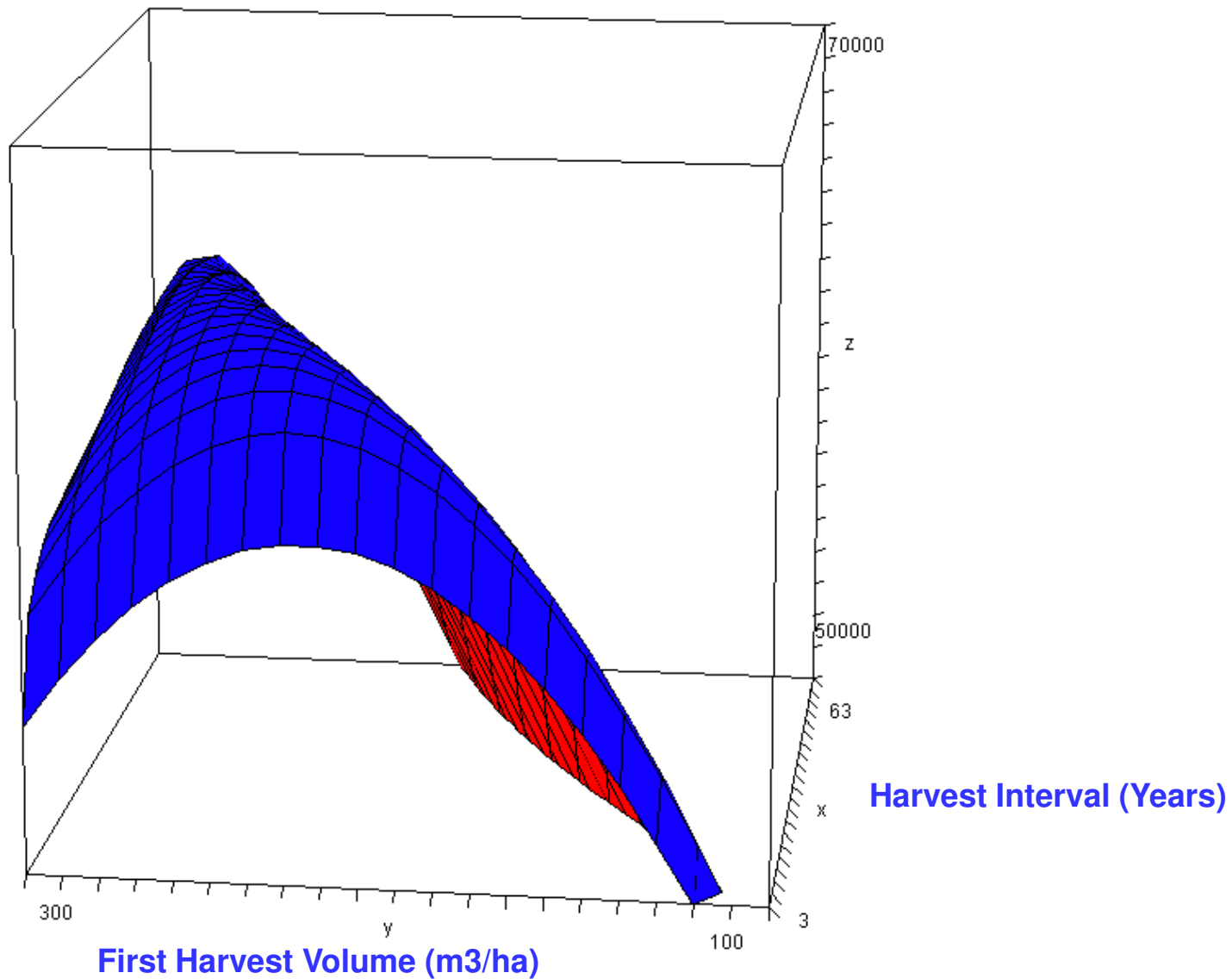
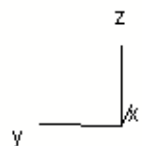
$$-500 + 200 \cdot 250 + \frac{-500 + 200 \cdot \left[ \frac{1}{400} + \left( \frac{1}{300 - 250} - \frac{1}{400} \right) \cdot \text{EXP}(-0.05 \cdot x) \right] - (300 - 250)}{\text{EXP}(0.03 \cdot x) - 1}$$

Present Value (SEK/ha)



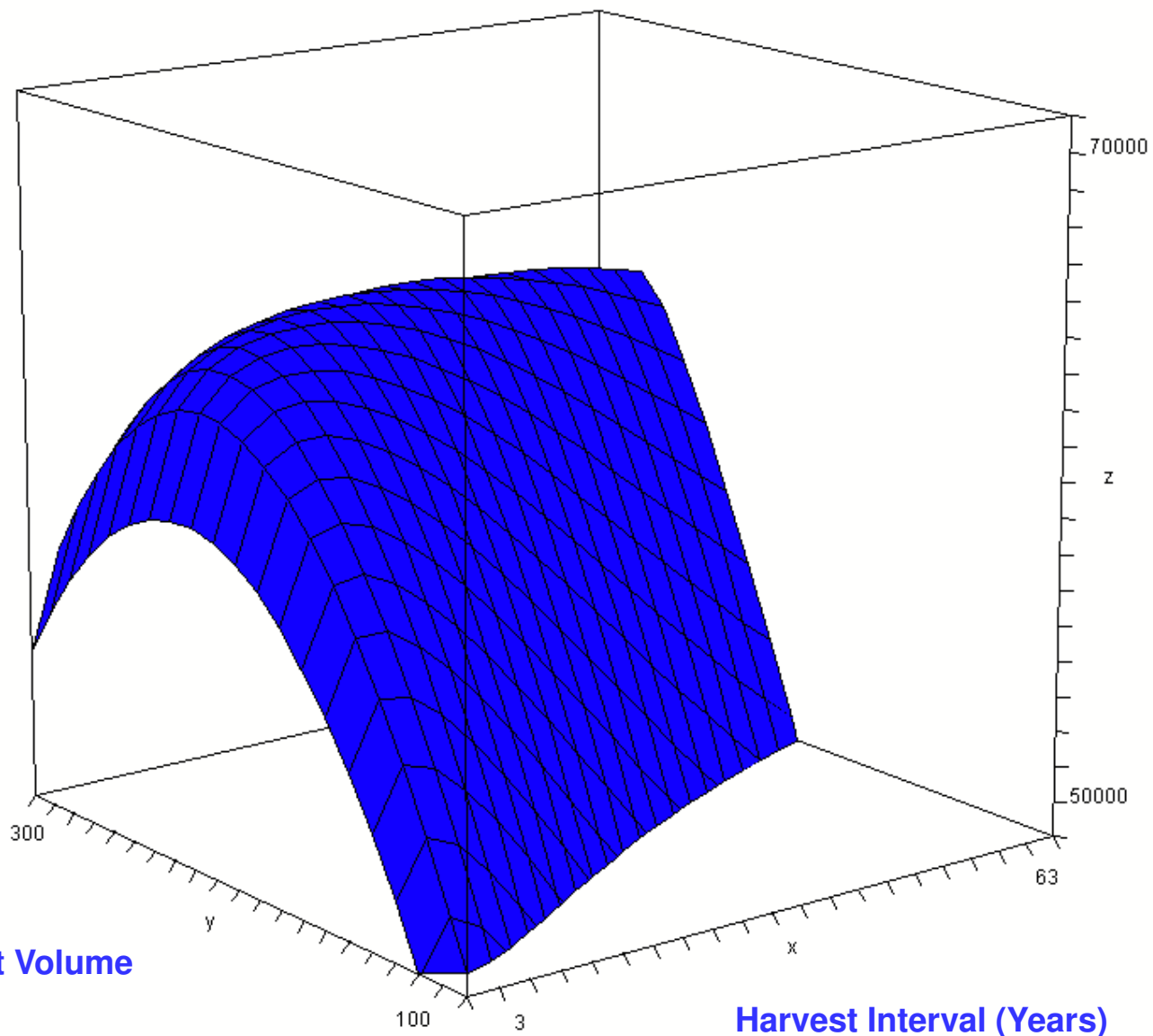
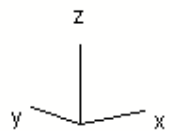
$C = 500$

Vertical Scale = Present Value (SEK/ha)



$C = 500$

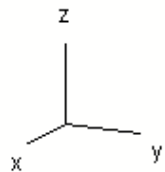
Vertical Scale = Present Value (SEK/ha)



First Harvest Volume  
(m3/ha)

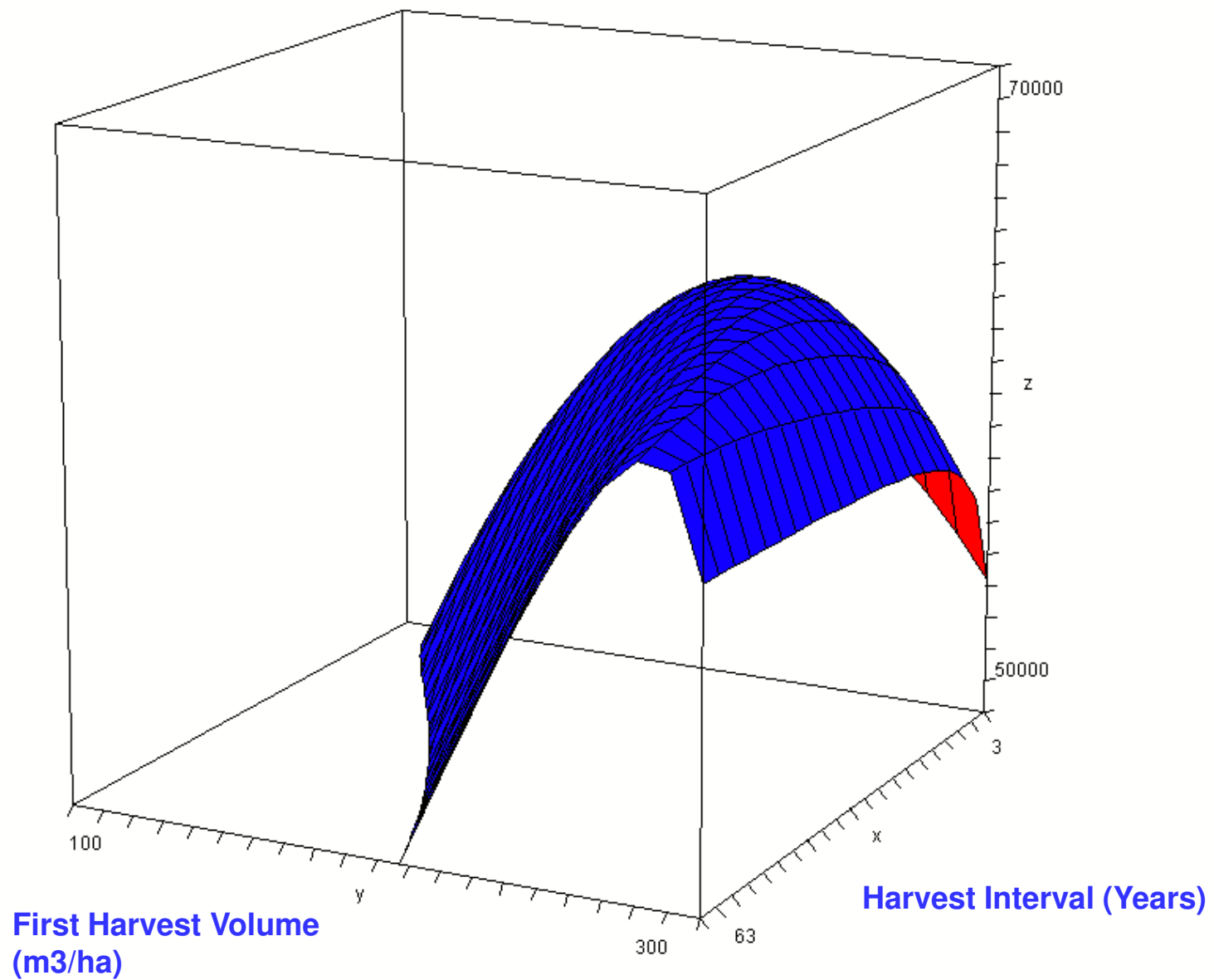
Harvest Interval (Years)





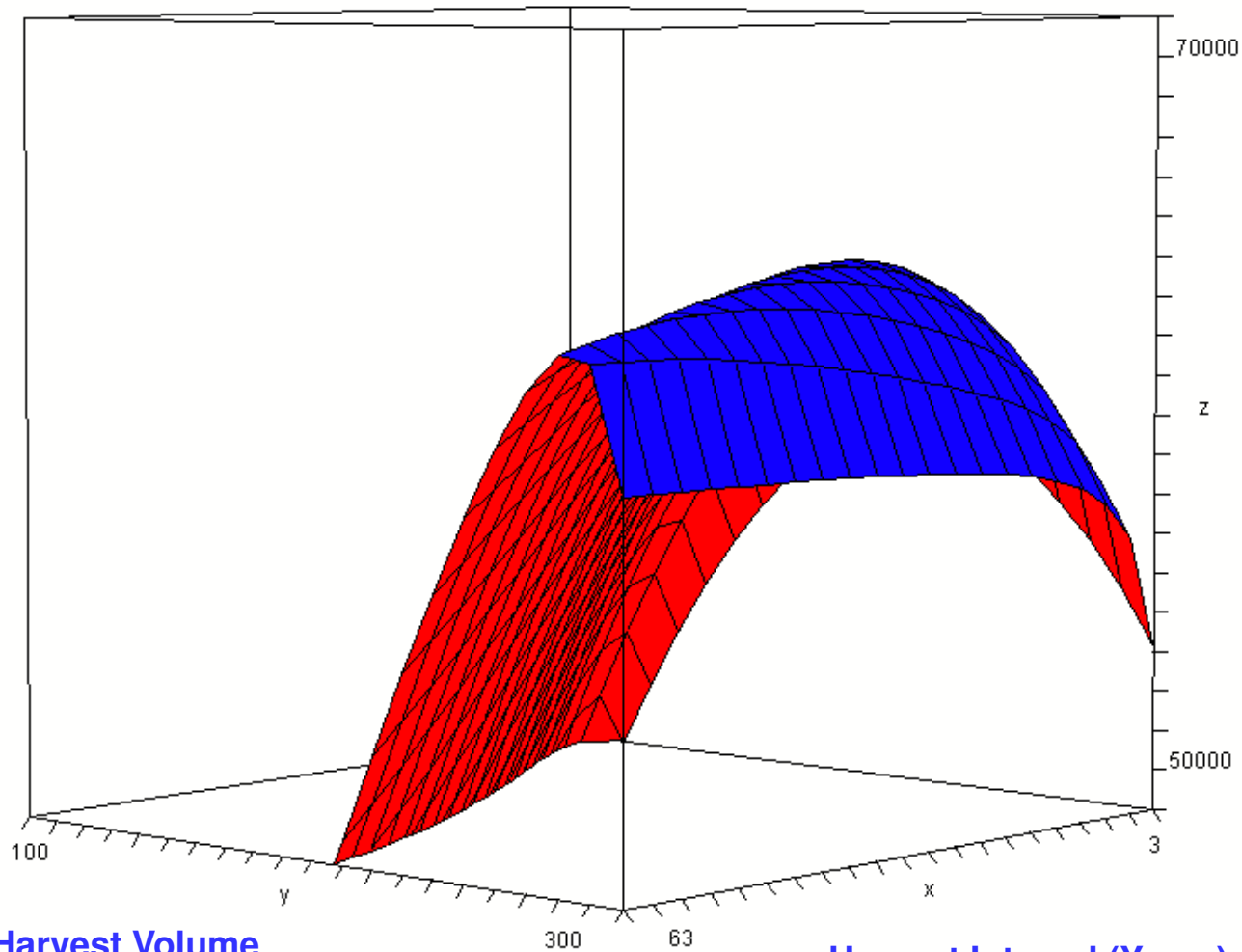
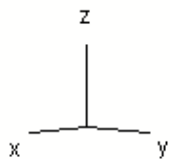
$C = 500$

Vertical Scale = Present Value (SEK/ha)



$C = 500$

Vertical Scale = Present Value (SEK/ha)



First Harvest Volume  
(m3/ha)

Harvest Interval (Years)

$C = 100$

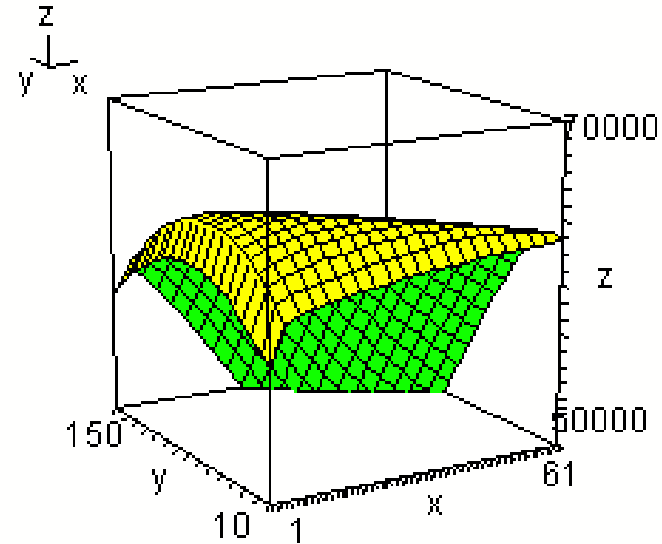
Stock Level Directly After Harvest (m<sup>3</sup>/ha)

$$-100 + 200 \cdot \left( \frac{1}{\frac{1}{400} + \left( \frac{1}{y} - \frac{1}{400} \right) \cdot \text{EXP}(-0.05 \cdot x)} - y \right)$$

$-100 + 200 \cdot (300 - y) +$

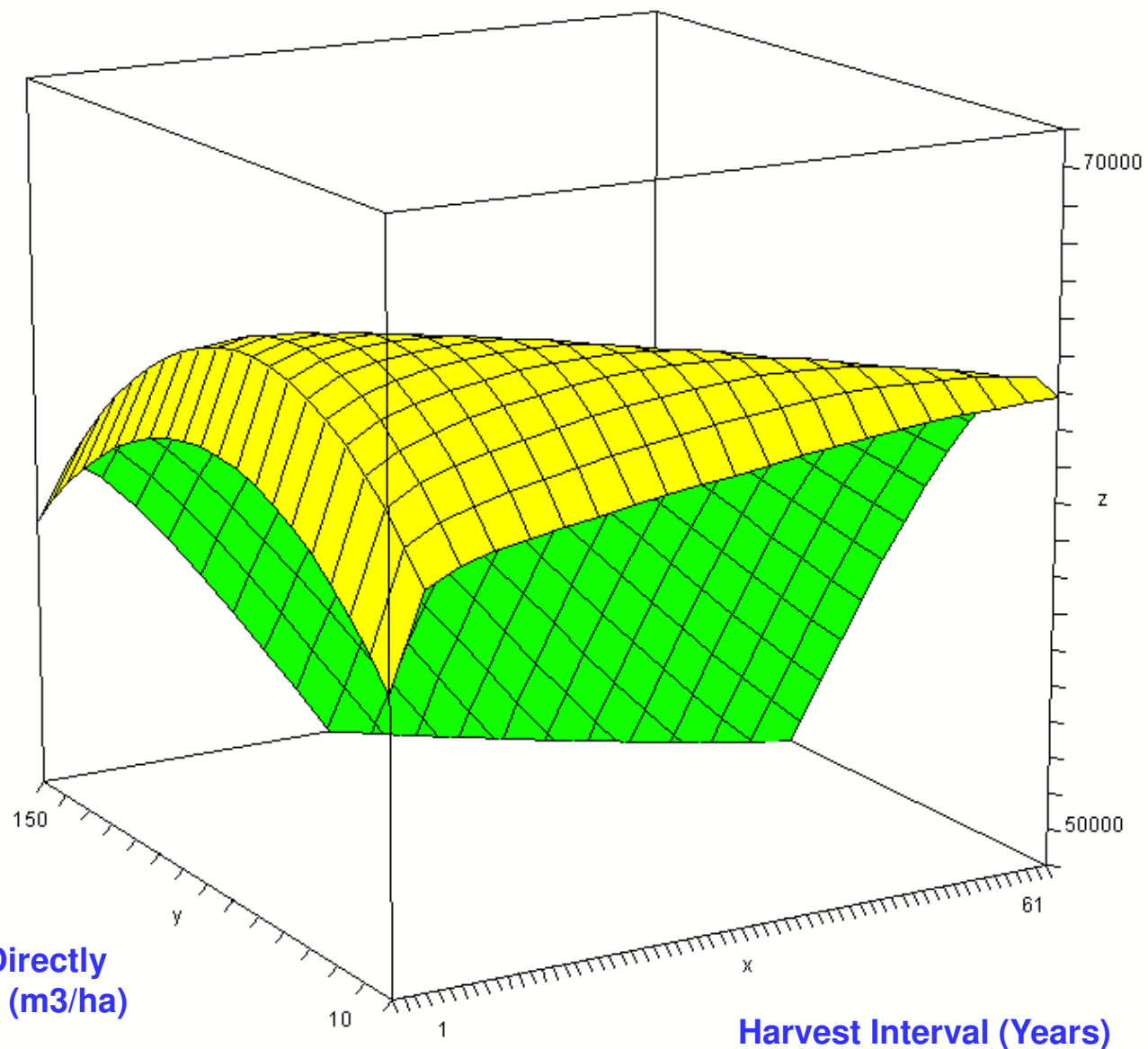
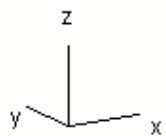
$\frac{\text{EXP}(0.03 \cdot x) - 1}{\text{EXP}(0.03 \cdot x) - 1}$

First Harvest Volume (m<sup>3</sup>/ha)



$C = 100$

Vertical Scale = Present Value (SEK/ha)

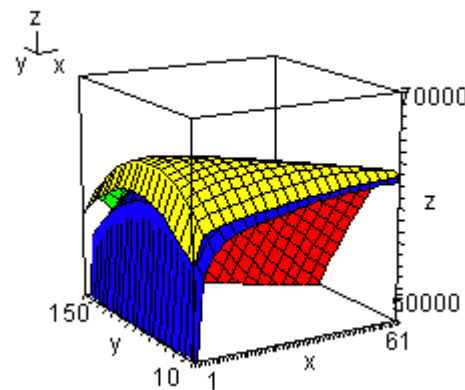


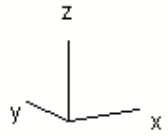
$$-500 + 200 \cdot (300 - y) + \frac{-500 + 200 \cdot \left( \frac{1}{\frac{1}{400} + \left( \frac{1}{y} - \frac{1}{400} \right) \cdot \text{EXP}(-0.05 \cdot x)} - y \right)}{\text{EXP}(0.03 \cdot x) - 1}$$

$$-100 + 200 \cdot (300 - y) + \frac{-100 + 200 \cdot \left( \frac{1}{\frac{1}{400} + \left( \frac{1}{y} - \frac{1}{400} \right) \cdot \text{EXP}(-0.05 \cdot x)} - y \right)}{\text{EXP}(0.03 \cdot x) - 1}$$

**C = 100**

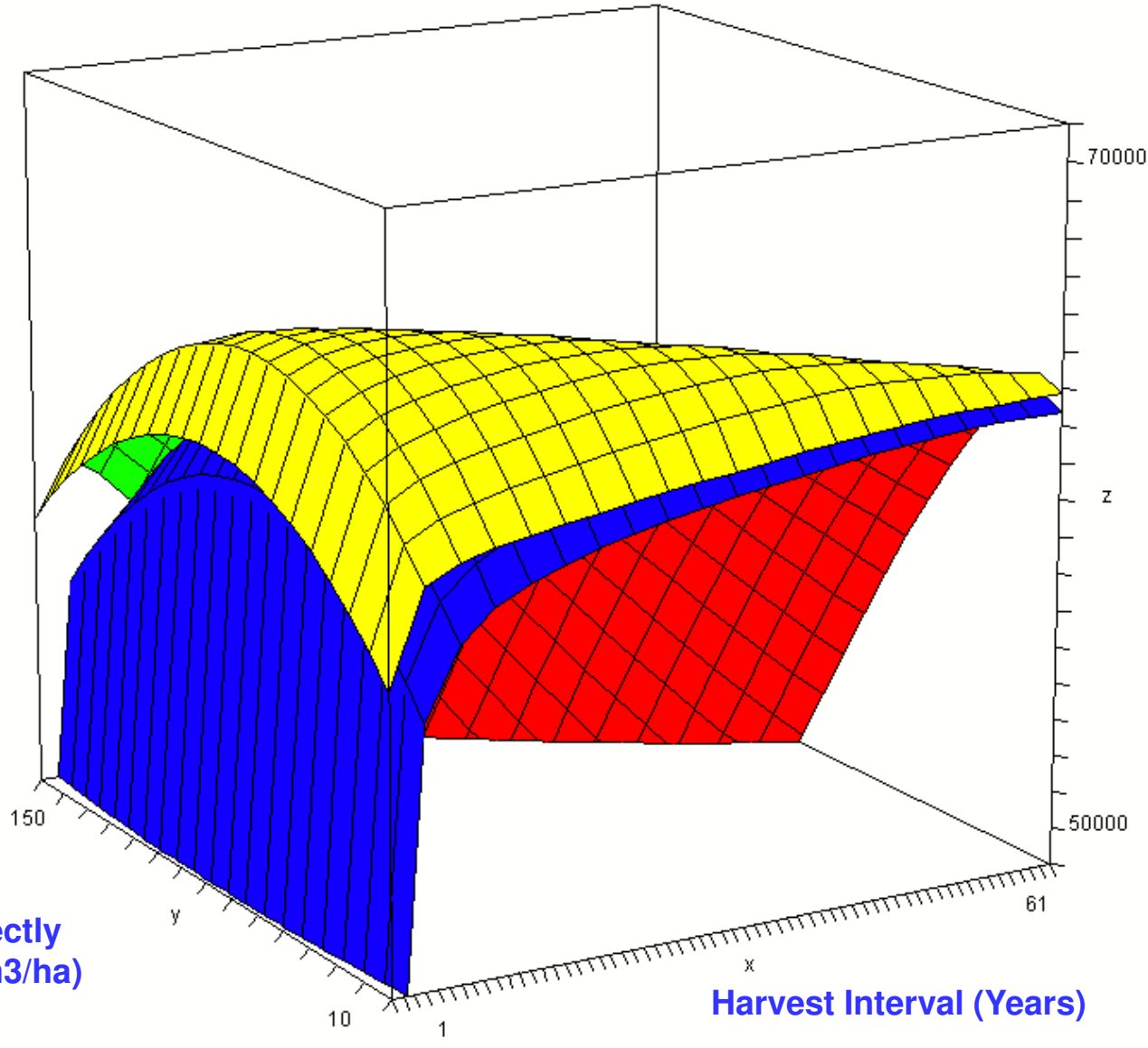
**C = 500**





Vertical Scale = Present Value (SEK/ha)

C = 100  
C = 500

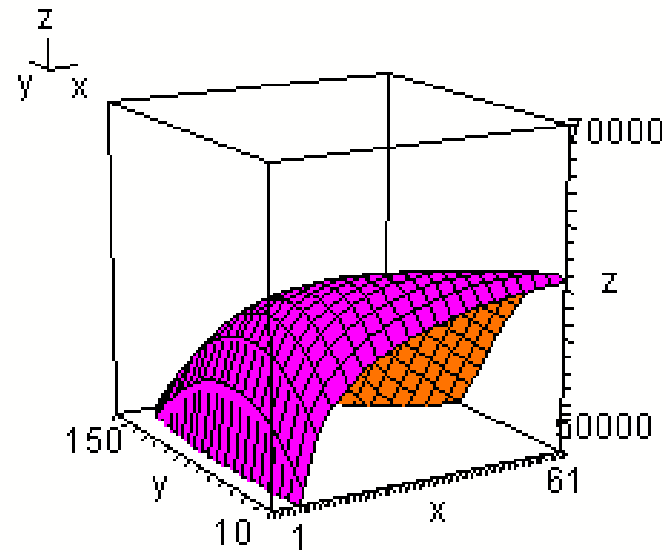


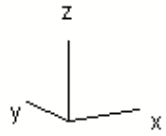
Stock Level Directly After Harvest (m<sup>3</sup>/ha)

Harvest Interval (Years)

$$-2000 + 200 \cdot (300 - y) + \frac{-2000 + 200 \cdot \left( \frac{1}{\frac{1}{400} + \left( \frac{1}{y} - \frac{1}{400} \right) \cdot \text{EXP}(-0.05 \cdot x)} - y \right)}{\text{EXP}(0.03 \cdot x) - 1}$$

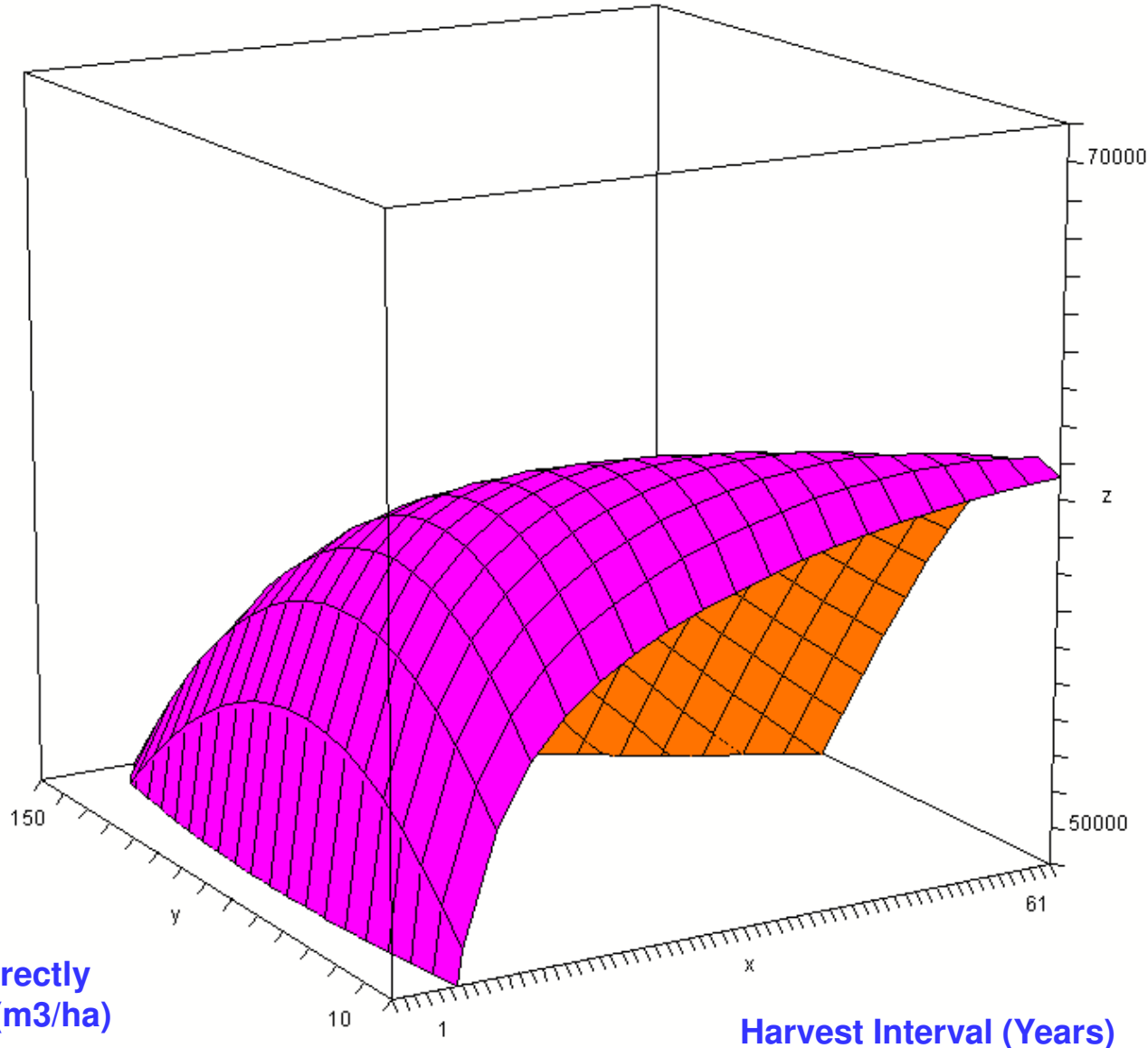
**C = 2000**





$C = 2000$

Vertical Scale = Present Value (SEK/ha)

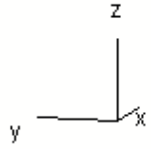


Stock Level Directly After Harvest (m<sup>3</sup>/ha)

Harvest Interval (Years)

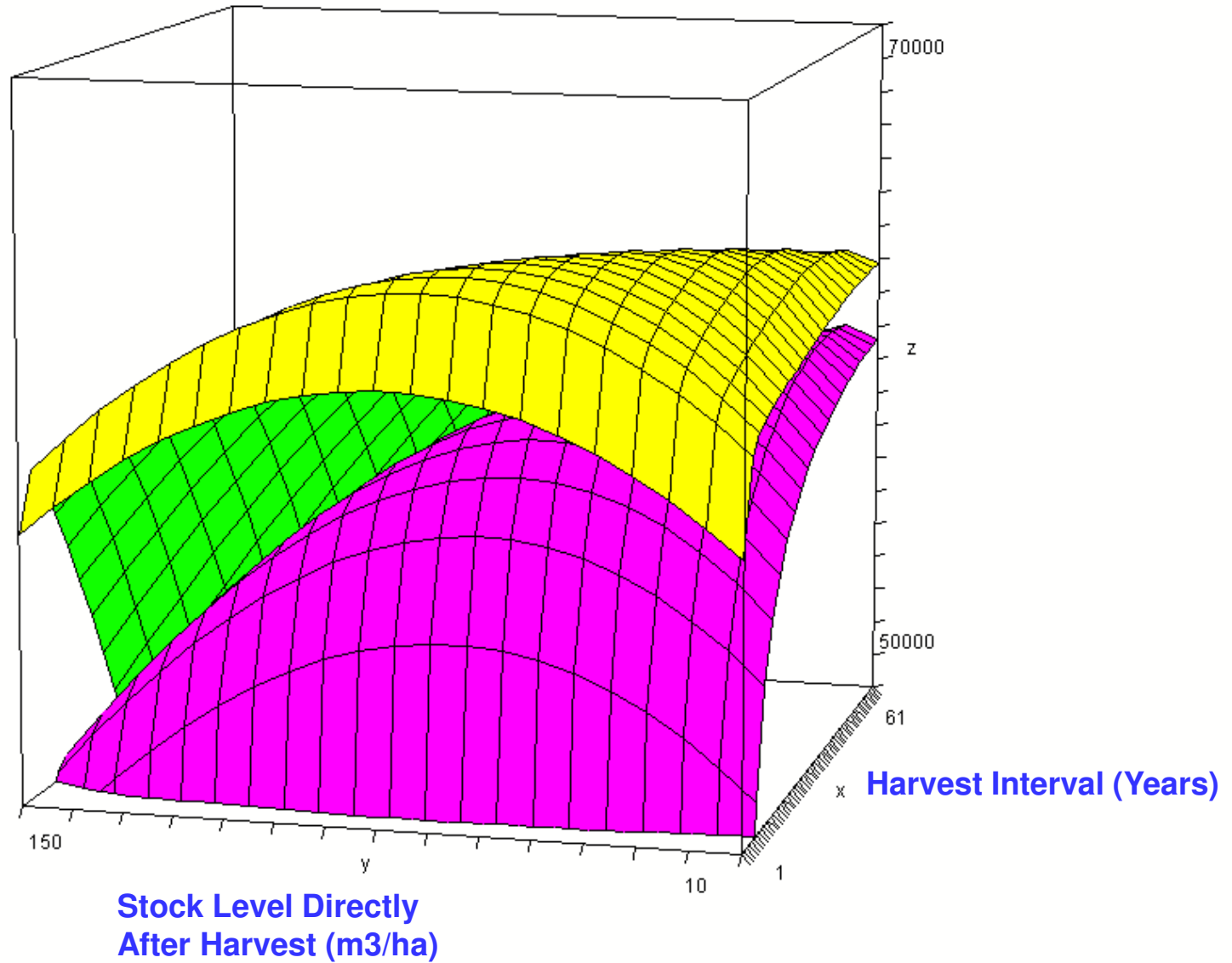


Vertical Scale = Present Value (SEK/ha)

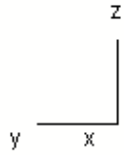


C = 100

C = 2000

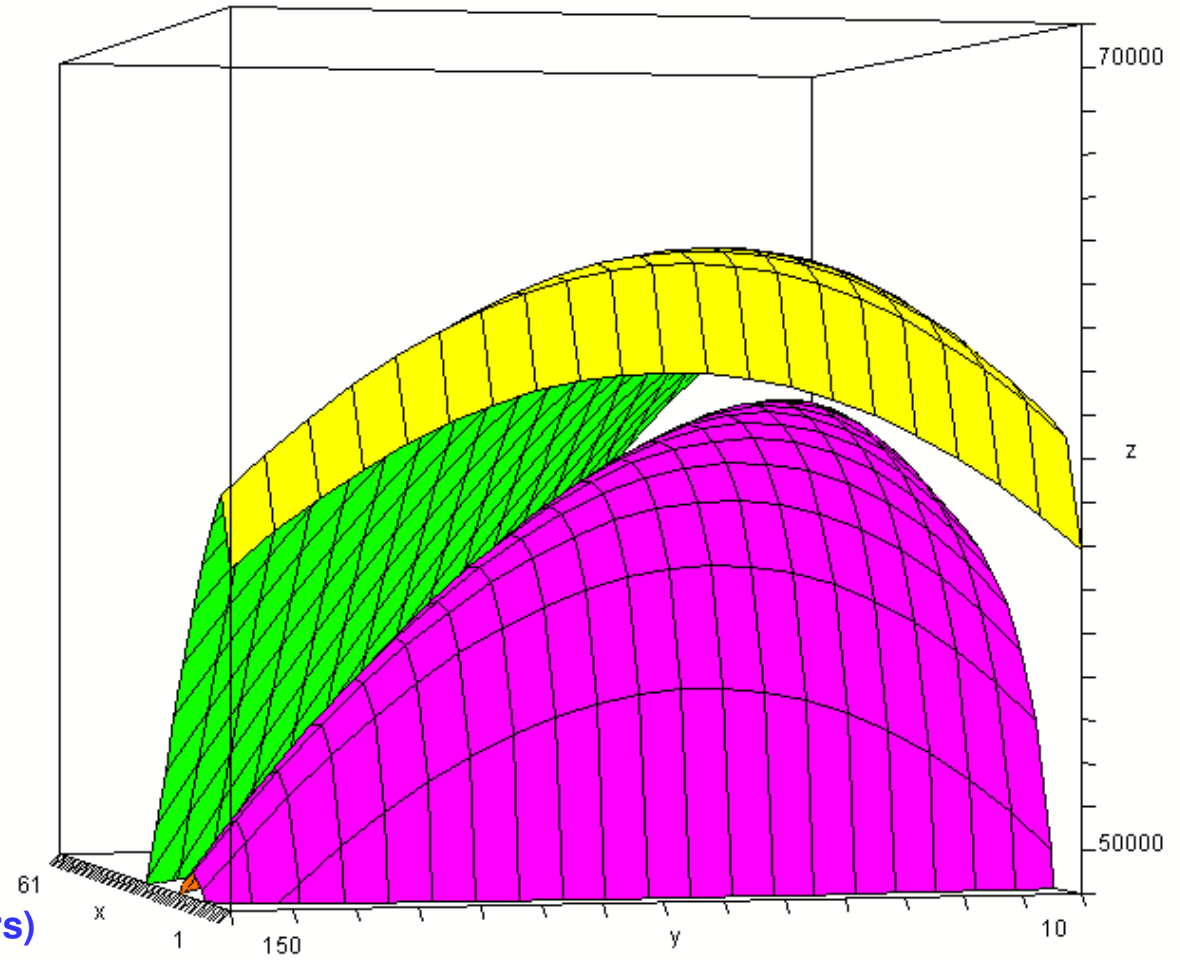


Vertical Scale = Present Value (SEK/ha)



C = 100  
C = 500

Harvest Interval (Years)



Stock Level Directly  
After Harvest (m3/ha)

```

REM
REM CCF0403
REM Peter Lohmander
REM
OPEN "outCCF.dat" FOR OUTPUT AS #1
PRINT #1, " x1  t  h1  PV"
  FOR x1 = 10 TO 150 STEP 20
    FOR t = 1 TO 31 STEP 5
      c = 500
      p = 200
      r = .03
      s = .05
      x0 = 300
      h0 = x0 - x1
      x2 = 1 / (1 / 400 + (1 / x1 - 1 / 400) * EXP(-.05 * t))
      h1 = x2 - x1
      multip = 1 / (EXP(r * t) - 1)
      pv0 = -c + p * h0
      pv1 = -c + p * h1
      PV = pv0 + pv1 * multip
      PRINT #1, USING "####"; x1;
      PRINT #1, USING "####"; t;
      PRINT #1, USING "####"; h1;
      PRINT #1, USING "#####"; PV
    NEXT t
  NEXT x1
CLOSE #1
END

```

x1	t	h1	PV
90	1	4	48299
90	6	23	61906
90	11	44	<b>62679</b>
90	16	67	62441
90	21	91	61755
90	26	116	60768
90	31	141	59561
110	1	4	47561
110	6	25	60776
110	11	49	<b>61115</b>
110	16	73	60419
110	21	98	59276
110	26	123	57858
110	31	146	56266
130	1	4	46144
130	6	28	<b>58919</b>
130	11	52	58802
130	16	77	57656
130	21	102	56094
130	26	125	54309
130	31	148	52414

```
REM
REM OCC0403
REM Peter Lohmander
REM
OPEN "outOCC.dat" FOR OUTPUT AS #1
PRINT #1, "  C X1opt topt PVopt h1opt x2opt"
FOR c = 0 TO 3000 STEP 100
  FOPT = -999999
  x1opt = 0
  topt = 0
  pvopt = 0
  h1opt = 0
  x2opt = 0
```

**FOR x1 = 10 TO 150 STEP 5**

**FOR t = 1 TO 61 STEP 1**

**p = 200**

**r = .03**

**s = .05**

**x0 = 300**

**h0 = x0 - x1**

**x2 = 1 / (1 / 400 + (1 / x1 - 1 / 400) \* EXP(-.05 \* t))**

**h1 = x2 - x1**

**multip = 1 / (EXP(r \* t) - 1)**

**pv0 = -c + p \* h0**

**pv1 = -c + p \* h1**

**PV = pv0 + pv1 \* multip**

**IF PV > pvopt THEN x1opt = x1**

**IF PV > pvopt THEN topt = t**

**IF PV > pvopt THEN h1opt = h1**

**IF PV > pvopt THEN x2opt = x2**

**IF PV > pvopt THEN pvopt = PV**

**NEXT t**

**NEXT x1**

**PRINT #1, USING "#####"; c; x1opt; topt; pvopt; h1opt; x2opt**

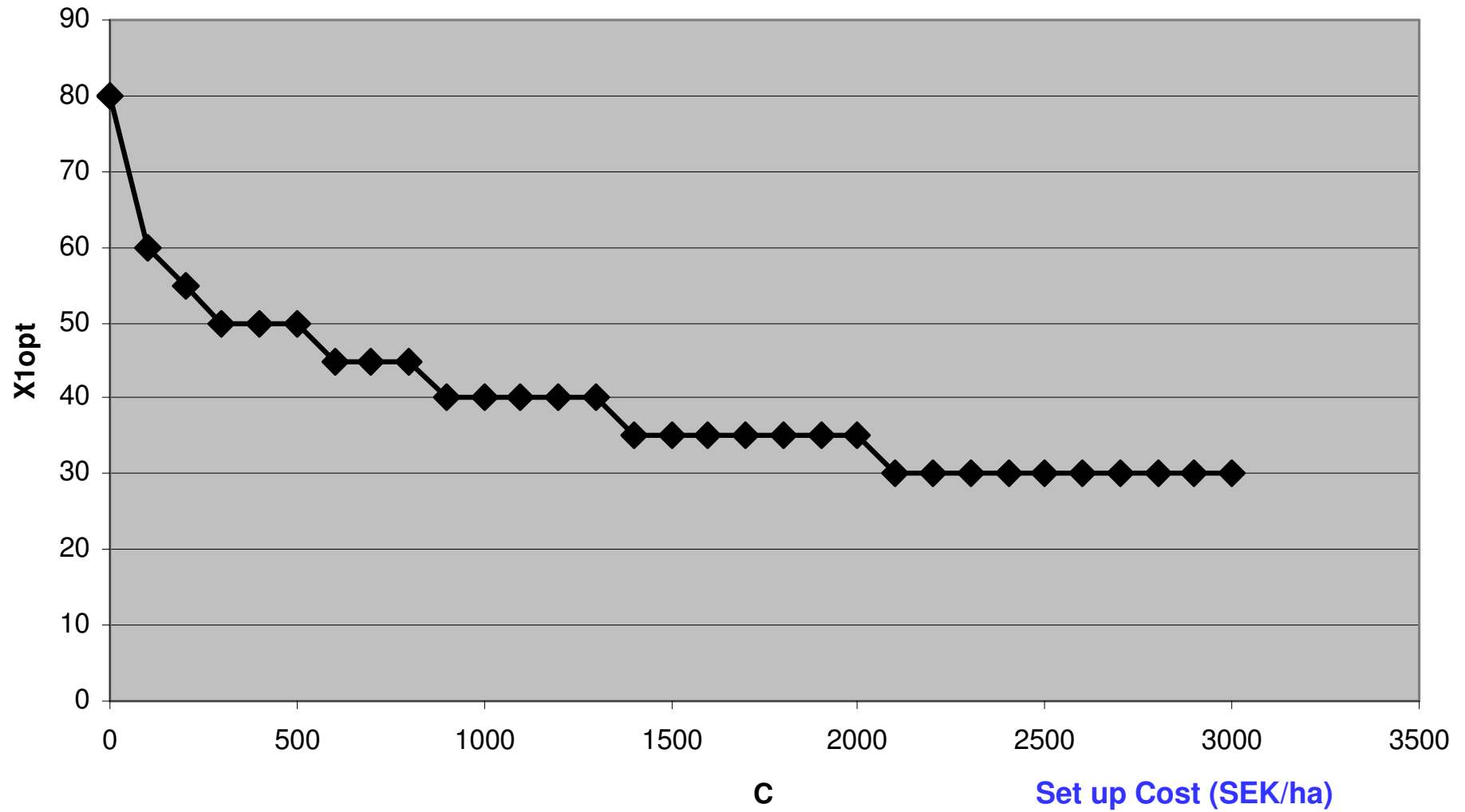
**NEXT c**

**CLOSE #1**

**END**

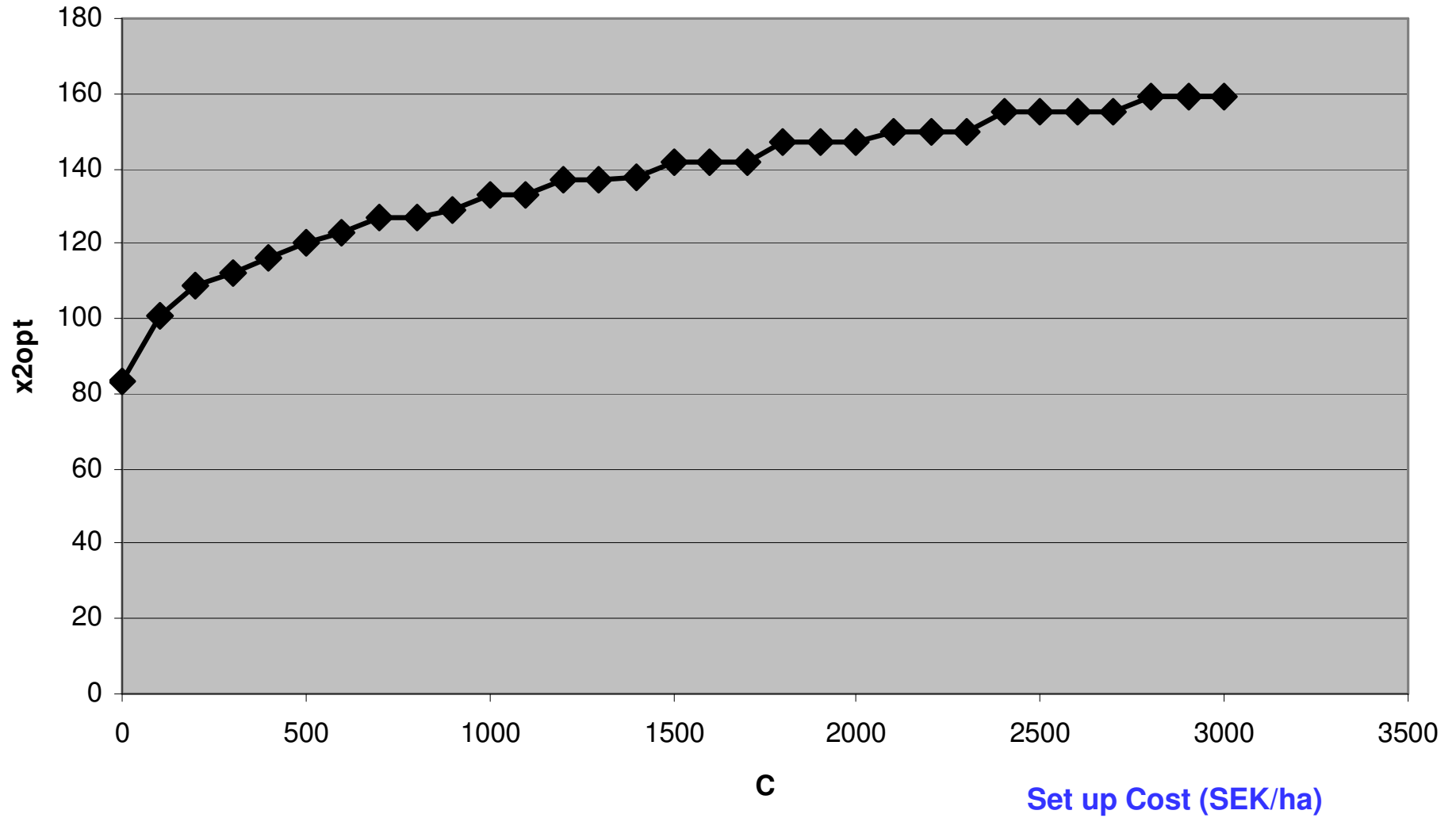
**Optimal Stock Level Directly  
After Harvest (m<sup>3</sup>/ha)**

**X1opt(C)**



**Optimal Stock Level Directly  
Before Thinning Harvest (m<sup>3</sup>/ha)**

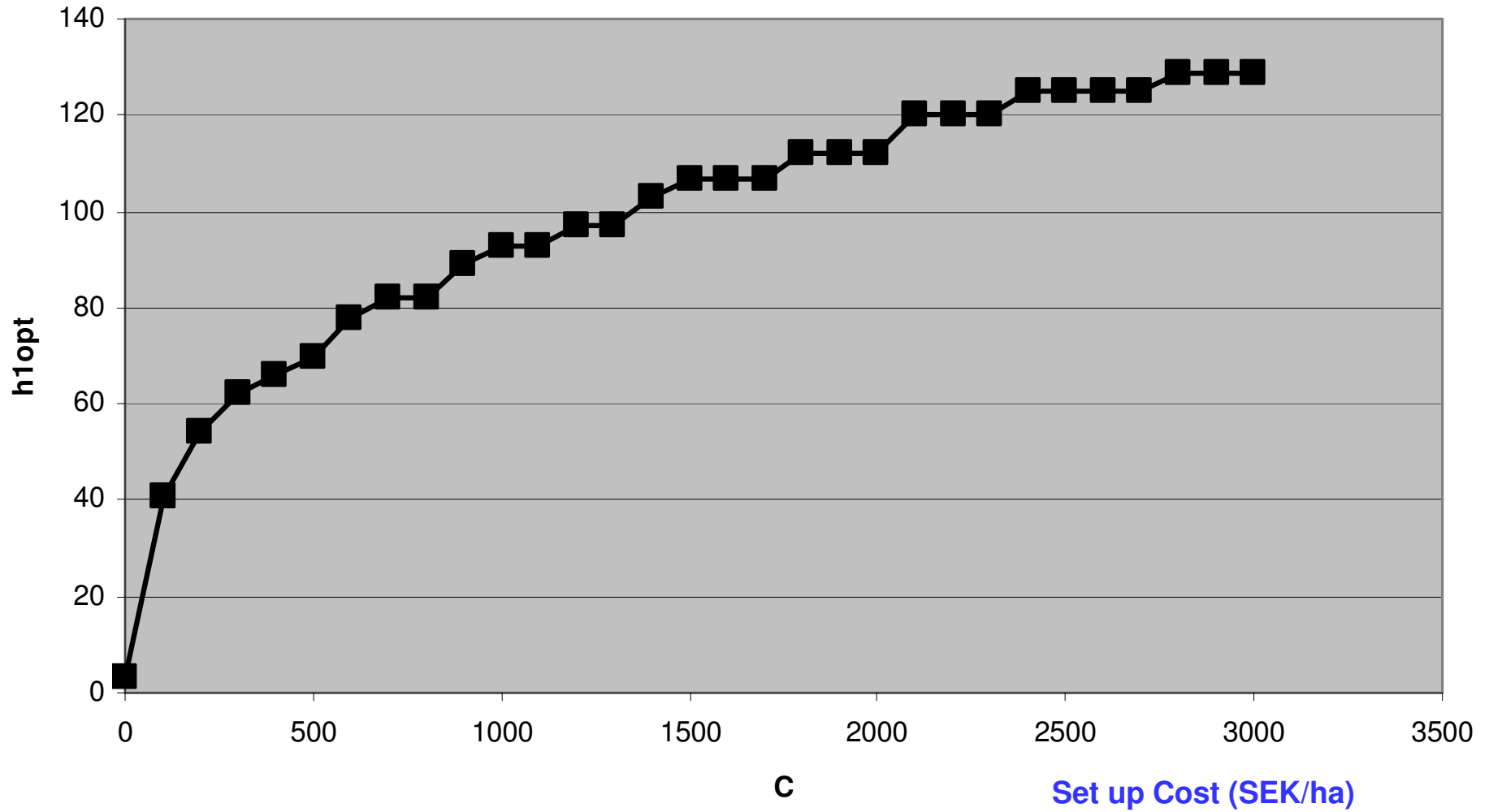
**x<sub>2opt</sub>(C)**





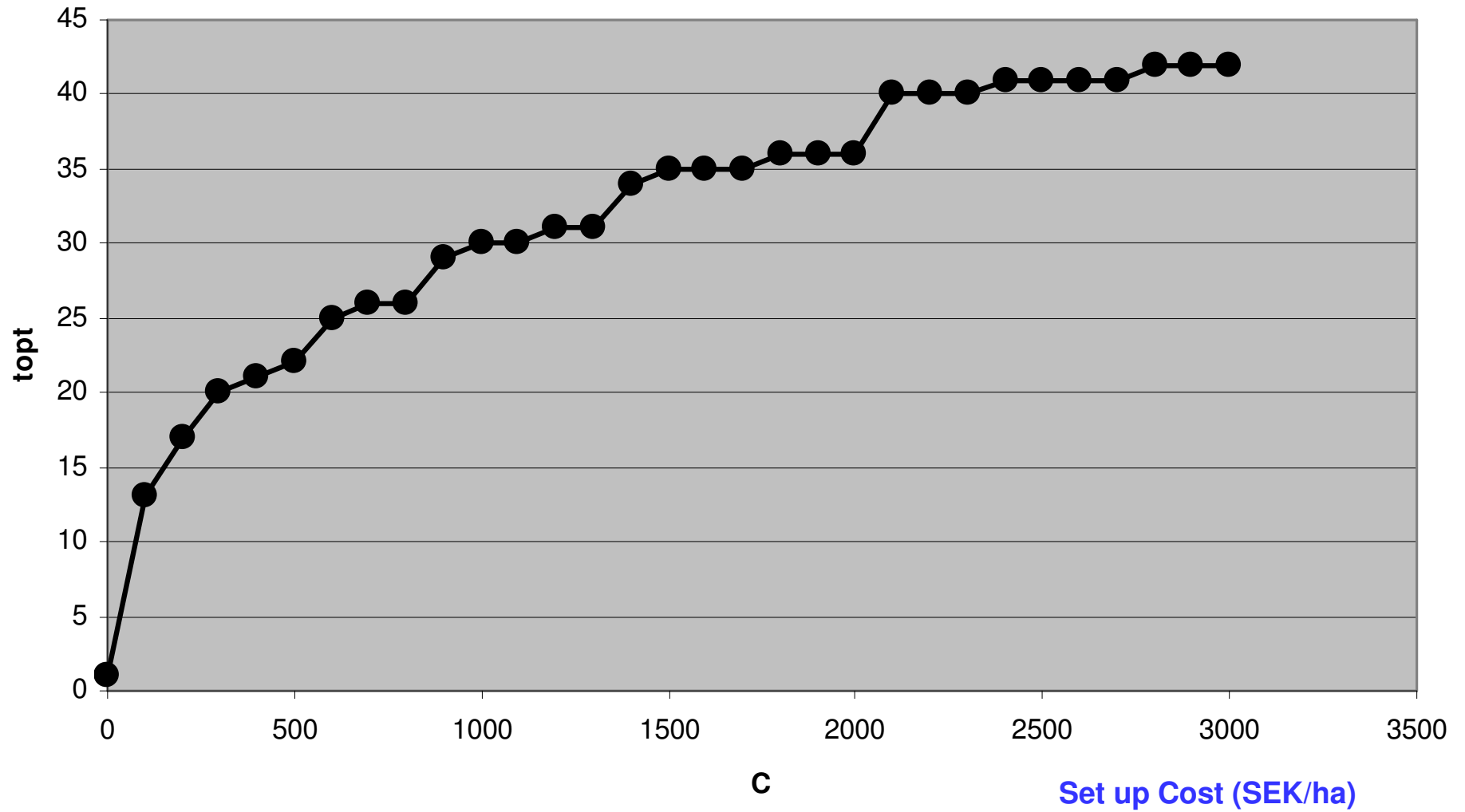
# h1opt(C)

Optimal Thinning Harvest Volumes per occasion (m<sup>3</sup>/ha)



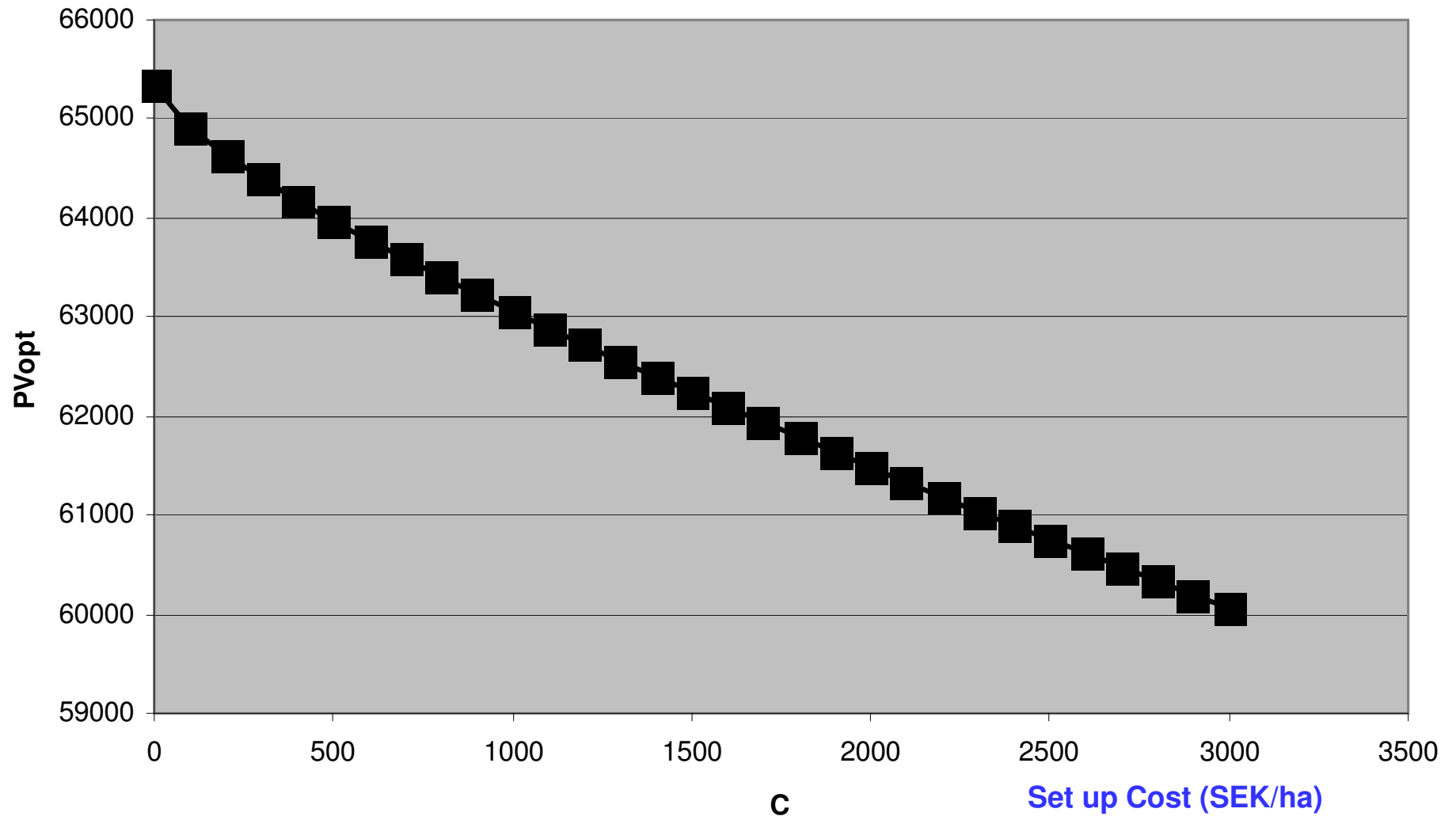
**Optimal Harvest Interval (Years)**

**topt(C)**



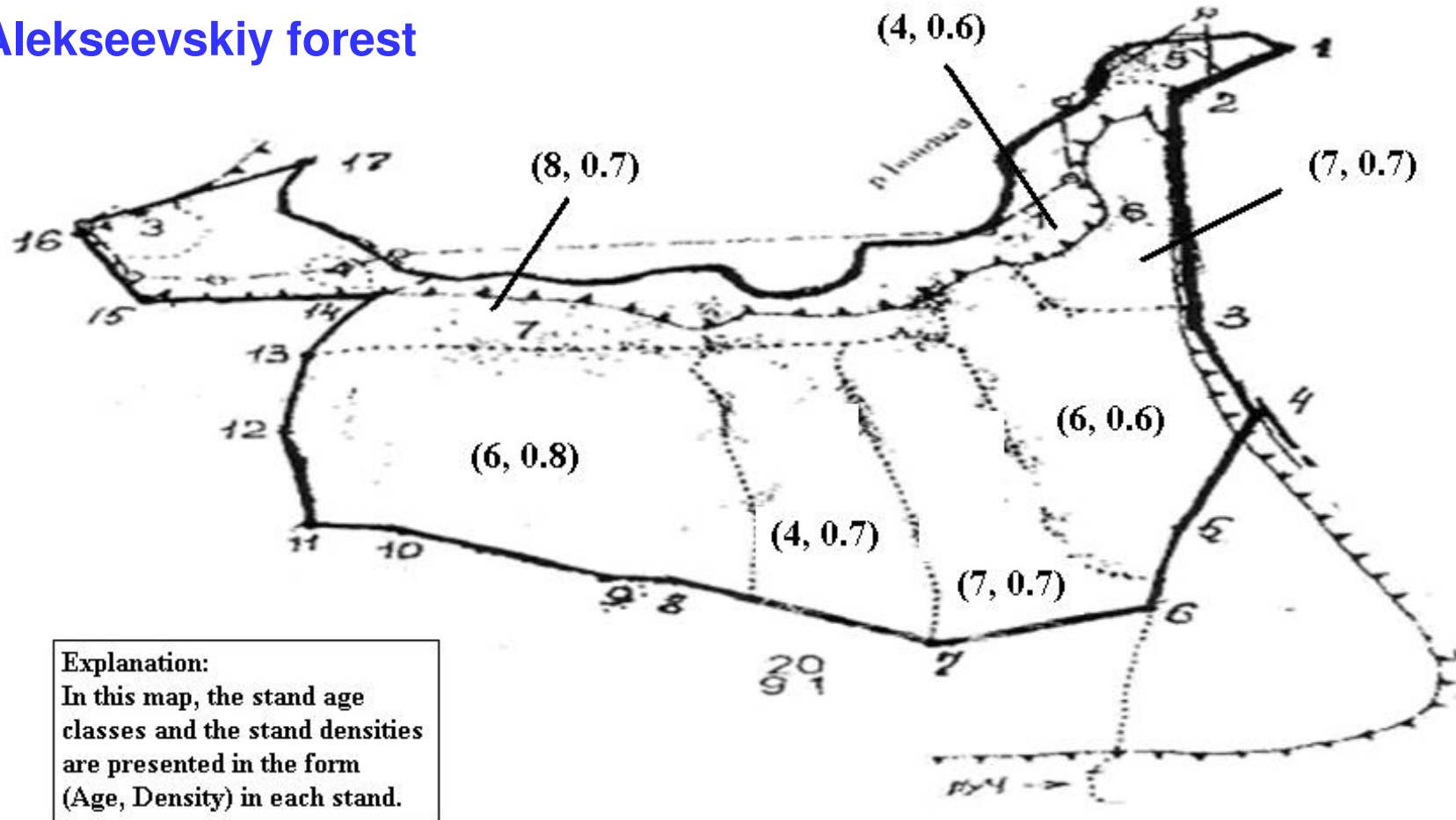
Optimal Present Value (SEK/ha)

PVopt(C)



***What happens to the optimal forest management schedule if we also consider the value of recreation?***

## Alekseevskiy forest



•Source: Zazykina, L., Lohmander, P., The utility of recreation as a function , of site characteristics: Methodological suggestions and a preliminary analysis, Proceedings of the II international workshop on Ecological tourism, trends and perspectives on development in the global world, Saint Petersburg Forest Technical Academy, April 15-16, 2010 , [http://www.Lohmander.com/SPb201004/Zazykina\\_Lohmander\\_SPbFTA\\_2010.pdf](http://www.Lohmander.com/SPb201004/Zazykina_Lohmander_SPbFTA_2010.pdf) [http://www.Lohmander.com/SPb201004/Zazykina\\_Lohmander\\_SPbFTA\\_2010.doc](http://www.Lohmander.com/SPb201004/Zazykina_Lohmander_SPbFTA_2010.doc)

**Table 1. Preferences of tourists concerning the forest density and forest age in Plot 1.**

<u>Question</u>	Which density would you prefer in Plot 1?			How old would you like the forest to be in Plot 1?			
<u>Answer</u>	<u>More density</u>	<u>Less density</u>	<u>Open area</u>	<u>1-20 years</u>	<u>21-49 years</u>	<u>50-100 years</u>	<u>More than 100 years</u>
<u>Yes</u>	3	17	9		3	13	14
<u>Could not answer</u>	1			0			

**Table 2.**

**Preferences of tourists concerning the forest density and forest age in Plot 2.**

<u>Question</u>	Which density would you prefer in Plot 2?			How old would you like the forest to be in Plot 2?			
<u>Answer</u>	<u>More density</u>	<u>Less density</u>	<u>Open area</u>	<u>1-20 years</u>	<u>21-49 years</u>	<u>50-100 years</u>	<u>More than 100 years</u>
<u>Yes</u>	5	16	11		11	10	8
<u>Could not answer</u>	0			1			

- Source: Zazykina, L., Lohmander, P., The utility of recreation as a function , of site characteristics: Methodological suggestions and a preliminary analysis, Proceedings of the II international workshop on Ecological tourism, trends and perspectives on development in the global world, Saint Petersburg Forest Technical Academy, April 15-16, 2010 , [http://www.Lohmander.com/SPb201004/Zazykina\\_Lohmander\\_SPbFTA\\_2010.pdf](http://www.Lohmander.com/SPb201004/Zazykina_Lohmander_SPbFTA_2010.pdf)  
[http://www.Lohmander.com/SPb201004/Zazykina\\_Lohmander\\_SPbFTA\\_2010.doc](http://www.Lohmander.com/SPb201004/Zazykina_Lohmander_SPbFTA_2010.doc)







# Interpretations and observations:

**#1: Several alternative interpretations are possible!**

**#2: Furthermore, the results are most likely sensitive to local conditions, weather conditions etc..**

# **Assumptions:**

The ideal average forest density, from a recreational point of view, is 0.5.

Directly before thinning, the density is 0.8 .

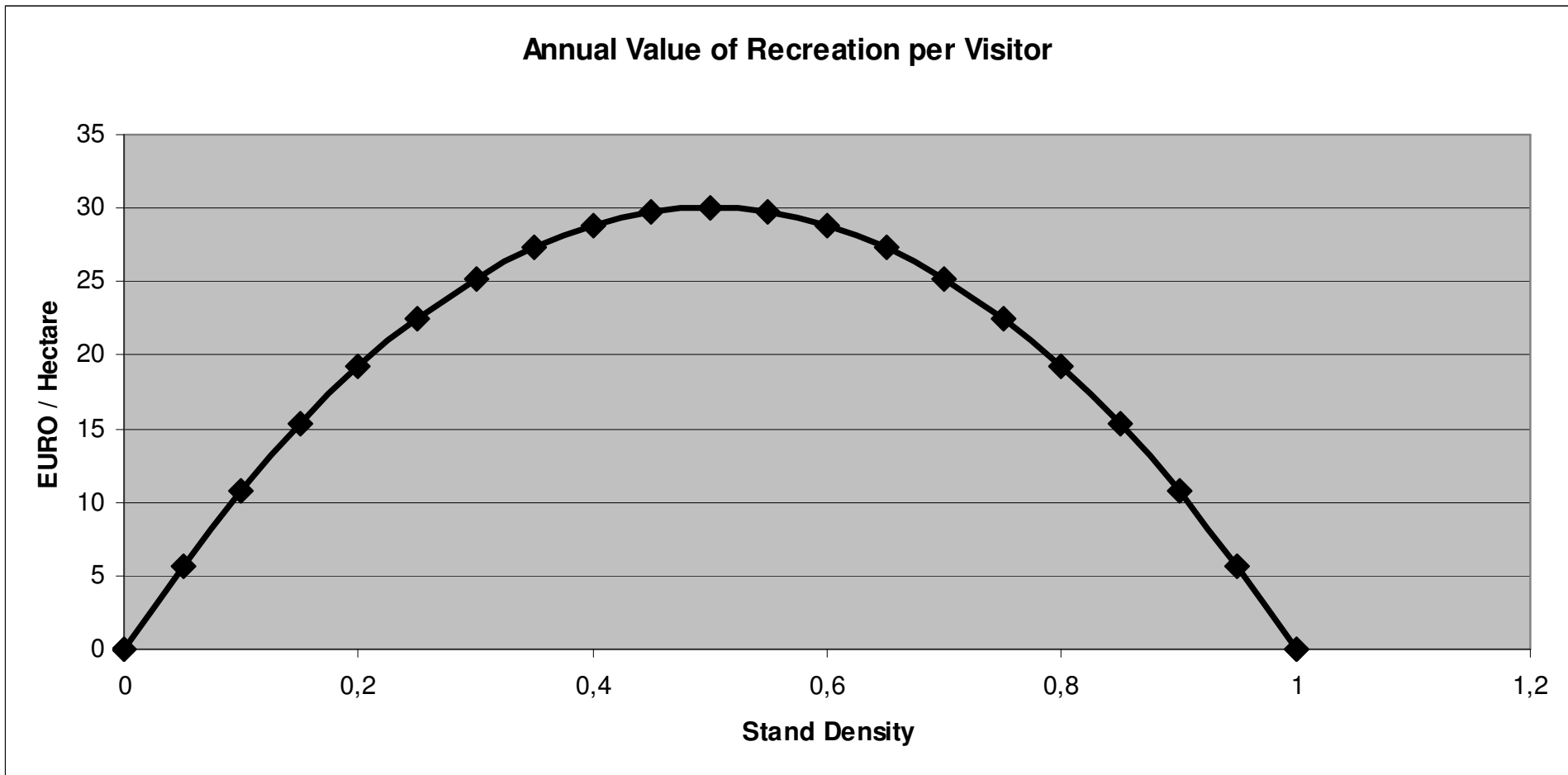
As a result of a thinning, the density in a stand is reduced in proportion to the harvest volume.

The density of a stand is a linear function of time between thinnings.

The value of recreation is a quadratic function of average stand density in the forest area.

The recreation value is zero if the density is 0 or 1.

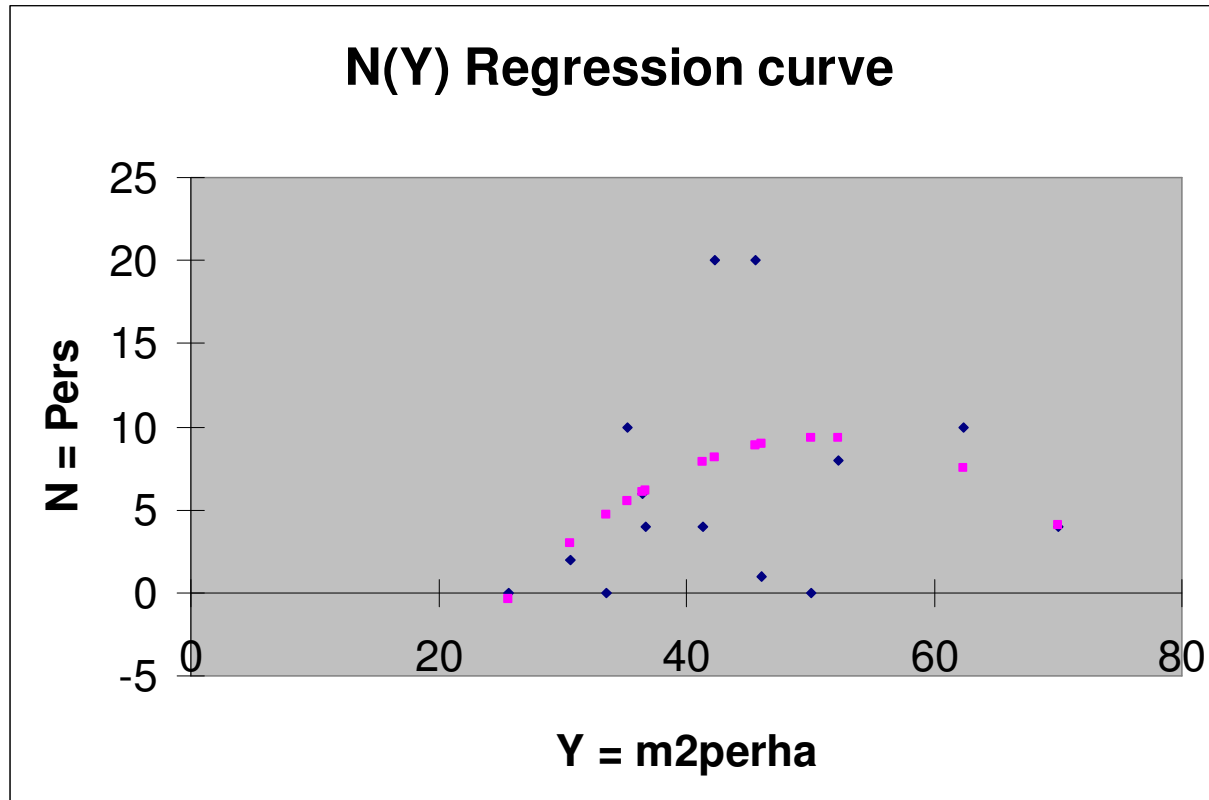
Under optimal density conditions, the value of recreation, per individual, hectare and year, is 30 EURO. (The value 30 has no empirical background.)



***Approximation in the software:***

- $D = .8 * ((x1 + x2) / (2 * x2))$
- IF  $D < 0$  THEN  $D = 0$
- IF  $D > 1$  THEN  $D = 1$
- $U = 120 * D - 120 * D * D$
- $PV_{totU} = n / r * U$

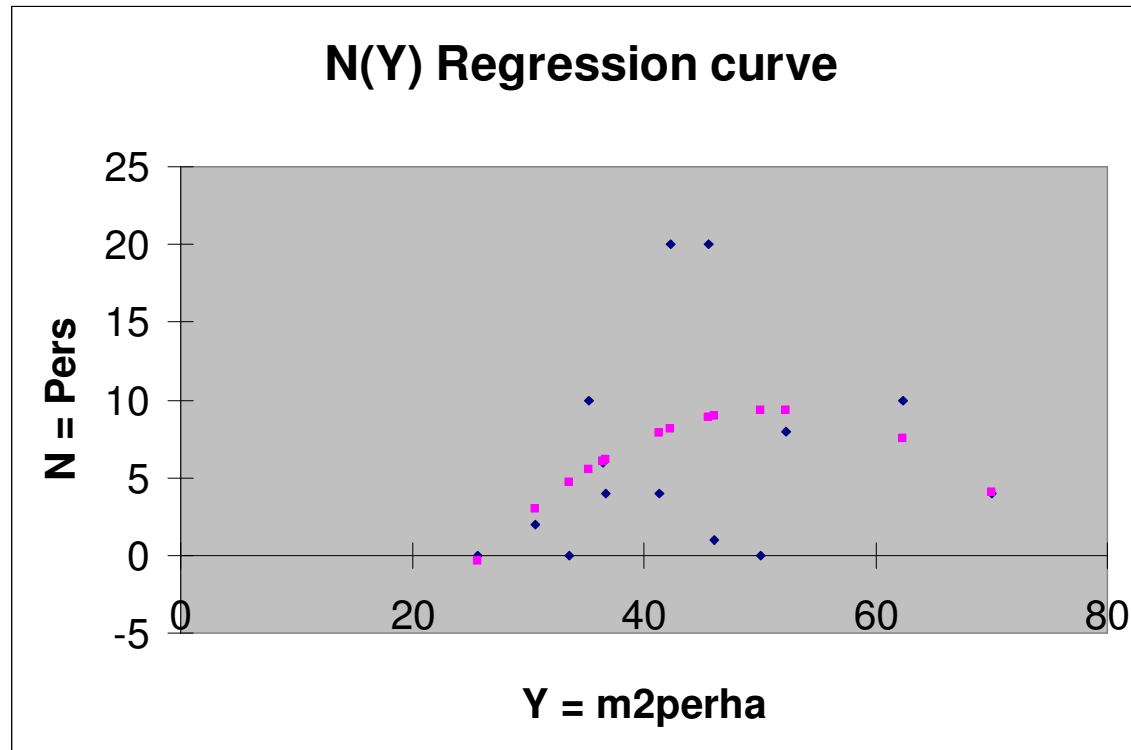
**Forest visitors seem to prefer forests with rather high basal area levels during hot periods.**



**N(Y) = Number of persons per 100 m2 a function of  
Y = Basal Area (m2/ha), Moscow**

***DURING THE VERY HOT SUMMER OF 2010***

$$N = -29.6 + 1.52 Y - 0.0148 Y^2$$



*N = Number of persons per 100 m<sup>2</sup>*

*Y = Basal area (m<sup>2</sup>/ha)*

<b>Pers</b>	<b>SSQ</b>	<b>SSQ2</b>	<b>m2perha</b>	<b>m2perha2</b>
10	79,4379	6310,38	62,39039	3892,56
4	52,63854	2770,815	41,34221	1709,179
2	39,02389	1522,864	30,64929	939,3791
0	32,72452	1070,894	25,70178	660,5816
10	44,95064	2020,56	35,30415	1246,383
4	46,73589	2184,243	36,70628	1347,351
0	63,83041	4074,322	50,1323	2513,247
0	42,76194	1828,584	33,58515	1127,963
1	58,73646	3449,972	46,13152	2128,117
20	53,79857	2894,287	42,25331	1785,342
8	66,57643	4432,421	52,28901	2734,141
6	46,4379	2156,478	36,47224	1330,225
20	58,10828	3376,572	45,63814	2082,84
4	89,09952	7938,725	69,97861	4897,006

SUMMARY OF RESULTS FROM THE REGRESSION

Regressionsstatistik

Multipel-R	0,415973
R-kvadrat	0,173034
Justerad R	0,022676
Standardfe	6,644249
Observatio	14

ANOVA

	<i>fg</i>	<i>KvS</i>	<i>Mkv</i>	<i>F</i>	<i>t-värde för F</i>
Regressior	2	101,6078	50,80392	1,150815	0,351709
Residual	11	485,6064	44,14604		
Totalt	13	587,2143			

	<i>Koefficiente</i>	<i>Standardfe.</i>	<i>t-kvot</i>	<i>p-värde</i>	<i>Nedre 95%</i>	<i>Övre 95%</i>	<i>Nedre 95,0%</i>	<i>Övre 95,0%</i>
Konstant	-29,6429	24,27584	-1,22109	0,24758	-83,0737	23,78786	-83,0737	23,78786
m2perha	1,520081	1,059533	1,434671	0,179195	-0,81194	3,852098	-0,81194	3,852098
m2perha2	-0,01483	0,010986	-1,34982	0,204197	-0,03901	0,009351	-0,03901	0,009351

**Observations:**

The parameter values estimated (above) are consistent with a strictly concave function  $N(Y)$ .

Note, however, that the number of observations is low.

With a larger number of observations and with similar general conditions

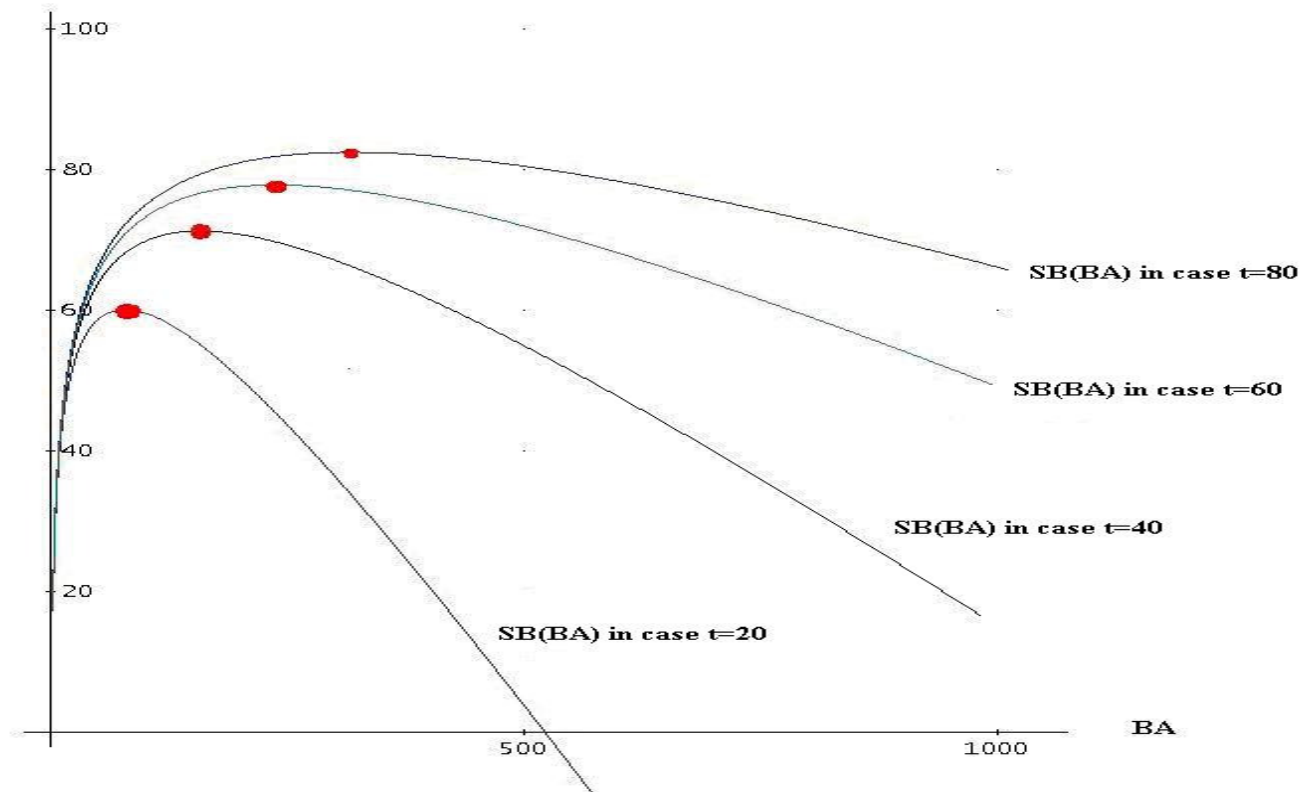
it is likely that the standard errors of the estimated parameters would decrease considerably and that the t-values would increase in absolute values.

# What was the **most popular basal area** during the hot summer of 2010 from a recreational point of view?

<p><b>N = Number of persons per 100 m<sup>2</sup></b>  <b>Y = Basal Area (m<sup>2</sup>/ha)</b></p>	<p>Definitions</p>
<p><math>N = -29,6 + 1,52 Y - 0,0148 YY</math></p>	<p><math>N(Y)</math></p>
<p><math>dN/dY = 1,52 - 0,0296 Y</math></p>	<p>Optimization of <math>N(Y)</math></p>
<p><math>dN/dY = 0</math></p>	<p>First order optimum condition</p>
<p><math>1,52 = 0,0296 Y</math></p>	
<p><math>Y = 1,52/0,0296</math></p>	
<p><math>Y = 51,4</math></p>	<p>Optimal value of Y</p>
<p><math>d^2N/dY^2 = -0,0296</math></p>	<p>Second order condition</p>
<p><math>d^2N/dY^2 &lt; 0</math></p>	<p>Unique maximum condition</p>
<p>Unique maximum</p>	



Scenic Beauty = SB(BA)



Scenic Beauty, SB, as a function of basal area, BA. The graph has been constructed using equation  $SB = 5.663 - 4.086 BA/t + 16.148 \ln (BA)$ , which is found in Hull & Buhyoff (1986).

**Optimization of present value of  
roundwood production and  
recreation**

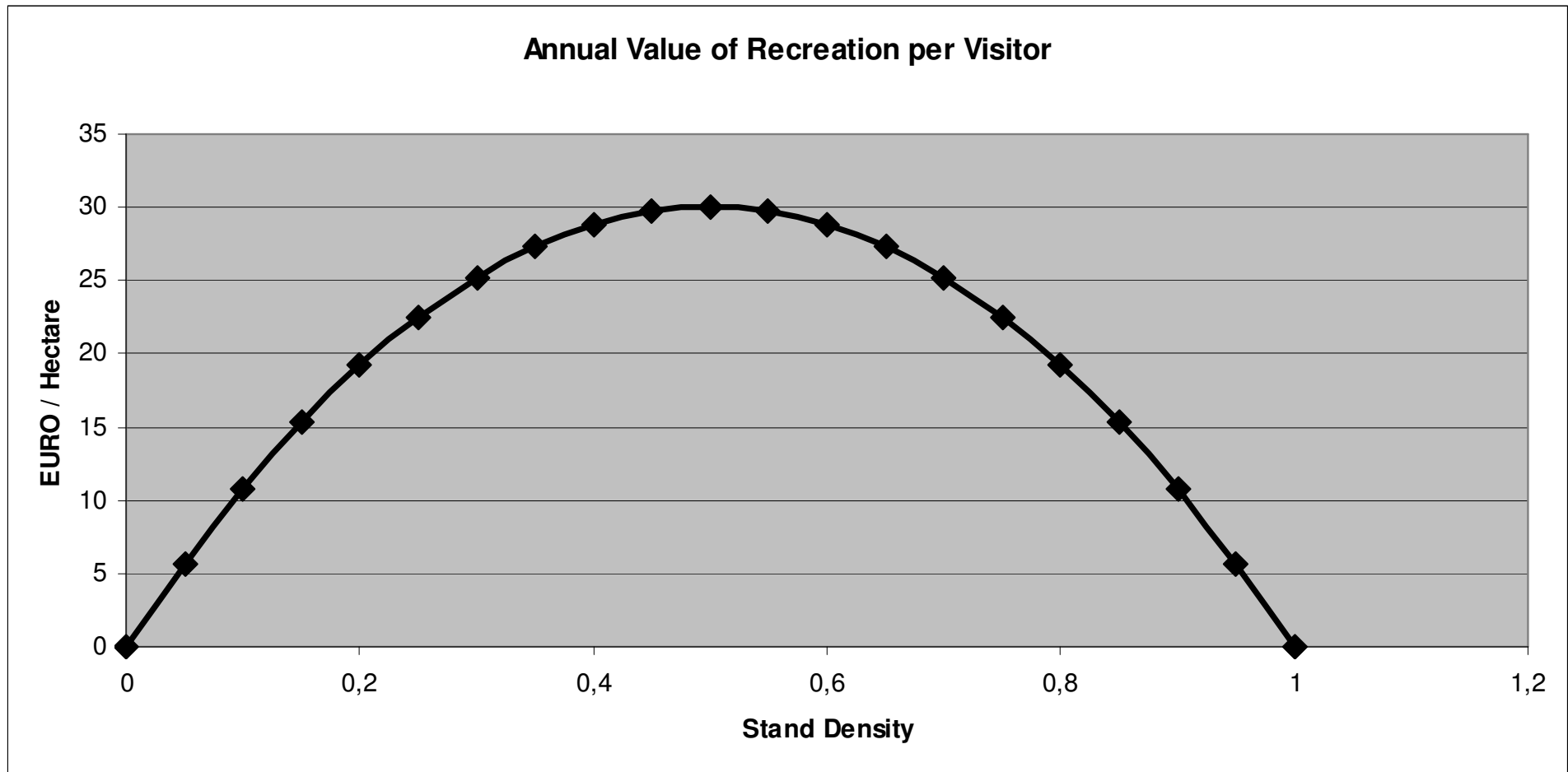
- REM OP100409
- REM Peter and Luba
- REM
- OPEN "outOP.txt" FOR OUTPUT AS #1
- PRINT #1, "    n x1opt topt h1opt x2opt  
pvopt    optPV    opttotU"

- **FOR n = 0 TO 550 STEP 55**

- **pvopt = -9999999**
- **optpv = -9999999**
- **x1opt = 0**
- **topt = 0**
- **h1opt = 0**
- **x2opt = 0**
- **c = 50**
- **p = 40**
- **r = .03**

- **FOR x1 = 10 TO 150 STEP 5**
- **FOR t = 1 TO 100 STEP 1**
- **x0 = 158**
- **h0 = x0 - x1**
- **x2 = 1 / (1 / 316 + (1 / x1 - 1 / 316) \* EXP(-.0848 \* t))**
- **h1 = x2 - x1**
- **multip = 1 / (EXP(r \* t) - 1)**
- **pv0 = -c + p \* h0**
- **pv1 = -c + p \* h1**
- **PV = pv0 + pv1 \* multip**

- $D = .8 * ((x1 + x2) / (2 * x2))$
- **IF  $D < 0$  THEN  $D = 0$**
- **IF  $D > 1$  THEN  $D = 1$**
- $U = 120 * D - 120 * D * D$
- $PV_{totU} = n / r * U$
  
- **$TPV = PV + PV_{totU}$**
  
- **IF  $TPV > p_{vopt}$  THEN  $x1_{opt} = x1$**
- **IF  $TPV > p_{vopt}$  THEN  $t_{opt} = t$**
- **IF  $TPV > p_{vopt}$  THEN  $h1_{opt} = h1$**
- **IF  $TPV > p_{vopt}$  THEN  $x2_{opt} = x2$**
- **IF  $TPV > p_{vopt}$  THEN  $opt_{pv} = PV$**
- **IF  $TPV > p_{vopt}$  THEN  $opt_{totU} = PV_{totU}$**
- **IF  $TPV > p_{vopt}$  THEN  $p_{vopt} = TPV$**
  
- **NEXT  $t$**
- **NEXT  $x1$**



***Approximation in the software:***

- $D = .8 * ((x1 + x2) / (2 * x2))$
- IF  $D < 0$  THEN  $D = 0$
- IF  $D > 1$  THEN  $D = 1$
- $U = 120 * D - 120 * D * D$
- $PV_{tot}U = n / r * U$

- **PRINT #1, USING "#####"; n; x1opt; topt;**
- **PRINT #1, USING "#####"; h1opt; x2opt;**
- **PRINT #1, USING "#####.##"; pvopt; optpv;  
opttotU**
  
- **NEXT n**
  
- **CLOSE #1**
- **END**

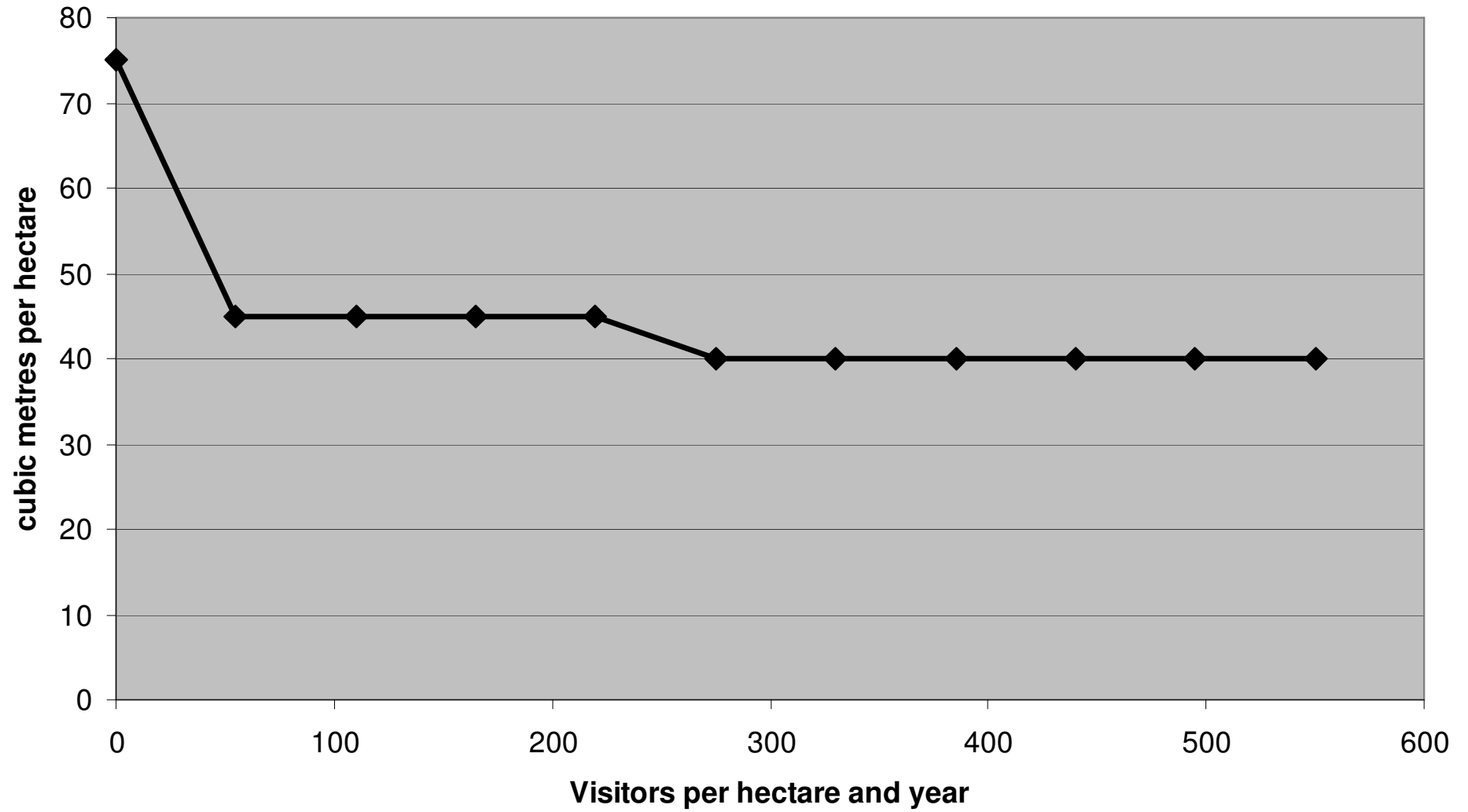
# Optimal results: (outOP.txt)

n	xlopt	topt	hlopt	x2opt	pvopt	optPV	opttotU
0	75	10	58	133	9756.82	9756.82	0.00
55	45	22	119	164	64466.54	9488.83	54977.71
110	45	23	125	170	119445.47	9460.00	109985.46
165	45	23	125	170	174438.19	9460.00	164978.19
220	45	23	125	170	229430.92	9460.00	219970.92
275	40	23	119	159	284429.47	9429.57	274999.91
330	40	23	119	159	339429.44	9429.57	329999.88
385	40	23	119	159	394429.44	9429.57	384999.88
440	40	23	119	159	449429.41	9429.57	439999.84
495	40	23	119	159	504429.38	9429.57	494999.81
550	40	23	119	159	559429.38	9429.57	549999.81



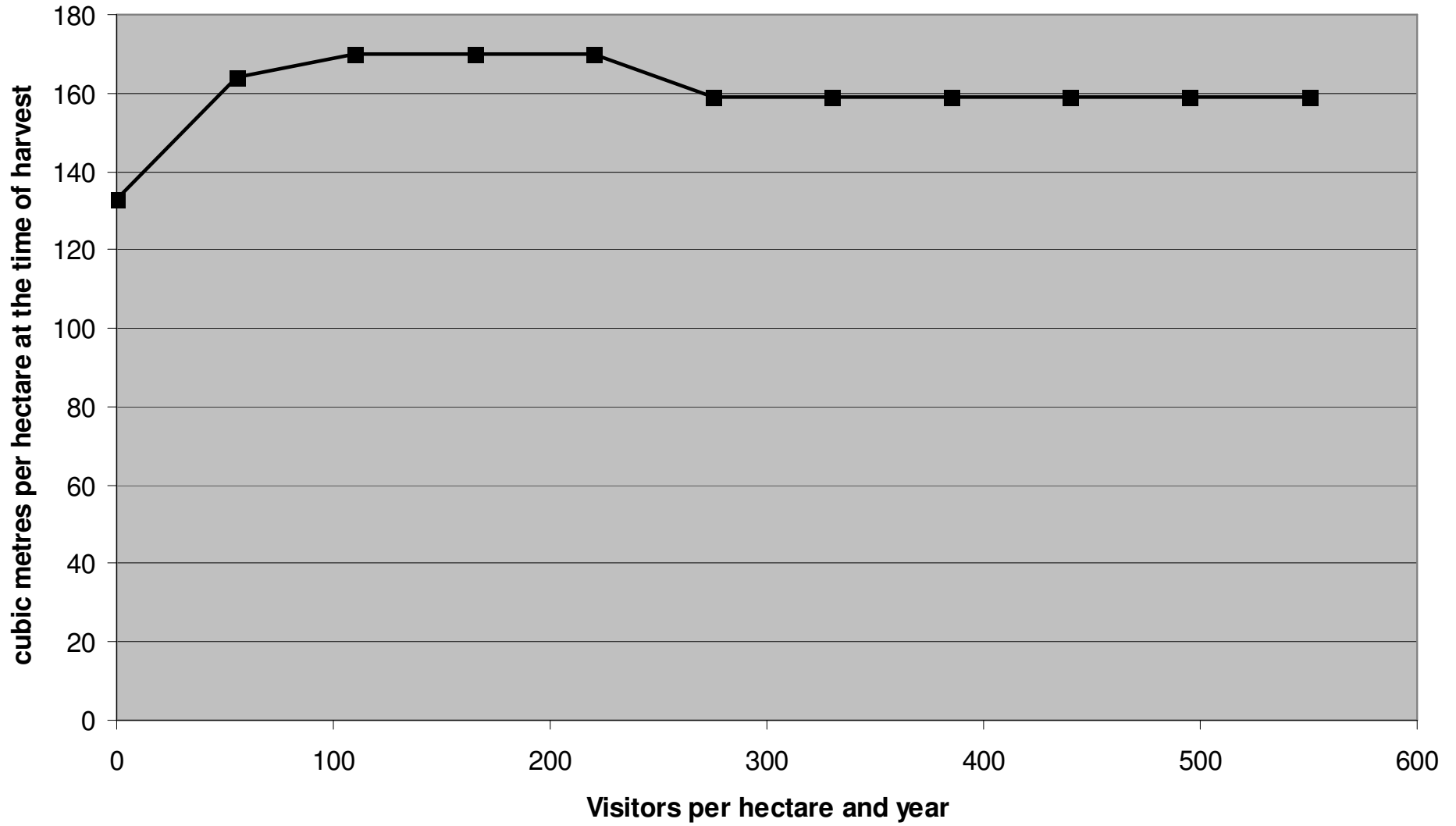
# x1opt

Optimal stock level directly after harvest



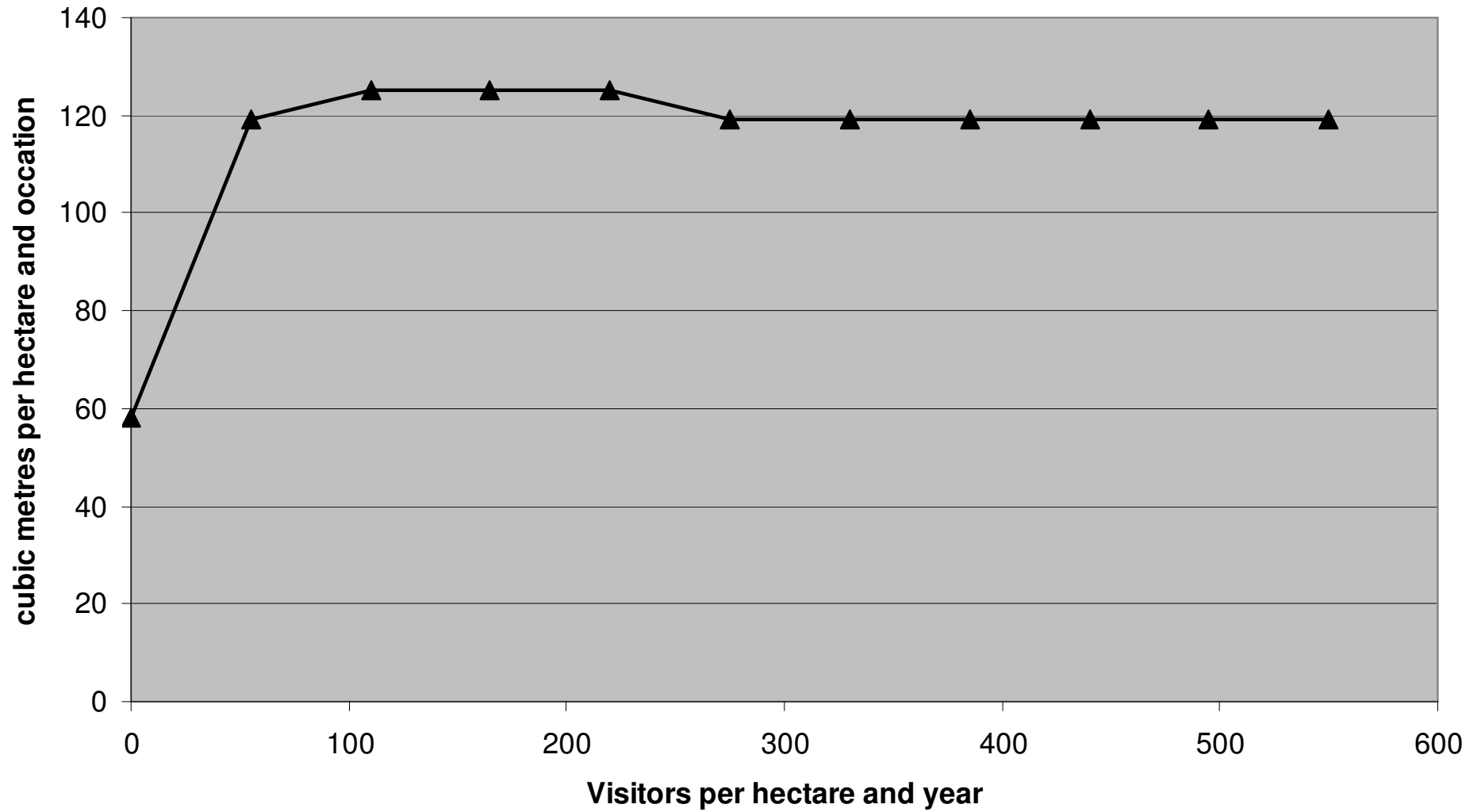
# x2opt

Optimal stock level directly before harvest



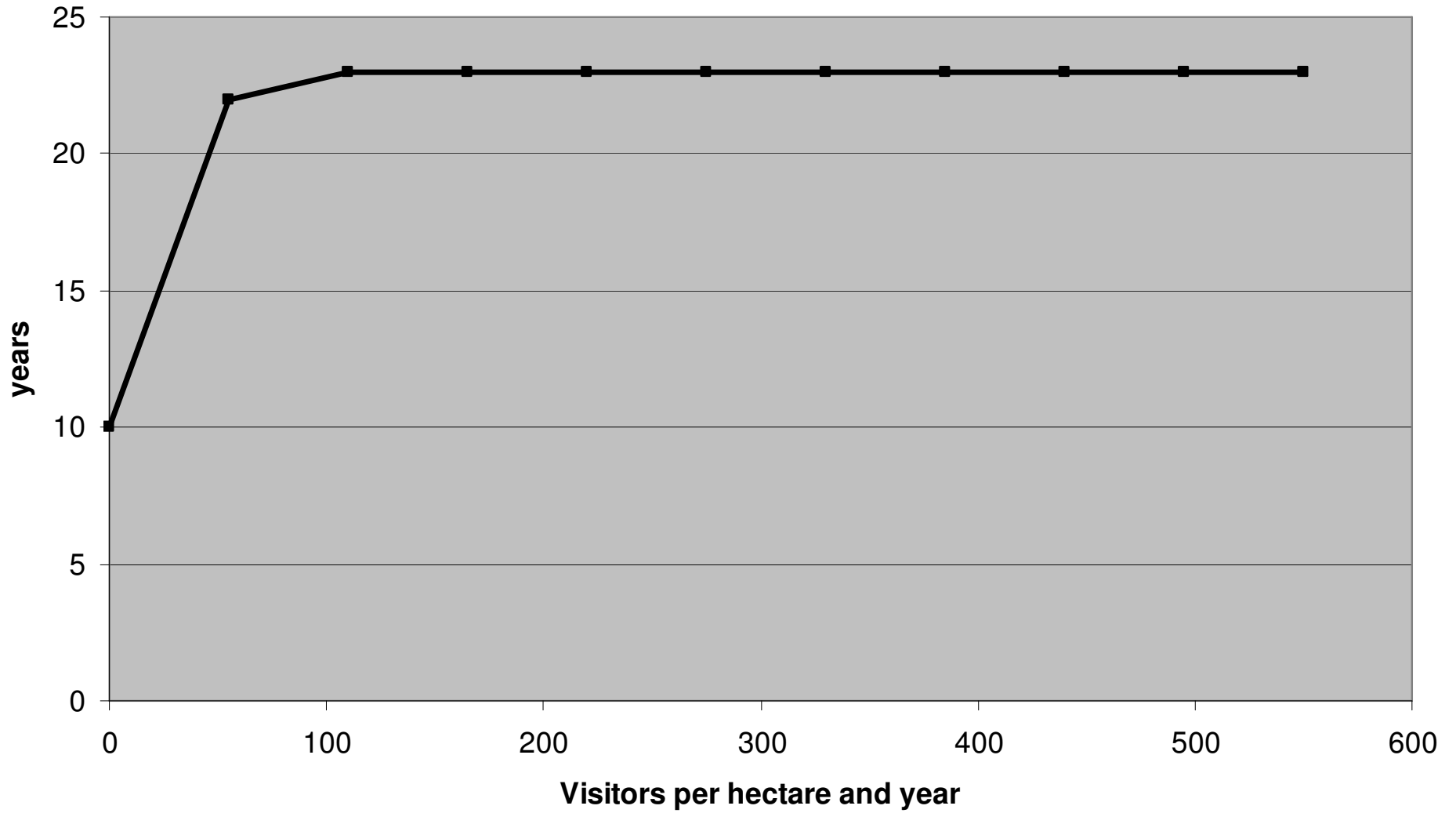
# h1opt

Optimal thinning volume per occation

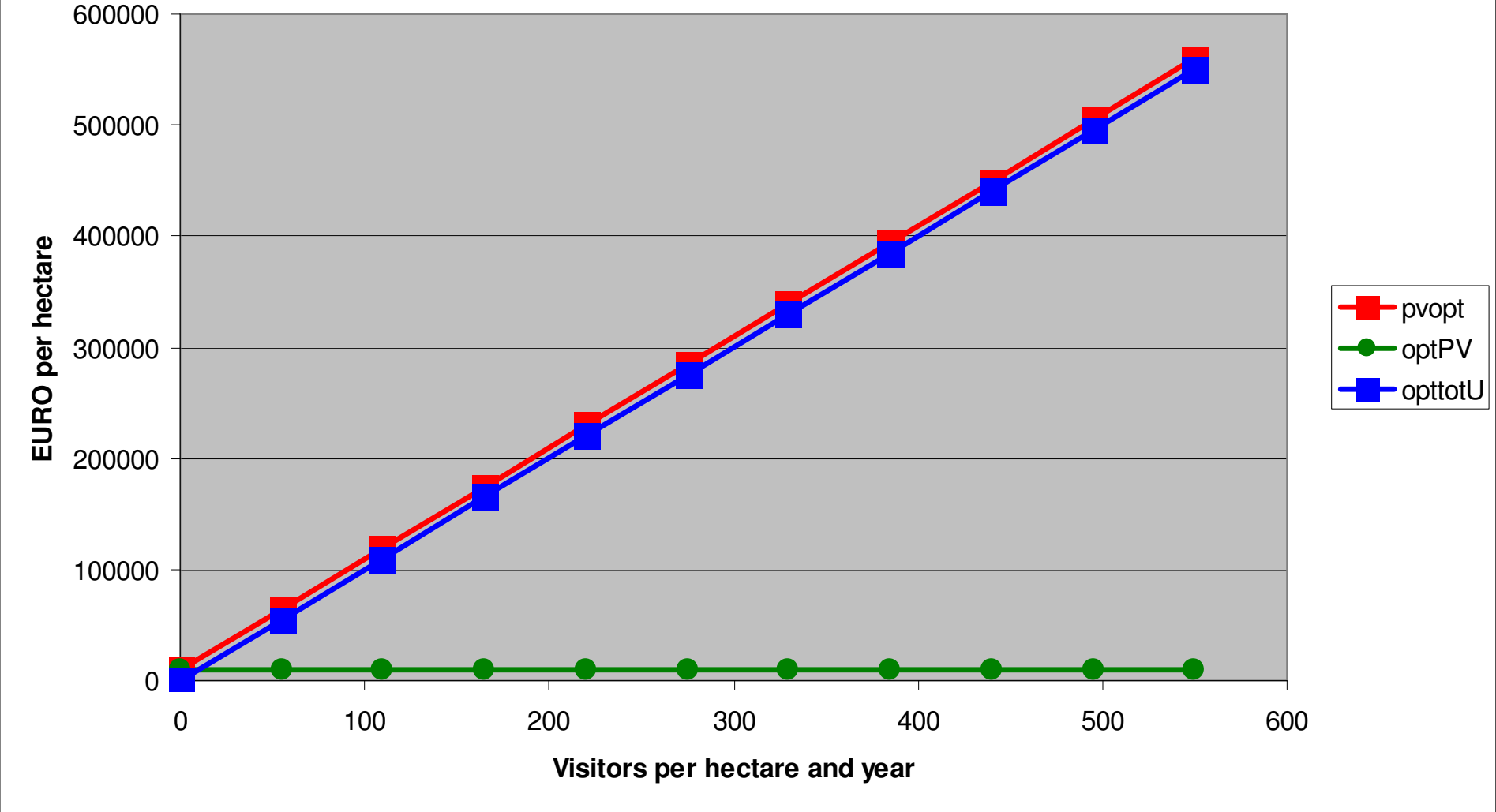


# topt

Optimal time interval between thinnings

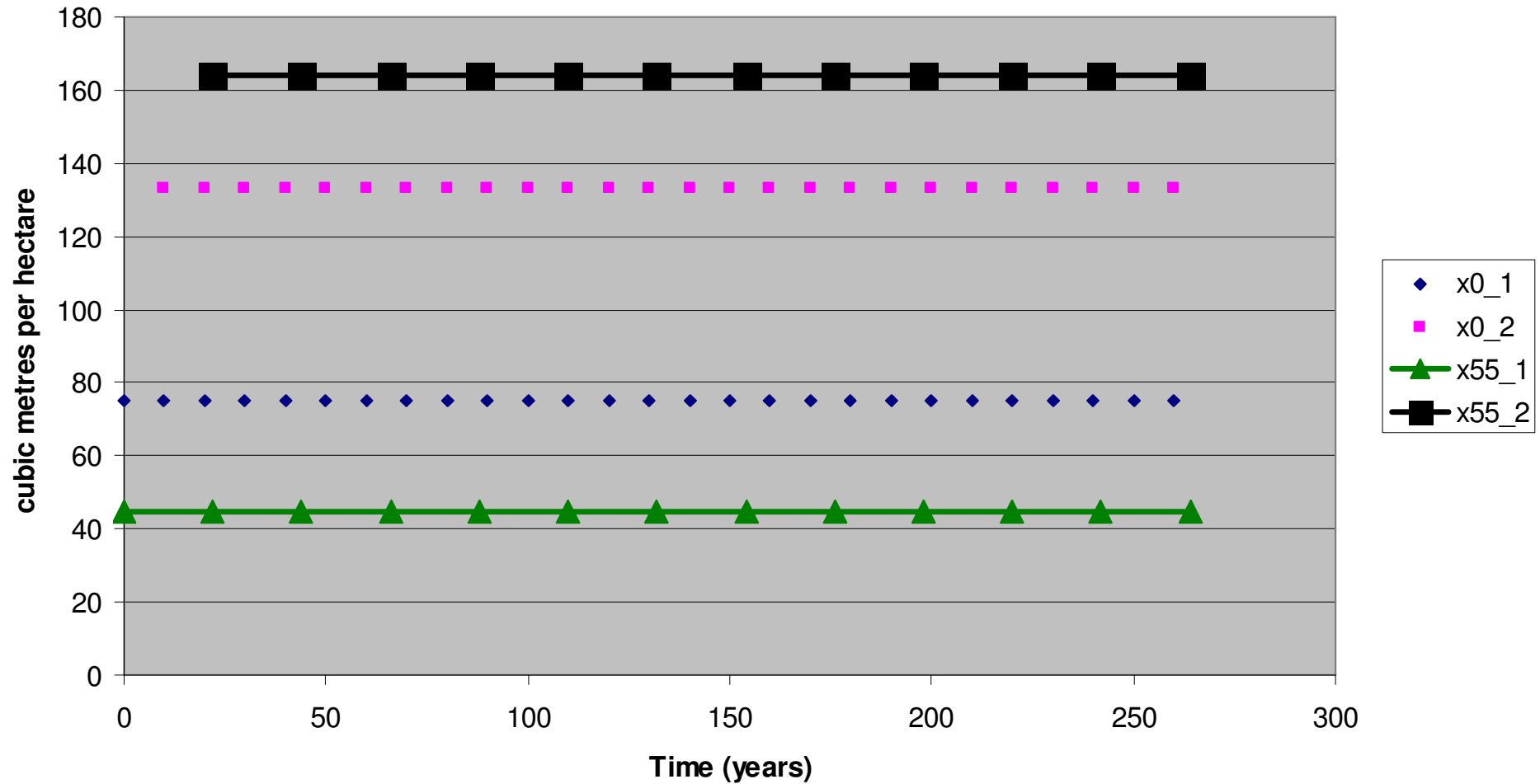


# Present values of forest harvesting and recreation

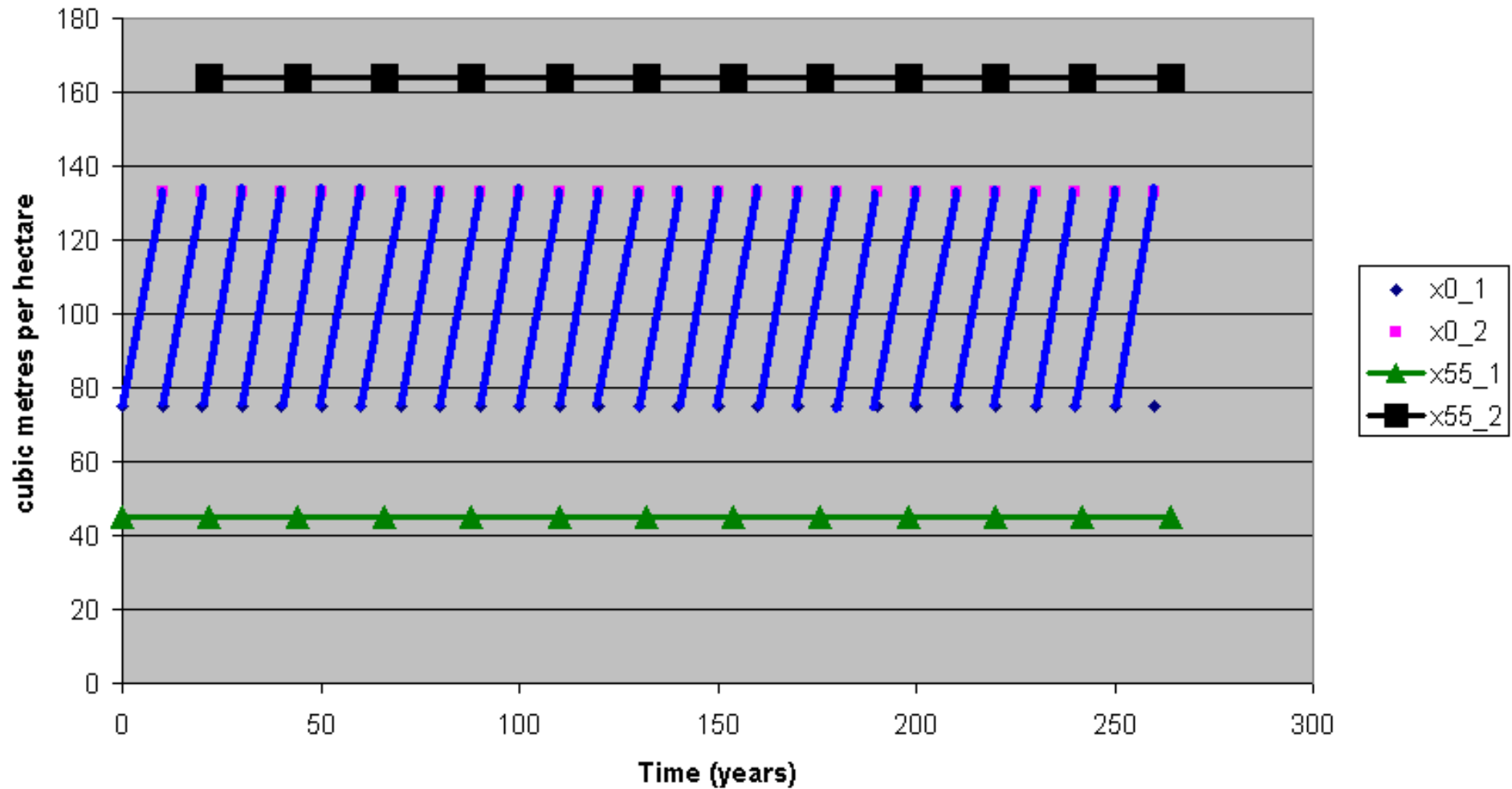


*In forest areas with many visitors, close to large cities, the present value of recreation may be much higher than the present value of timber production.*

# Upper and lower optimal stock levels as a function of time and the number of visitors

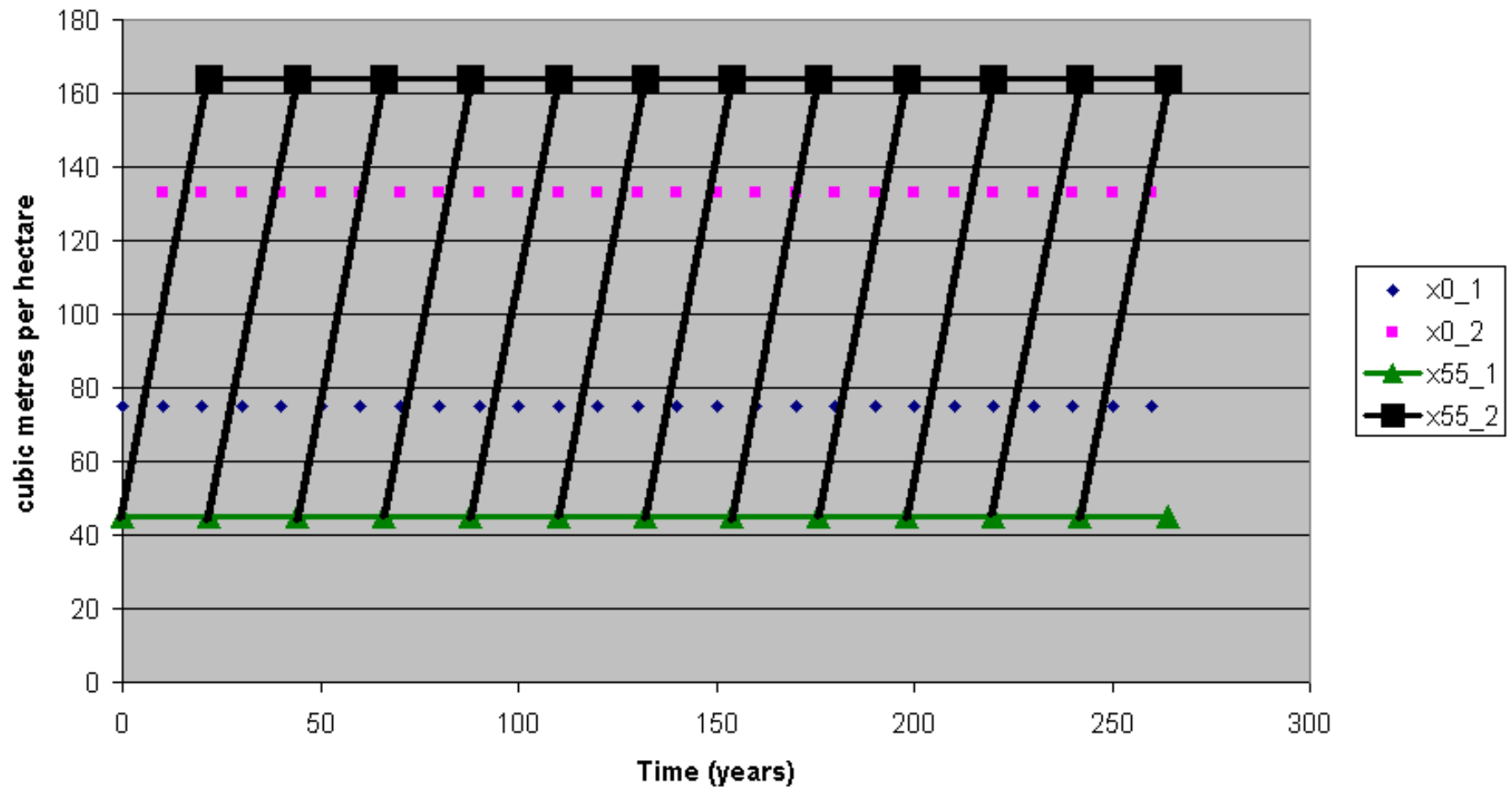


# Upper and lower optimal stock levels as a function of time and the number of visitors



The case with no visitors

## Upper and lower optimal stock levels as a function of time and the number of visitors



**The case with many visitors that prefer low density forests**



# ***Conclusions:***

**A new methodological approach to optimization of sustainable continuous cover forest management with consideration of recreation and the forest and energy industries has been developed.**

**It maximizes the total present value of continuous cover forest management and takes all relevant costs and revenues into account, including set up costs.**

**Optimal solutions to some investigated cases have been analysed and reported.**

## ABSTRACT Page 1(4)

Peter Lohmander and Liubov Zazykina:  
(Peter Lohmander, Professor, Swed.Univ.Agr.Sci., Sweden and Liubov Zazykina, PhD  
Student, Moscow State Forest University, Russia)

Title: **Dynamic economical optimization of sustainable forest harvesting in Russia with consideration of energy, other forest products and recreation.**

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[lyubovzazykina@rambler.ru](mailto:lyubovzazykina@rambler.ru)

**Forests are used for many different purposes. It is important to consider these simultaneously.**

A new methodological approach to optimization of forest management with consideration of recreation and the forest and energy industries has been developed. It maximizes the total present value of continuous cover forest management and takes all relevant costs and revenues into account, including set up costs.

**In several regions, in particular close to large cities, such as Paris and Moscow, the economic importance of recreation forestry is very high in relation to the economic results obtained from traditional “production oriented” forest management.**

## ABSTRACT Page 2(4)

This does however not automatically imply that production of timber, pulpwood and energy assortments can not be combined with rational recreation forestry. On the contrary:

**It is sometimes necessary to harvest and to produce some raw materials that can be utilized by the forest products industry and/or the energy industry, in order to avoid that the forest density increases to a level where most kinds of forest recreation becomes impossible, at least for large groups of recreation interested individuals.**

The optimization model includes one section where the utility of recreation, which may be transformed to the present value of net revenues from recreation, is added to the traditional objective function of the present value of the production of timber, pulpwood and energy assortments.

## ABSTRACT Page 3(4)

In several situations, individuals interested in recreation prefer forests with low density.

**This means that forest management that is optimal when all objectives are considered, typically is characterized by larger thinning harvests than forest management that only focuses on the production of timber, pulpwood and energy assortments.**

## ABSTRACT Page 4(4)

**The results also show that large set up costs have the same type of effect on optimal forest management as an increasing importance of typical forms of recreation, close to large cities.**

**Both of these factors imply that the harvest volumes per occasion increase and that the time interval between harvests increases.**

Even rather small set up costs imply that the continuous cover forest management schedule gives a rather large variation in the optimal stock level over time.

The general analysis of the optimization problems analysed within this study is based on differential equations describing forest growth, in combination with two dimensional optimization of the decisions “harvest interval” and “stock level directly after harvest”. All of the other variables are explicit functions of these decisions.

## ABSTRACT

Peter Lohmander and Liubov Zazykina:

(Peter Lohmander, Professor, Swed.Univ.Agr.Sci., Sweden and Liubov Zazykina, PhD Student, Moscow State Forest University, Russia)

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[lyubovzazykina@rambler.ru](mailto:lyubovzazykina@rambler.ru)

Forests are used for many different purposes. It is important to consider these simultaneously. A new methodological approach to optimization of forest management with consideration of recreation and the forest and energy industries has been developed. It maximizes the total present value of continuous cover forest management and takes all relevant costs and revenues into account, including set up costs. In several regions, in particular close to large cities, such as Paris and Moscow, the economic importance of recreation forestry is very high in relation to the economic results obtained from traditional "production oriented" forest management. This does however not automatically imply that production of timber, pulpwood and energy assortments can not be combined with rational recreation forestry. On the contrary: It is sometimes necessary to harvest and to produce some raw materials that can be utilized by the forest products industry and/or the energy industry, in order to avoid that the forest density increases to a level where most kinds of forest recreation becomes impossible, at least for large groups of recreation interested individuals.

The optimization model includes one section where the utility of recreation, which may be transformed to the present value of net revenues from recreation, is added to the traditional objective function of the present value of the production of timber, pulpwood and energy assortments. In several situations, individuals interested in recreation prefer forests with low density. This means that forest management that is optimal when all objectives are considered, typically is characterized by larger thinning harvests than forest management that only focuses on the production of timber, pulpwood and energy assortments.

The results also show that large set up costs have the same type of effect on optimal forest management as an increasing importance of typical forms of recreation, close to large cities. Both of these factors imply that the harvest volumes per occasion increase and that the time interval between harvests increases. Even rather small set up costs imply that the continuous cover forest management schedule gives a rather large variation in the optimal stock level over time. The general analysis of the optimization problems analysed within this study is based on differential equations describing forest growth, in combination with two dimensional optimization of the decisions "harvest interval" and "stock level directly after harvest". All of the other variables are explicit functions of these decisions.

**Thank you for listening!**  
**Questions?**

