Iranian Journal of Operations Research
Vol. 2, No.1, 2010, pp. 32-40

Decision making in forest management with consideration of stochastic prices

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The optimal harvesting policy is calculated as a function of the entering stock, the price state, the harvesting cost, and the rate of interest in the capital market. In order to determine the optimal harvest schedule, the growth function and stumpage price process are estimated for the Swedish mixed species forests. The stumpage price is assumed to follow a stochastic Markov process. A stochastic dynamic programming technique and traditional deterministic methods are used to obtain the optimal decisions. The expected present value of all future profits is maximized. The results of adaptive optimization are compared with results obtained by the traditional deterministic approach. The results show a significant increase in the expected economic values via optimal adaptive decisions.

Keywords: *Optimal harvesting, stochastic dynamic programming, forest growth, stumpage prices, Swedish forests.*

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1. Introduction

The stumpage price fluctuates over time and it is very difficult to predict it with high accuracy. Therefore, we can regard the stumpage price as a stochastic process. Clearly, some other phenomena, such as the growth of forest, may also be stochastic. However, price variation is the most important source of risk. The general assumptions are the following:

- The aim is to maximize the expected present value of all present and future profits from extraction.
- Price is a Markov process.
- The forest annual growth is assumed to be deterministic.

In this study, there is a stock level constraint, which means that the stock level may never go below 100 m^3/ha. Hence, this study can be considered as a continuous cover forest management or uneven aged forest management analysis.

Several past studies of uneven-aged forest management have dealt with the problem of finding the cutting schedules that maximize economic returns. The pioneering studies were based on deterministic approaches given by Duerr and Bond [4], Duerr et al. [5], Chang [2], Hall [10] and Michie [21]. Risk management in forestry decisions was discussed by Hool [1], who first used a Markovian framework to analyze the management of even-aged plantations. Hool [1] determined schedules that would maximize volume produced over a finite time period. Lembersky and Johnson [13] studied optimal policies for managed stands. Their approach resulted in optimal investments in timber production under price and growth uncertainties. Earlier studies of related problems can be found in Lohmander [14, 20]. Kaya and Buongiorno [12] studied economic harvesting of uneven-aged Northern hardwood stands under risk. Their method determined the harvesting policies under uncertain stand growths and prices. Haight [9] studied feedback thinning policies for uneven-aged stand management with stochastic prices. Buongiorno [1] developed a generalization of Faustmann's formula for stochastic forest growth and prices with a Markov

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decision process model. Rollin et al. [23] investigated the management strategy for uneven-aged forests in France with stochastic growth and price.

Zhou and Buongiorno [25] studied forest landscape management in a stochastic environment, with an application to mixed loblolly pine-hardwood forests. They used a Markov chain model to describe stand transition between pre-defined states, with high-frequency shocks and rare natural catastrophes. Stochastic optimization was then used with this model to study the trade-off between landscape diversity and other management objectives. Lohmander and Mohammadi Limaei [20] studied the optimal harvest decision in an uneven-aged forest in north of Iran. The results show that the expected present value increase by more than 26% via optimal adaptive decisions.

Zhou et al. [26] have studied the adaptive versus fixed policies for economic and ecological objective in forest management. In different articles, various mathematical programming methods have been used to determine the optimal solution.

In this paper, stochastic dynamic programming (SDP) is applied to determine the optimal harvest decisions. To achieve the goal, first of all a growth function for the Swedish mixed species forests will be estimated. There are 5.28 million ha of mixed-species forests in Sweden. Scots pine (*Pinus sylvestris*), Norway spruce (*Picea abies*) and Birch (*Betula pendula*) are the dominant tree species in these forests. In this study, a stumpage price process will be estimated for these forests. The next step is to use the estimated growth and price parameters to determine the optimal extraction level via SDP. The solutions will be discussed and compared with results obtained from deterministic optimization of a similar but simplified version of the problem.

2. Solution Method

2.1. Stochastic dynamic programming

The optimal decisions are determined by applying SDP, in discrete time. The periods are denoted by t, $t \in \{0,1,2,.,T\}$. The final period, the horizon, is denoted by by T.

$f_t(m)$ is the optimal expected present value(EPV of all profits (revenues–costs)) at the beginning of period t, when m is the entering state of the system in period t, $R_t(m,u)$ is the profit in period t as a function of the entering state in the same period and the control (or decision and action) u.

$U(m)$ denotes the set of feasible controls as a function of the entering state. In a generalized setting, $U(.)$ could also be defined as a function of time, which is however not necessary in this problem. In the final period, T, the optimal decisions and EPV's are determined by:

$$f_T(m) = \max_{u \in U(m)} \{R_T(m,u)\} \quad \forall m \in M \tag{1}$$

where M is the set of states. The optimal decisions and EPV s in the earlier periods $t, t \in \{0,1,2,.,T-1\}$, are determined recursively via the backward algorithm of stochastic dynamic programming:

$$f_t(m) = \max_{u \in U(m)} \left\{ R_t(m,u) + d \sum_n p(n|m,u) f_{t+1}(n) \right\} \quad \forall m \in M \ , \tag{2}$$

where $p(n|m,u)$ is the conditional probability of reaching state n in the next period if the entering state in the current period is m and the control is u, d is the one period discounting factor, and $d\sum_n p(n|m,u) f_{t+1}(n)$ is the EPV (expected in the beginning of period t) of all profits in the periods after period t in case the entering state in period t is m and decision u is made in period t.

Here, the state space is two dimensional. The general problem description using state index m is still relevant. One state dimension is the stock level (m^3/ha) and the other dimension is the price level. The stock level grows according to a deterministic growth function. If harvesting takes place, the stock level is reduced accordingly. The price is assumed to follow a stochastic Markov process. Decisions are sequentially optimized based on the latest information concerning the state, which means that the stock level and the price level are correctly observed and known in the beginning of each period.

2.2. Growth

In our numerical computations, the following formula was used to determine the growth in the presence of different stock levels and site indices, see [8].

$$G(B, c, k, p, x, y) = kp^{y} BC(x)\left[1 - \left[\frac{B}{(pc)}\right]^{x}\right], \tag{3}$$

and

$$C(x) = \frac{(x+1)^{\frac{(x+1)}{x}}}{x}, \tag{4}$$

where B is the stock (m^3/ha), c, k, x and y are estimated parameters as given in Table 1, p is the site index that varies between 4 and 10.

Table 1. Estimated parameters for mixed species (pine, spruce, birch).

	c	k	x	y
Mixed species	63.68309	0.008613	-0.39015	0.0813

After determining the growth under different stock levels and site indices, the following estimation was used to determine the growth function:

$$g = \alpha_0 V + \beta_0 V^2 + \varepsilon, \tag{5}$$

where g = growth (m^3/ha/year), α_0 and β_0 are estimated parameters, and V = stock level (m^3/ha), ε is assumed to be independent over time, identically distributed and Gaussian, with expected value 0 and standard deviations δ.

In all statistical tests discussed in this paper, the 95% level of significance is used.

The parameters (α_0, β_0 and ε) were estimated via regression analysis for site indices from 4 to 10. For example, the estimated parameters α_0, β_0 are found below for Swedish mixed species stands with site index 5 (t statistics in parentheses):

$$g = 0.04144 \text{ V} - 0.0001391 \text{ V}^2 + \varepsilon \tag{6}$$

(7.334) (–6.154)

δ_0 =0.6633.

Let us determine the stock level that maximizes growth:

$$\frac{\partial g}{\partial V} = \alpha_{.} + ۲\beta_{.} V \tag{7}$$

The first order condition is:

$$\alpha_0 + 2\beta_0 V = 0, \text{ which gives } V = -\frac{\alpha_{.}}{۲\beta_{.}} \tag{8}$$

If we use the estimates of α_0 and β_0 , we find that the growth is maximized if

$V = \dfrac{-0.04144}{2(-0.0001391)} = 148.96$ m^3/ha. This stock gives the maximum sustainable yield.

If we use the estimates of α_0 and β_0 and replace V by 148.96 in equation (5) and assume that $\varepsilon = 0$, we get:

$$g = (0.04144 \times 148.96) + 0.0001391 \times (148.96)^2 = 3.086 \text{ m}^3\text{/ha /year.} \qquad (9)$$

Hence, the maximum growth per hectare for the Swedish mixed species stands with site index 5 occurs when the stand density is approximately 149 m^3/ha and the maximum sustainable growth is 3.1 m^3/ha/year.

2.3. Stumpage price

It is economically optimal to adaptively adjust the harvest activities to the sequentially revealed stumpage prices since there is no method of perfect price prediction available. Theoretically, the stumpage price is determined by a balance of stumpage supply and demand. The ruling price would be at such a level that the total amount of stumpage that forest owners are willing to sell equals the total quantity that the buyers are willing to buy.

Stumpage price data for mixed species in Sweden was collected from Swedish Forest Agency . Then, the price observations were adjusted by Consumer Price Index of Sweden for the base year 2005, see Figure 1. Here, mixed species stumpage price data include the average prices on standing timber for sale of main Swedish forest species (Scots pine, Norway spruce and Birch species). It is assumed that harvesting is performed in a way that the proportions of the different species at are kept constant.

Regression analysis was used to determine the price process parameters. The estimated parameters (α_2, β_2) are found below: (t statistics in parentheses):

$$P_{t+1} = \alpha_2 + \beta_2 P_t + \varepsilon_{t+1},$$

$$\underset{(2.02681)}{P_{t+1} = 6.3061} + \underset{(8.52323)}{0.80954} P_t + \varepsilon_{t+1}, \qquad \sigma = 4.13743, \qquad (10)$$

where ε_{t+1} is assumed to be independent over time, identically distributed and Gaussian, with expected value 0 and standard deviations δ.

The equilibrium price was calculated based on the first order AR model parameters:

$$P_{eq} = \frac{\alpha_2}{1-\beta_2}. \qquad (11)$$

where P_{eq}, the equilibrium stumpage price, is 33.10 €/m^3. The equilibrium price will be used to determine the optimal harvest period in the deterministic case.

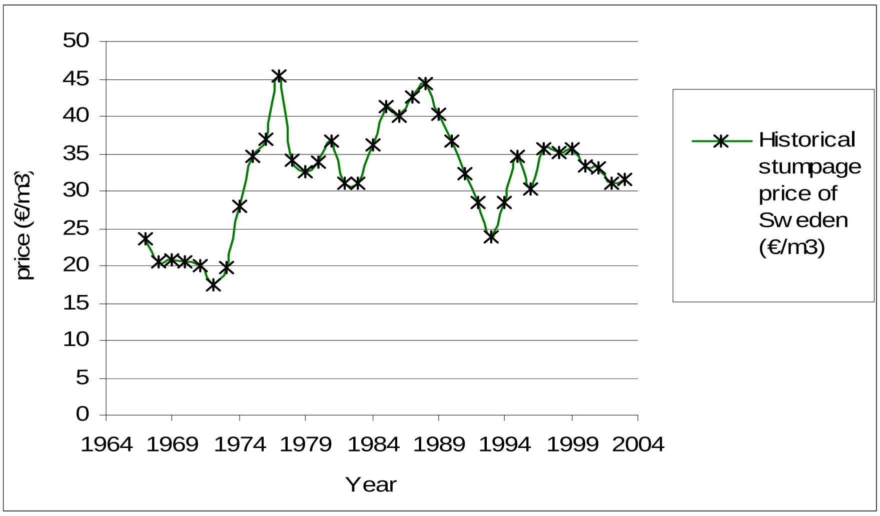

Figure 1. Historical stumpage prices (real price) of mixed species stumpage price adjusted to the price level to year 2005 in Sweden during the period 1967-2004 (€/m^3).

3. Optimization

The objective function is the EPV of all present and future harvestings. A deterministic version of the problem may be written as:

$$\pi = \sum_{t=0}^{\infty} e^{-rt}(P_t h_t - C_t), \tag{12}$$

where t is the time period, P is net price (price - variable harvesting cost), h is harvest level, r denotes the rate of interest, real prices and rates of interests will be used), and C_t is the set up cost (the cost of moving harvesting machines such as a harvester and a forwarder).

Fixed costs are not explicitly treated in this paper. Such costs are fixed and do not affect optimal behavior. One important reason to analyze the harvesting problem in discrete time is that this makes it possible to include set up costs (such as the cost of moving machines). If set up cost is not considered then it is mostly optimal to harvest very small quantities at any moment. When we have set up costs, which must be considered in real cases, it is optimal to harvest larger quantities more seldom. Then, the optimal stock level will have a saw tooth shape, Figure 2.

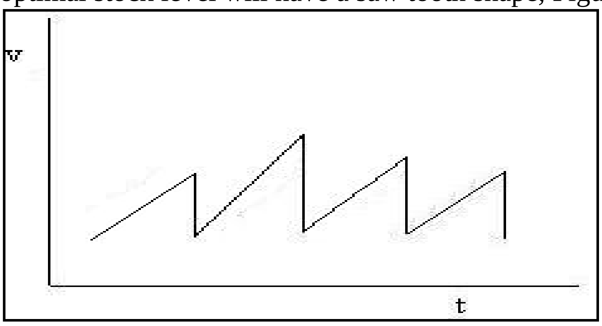

Figure 2. Relation between time (*t*) and the stock level (***V***)in the presence of setup costs.

3.1 Optimal harvest decision via SDP

Here, we study the decision making under risk. We determine the optimal harvest level in case price is stochastic and growth is deterministic. The optimization software created by Lohmander was used to determine the optimal harvest decisions in this case. We need to specify the parameters for the software representing: the growth function, the price function, the lowest feasible stock (100 m^3/ha), the period length (1 year), the horizon (100 years) , the set up cost (50 €/ha), the rate of interest(3%) , the highest stock level index (100) , the highest harvest level (20 m^3), the number of price states (20) and the difference between price states (10 €/m^3). For site indices 4 to 10, the parameters were specified and given to the software. After executing the program, the following results were obtained.

The transition probability matrix (systematic sample):

This probability matrix shows the probability distribution of prices in the next period when the price in the first period is known (Table 2). In each row, the sum of transition probabilities will be 1, when all columns are included.

Table 2: Transition probability matrix
(only 3 rows and 8 columns are shown).

		S2=	6	8	10	12	14	16	18	20
		P2=	-69.85	13.015	33.015	53.015	73.015	93.015	113.015	133.015
S1=6	P1=-69.85		0.095	0.0902	0.0699	0.0496	0.0294	0.0091	0	0
S1=8	P1=13.015		0.076	0.0956	0.0835	0.0639	0.0443	0.0246	0.005	0
S1=10	P1=33.015		0.0596	0.0791	0.0985	0.0791	0.0596	0.0402	0.0207	0.0013

3.2. The optimal harvest volumes in different states:

Here, the optimal harvest volumes are determined for different price and stock states. For example, for mixed species with site index 5, we have the following outcome: If the stock is higher than 100 m^3/ha and the present price is 123.015 €/m^3 or more, then the stock should instantly be harvested down to the lowest feasible level (100 m3/ha). In other situations, when the price is lower than 123.015 €/m^3, we should wait longer. For example, when the present price is 93.015 €/m^3, we should wait until the stock reaches 128 m^3/ha. Then, we should harvest 28 m^3/ha. In Figure 3, the grey area represents the states for which it is optimal to postpone the harvest and the black area depicts those for which it is optimal to harvest immediately. The numbers in the black boxes show the harvest volume per hectare.

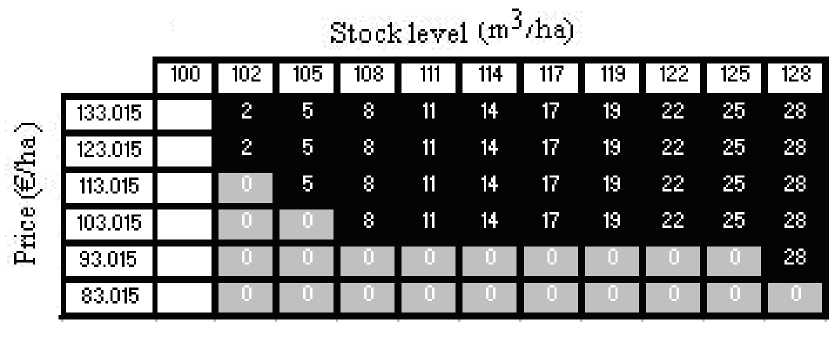

Figure 3. Optimal harvest decisions under stochastic stumpage price and different stock states.

The total EPV when we consider price variation and adaptive harvesting is calculated for each stock level and price state combination. The total EPV is 7305 €/ha when the stock and price are 100 m3/ha and 33.015 €/m^3, respectively, Table 3.

Table 3: Total expected present value (€/ha) for mixed species with site index 5. (only 3 rows and 9 columns are shown).

		Stock level (*i*)	*i*=0	*i*=1	*i*=2	*i*=3	*i*=4	*i*=5	*i*=6	*i*=7	*i*=8
price state (*j*)	Price (€/m^3)	Stock (m^3/ha)	100	102	105	108	111	114	117	119	122
		EPV (€/ha)									
j=12	53.015		7322	7559	7799	8040	8282	8525	8768	9013	9257
j=11	43.015		7314	7546	7781	8017	8254	8492	8730	8968	9207
j=10	33.015		7305	7533	7763	7994	8225	8458	8690	8923	9156

According to Table 3, EPV is an increasing function of the net price and of the stock level.

3.3. Optimal series of harvest decisions in case price and growth are deterministic

In case there is no price variation and the first harvesting begins t_1 years from now, the present value is:

$$\pi = \frac{\overline{P}gt_1 - C}{((1+i)^{t_1} - 1)}, \qquad (13)$$

where g is the annual growth(m^3/ha), C is the set up cost per hectare and occasion. When the initial stock is 100 m^3/ha and site index is 5, then from the growth function (5):

$g = 0.0414365(100) - 0.0001391(100)^2 = 2.753$ m^3/ha /year:

where $\overline{P}$ is the equilibrium net price, $\overline{P}$ = 33.10 €/m^3, t_1 is the harvest time interval and i is the rate of interest in the capital market (3%).

The optimal harvest interval is 6 (9) years, if the set up cost is 50 (100) €/ha, and the present value is 2559.85 (2362.80) €/ha, Figure 4.

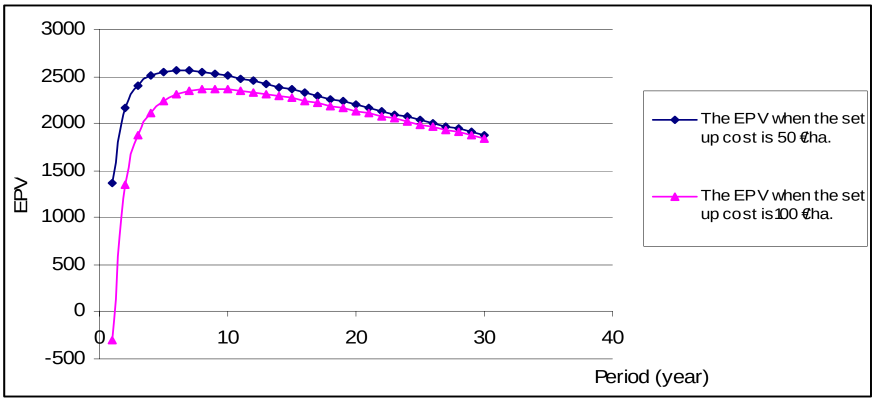

Figure 4. EPV for different harvesting intervals when the set up costs are 50 €/ha and 100 €/ha.

The EPV in the deterministic case (δ = 0) and in the real case, where price is stochastic ($\sigma_2 = 4.13743$ €/m^3) and harvesting is adaptive, were compared. Table 4 shows the result for a case that the standard deviation is 100% higher (δ = 8.2748 €/m^3) than the one with the empirical data. The results showed that the EPV in the stochastic case increased by 79.03% (3).

Table 4. Optimal expected present value for deterministic and stochastic cases

Standard deviation (δ, €$/m^3$)	EPV or π (€$/ha$)
0	2559.8
4.1374	4583
8.2748	6064

4. Results and discussions

The aim of this study was to determine the optimal harvest level under different price and stock level states. Growth functions and stumpage price process functions were estimated for mixed species stands in Sweden. A logistic growth function was estimated where the growth increased monotonically to up some critical level of the stock, the maximum sustainable yield, and then decreased monotonically.

It was shown that the stumpages price in Sweden during the period 1967-2004 fluctuated over time. The investigation of the autocorrelation function for different lags showed that as the number of lags increased, the autocorrelation tended to zero. When the number of years between two observations increased, the correlation between two prices decreased. There are many factors affecting the stumpage prices which are not predictable and depend on socio-economic conditions in the future. Changes in forest policies and regulations may also have influences on future supplies and/or demands for stumpage. Since such possible factors are not known in advance, it is reasonable to consider future stumpage prices to be stochastic variables.

The expected present values were determined for deterministic and the real stochastic cases. Under deterministic assumptions, the optimal harvest interval was set to be 6 (9) years, if the set up cost was 50 € (100 €) per hectare, and the present value was 2559.85 (2362.80) €/ha.

As the next step, a stochastic dynamic programming model was used to determine the optimal cutting rule for different price and stock states. The results showed that in the economically optimal harvest decisions, when the price was stochastic, we should in general wait for the latest information before we take the final decisions. It was found that the optimal harvest levels and the expected present values were increasing functions of the net price and of the stock level. The expected present values of deterministic and stochastic cases were compared. The result showed that one can expect to gain from adaptive harvesting. The EPV increased by 79.03%. This is reasonable, since one can mostly select to harvest during years when prices are at least one standard deviation above the expected value.

Acknowledgment

The authors are grateful to the Swedish Research Council for Environment, Agricultural Sciences and Spatial Planning (FORMAS) for the financial support and forestry program at the International Institute for Applied Systems Analysis (IIASA), Laxenburg, Austria.

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