Rational Control of Global Warming Dynamics via the CO_2 level, Emission Reductions and Forestry Expansion

by

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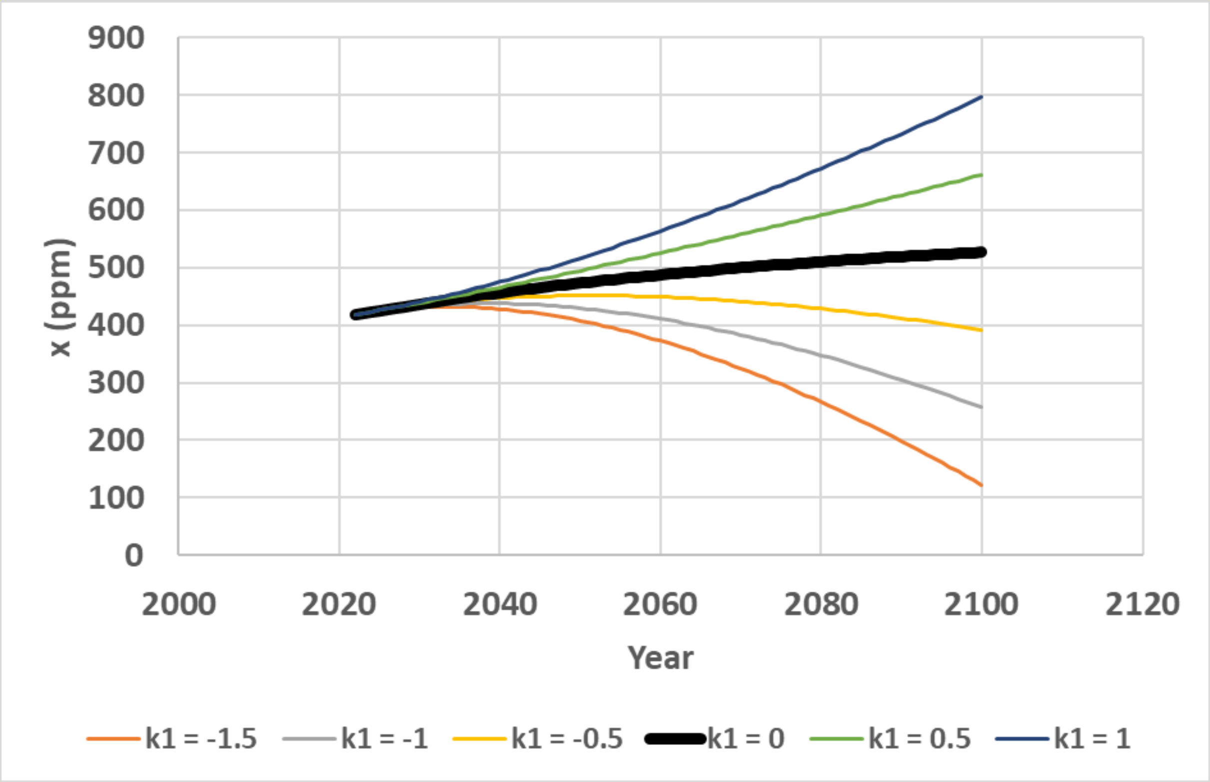

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Abstract

First, the observed CO_2 level in the atmosphere, recorded by NOAA (2022) at the Mauna Loa observatory, and the global industrial CO_2 emissions, reported by EDGAR (2021), European Commission, are investigated, from 1990 until 2021. Then, a differential equation model is developed, based on two hypotheses, that explains how these time series interact. The hypotheses of the explaining model are tested with regression analysis, and it is demonstrated that no hypothesis can be rejected on statistical grounds. The parameters of the CO_2 concentration model are determined with high t-values and low p-values. The model is used to determine the time path of the CO_2 concentration of the natural system without industrial emissions, for arbitrary initial conditions. This system has a unique and stable equilibrium at 262 ppm. With constant industrial emissions, the equilibrium is found at a higher level, which is shown with an explicit equation. Comparative statics analysis shows how the equilibrium is affected by alternative parameter adjustments. An extended version of the natural differential equation, with a forcing function, representing the time paths of industrial emissions, is developed. The industrial emissions are modeled as a quadratic function of time. The general function of the time path of the CO_2 concentration of the natural system under the influence of industrial emissions, is determined for arbitrary initial conditions and parameters of the industrial emission function. The CO_2 time path function is analytically verified. Then, it is also empirically tested and found to be able to reproduce the historical CO_2 observations with high precision. Then, the time paths of the future CO_2 concentrations are calculated, for six alternative levels of change of the industrial emissions, from -1.5 Gt/year to +1.0 Gt/year, from the year 2022 until 2100. These results are presented as a function and as graphs. The net CO_2 emissions can also be reduced over time, if forestry is gradually intensified. The rational intensity of this investment process is determined, taking the time path of the CO_2 level into consideration, during an arbitrary time interval. An explicit function for the optimal forestry intensification level, based on all CO_2 time path function parameters, the marginal cost of the CO_2 concentration, time interval parameters, rate of interest and different cost function parameters, is derived and presented.

Contents:

The differential equation of the CO_2 concentration in the atmosphere:

Fundamental theory, mathematics and statistical estimation.

Time path of the CO_2 concentration:

Determined without and with arbitrary industrial emissions.

Historical CO_2 observations:

Reproduced by the model.

The CO_2 concentration equilibrium:

Exists, is unique and stable.

Intensified sustainable forestry:

Reduces the future CO_2 concentration.

The optimal forestry intensification level:

Is determined as an explicit function of all parameters.

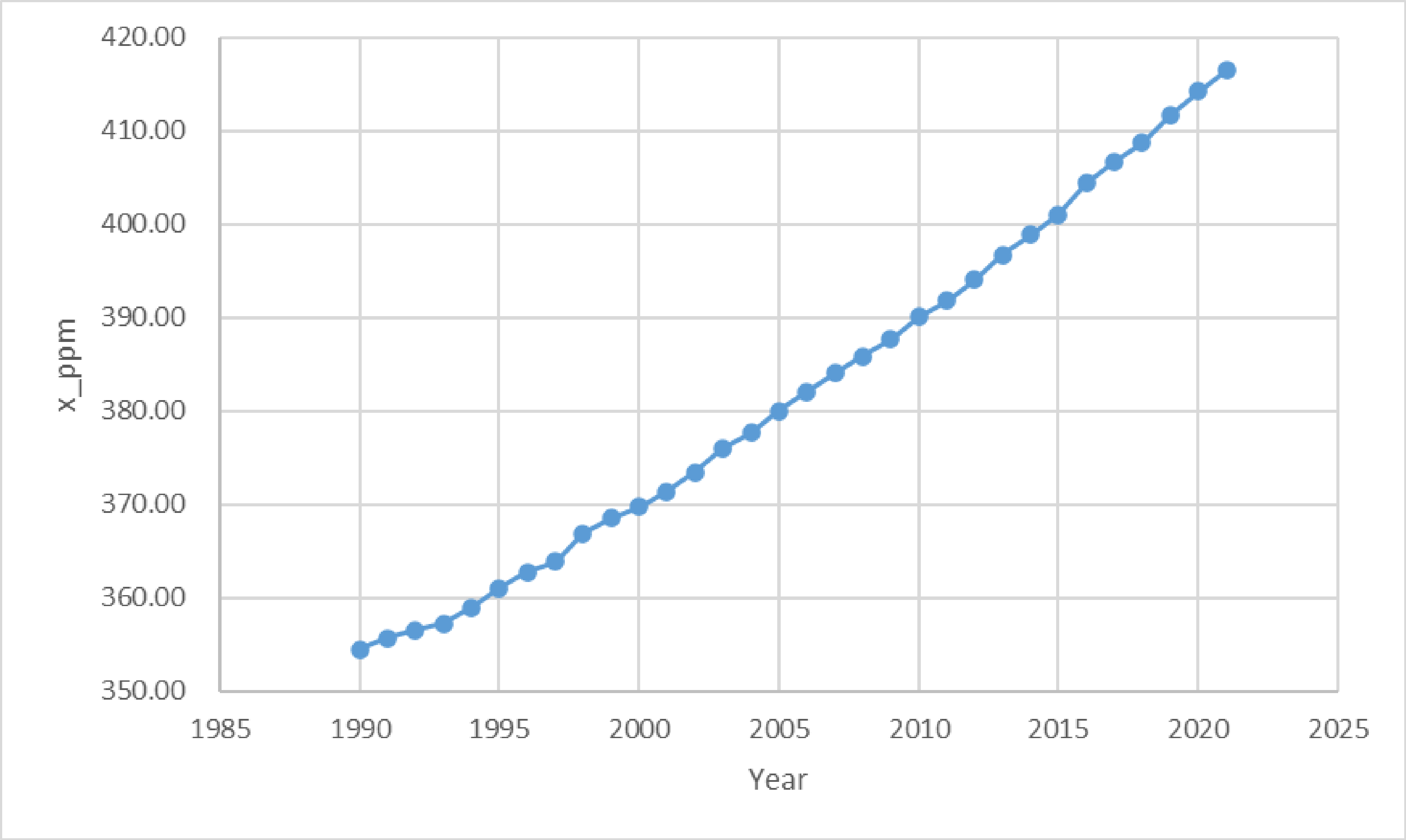


Figure 1.

The CO_2 concentration in the atmosphere, in the unit ppm, according to the observations from the Mauna Loa observatory. Source: NOAA (2022).

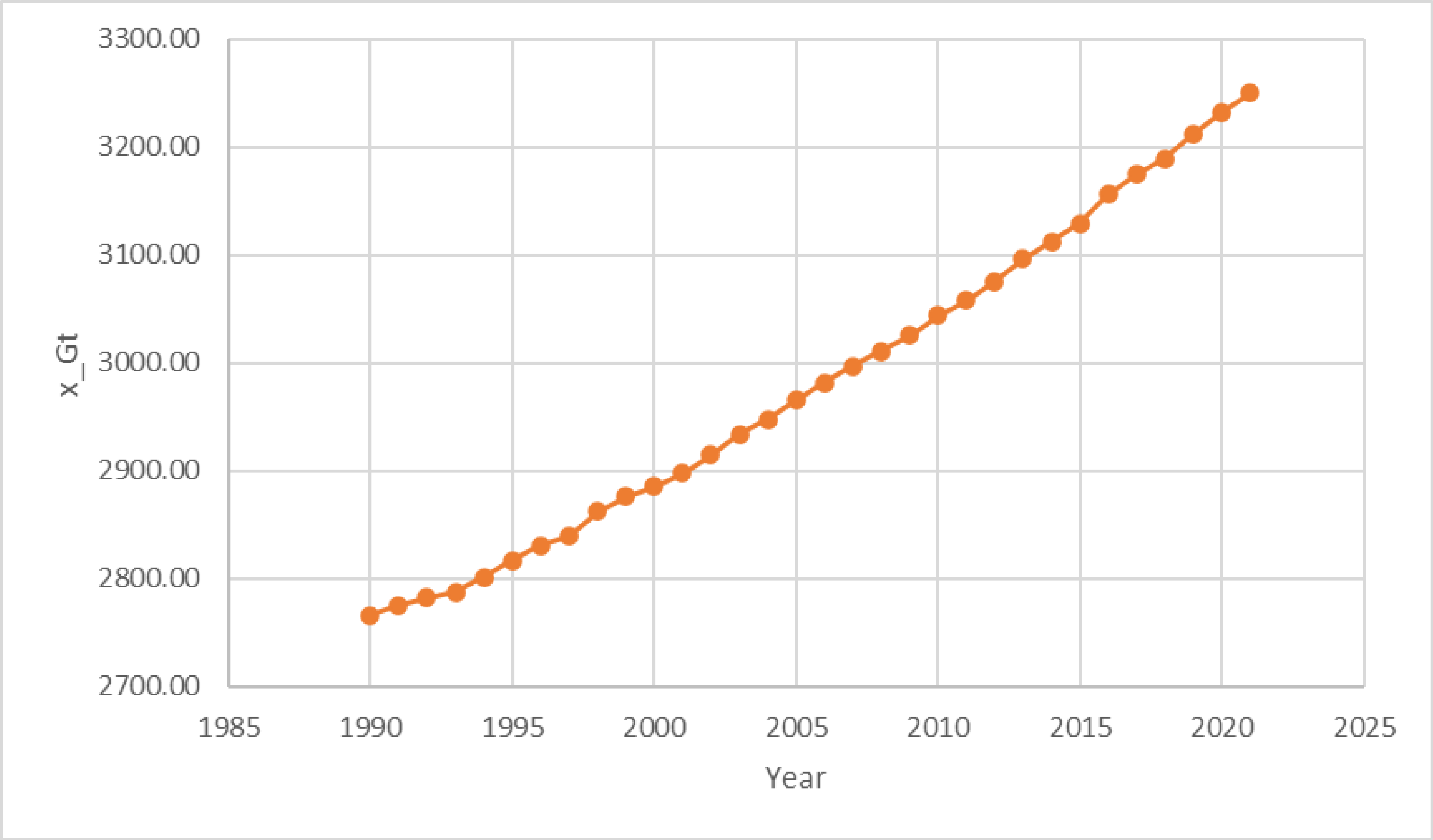


Figure 2.

The CO_2 level in the atmosphere, in the unit Gt, according to the observations from the Mauna Loa observatory and variable transformations. Sources: NOAA (2022) and O’Hara (1990).

The natural system:

CO_2 concentration in the atmosphere

$$\dot{x} = a + bx \quad , \quad a > 0, b < 0$$

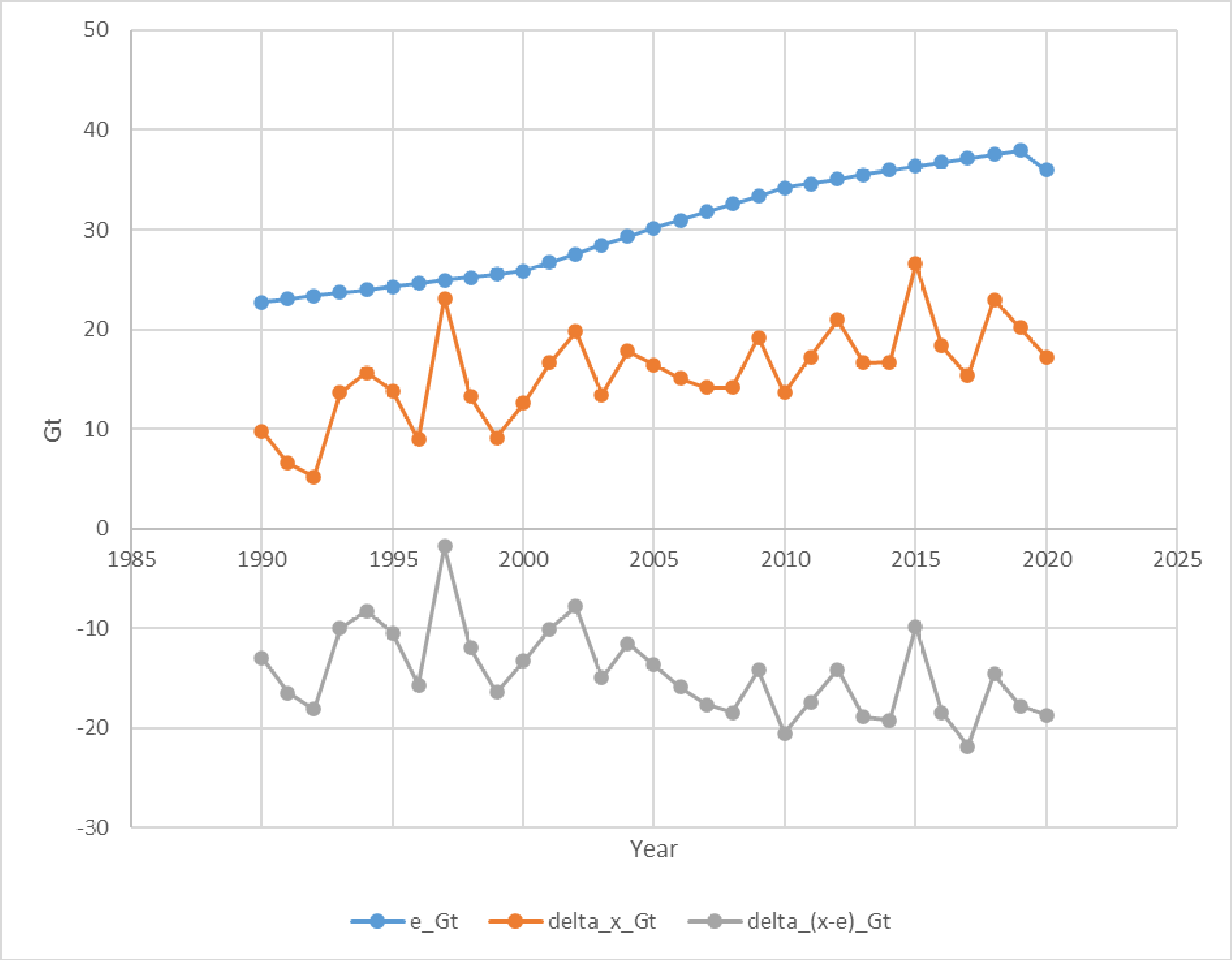
The natural system + industrial net emissions:

$$\dot{x} = a + bx + f(t)$$

Empirical data and transformations

Column	Variable	Source
1	Year	
2	CO2 concentration in the atmosphere (ppm)	NOAA (2022).
3	CO2 mass in atmosphere (Gt)	NOAA (2022). O´Hara (1990).
4	Industrial emissions, CO2, observations (Mt)	EDGAR (2021).
5	Change per year of the Industrial emissions, until the next observation	EDGAR (2021).
6	Industrial emissions, CO2, observations and values determined via linear interpolation (Mt)	EDGAR (2021).
7	Industrial emissions, CO2, observations and values determined via linear interpolation (Gt)	EDGAR (2021).
8	Differences of CO2 mass in atmosphere (Gt)	NOAA (2022). O´Hara (1990).
9	Differences of CO2 mass in atmosphere (Gt) - Industrial emissions, CO2, observations and values determined via linear interpolation (Gt)	NOAA (2022). O´Hara (1990). EDGAR (2021).
10	Differences of CO2 mass in atmosphere (ppm) - Industrial emissions, CO2, observations and values determined via linear interpolation (ppm)	NOAA (2022). O´Hara (1990). EDGAR (2021).

year	x_ppm	x_Gt	e_Mt_obs	delta_e_Mt	e_Mt	e_Gt	delta_x_Gt	delta_(x-e)_Gt	delta_(x-e)_ppm
1990	354,45	2766,24	22728,88	311,844	22728,88	22,72888	9,76	-12,97	-1,662345983
1991	355,70	2776,00			23040,72	23,04072	6,56	-16,49	-2,112303852
1992	356,54	2782,55			23352,57	23,35257	5,23	-18,12	-2,322261722
1993	357,21	2787,78			23664,41	23,66441	13,66	-10,01	-1,282219591
1994	358,96	2801,44			23976,26	23,97626	15,69	-8,29	-1,062177461
1995	360,97	2817,13			24288,1	24,2881	13,81	-10,47	-1,34213533
1996	362,74	2830,94			24599,94	24,59994	8,90	-15,70	-2,0120932
1997	363,88	2839,84			24911,79	24,91179	23,10	-1,81	-0,232051069
1998	366,84	2862,94			25223,63	25,22363	13,27	-11,96	-1,532008939
1999	368,54	2876,20			25535,48	25,53548	9,13	-16,40	-2,101966808
2000	369,71	2885,34	25847,32	864,542	25847,32	25,84732	12,56	-13,28	-1,701924678
2001	371,32	2897,90			26711,86	26,71186	16,62	-10,09	-1,292702042
2002	373,45	2914,52			27576,4	27,5764	19,74	-7,83	-1,003479406
2003	375,98	2934,27			28440,95	28,44095	13,42	-15,02	-1,924256771
2004	377,70	2947,69			29305,49	29,30549	17,79	-11,51	-1,475034135
2005	379,98	2965,49	30170,03	802,14	30170,03	30,17003	16,47	-13,70	-1,755811499
2006	382,09	2981,95			30972,17	30,97217	15,06	-15,91	-2,038593036
2007	384,02	2997,01			31774,31	31,77431	14,13	-17,65	-2,261374572
2008	385,83	3011,14			32576,45	32,57645	14,13	-18,45	-2,364156108
2009	387,64	3025,27			33378,59	33,37859	19,20	-14,18	-1,816937645
2010	390,10	3044,47	34180,73	441,044	34180,73	34,18073	13,66	-20,52	-2,629719181
2011	391,85	3058,12			34621,77	34,62177	17,25	-17,37	-2,226231984
2012	394,06	3075,37			35062,82	35,06282	20,92	-14,15	-1,812744788
2013	396,74	3096,29			35503,86	35,50386	16,62	-18,88	-2,419257591
2014	398,87	3112,91			35944,91	35,94491	16,70	-19,24	-2,465770394
2015	401,01	3129,61	36385,95	381,355	36385,95	36,38595	26,53	-9,85	-1,262283197
2016	404,41	3156,15			36767,31	36,76731	18,34	-18,43	-2,3611478
2017	406,76	3174,49			37148,66	37,14866	15,30	-21,85	-2,800012403
2018	408,72	3189,78			37530,02	37,53002	22,94	-14,59	-1,868877007
2019	411,66	3212,73	37911,37		37911,37	37,91137	20,14	-17,78	-2,27774161
2020	414,24	3232,86	35962,87		35962,87	35,96287	17,25	-18,72	-2,398072196
2021	416,45	3250,11		0 (Guess)	35962,87	35,96287			



The industrial net CO_2 emissions to the atmosphere per year.

Change during one year of the CO_2 level in the atmosphere.

Change during one year of the CO_2 level in the atmosphere reduced by the industrial net emissions.

(These negative values also have a negative time trend. More CO_2 is absorbed now than earlier, most likely because of the increasing CO_2 level.)

Figure 3.

The CO_2 emissions to the atmosphere, e_Gt, in the unit Gt, the change during one year of the CO_2 level in the atmosphere, delta_x_Gt, and the yearly change of the CO_2 level in the atmosphere reduced by the emissions, delta_(x-e)_Gt, in the unit Gt. Sources: NOAA (2022), EDGAR (2021) and O'Hara (1990).

$$\dot{x} = a + bx \quad , \quad a > 0, b < 0 \tag{1}$$

In the empirical data, the time derivative of x is also affected by f(t), which represents the total industrial emissions as a function of time.

$$\dot{x} = a + bx + f(t) \tag{2}$$

We may reformulate (2) to get (3) and (4).

$$\dot{x}(t) - f(t) = a + bx(t) \tag{3}$$

In regression analysis, we estimate y(t) as a function of the parameters a, b and the value of x(t). Compare (4) and (5) and the Appendix.

$$y(t) = \dot{x}(t) - f(t) \tag{4}$$

$$y(t) = a + bx(t) \tag{5}$$

Table A2.

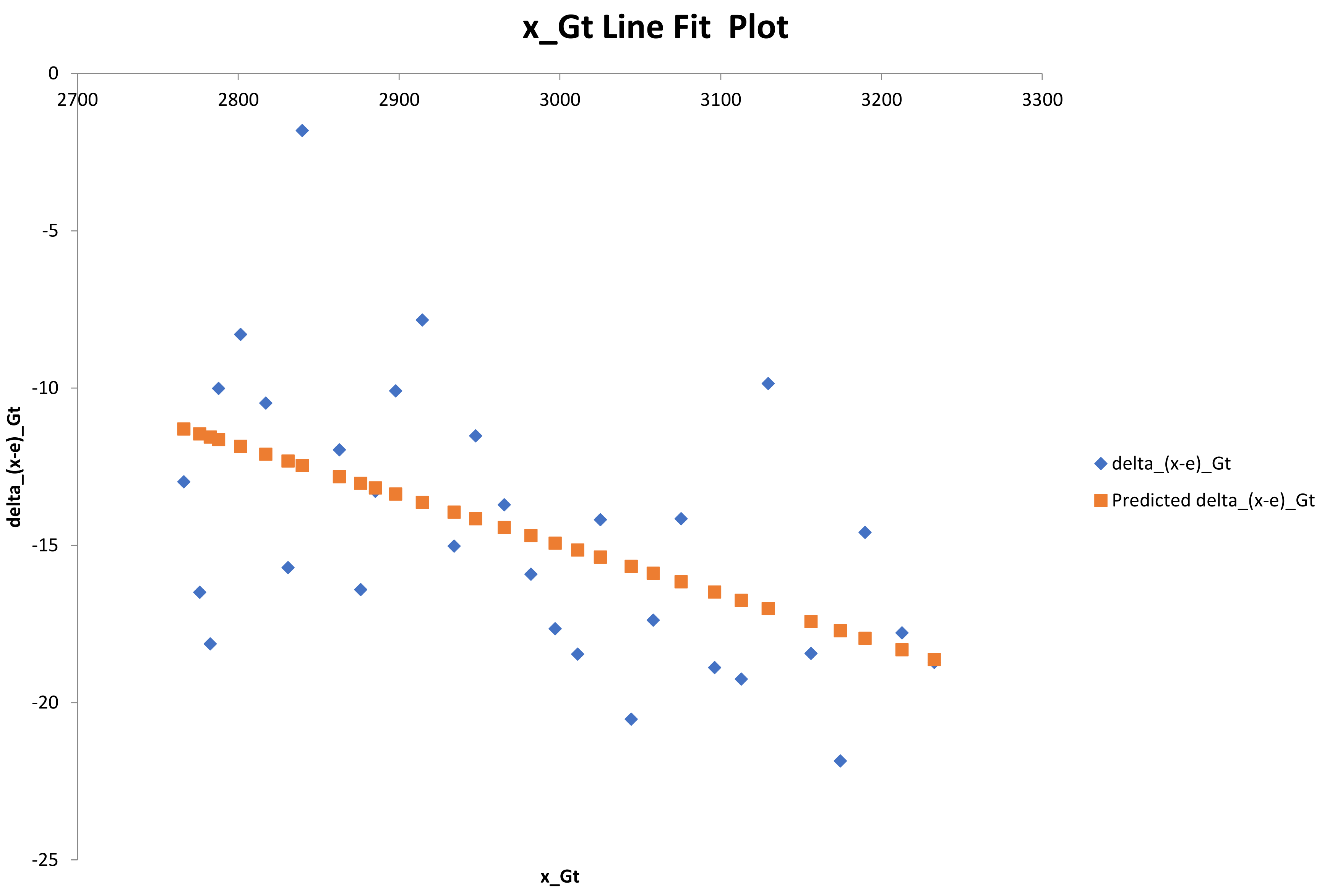
Regression in Gt.

Regression Statistics	
Multiple R	0,514535803
R Square	0,264747092
Adjusted R Square	0,239393544
Standard Error	3,820257244
Observations	31

ANOVA

	df	SS	MS	F	Significance F
Regression	1	152,3974368	152,3974368	10,44221057	0,003061931
Residual	29	423,236597	14,59436541		
Total	30	575,6340338			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 95,0%	Upper 95,0%
Intercept	32,1679952	14,47537138	2,222256988	0,034225527	2,562536581	61,77345382	2,562536581	61,77345382
x_Gt	-0,015712373	0,004862343	-3,231440943	0,003061931	-0,025656981	-0,005767766	-0,025656981	-0,005767766



x_Gt Line Fit Plot
0
-5
-10
-15
-20
-25
2700
2800
2900
3000
3100
3200
3300
delta_(x-e)_Gt
x_Gt
delta_(x-e)_Gt
Predicted delta_(x-e)_Gt

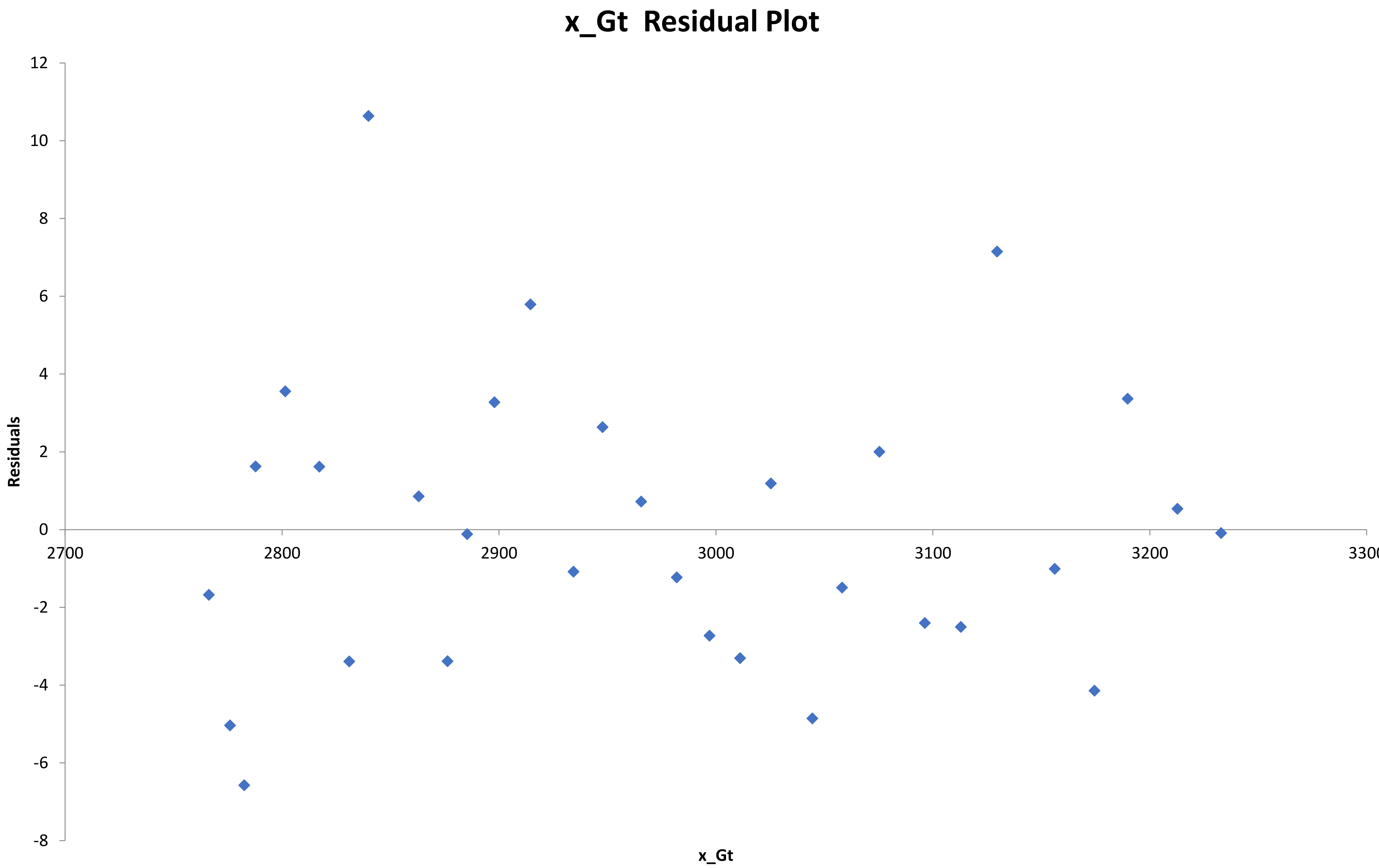

x_Gt Residual Plot
12
10
8
6
4
2
0
-2
-4
-6
-8
Residuals
2700
2800
2900
3000
3100
3200
3300
x_Gt

Table A3.

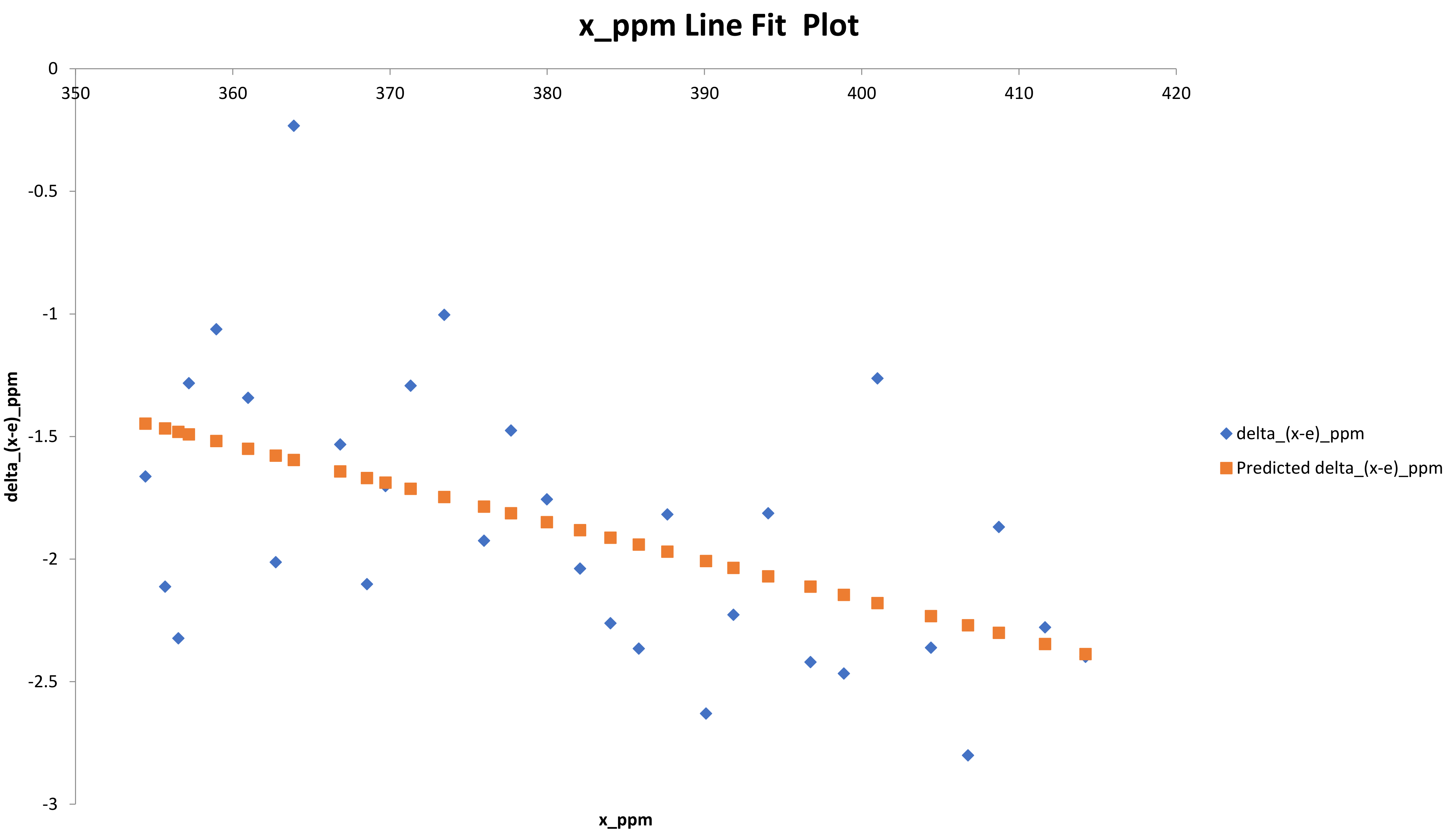
Regression in ppm.

Regression Statistics	
Multiple R	0,514535803
R Square	0,264747092
Adjusted R Square	0,239393544
Standard Error	0,489505459
Observations	31

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	2,502116493	2,502116493	10,44221057	0,003061931
Residual	29	6,94885224	0,239615594		
Total	30	9,450968733			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95,0%*	*Upper 95,0%*
Intercept	4,121819095	1,854789575	2,222256988	0,034225527	0,328348476	7,915289714	0,328348476	7,915289714
x_ppm	-0,015712373	0,004862343	-3,231440943	0,003061931	-0,025656981	-0,005767766	-0,025656981	-0,005767766



x_ppm Line Fit Plot
0
-0.5
-1
-1.5
-2
-2.5
-3
350
360
370
380
390
400
410
420
delta_(x-e)_ppm
x_ppm
delta_(x-e)_ppm
Predicted delta_(x-e)_ppm

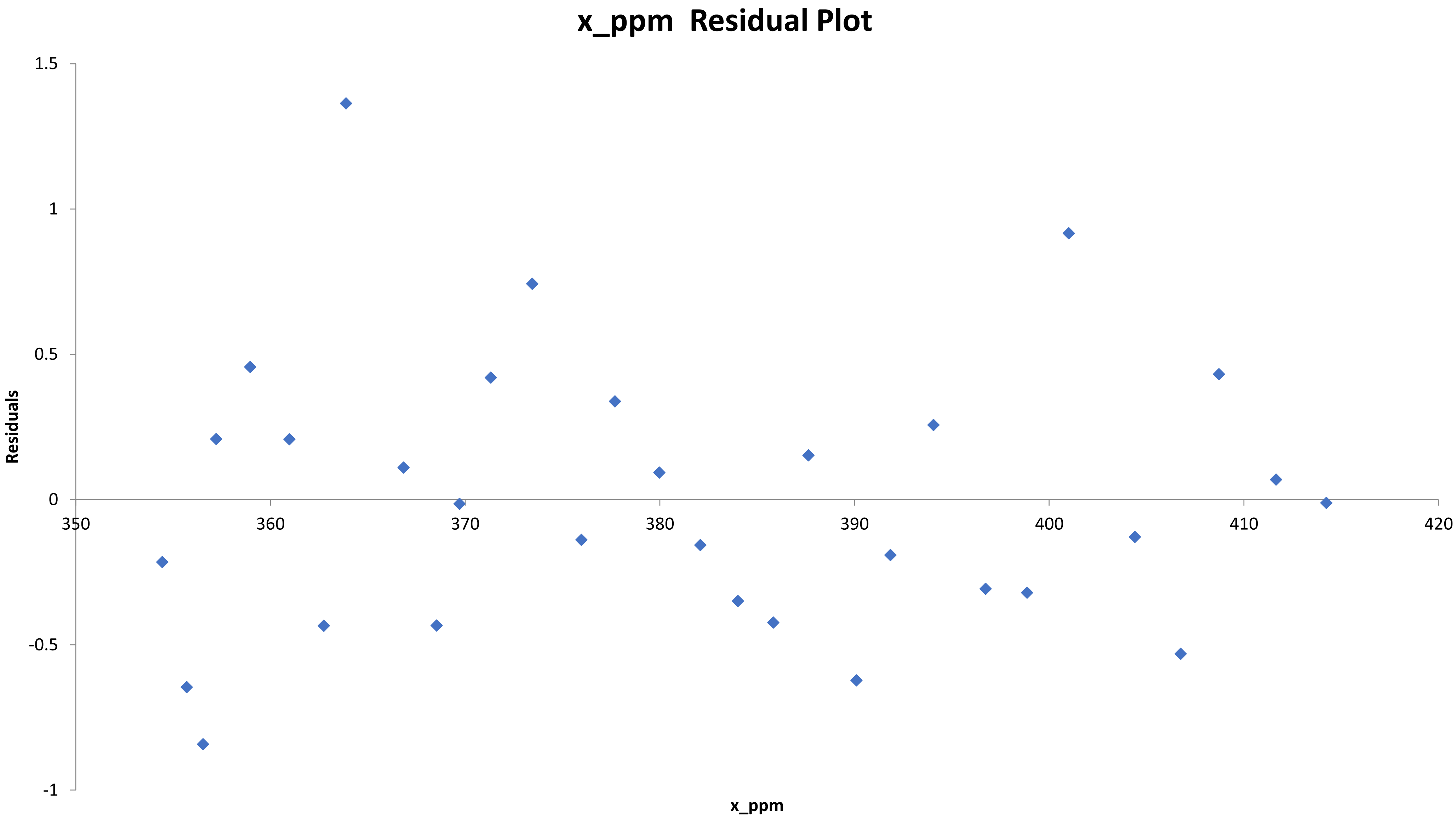

x_ppm Residual Plot
1.5
1
0.5
0
-0.5
-1
Residuals
350
360
370
380
390
400
410
420
x_ppm

Table 1.
Estimated regression parameters when the unit of x(t) is Gt. y(t) and the time derivative of x(t), have the unit Gt/year. The parameters have five value figures in this table. All of the regression results and the data are found in the Appendix.

Parameter	Estimated Value	Standard Error	P-value
a	32.168	14.475	0.034225
b	-0.015712	0.0048623	0.0030619

Table 2.
Estimated regression parameters when the unit of x(t) is ppm. y(t) and the time derivative of x(t), have the unit ppm/year. The parameters have five value figures in this table. All of the regression information and the data are found in the Appendix.

Parameter	Estimated Value	Standard Error	P-value
a	4.1218	1.8548	0.034226
b	-0.015712	0.0048623	0.0030619

In case the industrial emissions would be constant and equal to f_c, then in CO_2 equilibrium, x_c, equation (6) is satisfied, which leads to (7).

$$\left(\dot{x} = 0 \wedge f(t) = f_c \big|_{\forall t} \right) \Rightarrow \left(a + bx + f_c = 0 \right) \qquad (6)$$

$$x_c = x = -\left(\frac{a + f_c}{b} \right) \qquad (7)$$

Without industrial emissions, the equilibrium is (8).

$$x_c \big|_{f_c = 0} = -\left(\frac{a}{b} \right) > 0 \qquad (8)$$

We can now use equation (8) to determine the equilibrium value of the natural system. When we use the unit Gt, as in Table 1., we get the result in equation (9).

$$x_c\Big|_{f_c=0} = -\frac{32.168}{(-0.015712)} \approx 2047.4(Gt) \qquad (9)$$

If we use the unit ppm, as in Table 2, we get the result in equation (10).

$$x_c\Big|_{f_c=0} = -\frac{4.1218}{(-0.015712)} \approx 262.33(ppm) \qquad (10)$$

Pre industrial equilibrium

Following the principles by O’Hara (1990), the following transformation rules have been applied: 1 ppm (CO_2) can be transformed to 2.13*3.664=7.80432 Gt CO_2. 1 ppm by volume of atmosphere CO_2 =2.13 Gt C. 1 g C=0.083 mole CO_2 =3.664 g CO_2. Hence, we may change units, and go to the unit ppm from the unit Gt, if we divide the figure in Gt by 7.80432.

In equation (11), we find that the ratio is very close to the correct figure, also if we only use five value figures.

$$\frac{x_c\big|_{f_c=0}(Gt)}{x_c\big|_{f_c=0}(ppm)} \approx \frac{2047.4}{262.33} \approx 7.8047 \qquad (11)$$

Comparative statics analysis of the CO_2 equilibrium with constant emissions:

If the CO_2 level in the atmosphere is in equilibrium and the exogenous industrial emissions are constant over time, for instance zero, then equation (12) is satisfied.

$$G(a,b,x_c, f_c) = \dot{x}(.) = a + bx_c + f_c = 0 \tag{12}$$

Total differentiation of the equilibrium condition gives (13).

$$dG = da + db \times x_c + b \times dx_c + df_c = 0 \tag{13}$$

$$\left.\frac{dx_c}{da}\right|_{db=df_c=0} = -b^{-1} > 0$$

The equilibrium CO_2 level in the atmosphere is a strictly increasing function of the level of the natural emissions.

$$\left.\frac{dx_c}{db}\right|_{da=df_c=0} = -b^{-1}x_c > 0$$

The CO_2 equilibrium level in the atmosphere is a strictly increasing function of b, which means that it is a strictly decreasing function of the natural absorption level.

$$\left.\frac{dx_c}{df_c}\right|_{da=db=0} = -b^{-1} > 0$$

The equilibrium CO_2 level in the atmosphere is a strictly increasing function of the exogenous industrial emission level.

Explicit dynamics analysis of the natural system:

Now, we will derive the CO_2 level in the atmosphere as an explicit function of time, in case we have a natural system, (23), without any exogenous industrial emissions.

$$\dot{x} = a + bx \quad , \quad a > 0, b < 0 \tag{23}$$

(23) can be written as (24).

$$\dot{x} - bx = a \tag{24}$$

First, we study the homogenous equation, (25) and try to find an explicit solution.

$$\dot{x} - bx = 0 \tag{25}$$

We assume that the homogenous solution, $x_h(t)$ has the functional form found in (26). Z and λ are two parameters.

$$x_h(t) = Ze^{\lambda t} \tag{26}$$

The time derivative of the homogenous solution is found in (27).

$$\dot{x}_h(t) = \lambda Ze^{\lambda t} \tag{27}$$

Equations (28) to (31) give the value of λ .

$$\left(\dot{x} - bx = 0\right) \Rightarrow \left(\lambda Ze^{\lambda t} - bZe^{\lambda t} = 0\right) \tag{28}$$

$$\left(\lambda - b\right) Ze^{\lambda t} = 0 \tag{29}$$

$$\left(Z \neq 0 \wedge \lambda t > -\infty\right) \Rightarrow \left(\lambda - b = 0\right) \tag{30}$$

$$\lambda = b \tag{31}$$

The homogenous solution is reported in (32). This now contains one parameter that has not yet been determined, namely Z . Considerable efforts will be used to determine this parameter as a function of other relevant parameters, in the later part of this paper.

$$x_h(t) = Ze^{bt} \tag{32}$$

Particular solution:

It is also necessary to determine the particular solution. We start with equation (33).

$$\dot{x}(t) - bx(t) = a \tag{33}$$

We assume that the particular solution is an arbitrary constant, as in (34).

$$x_p(t) = m \tag{34}$$

As we see in (35), the time derivative of the particular solution is zero.

$$\dot{x}_p(t) = 0 \tag{35}$$

(33), (34) and (35) give (36) and (37).

$$\dot{x}_p(t) - bx_p(t) = -bm \tag{36}$$

$$-bm = a \tag{37}$$

(37) leads to the particular solution, namely equation (38).

$$x_p(t) = m = \frac{-a}{b} \tag{38}$$

The complete solution to the differential equation is the sum of the homogenous solution and the particular solution. Compare (39).

$$x(t) = x_h(t) + x_p(t) \tag{39}$$

The explicit form of the solution to the natural system is found in (40).

$$x(t) = Ze^{bt} - \frac{a}{b} \tag{40}$$

Since we already know that the value of b is strictly negative, it is clear that the homogenous solution (32) goes to zero as t goes to infinity. As a consequence, as we see in (41), the equilibrium is stable. Equation (40) also reveals that we have monotone convergence to the equilibrium. Furthermore, since the natural emission level is strictly positive, the equilibrium is strictly positive. Note that this equilibrium was also found in equation (8) and that the numerical values were derived in different units, in equations (9) and (10).

$$\lim_{\substack{t\to\infty \\ (a>0,b<0)}} x(t) = x(t) = -\frac{a}{b} > 0 \qquad (41)$$

The natural system converges to the "Pre industrial equilibrium".

Explicit dynamics analysis of the natural system with added exogenous forcing:

Now, we introduce exogenous (industrial) net emissions as an explicit function of time, in equation (42). This makes it possible to represent the future time path of alternative global CO_2 emissions as a quadratic function of time. Certainly, also other functional forms could be used. The polynomial is however a flexible tool with suitable properties for the relevant applications that will follow. We may also use the function to model alternative forestry expansion strategies, where intensified forestry can lead to increased absorption of CO_2.

$$f(t) = k_0 + k_1 t + k_2 t^2 \tag{42}$$

When (42) is added to the natural system differential function, we get (43).

$$\dot{x}(t) = a + bx(t) + f(t) \quad , \quad a > 0, b < 0 \tag{43}$$

The explicit version of (43) is (44). This may also be transformed to (45).

$$\dot{x}(t) = a + bx(t) + k_0 + k_1 t + k_2 t^2 \tag{44}$$

From (45) we understand that the solution to the homogenous solution, (32), will be useful also when we solve (45).

$$\dot{x}(t) - bx(t) = a + k_0 + k_1 t + k_2 t^2 \tag{45}$$

Determination of the particular solution:

Now, we need a more complicated functional form of the particular solution than when we only had to deal with the natural system, without changing exogenous emissions, as in equation (34).

$$x_p(t) = c_0 + c_1 t + c_2 t^2 \tag{46}$$

The time derivative of the particular solution is (47).

$$\dot{x}_p(t) = c_1 + 2c_2 t \tag{47}$$

(45), (46) and (47) lead to (48). We have to determine the three parameters in the particular solution, found in (46), from equation (48).

$$c_1 + 2c_2 t - b\left(c_0 + c_1 t + c_2 t^2\right) = a + k_0 + k_1 t + k_2 t^2 \tag{48}$$

It is clear that (48) has to be satisfied for every possible value of t. In (49), the LHS and RHS expressions are written in a more convenient form.

$$\left(c_1 - bc_0\right) + \left(2c_2 - bc_1\right)t + \left(-bc_2\right)t^2 = \left(a + k_0\right) + k_1 t + k_2 t^2 \tag{49}$$

We realize that the equation system found in (50) has to be satisfied. That system will hopefully be useful to derive the correct parameters of the particular solution (46).

$$\begin{cases} c_1 - bc_0 = a + k_0 \\ 2c_2 - bc_1 = k_1 \\ -bc_2 = k_2 \end{cases} \tag{50}$$

In (51), the equation system in (50) is represented in matrix format.

$$\begin{bmatrix} -b & 1 & 0 \\ 0 & -b & 2 \\ 0 & 0 & -b \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} a + k_0 \\ k_1 \\ k_2 \end{bmatrix} \tag{51}$$

We find that the system has a structure that makes it possible to solve it with a sequence of substitutions. If that would not have been the case, we could have used other methods from matrix algebra. From row 3, we instantly get the value of c_2. This is shown in (52).

$$(row3) \Rightarrow c_2 = \frac{-k_2}{b} \tag{52}$$

Then we move to row 2, which can be used to derive c_1 , using c_2 and some parameters. Compare equation (53).

$$\left(row\,2\right) \Rightarrow -bc_1 + 2c_2 = k_1 \tag{53}$$

Equations (54), (55) and (56) lead to the value of c_1.

$$-bc_1 + 2\left(\frac{-k_2}{b}\right) = k_1 \tag{54}$$

$$-bc_1 = k_1 + \frac{2k_2}{b} \tag{55}$$

$$c_1 = \frac{-k_1}{b} - \frac{2k_2}{b^2} \tag{56}$$

Finally, we can derive c_0 via equations (57) to (60).

$$\left(row1\right) \Rightarrow -bc_0 + c_1 = a + k_0 \tag{57}$$

$$-bc_0 = a + k_0 - c_1 \tag{58}$$

$$-bc_0 = a + k_0 + \frac{k_1}{b} + \frac{2k_2}{b^2} \tag{59}$$

$$c_0 = \frac{-\left(a + k_0\right)}{b} - \frac{k_1}{b^2} - \frac{2k_2}{b^3} \tag{60}$$

The particular solution (61) is found in explicit form in (62).

$$x_p(t) = c_0 + c_1 t + c_2 t^2 \tag{61}$$

$$x_p(t) = \left(\frac{-\left(a + k_0\right)}{b} - \frac{k_1}{b^2} - \frac{2k_2}{b^3} \right) + \left(\frac{-k_1}{b} - \frac{2k_2}{b^2} \right) t + \left(\frac{-k_2}{b} \right) t^2 \tag{62}$$

The CO2 level in the atmosphere, (63), is the sum of the homogenous solution and the particular solution.

$$x(t) = x_h(t) + x_p(t) \tag{63}$$

The homogenous solution is found in (64) and the CO_2 level in the atmosphere is shown in (65).

$$x_h(t) = Ze^{bt} \tag{64}$$

$$x(t) = Ze^{bt} + \frac{-(a+k_0)}{b} - \frac{k_1}{b^2} - \frac{2k_2}{b^3} + \left(\frac{-k_1}{b} - \frac{2k_2}{b^2}\right)t - \frac{k_2}{b}t^2 \tag{65}$$

Clearly, before we move further and apply function (65), it is important to verify that the function is correct. We use the following procedure: First we derive the time derivative of (65), namely (66).

$$\dot{x}(t) = bZe^{bt} - \frac{k_1}{b} - \frac{2k_2}{b^2} - \frac{2k_2}{b}t \qquad (66)$$

Then, we remember the original differential equation (67).

$$\dot{x}(t) = a + bx(t) + k_0 + k_1 t + k_2 t^2 \qquad (67)$$

Equation (66) is defined from equation (68).

$$\phi = bZe^{bt} - \frac{k_1}{b} - \frac{2k_2}{b^2} - \frac{2k_2}{b}t \qquad (68)$$

Equation (69) is defined from equation (67).

$$\varphi = a + bx(t) + k_0 + k_1 t + k_2 t^2 \qquad (69)$$

We denote the difference between the expressions (68) and (69) by (70). Then, we find that the difference is zero, which means that the expressions are equal, as in equation (70).

$$\phi-\varphi = bZe^{bt} - \frac{k_1}{b} - \frac{2k_2}{b^2} - \frac{2k_2}{b}t - a$$
$$-b\left(Ze^{bt} + \frac{-(a+k_0)}{b} - \frac{k_1}{b^2} - \frac{2k_2}{b^3} + \left(\frac{-k_1}{b} - \frac{2k_2}{b^2} \right) t - \frac{k_2}{b} t^2 \right) - k_0 - k_1 t - k_2 t^2 \tag{70}$$

$$\phi-\varphi = 0 \quad Q.E.D. \tag{71}$$

In other words, the CO_2 level in the atmosphere should really follow equation (65).

$$x(t) = Ze^{bt} + \frac{-(a+k_0)}{b} - \frac{k_1}{b^2} - \frac{2k_2}{b^3} + \left(\frac{-k_1}{b} - \frac{2k_2}{b^2} \right) t - \frac{k_2}{b} t^2 \tag{65}$$

The general solution to the differential equation has been determined. Furthermore, we have already determined the empirically relevant values of two parameters, a and b, from the empirical data.

Now, we will study the future of the CO_2 level in the atmosphere, x(t), as a function of alternative levels of emissions and forestry activities. We define time zero as “the middle of year 2022”, namely July 1, 2022. Then, t=0. At that time, x(0) = x_0.

For alternative assumptions concerning emissions and forestry activities, we can also determine the parameters of the exogenous forcing function, f(.), namely k_0, k_1 and k_2. With all of this information available, we can determine the final free parameter of the differential function, namely Z.

From (65), we get the general function of x(t):

$$x(t) = Ze^{bt} + \frac{-(a+k_0)}{b} - \frac{k_1}{b^2} - \frac{2k_2}{b^3} + \left(\frac{-k_1}{b} - \frac{2k_2}{b^2}\right)t - \frac{k_2}{b}t^2 \tag{72}$$

We introduce the initial condition, the value of x(t) at t=0.

$$x_0 = Ze^{b\times 0} + \frac{-(a+k_0)}{b} - \frac{k_1}{b^2} - \frac{2k_2}{b^3} + \left(\frac{-k_1}{b} - \frac{2k_2}{b^2}\right)\times 0 - \frac{k_2}{b}\times 0^2 \tag{73}$$

Now, we can determine Z as a function of the parameters:

$$Z = 1 \cdot \left(x_0 - \left(\frac{-\left(a + k_0\right)}{b} - \frac{k_1}{b^2} - \frac{2k_2}{b^3} \right) \right) \tag{74}$$

$$Z = x_0 + \frac{\left(a + k_0\right)}{b} + \frac{k_1}{b^2} + \frac{2k_2}{b^3} \tag{75}$$

In the special case when $k_2=0$, we have:

$$Z_1 = Z\big|_{k_2=0} = x_0 + \frac{\left(a + k_0\right)}{b} + \frac{k_1}{b^2} \tag{76}$$

Now, with this information about Z as a function of the parameters, we have:

$$x(t) = Ze^{bt} + \frac{-\left(a + k_0\right)}{b} - \frac{k_1}{b^2} - \frac{2k_2}{b^3} + \left(\frac{-k_1}{b} - \frac{2k_2}{b^2} \right) t - \frac{k_2}{b} t^2 \tag{77}$$

$$x(t) = \left(x_0 + \frac{(a+k_0)}{b} + \frac{k_1}{b^2} + \frac{2k_2}{b^3} \right) e^{bt}$$
$$- \frac{(a+k_0)}{b} - \frac{k_1}{b^2} - \frac{2k_2}{b^3} + \left(\frac{-k_1}{b} - \frac{2k_2}{b^2} \right) t - \frac{k_2}{b} t^2 \tag{78}$$

$$x(t) = x_0 e^{bt} + \left(\frac{(a+k_0)}{b} + \frac{k_1}{b^2} + \frac{2k_2}{b^3} \right) \left(e^{bt} - 1 \right) - \left(\frac{k_1}{b} + \frac{2k_2}{b^2} \right) t - \frac{k_2}{b} t^2 \tag{79}$$

and the special case:

$$x_1(t) = x(t)\Big|_{k_2=0} = x_0 e^{bt} + \left(\frac{(a+k_0)}{b} + \frac{k_1}{b^2} \right) \left(e^{bt} - 1 \right) - \frac{k_1}{b} t \tag{80}$$

In Figure 4., equation (80) is used to predict the CO_2 time path from 1990 to 2021. It is compared to the real observations.

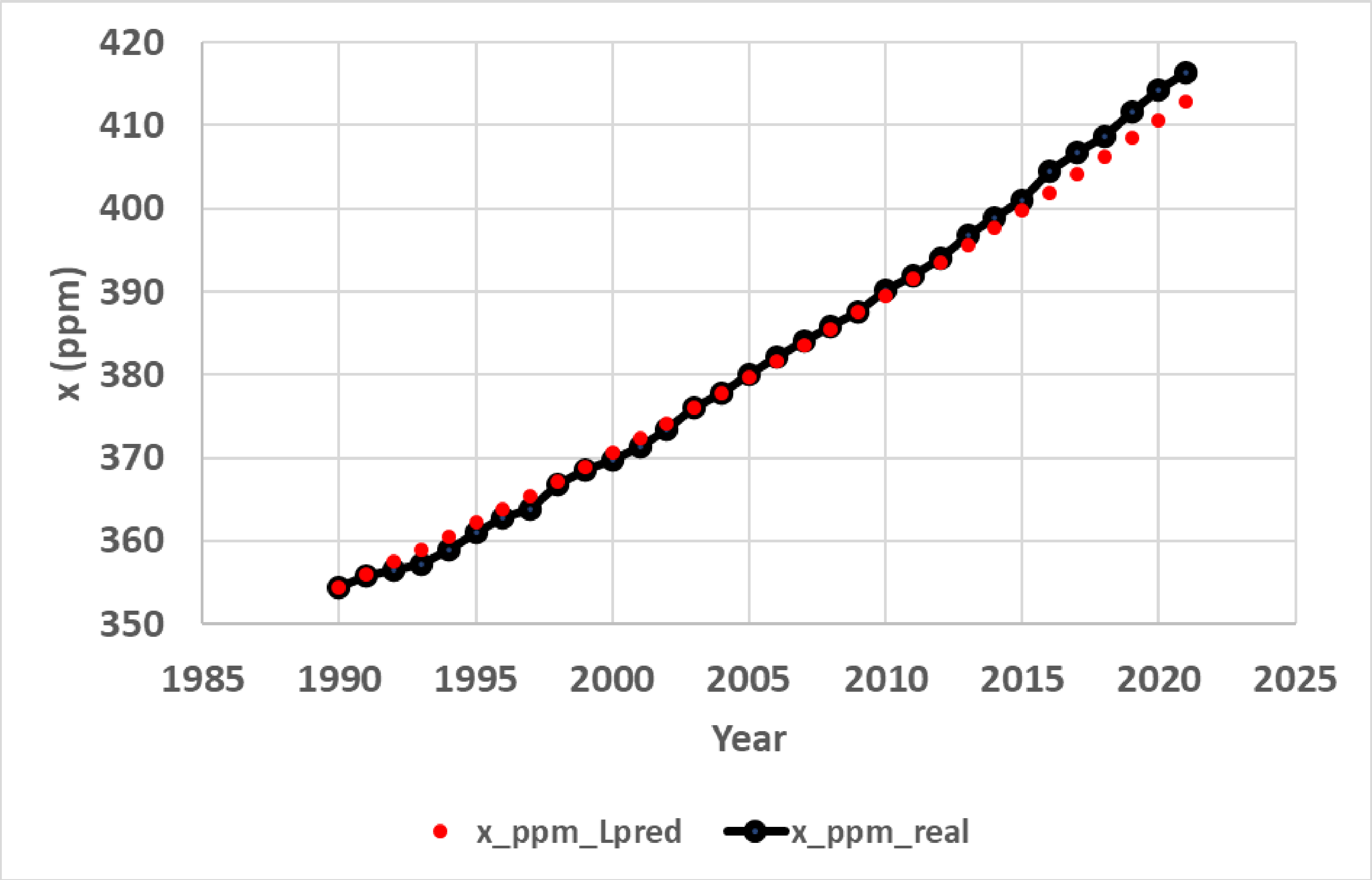


Figure 4.

The time path of the CO_2 level in the atmosphere, x, from year 1990 until 2021, in the empirical data, x_ppm_real, and the prediction, x_ppm_Lpred, via the solution to the differential equation (80), based on the assumptions that the initial CO2 level in year 1990 and all other parameters are known. In year 1990, t = 0. Parameter k_0 is the emission level in 1990, namely 22.729 (Gt) and k_1 = (the emissions in 2021 – the emissions in 1990) / (31 years). This calculation gives a k_1 value of approximately 0.4269 (Gt per year). Tables 1 and 2 contain the other parameters. The differential equation prediction follows the true development rather well, but underestimates the latest CO_2 levels slightly. One reason may be variations in the industrial emission increments. Still, the prediction model results replicate the true history rather well and we may believe in equation (80).

In Figures 5 and 6, we see the predictions of the future CO_2 development, from year 2022 until year 2100, based on equation (80) and alternative emission strategies. It is clearly possible to reduce or increase the future CO_2 concentrations very much, depending on the selected emission strategy. Note that, even if we select to reduce the emissions by 1.5 Gt/year, the CO2 concentration in the atmosphere will continue to increase from the 2022 level during several years, before it starts to decrease.

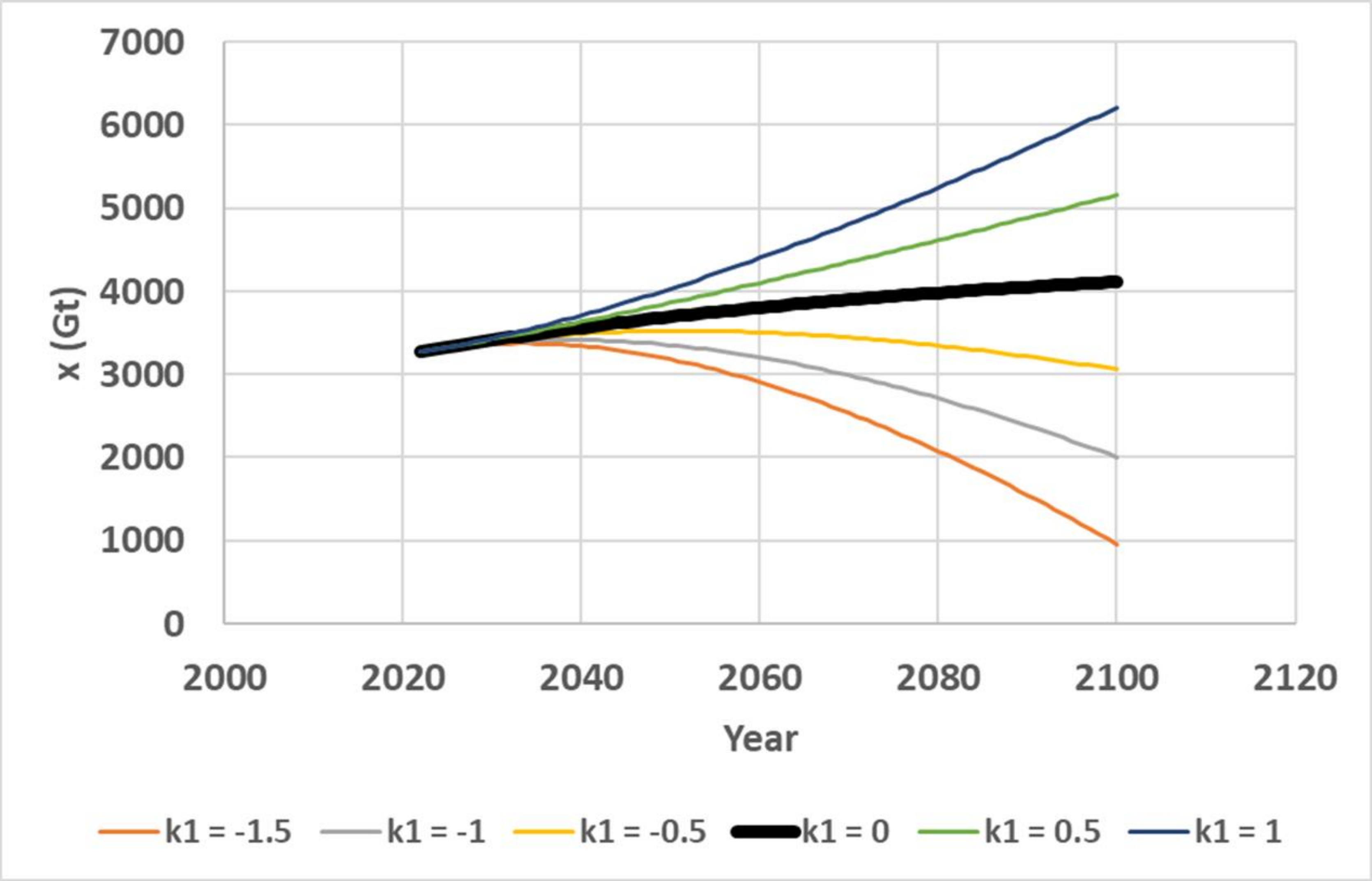


Figure 5.

Emission strategy conditional predictions of the CO_2 level in the atmosphere, x (Gt), from year 2022 until 2100, via equation (80), based on the assumptions that the initial CO_2 level in year 2022 and all other parameters are known. In year 2022, t = 0, and the initial value x_0 is estimated from the values in 2020 and 2021. (3250.11-3232.86) + 3250.11 = 3267.36 (Gt). Parameter k_0 is the estimated emission level in 2022, assumed to be identical to the level in 2019, directly before the Corona pandemic, namely 37.911 (Gt). The time derivative of the emissions, k_1, takes alternative values, from -1.5 to + 1.0 (Gt per year). Tables 1 and 2 contain the other parameters.

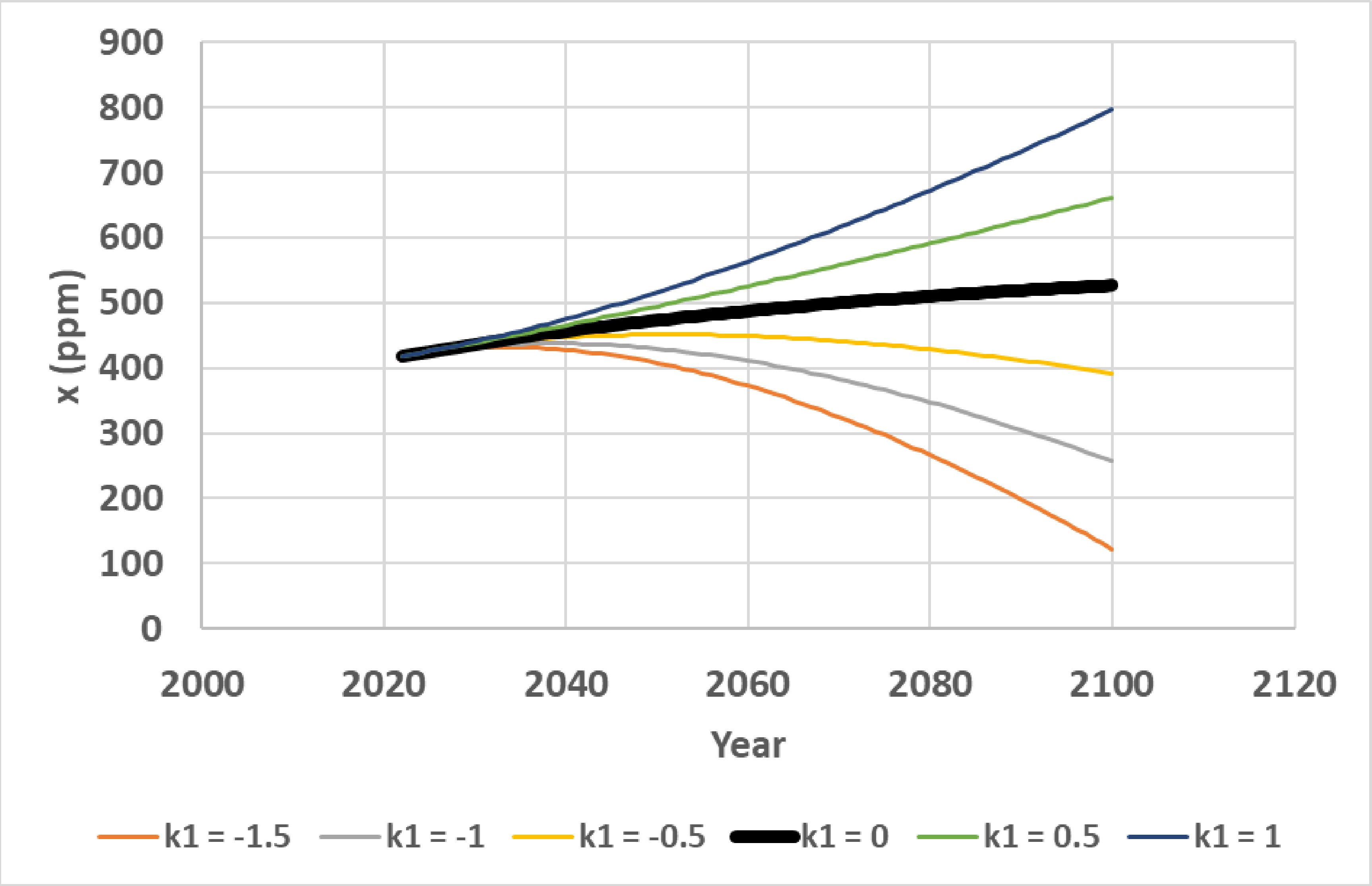


Figure 6.

Emission strategy conditional predictions of the CO2 concentration in the atmosphere, x (ppm), from year 2022 until 2100, via equation (80), based on the assumptions that the initial CO2 level in year 2022 and the parameters were known. In year 2022, t = 0, and the initial value x_0 is estimated from the values in 2020 and 2021. (3250.11-3232.86) + 3250.11 = 3267.36 (Gt). Parameter k_0 is the estimated emission level in 2022, assumed to be identical to the level in 2019, directly before the Corona pandemic, namely 37.911 (Gt). The time derivative of the emissions, k_1, takes alternative values, from -1.5 to + 1.0 (Gt per year). Tables 1 and 2 contain the other parameters.

The decision k_1 and effects on the climate:

$$x(t) = x_0 e^{bt} + \left(\frac{(a+k_0)}{b} + \frac{k_1}{b^2} \right) \left(e^{bt} - 1 \right) - \frac{k_1}{b} t \qquad (81)$$

The derivative of x with respect to k_1 is found in (82).

$$\frac{dx}{dk_1} = b^{-2} \left(e^{bt} - 1 \right) - b^{-1} t \qquad (82)$$

It is important to know if this derivative can be signed. The following procedure makes this possible:

$$\left. \frac{dx}{dk_1} \right|_{t=0} = 0 \qquad (83)$$

$$\frac{d^2x}{dk_1dt} = b^{-1}\left(e^{bt} - 1\right) \tag{84}$$

$$\left.\frac{d^2x}{dk_1dt}\right|_{t>0} > 0 \tag{85}$$

Now, we know that the derivative is zero for t=0 and increases with t. As a result, we get (86).

$$\left(\left(\left.\frac{dx}{dk_1}\right|_{t=0} = 0\right) \wedge \left(\left.\frac{d^2x}{dk_1dt}\right|_{t>0} > 0\right)\right) \Rightarrow \left(\left.\frac{dx}{dk_1}\right|_{t>0} > 0\right) \tag{86}$$

Hence, we know that the derivative of the CO_2 level with respect to k_1 is strictly positive, at every future point in time. As we see in equation (87), the derivative of the CO_2 level with respect to k_1 is a strictly increasing function of time.

$$\frac{d^3x}{dk_1dt^2} = e^{bt} > 0 \tag{87}$$

If we are interested to control the climate via the CO_2 level, we should have some objective function that makes it possible to know how the utility is affected by the CO_2 level at different points in time. Let us consider a CO_2 path dependent utility function, which is scaled in such a way that it can be expressed in economic terms. In the rest of this paper, the expression "utility" should be understood as the total economic value of the utility, from t = 0 until t = T. Note that we are interested in the climate from t = 0 until some future point time, T.

$$U = \int_0^T v\left(x\left(t, k_1\right)\right) dt \qquad (88)$$

The marginal utility of increasing k_1 is found in (89). We assume that we prefer to have a colder climate, and for that reason want to have a lower CO_2 level in the atmosphere. In other words; The derivative of v with respect to x should be strictly negative.

$$\frac{dU}{dk_1} = \int_0^T \frac{dv}{dx}(.)\frac{dx}{dk_1}(t,k_1)\,dt \quad , \quad \frac{dv}{dx}(.) < 0 \tag{89}$$

We assume that k_1 is a function of $k_{1,I}$ and k_3, where the first part, $k_{1,I}$, is caused by changes in industrial emissions and the second part, k_3, is caused by increased net absorption of CO_2 in forests, because of investments in more productive and sustainable forestry.

$$k_1 = k_{1,I} - k_3 \tag{90}$$

Obviously, the marginal utility of forestry investments (91), has the opposite sign compared to (89).

$$\frac{dU}{dk_3} = -\int_0^T \frac{dv}{dx}(.)\frac{dx}{dk_1}(t,k_1)\,dt \tag{91}$$

The forestry investment optimization problem is found in (92). $C(k_3)$ is the investment cost function at a particular point in time during the investment process. The interest rate in continuous time is r. It is assumed that a particular level of "continuous investment" is selected, which for instance can mean that some new active forestry is started, each year, from time 0 until time T. Each year within this time interval, the area of "new forestry" increases with the same number of hectares. This type of action can for instance be made in very large regions in Canada and Russian Federation, where presently no active forestry can be found. It is natural to distribute such investments over time, in this way, since it takes considerable time to construct new infrastructure and to expand the capacities of industrial facilities and the labor force. The variable cost at time t is jk_3t, where j is a new cost parameter.

$$\max_{k_3} \pi = -\int_0^T e^{-rt} C(k_3) dt - \int_0^T e^{-rt} jk_3 t \, dt + \int_0^T v\left(x\left(t, k_{1,I} - k_3\right)\right) dt \qquad (92)$$

Clearly, parts of the cost functions can be placed outside the integrals.

$$\max_{k_3} \pi = -C(k_3)\int_0^T e^{-rt} dt - jk_3 \int_0^T e^{-rt} t\, dt + \int_0^T v\Big(x\big(t, k_{1,I} - k_3\big)\Big) dt \tag{93}$$

As we see in (94), the first two integrals can be explicitly calculated.

$$\max_{k_3} \pi = -C(k_3)\left(-\frac{e^{-rt}}{r}\Bigg]_0^T \right) - jk_3 \left(\frac{e^{-rt}}{r^2}\left(-rt-1\right)\Bigg]_0^T \right) + \int_0^T v\Big(x\big(t, k_{1,I} - k_3\big)\Big) dt \tag{94}$$

In (95), we have the optimization problem in explicit form.

$$\max_{k_3} \pi = -C(k_3)\left(\frac{1-e^{-rT}}{r} \right) - jk_3 \left(\frac{1-e^{-rT}\left(1+rT\right)}{r^2} \right) + \int_0^T v\Big(x\big(t, k_{1,I} - k_3\big)\Big) dt \tag{95}$$

The first order optimum condition is:

$$\frac{d\pi}{dk_3} = -\left(\frac{1-e^{-rT}}{r}\right)\frac{dC}{dk_3} - j\left(\frac{1-e^{-rT}\left(1+rT\right)}{r^2}\right) - \int_0^T \frac{dv}{dx}(.)\frac{dx}{dk_1}\left(t,k_1\right)dt = 0 \qquad (96)$$

The second order derivative of the objective function with respect to the forestry investment level is shown in (97).

$$\frac{d^2\pi}{dk_3{}^2} = -\left(\frac{1-e^{-rT}}{r}\right)\frac{d^2C}{dk_3{}^2} \qquad (97)$$

We assume that the rate of interest is strictly positive, that the investment process continues during a strictly positive time interval and that the cost function is strictly convex. In (98), we see that the objective function is a strictly concave function of the forestry investment level.

$$\left(r > 0 \wedge T > 0 \wedge \frac{d^2C}{dk_3{}^2} > 0\right) \Rightarrow \left(\frac{d^2\pi}{dk_3{}^2} < 0\right) \qquad (98)$$

Hence, we have a unique maximum. Let us determine an explicit expression for the unique maximum. We assume that the derivative of the utility function with respect to the CO_2 level is constant, namely v_1. Alternative assumptions can of course be made, if some empirical facts can be shown to support such assumptions. The resulting first order optimum condition is found in (99).

$$\left(v_1 = \frac{dv}{dx}(.) = const \right) \Rightarrow$$
$$\left(\frac{d\pi}{dk_3} = -\left(\frac{1-e^{-rT}}{r} \right) \frac{dC}{dk_3} - j\left(\frac{1-e^{-rT}\left(1+rT\right)}{r^2} \right) - v_1 \int_0^T \frac{dx}{dk_1}\left(t, k_1\right) dt = 0 \right) \tag{99}$$

Now, since we already know the derivative of the CO_2 level with respect to k_1, from (82), we get (100).

$$\frac{d\pi}{dk_3} = -\left(\frac{1-e^{-rT}}{r} \right) \frac{dC}{dk_3} - j\left(\frac{1-e^{-rT}\left(1+rT\right)}{r^2} \right) - v_1 \int_0^T \left(b^{-1}\left(b^{-1}\left(e^{bt} - 1 \right) - t \right) \right) dt = 0 \tag{100}$$

(100) can be further developed to (101), (102) and (103).

$$\frac{d\pi}{dk_3} = -\left(\frac{1-e^{-rT}}{r}\right)\frac{dC}{dk_3} - j\left(\frac{1-e^{-rT}\left(1+rT\right)}{r^2}\right) - v_1\left(b^{-2}\int_0^T e^{bt}dt - b^{-2}\int_0^T 1dt - b^{-1}\int_0^T t\,dt\right) = 0 \quad (101)$$

$$\frac{d\pi}{dk_3} = -\left(\frac{1-e^{-rT}}{r}\right)\frac{dC}{dk_3} - j\left(\frac{1-e^{-rT}\left(1+rT\right)}{r^2}\right) - v_1\left(b^{-2}\left.\frac{e^{bt}}{b}\right]_0^T - b^{-2}t\Big]_0^T - b^{-1}\left.\frac{t^2}{2}\right]_0^T\right) = 0 \quad (102)$$

$$\frac{d\pi}{dk_3} = -\left(\frac{1-e^{-rT}}{r}\right)\frac{dC}{dk_3} - j\left(\frac{1-e^{-rT}\left(1+rT\right)}{r^2}\right) - v_1\left(\frac{\left(e^{bT}-1\right)}{b^3} - \frac{T}{b^2} - \frac{T^2}{2b}\right) = 0 \quad (103)$$

The last part of the expression (103) deserves a special treatment. We define that as W in equation (104). We are interested to determine the sign of W.

$$W = \frac{\left(e^{bT}-1\right)}{b^3} - \frac{T}{b^2} - \frac{T^2}{2b} \quad (104)$$

$$W = \frac{\left(e^{bT} - 1\right)}{b^3} - \frac{T}{b^2} - \frac{T^2}{2b} \tag{104}$$

In (105), we find that W = 0 for T = 0. Equations (106), (107) and (108) make sure that W is strictly positive for strictly positive T.

$$W\big|_{b \neq 0, T = 0} = 0 \tag{105}$$

$$\frac{dW}{dT} = b^{-2}\left(e^{bT} - 1 - bT\right) \tag{106}$$

$$\left(\gamma = bT \neq 0\right) \wedge \left(e^{\gamma} > (1 + \gamma)\ \forall\ \gamma\big|_{\gamma \neq 0}\right) \Rightarrow \left(\frac{dW}{dT} > 0\right) \tag{107}$$

$$\left(W\big|_{b \neq 0, T = 0} = 0\right) \wedge \left(\frac{dW}{dT} > 0\right) \Rightarrow \left(W\big|_{b \neq 0, T > 0} > 0\right) \tag{108}$$

From equation (103), we get the optimal value of the marginal cost of the investment level, in (109). Since this marginal cost is a monotonically increasing function of the investment level, it will soon be possible to determine the optimal investment level from this value.

$$\left(\frac{d\pi}{dk_3}=0\right)\Rightarrow\frac{dC}{dk_3}=\frac{-v_1\left(\frac{\left(e^{bT}-1\right)}{b^3}-\frac{T}{b^2}-\frac{T^2}{2b}\right)-j\left(\frac{1-e^{-rT}\left(1+rT\right)}{r^2}\right)}{\left(\frac{1-e^{-rT}}{r}\right)} \qquad (109)$$

We assume that the optimal value of the marginal investment cost function is strictly positive (110).

$$\frac{dC}{dk_3}=\frac{-v_1\left(W\right)-j\left(\frac{1-e^{-rT}\left(1+rT\right)}{r^2}\right)}{\left(\frac{1-e^{-rT}}{r}\right)}>0 \qquad (110)$$

We assume that the investment cost function can be approximated as a quadratic function. When empirical data becomes available, the parameters can be estimated.

$$C = g_0 + g_1 k_3 + g_2 k_3^{\,2} \quad , \quad g_0 \geq 0, g_1 > 0, g_2 > 0 \tag{111}$$

We also assume that the fix cost, g_0, is comparatively small and does not make it more profitable to avoid all investments studied in this article. This is of course an empirical question, but in typical cases, g_0 should be small. The marginal cost is found in (112).

$$\frac{dC}{dk_3} = g_1 + 2g_2 k_3 \tag{112}$$

(109) and (112) lead to (113).

$$g_1 + 2g_2 k_3 = \frac{-v_1\left(\frac{\left(e^{bT}-1\right)}{b^3} - \frac{T}{b^2} - \frac{T^2}{2b}\right) - j\left(\frac{1-e^{-rT}\left(1+rT\right)}{r^2}\right)}{\left(\frac{1-e^{-rT}}{r}\right)} \tag{113}$$

We assume that the parameter g_1 is sufficiently small to motivate a strictly positive investment level. (It is easy to show that, in an earlier "forestry investment equilibrium", obtained when the utility of climate change was not considered, the investments took place until the marginal cost of expansion was equal to the marginal revenue of increased access to forest areas. Hence, this argument tells us that g_1 can be expected to be very close to zero. This makes it highly probable that (114) is relevant.

$$g_1 < \frac{-v_1(W) - j\left(\frac{1-e^{-rT}(1+rT)}{r^2}\right)}{\left(\frac{1-e^{-rT}}{r}\right)} \tag{114}$$

Then, the unique and strictly positive optimal investment in higher CO_2 absorption via forestry is given in (115).

$$k_3^* = k_3 = \frac{\left(\dfrac{-v_1\left(\dfrac{\left(e^{bT}-1\right)}{b^3}-\dfrac{T}{b^2}-\dfrac{T^2}{2b}\right)-j\left(\dfrac{1-e^{-rT}\left(1+rT\right)}{r^2}\right)}{\left(\dfrac{1-e^{-rT}}{r}\right)}-g_1 \right)}{2g_2} > 0 \tag{115}$$

OBSERVATIONS 1.

1.a. **It is possible to model the dynamics of the CO_2 level in the atmosphere via a differential equation. The formulated hypotheses, of how the CO_2 level is affected by natural emissions and concentration dependent absorption, could not be rejected.**

1.b. **The parameters were empirically estimated, with high precision, from the latest available empirical time series of observations of CO_2 concentration in the atmosphere, and industrial emissions.**

1.c. **It is possible to determine the time path of the CO_2 concentration of the natural system without industrial emissions, for arbitrary initial conditions.**

1.d. **This system has a unique and stable equilibrium, with an expected estimated value of 262 ppm. With constant industrial emissions, the equilibrium would be found at a higher level, according to an explicit equation.**

OBSERVATIONS 2.

2.a. Comparative statics analysis shows how the equilibrium is affected by alternative parameter adjustments.

2.b. An extended version of the natural differential equation, with a forcing function, a quadratic function of time, representing the time paths of industrial emissions, has been developed.

2.c. The general function of the time path of the CO_2 concentration of the natural system under the influence of industrial emissions, has been determined for arbitrary initial conditions and parameters of the industrial emission function.

OBSERVATIONS 3.

3.a. The CO_2 time path function has been analytically verified and empirically tested and found to be able to reproduce the historical CO_2 observations with high precision.

3.b. The time paths of the future CO_2 concentrations have also been calculated, for six alternative levels of change of the industrial emissions, from -1.5 Gt/year to +1.0 Gt/year, from the year 2022 until 2100.

3.c. The net CO_2 emissions can be reduced over time, if sustainable forestry is gradually intensified. The rational intensity of this investment process has been determined.

3.d. An explicit function for the optimal forestry intensification level, based on all CO_2 time path function parameters, the marginal cost of the CO_2 concentration, time interval parameters, rate of interest and cost function parameters, has been derived.

Optimal management of the climate: FIRST STEP:

As a *first step*, we have to know, and agree about, the economic value of decreasing the CO_2 concentration by 1 ppm.

(If we can not agree about that, we will not agree about rational investment levels in emission reductions and/or optimal areas of intensified forestry.)

For these reasons, the author hopes and suggests that United Nations initiates an international research and negotiation process where the fundamental principles and facts of relevance to managing the CO_2 concentration problem are in focus. In this work, the analyses and results presented in this paper can hopefully be useful as a starting point.

Conclusions:

The differential equation of the CO_2 concentration in the atmosphere:

Fundamental theory, mathematics and statistical estimation.

Time path of the CO_2 concentration:

Determined without and with arbitrary industrial emissions.

Historical CO_2 observations:

Reproduced by the model.

The CO_2 concentration equilibrium:

Exists, is unique and stable.

Intensified sustainable forestry:

Reduces the future CO_2 concentration.

The optimal forestry intensification level:

Is determined as an explicit function of all parameters.

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