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THE OPTIMAL DYNAMIC PRODUCTION AND STOCK LEVELS UNDER THE INFLUENCE OF STOCHASTIC DEMAND AND PRODUCTION COST FUNCTIONS:
Theory and Application to the Pulp Industry Enterprise

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It is tempting to analyze production and stock level problems with deterministic and other simple assumptions. Several versions of simple and partial optimal inventory formulae are available in the literature. However, these formulae seldom give the optimal answer to the relevant problem. A critical discussion of economic predictions is included. The optimal dynamic production and stock levels are determined under the influence of stochastic demand and production cost functions. The sensitivity of the optimal solution to the parameters is derived. A forest industry example is constructed and discussed. The approach used is stochastic dynamic programming in Markov chains with an infinite horizon where the relevant large dimension linear programming problem is solved via policy iteration. The solution converges to the global optimum in a finite number of iterations and the necessary size of the computer memory is approximately N times smaller than if the simplex method would have been used (where $N = 2 * K * K$, K = number of feasible decisions per state). An optimization program for the personal computer is constructed and included in the numerical appendix.

KEY WORDS Dynamic production, pulp industry, stock level, stochastic dynamic programming problem, forest industry.

1. INTRODUCTION

1.1. The Questions and the Content

How much should we produce today when we may store one part (for future sales) and sell one part (of the production and the entering stock)?

This general question will be asked in this paper under the assumptions of stochastic demand and production cost functions. Empirical data relevant to a pulp producing enterprise active in the international pulp and Swedish pulp wood markets will be presented and discussed. A pulp producing enterprise will be defined and the optimal market dependent dynamic production, stock level and sales decisions will be determined.

1.2. Predictions in Economics—A Discussion of Recent Contributions

One of the more important questions is if there are any theoretical reasons why we should not use traditional deterministic inventory formulae. One may argue that there are reasons for every event that takes place. If we know how things affect each other and know the initial state of the world, then we should be able to predict the future. Unfortunately, and for many reasons fortunately, this is not possible. One simple reason is that we do not know much enough about the present state of the world. Complete information would be too expensive and for many reasons not possible to obtain. The other reason is that many systems give deterministic chaos. Even in very simple nonlinear determinisitc systems, it has been shown that the future state can not be predicted. We should not expect to be able to predict the business cycles or the prices of specific products very well in the real world since the economic system is not only nonlinear but also very complex. The interested reader is suggested to read Puu (1991), chapter 4, where these issues are discussed in detail.

Brännäs and De Gooijer (1991) introduce autoregressive assymetric moving average models in the modelling of business cycle data. In the introduction they discuss results reported in the the business cycle literature. They write that some authors have and some have not found evidence of chaos and/or nonlinearity in tested GNP series.

In oligopolistic and oligopsonistic markets, there are often game theoretic reasons why firms take decisions that can not be perfectly predicted by the competitors. We may say that the firms introduce quasi-random behavior because they find it profitable. von Stackelberg (1934) and (1938) was the pioneer in this field. A recent contribution to the theory and application is found in Lohmander (1991).

We should be aware that most dynamical systems in the literature that have been shown to give chaotic solutions, have parameters that are fixed over time. This may give the impression that all variables in chaotic systems stay within some known intervals for ever and that a probability density function can be estimated during some time interval and used in later time intervals. This is not necessarily true in real world systems since the parameters can be expected to vary over time in ways that are not necessarily predictable. Hence, it is not obvious how and if we could detect chaotic behaviour in real world time series. Maybe we will never be sure if a time series is generated by a deterministic chaotic system or really is "truely" stochastic (if such phenomena can exist).

Löfgren, Ranneby and Sjöstedt (1989) investigate the business cycle forecasting problem. They write that business cycles have been defined as the:—"... recurrent fluctuations of output about trend and the co-movements among other aggregate time series." They quote Burns and Mitchell (1946) who write:—"... in duration business cycles vary from more than one year to ten or twelve years; they are not divisible into shorter cycles of similar character with amplitudes approximating their own." Löfgren, Ranneby and Sjöstedt suggest a forecasting method which they claim should work well when the time series are stationary. They write that trends and seasonal variation in time series must be taken away before the methods can be used. Different methods are discussed to handle this problem. Finally they select to decompose the original time series into two components. One series contains trend and seasonal variation and the other series contains the deviation from the first series, something which may be regarded as a stationary time series. The series containing trend and seasonal variation is estimated as a polynomial with dummy variables for the different seasons.

One problem with the approach of Löfgren, Ranneby and Sjöstedt, and of course with most forecasting models, is that we do not know if the trend that we think that we can isolate from the original time series really is a trend that tells us something about the future. Maybe the series simply is a nonstationary Martingale. Note that even the time path of a Martingale, a stochastic process with statistically independent increments with expected value zero in every time period, gives time segments where we could easily find "trends". These "trends" are however completely random (they could not be predicted before the increments were revealed) and tell us nothing about the future behaviour of the time path. (Löfgren, Ranneby and Sjöstedt carefully write that their method presupposes that the regression model is correct. Most likely, one should here include the assumption of the predictable trend.)

It has always been tempting to predict the future state of the world. This has never been easy. Two recent examples from the Swedish forest sector will be mentioned here. In 1988, Wibe (1988) predicts (translations from Swedish by the author):

—"Until the year 2000, the demand for pulp is expected to increase about 2% annually, while the growth in sawn wood demand stays at about 0.5%."

—The expected increase in demand (about 2% annually) is of approximately the same magnitude as the expected increase in supply (in the raw materials market). Thus, there is no reason to expect a general shortage of round wood (with increasing prices) or an excess supply (with falling prices). The likely development is that these two (demand and supply) will be balanced in the time period under consideration (until the year 2000)."

—"At least until the end of the century, we can make the certain prediction that our competitive power is not seriously threatened."

—"The forecasts are maybe not extremely good but they are not particularly dark either. There are threats, but no threats that mean that we should make sudden and drastic changes in the course."

The reader should observe that these predictions were not made very long ago. The reader may also compare the predictions with the present state of the Swedish and the international economies. The Swedish forest industry has this year already had to close down several pulp mills and the severe economic conditions in the forest industry are under debate in the press every day.

Lönnstedt (1991) made his predictions even more recently, namely January 21, 1991. He writes (translations from Swedish by the author):

—"I am convinced that we will soon experience a new raise for the Swedish forest sector. However, I had better make a reservation concerning the time it takes before the business cycle turns."

—"To me, the development of the Swedish forest industry looks bright (even if there may be a certain pause)."

—"The forest sector and in particular forestry lives with a high degree of uncertainty".

Note that these predictions are rather ambiguous. As long as you do not say when things will become better, it is impossible to say that your prediction was wrong. On the other hand, it is possible that, some time in the future, things will become better. Then, you know that you made a correct prediction!

Of course, people who make positive predictions can afterwards say:

—My predictions were correct (if they happened to be correct) and you should believe in my predictions also in the future.
—I made my best to give a positive perspective and courage to the decision makers in the markets. That is why I made a very positive prediction. Unfortunately, this was not enough. (If they happened to be wrong.)

Most people dislike a person who makes a negative prediction. Thus, we should often expect predictions (in particular predictions without strong theoretical and empirical documentation) to be biased towards the positive side.

In this paper, the general market hypothesis made is that the prices of the pulp and the pulp wood are determined by two different quantity dependent functions. These functions contain stochastic components that can not be predicted in any other way than in the sense that the probability distribution is known.

1.3. The Inventory Level in the Forest Industry

Bergman and Löfgren (1989) aim at investigating whether inventory policy is mainly a means to handle uncertain supply, without dealing explicitly with the complications created by the determination of an optimal inventory policy. They statistically find support for the idea that the Swedish forest industry increases the inventory when the round wood supply in the domestic market is greater than expected and vice versa. Bergman and Löfgren write that a partial adjustment process is not inconsistent with their empirical findings. They claim that a partial adjustment mechanism would be the most "economic" way to restore the optimal inventory level.

Here, we should note that Bergman and Löfgren assume the existence of an optimal inventory level. In this paper, a stochastic dynamic optimization model will be developed. It will be shown that the optimal inventory level in a model enterprise is highly market dependent. The price levels in the pulp wood market and in the pulp market together with the entering stock level from the previous period determine the optimal level of the stock until the next period.

1.4. Multi Stage Adaptive Decisions

If we agree that the future can not be perfectly predicted, we should not neglect the temporal structure of decisions in real world problems. Later decisions can and should be based on later information than the earlier decisions. The initial decisions should take the economically valuable future level of flexibility into consideration. In the last years, the stochastic and adaptive decision theory with applications to resource management has exploded. Adaptive economic theory is not a strange and restrictive branch of only theoretical interest. Adaptive economic theory is a more general theory than the traditional deterministic theory. It has strong implications to practical economic decisions. It should, if possible, be used in the planning of all economic decisions that have effects on the future state of the world. In the reference list, several examples by Lohmander and by Gong from the forest sector are included. In the analysis in this paper, the adaptive approach will be used.

1.5. Assumptions, Consequences and Aspects on Implementation

In the early university courses of business and management, most students learn how to calculate the optimal reorder quantity and the optimal average inventory level under specific deterministic assumptions. Most management books, for instance Sasieni, Yaspan and Friedman (1965), Ackoff and Sasieni (1968), Wagner (1975), Ullman (1976) and Baumol (1977), contain such sections.

In some courses, the students learn to derive the formulae via the first order optimization conditions. This is of course a valuable exercise in calculus and optimization. However, in real world problems, the assumptions made in the inventory calculations of the course are seldom relevant.

In several publications including many of the mentioned course books, much more general inventory models are suggested and discussed. Wagner (1975) combines the general theory of stochastic dynamic programming in Markov chains with probabilistic inventory models in an excellent way. Unfortunately, many students of management never have the time and/or the capacity to discover the richness of this field and the fundamental importance to economic inventory decisions. When they leave the university and become directors of real enterprises, more problems appear:

—It is very difficult to communicate the principles of stochastic dynamic programming in Markov chains within the typical enterprise.

In this situation, there are three options open to the new director in case he knows about the advanced and often highly relevant methods:

—He may introduce the methods and (re)educate the personnel. This may however be very expensive and time consuming.

—He may introduce the methods and control the inventory decisions personally. This may occupy his personal time and thus be costly in several ways.

—He may introduce, or stay with some already existing, simple decision rules concerning inventory and production. These rules, and the economic principles behind them, may easily be communicated within most enterprises. However, it is likely that the rules will often suggest production and inventory decisions that are not optimal. The rules are based on a simple and partly irrelevant description of the true problem.

The value of some simple rules may be that they create a "robust enterprise". Everyone understands the simple rules and no strange errors in some calculations make the enterprise system break down.

Nevertheless, the simple rules may suggest decisions that are completely wrong (or completely correct in some situations but because of the wrong reasons!). In this paper, we will investigate the sensitivity of the optimal production and stock level decisions to some parameter changes. In several inventory models, these parameters are not at all found!

Since the optimal decisions in the "more general" model of this paper are strongly affected by these parameter changes, it is clear that the simple models will suggest decisions that frequently are far from optimal in a world where these changes often take place.

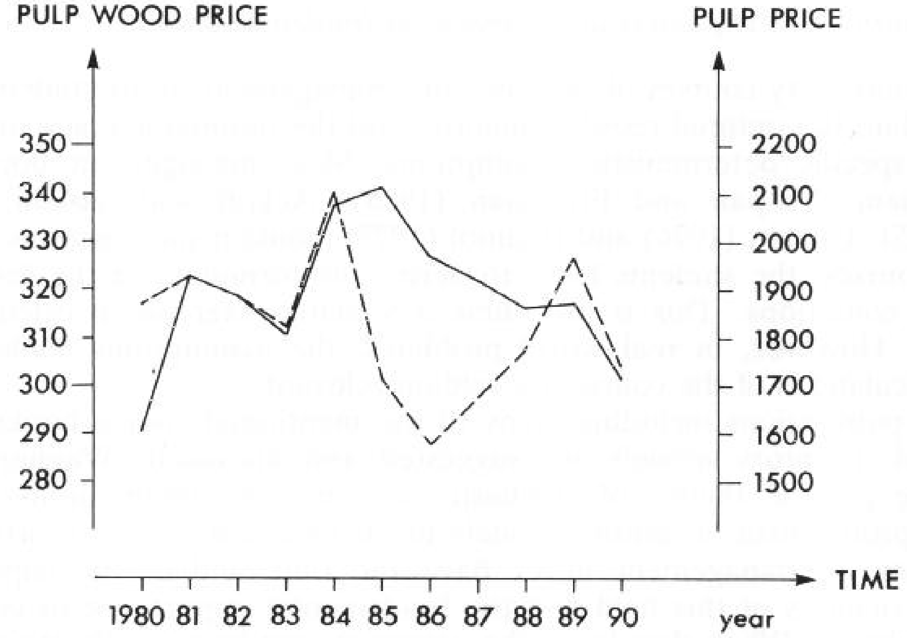

Figure 1 The pulp wood price (solid line) and the pulp price (dotted line) series in Sweden. Pulp price = Export price (from Sweden) of unbleached sulphite pulp. SEK per ton, dry weight. Pulp wood price = Price index for all pulp wood in Sweden where the harvest season 1967/68 is given the value 100. Both price series are represented as yearly averages and multiplied by the series $\mathrm{EXP}(-s*(\mathrm{YEAR}-1980))$ in order to take away the trend (partly because of inflation). $s=8\%$. Source: Statistical Yearbook of Forestry, 1991.

1.6. The Empirical Reality of the Pulp Industry

Now, let us look at a fraction of the empirical reality of the enterprises in the forest sector. Figure 1 shows that the pulp wood price and the pulp price have varied considerably over time during the last 10 years. Note that the correlation is strongly positive in the period 1981 to 1984. In the rest of the time interval, the picture is completely different. The 1991 observation is not yet available but it is already well known that this year is one of the worst in modern time with respect to profitability in most sectors of the economy.

Figure 2 contains a plot of the pulp price and the pulp wood price. Again, we may observe that the time period 1981 to 1984 is a period of positive correlation. The complete period, however, seems much more random. The correlation of the pulp price and the pulp wood price is surprisingly low. The data series plotted in Figure 2 give the correlation coefficient 4%.

The stock in the forest industry of pulp wood and chips has varied very much and very rapidly during the last 20 years. Figure 3 tells us that the stock represented the consumption of approximately 8 months in 1971. In 1990, the stock was almost the same as the consumption of 2 months. Even if the yearly variation is high, it is obvious that the general trend is negative: The general stock/consumption ratio has decreased very much during the last two decades.

The value of the stock of the output, the pulp, is found in Figure 4. Estimated values can be expected to vary in the statistics for several reasons, one is the price level and one is the physical quantity level. Hence, the level of precision with respect to the physical stock level is not as high in Figure 4 as in Figure 3. Approximative calculations show that the ratio "(pulp stock)/(pulp sales)"

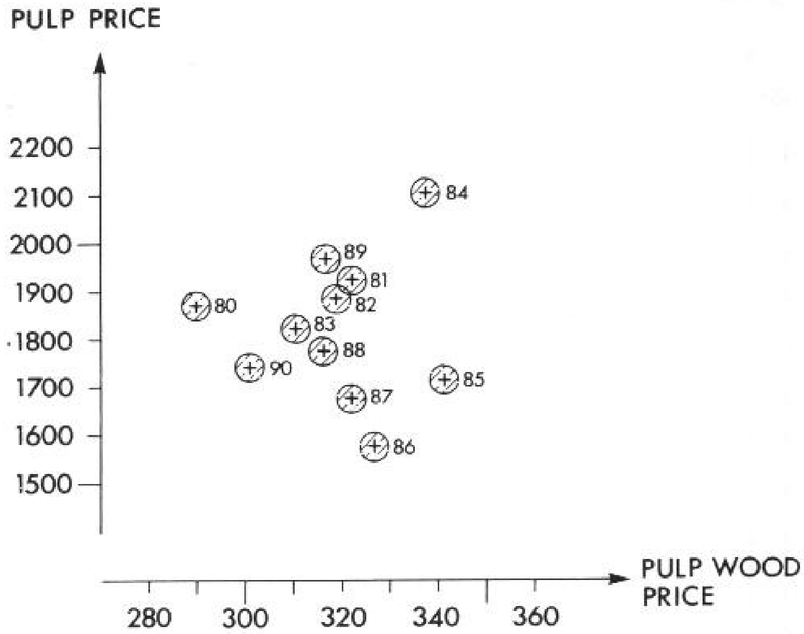

Figure 2 Combinations of pulp price and pulp wood price different years. The definitions and the source are the same as in Figure 1.

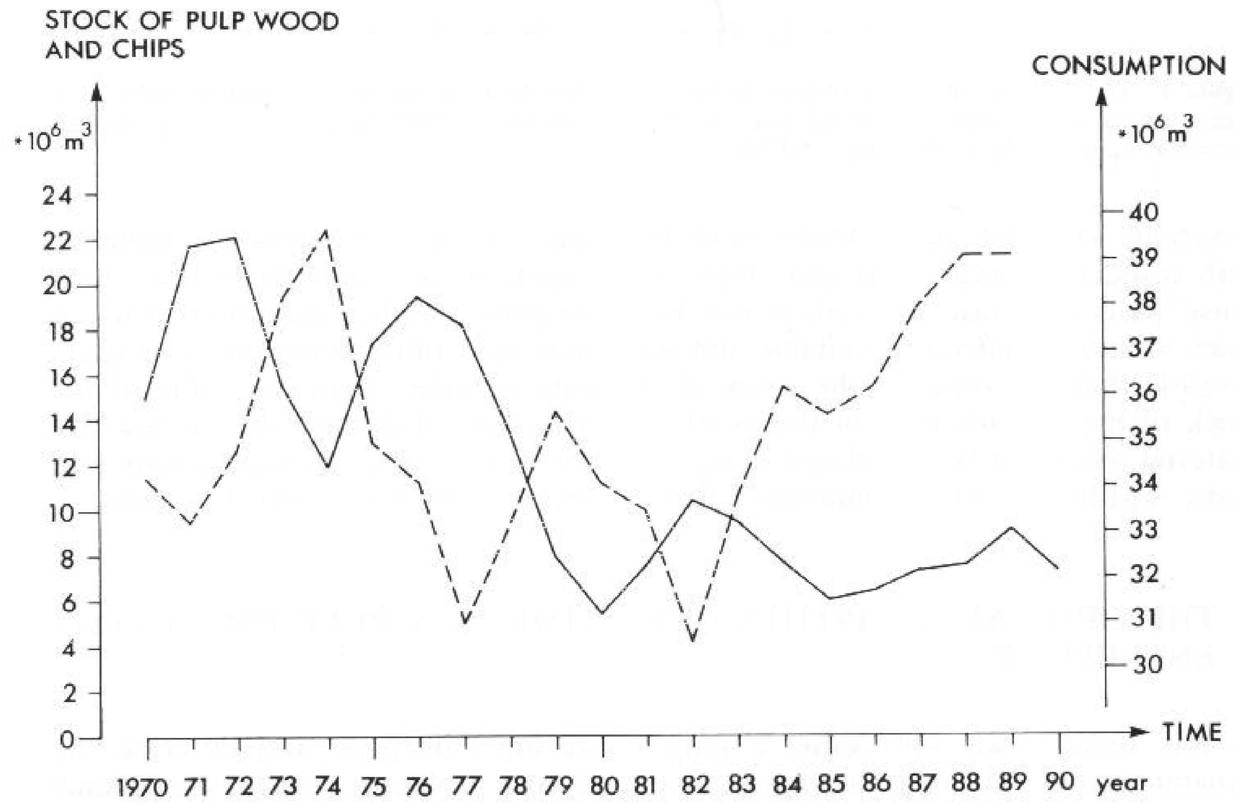

Figure 3 The Swedish stock of pulp wood and chips in the end of the years (solid line) and the industrial consumption of wood raw-material in the Swedish pulp and paper industry (dotted line). Source: Statistical Yearbook of Forestry, 1991 (and earlier years).

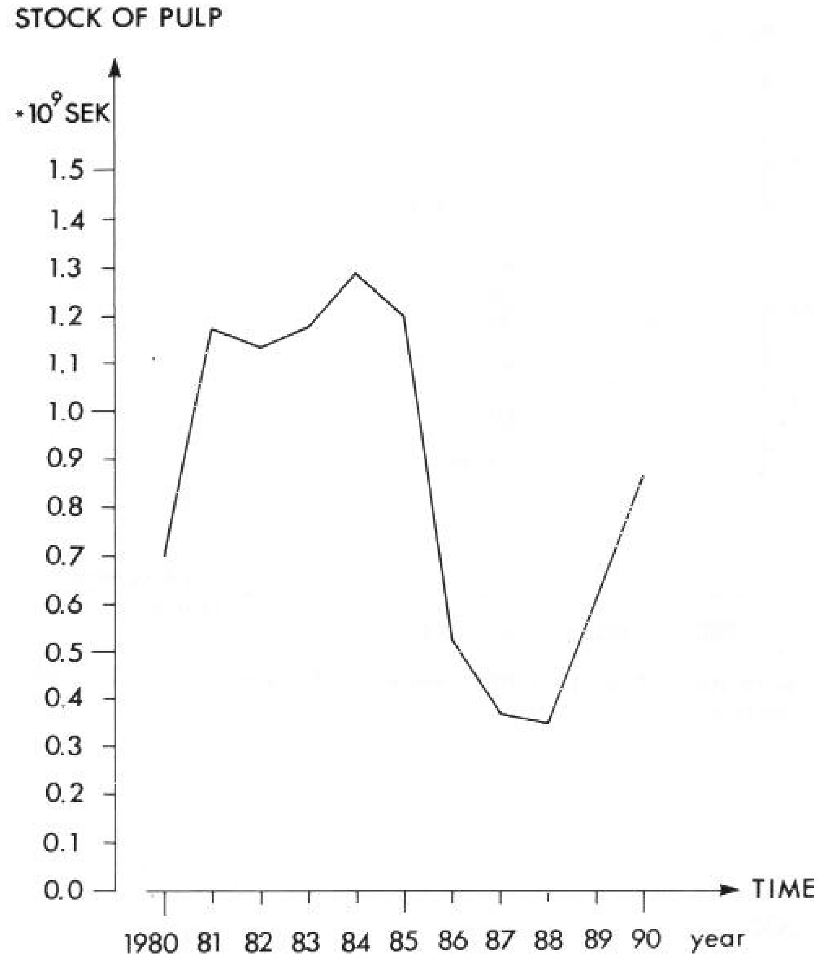

Figure 4 The value of the stock of pulp in the end of the years in the Swedish pulp industry. The values are calculated according to the price level in the middle of 1980. Source : Statistics Sweden, Statistical Reports, Stocks Industry, SNI 34111.

generally has been much lower than the ratio "stock/consumption" discussed with respect to pulp wood and chips. The "main stock", defined in this "time sense", has been raw materials in the discussed period. Other definitions, such as stock values in officially available statistics, may give other relations. It is quite possible that the value of the stock of the pulp is higher than the value of the stock of the raw material in the yearly reports. Note that the value of the raw material stock can be calculated many different ways and tax considerations may make it profitable for the individual firm to deviate from the "correct" values.

2. THE OPTIMAL ACTIVITIES OVER TIME IN A PULP PRODUCING ENTERPRISE

In this section, we will define a pulp producing enterprise and describe the dynamic economic production, stock level and sales problem. Finally, the optimal production, stock level and sales decisions will be calculated. The price series presented above serve as the empirical background and motivation. Numerical assumptions are made concerning conditions that are usually not the same in different enterprises. Every assumption is described in full detail in the graphs and/or in the numerical appendix.

The problem is defined as an infinite horizon finite state stochastic Markov chain problem. The resulting linear programming problem is solved via the policy iteration method discussed in Wagner (1975), chapter 12. A problem specific optimization code for the personal computer has been constructed and included in the numerical appendix. In the main text, all results essential to the qualitative discussion are presented in the form of graphs.

2.1. General Assumptions Concerning the Pulp Enterprise

—The objective of all activities including production, storage and sales, is to maximize the total expected present value.

—The time horizon is infinite.

—The demand quantity in a particular time period is not fixed and exogenous: More generally, the price of the quantity sold in a particular time period to the consumers is a function of the sold quantity, which is endogenous, and a stochastic price function parameter. Compare Figure 7.

—The production cost in a particular period is a function of the production volume and a stochastic production cost parameter. Compare Figure 8.

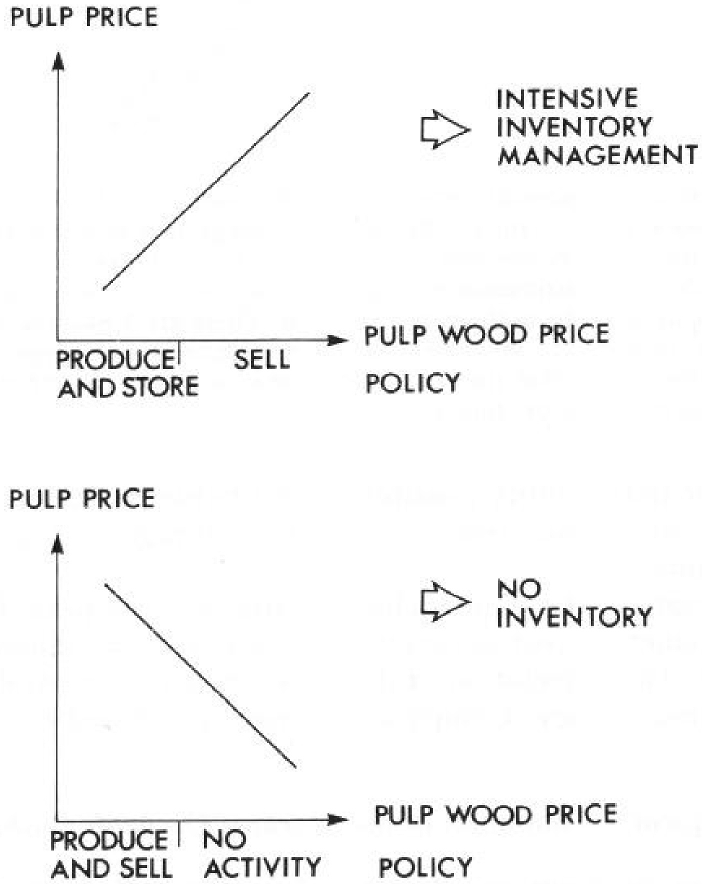

Figure 5 A simple hypothesis concerning the optimal production and inventory policy under different kinds of parameter correlation. Positive (upper graph) and negative (lower graph) correlation between the pulp price and the pulp wood price are shown in the graphs together with the corresponding policy effects.

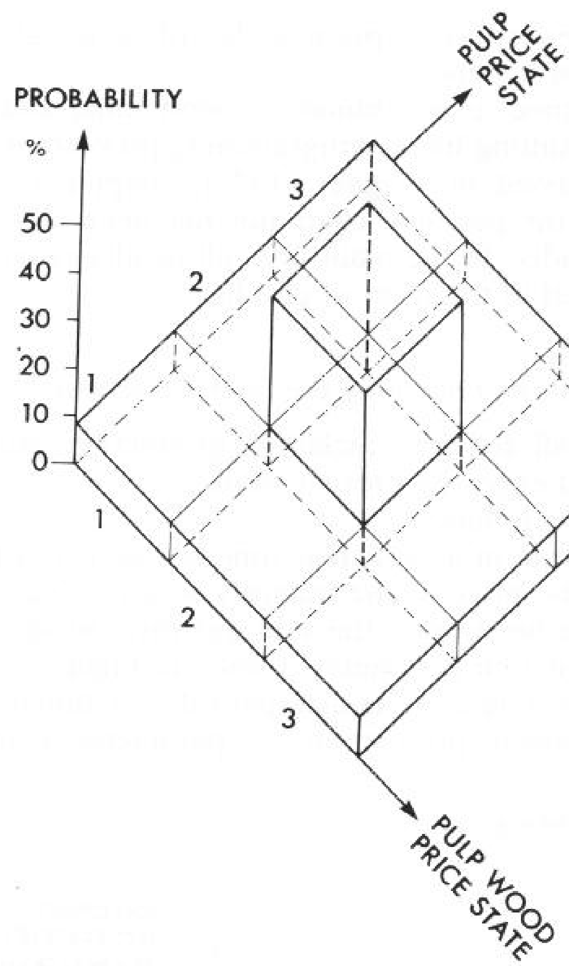

Figure 6 A possible and numerically practical two dimensional probability distribution of the pulp price and the pulp wood price. Compare the empirical background shown in Figure 2. The number of observations in Figure 2. is very low and it is difficult to reject the suggested probability distribution on statistical grounds. The distribution is suggested since we need an explicitly defined probability distribution in order to solve the optimization problem. There are 9 possible outcomes. The outcome (2, 2) is given the probability 36% and all other outcomes are given the probabilities 8%. ($36\% + 8 * 8\% = 100\%$). Note that the price levels associated with the different states in the graph have not yet been decided, just the functional form.

—The level of the quantity possibly stored between periods is restricted from above and, of course, from below. The storage cost is a function of the storage volume.

—Most importantly, the two stochastic variables, the price function parameter and the production cost parameter, are given a two dimensional probability distribution. The correlation of these variables is of central importance to the derived optimal policy. Compare the Figures 2, 5 and 6.

2.2. A Mathematical Formulation of the Markov Decision Problem

The optimization problem can be stated in the following way: We want to maximize the expected present value of the sum of all present and future profits. For each initial state (for each combination of the entering stock level, the cost function state and the price function state), we want to select the optimal

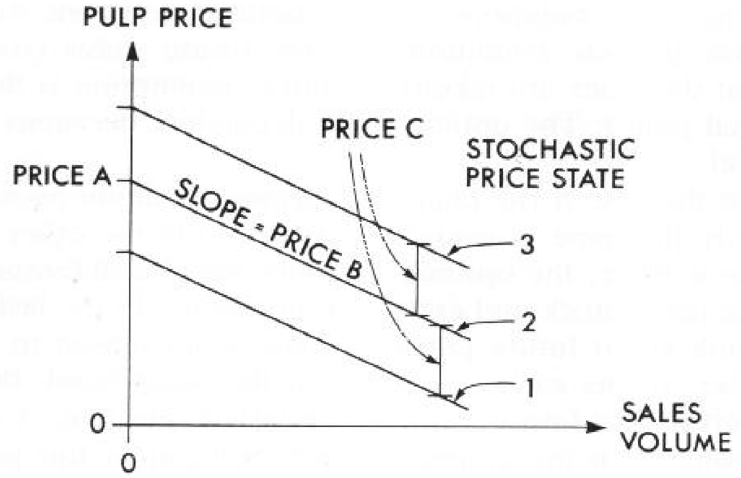

Figure 7 The stochastic pulp price function used in the optimization model. The pulp price is a linear function of the sales volume in the same time period. Stochastic shifts in demand are introduced via vertical shifts in the price function. The user determined price parameters are:

PRICEA = the price level when the sales volume is zero and the price level state takes the medium value (=2).

PRICEB = the derivative of price with respect to the sales volume in the same period.

PRICEC = level of change in the price function when the price function state increases or decreases one step.

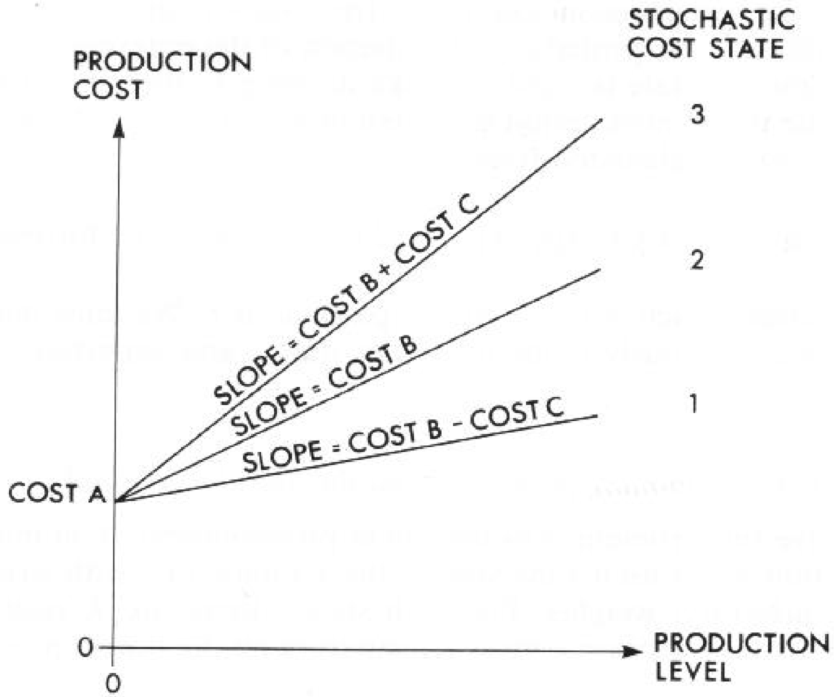

Figure 8 The stochastic production cost function used in the optimization model. The user determined cost parameters are:

COSTA = the set up cost (the production dependent fix cost per period).

COSTB = the marginal product cost when the cost function state takes the medium value (=2).

COSTC = level of change in the marginal cost function when the cost function state increases or decreases one step.

STOC = marginal cost of storing one unit of the stock from one period to the following period.

FIXC = the production independent fix cost per period.

decision. One important assumption is that optimal decisions will be taken also in the future time periods, conditional on the future states (states that will be known when that decisions are taken). Another assumption is the existence of a stationary optimal policy: The optimal state dependent decisions are the same in every time period.

This should be the case if the planning horizon is infinite because the decision problem is exactly the same in every time period. On the other hand, when the planning horizon is finite, the optimal decisions may be different in the first and the last period in many stock and extraction problems. In the last period, there is no reason to think about future periods. There is no reason to save stocks that may be used later. In the earlier periods, on the other hand, the optimal stock that should be left for the future frequently is strictly positive. A stationary policy thus seems reasonable in the infinite horizon problem of this paper if the state description used in the model really approximates the relevant state space in a sufficiently good way.

If important exogenous conditions, not described through the state space in the model, change over time, it is not unlikely that the "model state dependent" decisions should also change over time. We will assume that the state description of the model really represents the relevant state space. We assume the existence of a stationary optimal policy.

Let $Y(i)$ denote the expected present value if the initial state is i. There are I possible states. For each state, i, there are K different possible decisions. A particular decision is denoted by k. If you make decision k when you are in state i, you will get the instant economic profit (revenues–costs) or loss $c(i, k)$. The decision will influence the probability distribution of the entering state in the next period. If the entering state is i and you take decision k, then the probability that the entering state in the next period is j is denoted by $T(i, j, k)$. Now, it should be clear that $Y(i)$ can be calculated from:

$$Y(i) = \max_k \left(c(i, k) + \exp(-r) * \sum_j (T(i, j, k) * Y(j)) \right) \quad \text{for every } i \tag{1}$$

The rate of interest, which must be strictly positive, is r. We must make sure that this equation simultaneously holds for every state i and expected present value $Y(i)$.

2.3. *The Linear Programming Approach and the Simplex Method*

One way to solve this problem is to use linear programming. You minimize (note that maximization is not used!) the sum of the I values $Y(i)$ with strictly positive, but otherwise arbitrary, weights. For each state i there are K restrictions from below. Every possible decision k gives a restriction on the lowest possible value of $Y(i)$.

$$Y(i) \geq c(i, k) + \exp(-r) * \sum_j (T(i, j, k) * Y(j)) \quad \text{for every } i, k \tag{2}$$

Since the problem is to minimize the positively weighted sum of the values, the restrictions of the decisions which give the highest values $Y(i)$ will be efficient in the optimal solution. The dual variables of that restrictions, the shadow prices, will be positive (strictly positive if the optimum is unique). Note that the number of restrictions becomes $I*K$. This often becomes a very large number in "realistic" models. In the numerical model of this paper, there are 5 stock levels, 3 price function states and 3 cost function states. Hence, $I = 5*3*3 = 45$.

The number of possible decisions in each state, K, is 18 since there are 3 levels of production and 6 levels of sales. For very low levels of the entering stock, the number of possible decisions is lower than 18. The reason is that you can not sell more than the sum of the entering stock and the present production. Furthermore, for every high levels of the entering stock, the number of possible decisions is lower than 18, since the maximum stock level is restricted. Hence, the number of restrictions is a little lower than $45*18 = 810$. For simplicity, we assume that the number of restrictions is 700. Note that the size of the simplex matrix becomes considerable even in this rough model.

We assume that we use the simplex algorithm with the Big-M method in order to handle the $\geq$ constraints. The number of rows is 701 since there are 700 restrictions and 1 objective function.

The number of columns is $45 + 2*700 + 1 = 1446$ since each $Y(i)$ must have one column and each restriction must include one column for the surplus variable and one column for the artificial variable. Finally, the right hand side of the restrictions needs one column. The total number of elements in the matrix becomes $701*1446 = 1\,013\,646$. In order to avoid severe errors in the calculations, we must use a double precision matrix. The reader realizes that not even this rough problem can be solved in a typical personal computer. Furthermore, the number of elements in the matrix is approximately a quadratic function of the resolution in each dimension. For instance, if the number of possible production levels increases from 3 to 6, then the number of elements in the matrix is approximately 4 000 000.

If the number of possible production levels is 6 and the number of possible sales levels is 12, then the number of elements is approximately 16 000 000. (A normal personal computer can today handle one simplex matrix in double precision with approximately 20 000 elements.) Simplex calculations of the suggested dignity will not only need a very large internal memory capacity of the computer but also often imply that numerical errors become large and that the computation takes a long time.

2.4. The Policy Iteration Approach

Is there a way to avoid the enormous size of the internal computer memory required by the simplex method? Yes, policy iteration is a very tempting approach in this case. The method can be described the following way:

The constraints are exactly those discussed above. However, the method to find the efficient constraints, in other words the optimal decisions, is different. Clearly, we need one decision for each state i. In the above problem, we have to determine 45 values $Y(i)$ via 45 efficient constraints (equations). In other words, we have to find the correct linear equation system with 45 variables and 45 equations. This means that we do not have to use a very large size internal computer memory even if we use a double precision matrix. We do, however, often have to solve many such linear equation systems. One can show that the solution monotonically converges to the optimal solution in a finite number of iterations since there is only a finite number of possible equation systems and we always change equations in the system in such a way that all values $Y(i)$ increase nonstrictly and at least one value $Y(i)$ increases strictly.

The method can be presented the following way: Guess what 45 decisions are optimal in the 45 different states. Solve the corresponding equation system. The solution is the first guess of the values $Y(i)$. Assume that the values $Y(2), Y(3), \ldots, Y(45)$, of the first guess are optimal. Check all of the 18 possible decisions and corresponding value restrictions for state 1. One of these decisions maximizes $Y(1)$ conditional on the other values in the vector Y. We may here denote this new conditional value of $Y(1)$ by $Y_{\text{new}}(1)$.

Note that if this decision is selected to replace the original decision, every $Y(i)$ increases or is unchanged. This is obvious from the restrictions since all probabilities and discount factors are nonstrictly positive. Hence, if the new decision is selected instead of the old guess, then the objective function of the linear programming problem where every value $Y(i)$ has a positive weight increases. Note that $Y_{\text{new}}(1)$ denotes the expected present value in case the initial state is 1, the new decision is selected for the first time period and the old strategy guess is used in every future time period. In the same way, we may calculate $Y_{\text{new}}(2), Y_{\text{new}}(3), \ldots, Y_{\text{new}}(45)$.

These values are of course calculated conditional on the initial guesses concerning the other 44 decisions. Now, we want to make a new qualified guess concerning the decisions and equation system that give the optimal values $Y(i)$.

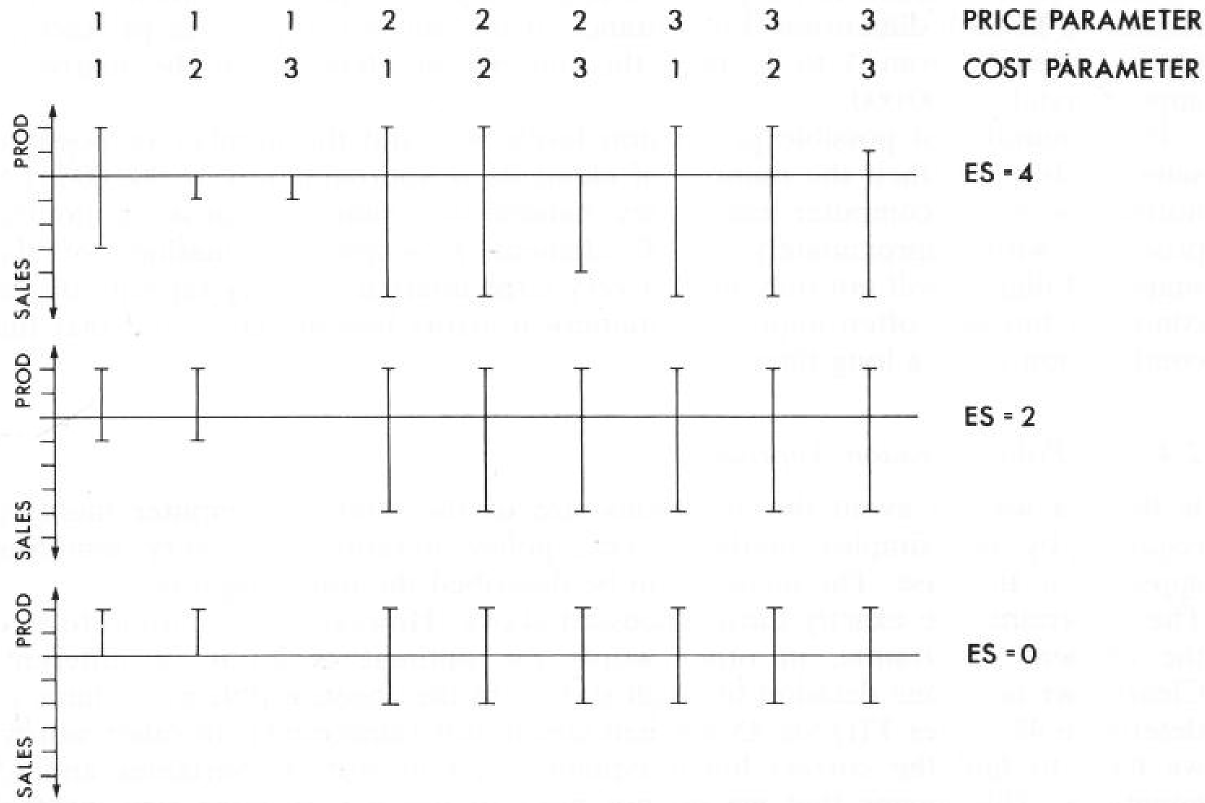

Figure 9 The optimal production and sales decisions as functions of the price function state, the production cost function state and the entering stock level, ES. The price function state and cost function state probabilities are those presented in Figure 6. Parameters: The real rate of interest = 5%, FIXC = 0, STOC = 1. (COSTA, COSTB, COSTC) = (1, 10, 2). (PRICEA, PRICEB, PRICEC) = (14, −0.2, 3).

Select to replace one of the equations of the original system, namely the equation of the state (row) which maximizes the difference $Y_{new}(i) - Y(i)$, where $Y(i)$ denotes the old solution. In this row, i, introduce the "partially" best decision according to the rules above. Solve the new equation system and continue the same way until it is not possible to improve the solutions anymore. Then, the optimum is found.

3. OPTIMAL PRODUCTION, STOCK LEVEL AND SALES DECISIONS IN DIFFERENT MARKET STATES

The Figures 9–12 show how sensitive the optimal production, stock level and sales decisions are to the parameters. We make the observation that the optimal stock change (production–sales) is a nonstrictly decreasing function of the entering stock level. This observation is consistent with the partial stock adjustment process suggested by Bergman and Löfgren (1989). We also find, however, that the optimal production and sales decisions are highly market dependent. The "optimal stock level" must be calculated as a function of the states also in the two stochastic markets. Maybe, the "optimal inventory level" suggested by Bergman and Löfgren (1989) could be interpreted and defined as the expected optimal stock level. With that definition, the results presented here suggest an expected stock adjustment towards that level. The time path of the optimal stock will of course be stochastic.

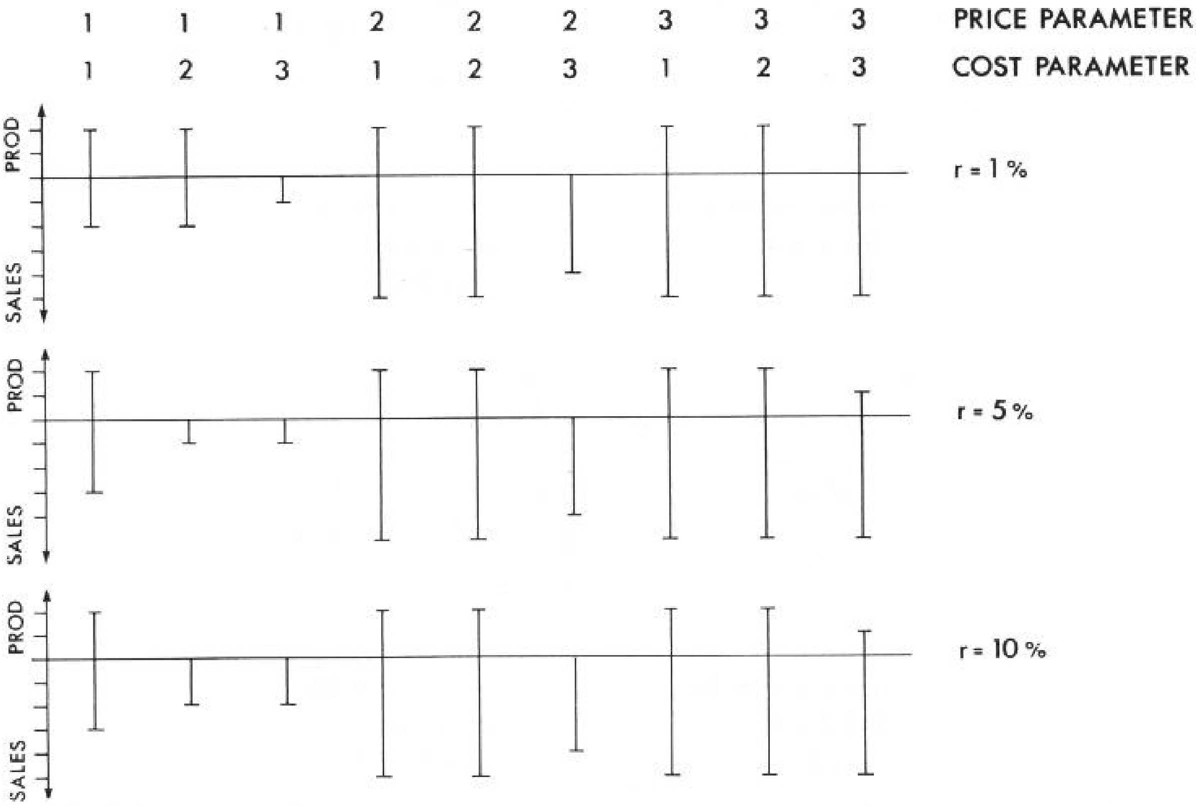

Figure 10 The optimal production and sales decisions as functions of the price function state, the production cost function state and the real rate of interest. The entering stock is 4 units. The parameters are the same as in Figure 9 except for that the rate of interest varies in this graph.

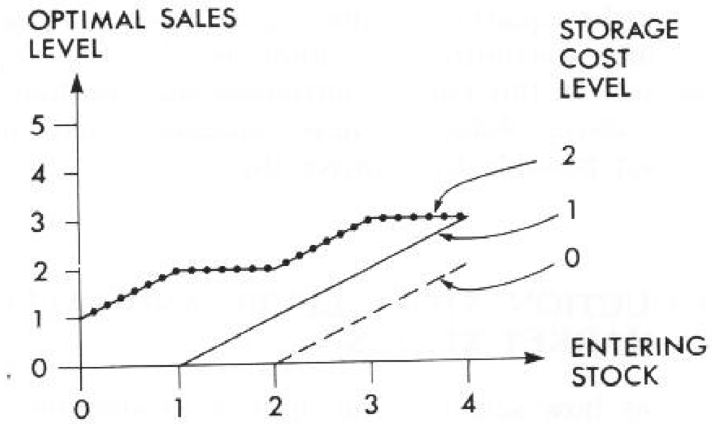

Figure 11 The optimal sales level as a function of the entering stock level and the storage cost level. The price function state = 1, the production cost function state = 1. The parameters are the same as in Figure 9 except for that the storage cost level varies in this graph.

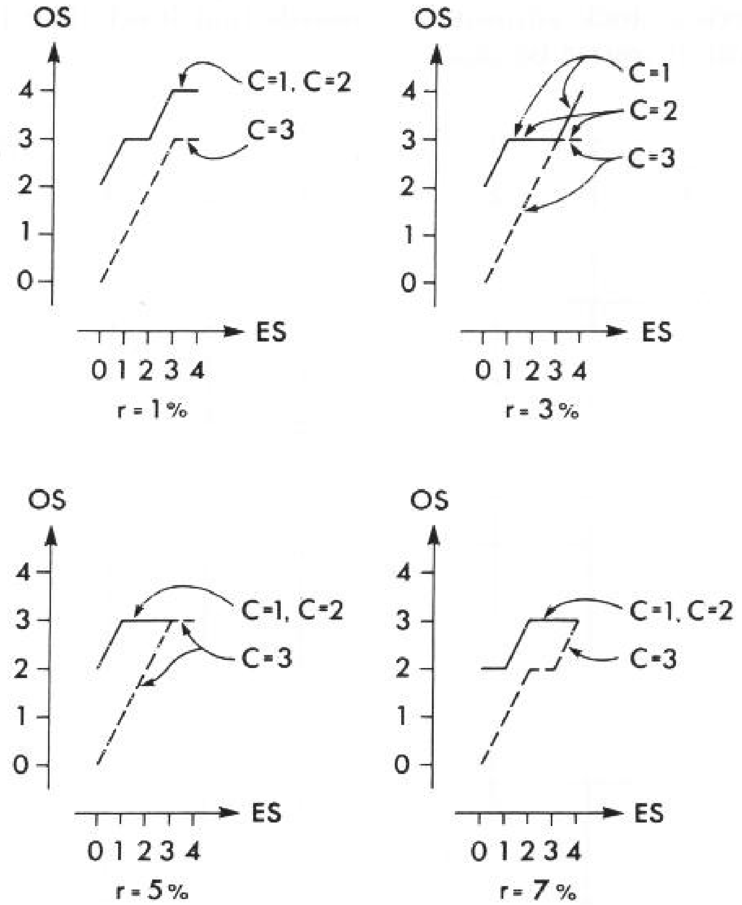

Figure 12 The optimal stock level, OS, until the next period as a function of the entering stock level, ES, the real rate of interest and the cost function state. The price function state = 1. The parameters are the same as in Figure 9 except for that the rate of interest varies in this graph. C denotes cost function state.

The optimal stock level decreases nonstrictly with the rate of interest and the storage cost level. These results are consistent with the corresponding results in most partial and simple inventory models. In the graphs, it is clear that the illustrated changes in the two stochastic markets generally affect the optimal decisions more than the illustrated changes in the rate of interest and the storage cost.

We must have detailed information about the specific economic conditions in the specific enterprise in order to be able to say if typical variations in the markets or typical variations in the rate of interest and the storage cost mean more to the optimal changes in the stock level. In the illustrated enterprise example, however, market changes are more critical to the optimal stock level decisions than changes in the other parameters under discussion.

4. OPTIMAL STOCHASTIC MARKOV CHAIN NETWORKS AND STOCK STATE PROBABILITIES: PARAMETER DEPENDENCE AND SENSITIVITY ANALYSIS

Now, when we know the optimal production and sales decisions for every possible state of the entering stock, the stochastic parameters of the price function and of the cost function, we also know the optimal level of the stock after sales for every state. Since we know the probabilities of the different price function state and cost function state combinations, we know the probability that it is optimal to move from one stock level to another. This is shown in Figure 13.

The arcs in Figure 13 show transitions that are optimal with strictly positive probabilities. The probabilities are shown for each transition. Note that the graphs do not reveal everything about the production and sales activities. As an example: The stock transition graphs are identical for the cases STANDARD and SETUPC = 0 but the production and sales decisions are not. The production and sales levels are more sensitive to the states of the markets if the set up cost is reduced. In Figure 13, we sometimes find that the probability of moving from some state, S, to some other states are strictly positive and the probability of moving to some state, S, from other states is zero. Then, it is clear that the probability that the stock level is in state S approaches zero as time approaches infinity. As one example, we may investigate the case "STANDARD" in Figure 13. If the initial state is 5, then the stock has a lower level than the initial state with probability 1 in the next period. The stock level may take the value 2 or some other values. If we go to state 2, the probability is strictly positive that we go to other states in the following period. The probability of going to state 2 from some other state is zero for all other states than 5. Hence, as time approaches infinity, the probability that the stock level is in one of the states 2 or 5 approaches zero.

Grimmett and Stirzaker (1985), chapter 6, discuss the qualitative properties and conceptually important definitions of Markov chains and states. In particular, a state is called persistent (or recurrent) if the probability of returning to it, having started from it, is 1. If this probability is strictly less than 1, the state is transient. Quite clearly, the states 2 and 5 in the case "STANDARD" are transient. Grimmett and Stirzaker also write that state i communicates with state j if the state may ever visit state j with positive probability, starting from i. State

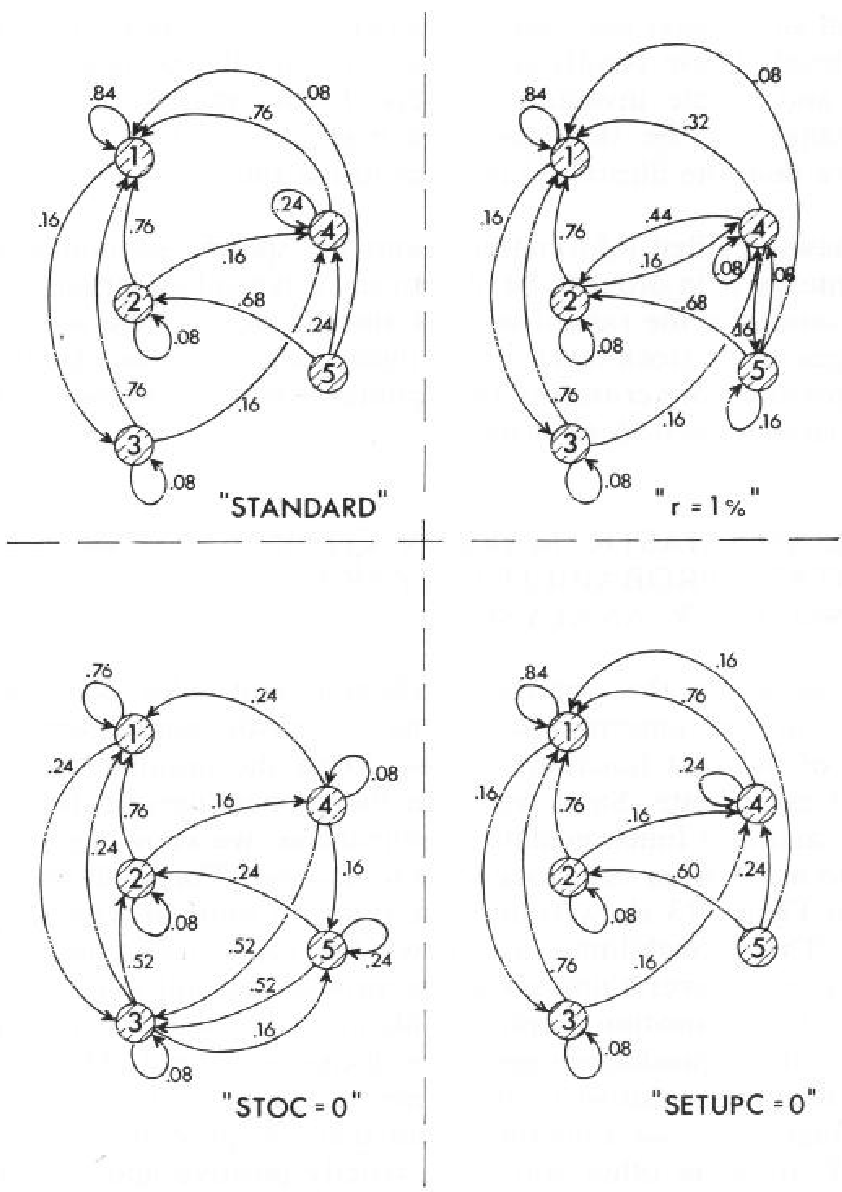

Figure 13 The stock state network in four different cases. The stock states 1, 2, 3, 4 and 5 correspond to the pulp wood consumption levels of 0, 6, 12, 18 and 24 month production at full capacity utilization.

CASE	DEFINITION
STANDARD	Parameter assumptions according to Figure 9.
$r = 1\%$	As STANDARD except for that the real rate of interest = 1%.
STOC = 0	As STANDARD except for that the storage cost = 0.
SETUPC = 0	As STANDARD, but the set up cost = COSTA = 0.

i and state *j* intercommunicate if *i* communicates with *j* and *j* communicates with *i*. In the case "STANDARD" we find that all states communicate with the states 1, 3 and 4. Furthermore, these states intercommunicate with each other. No state communicates with state 5 and only state 5 communicates with state 2. We make use of the knowledge that, as time approaches infinity, the stock will be in one of the states that intercommunicate, 1, 3 or 4 with a probability which approaches 1.

In case "STANDARD", we have 3 variables to determine; the probabilities

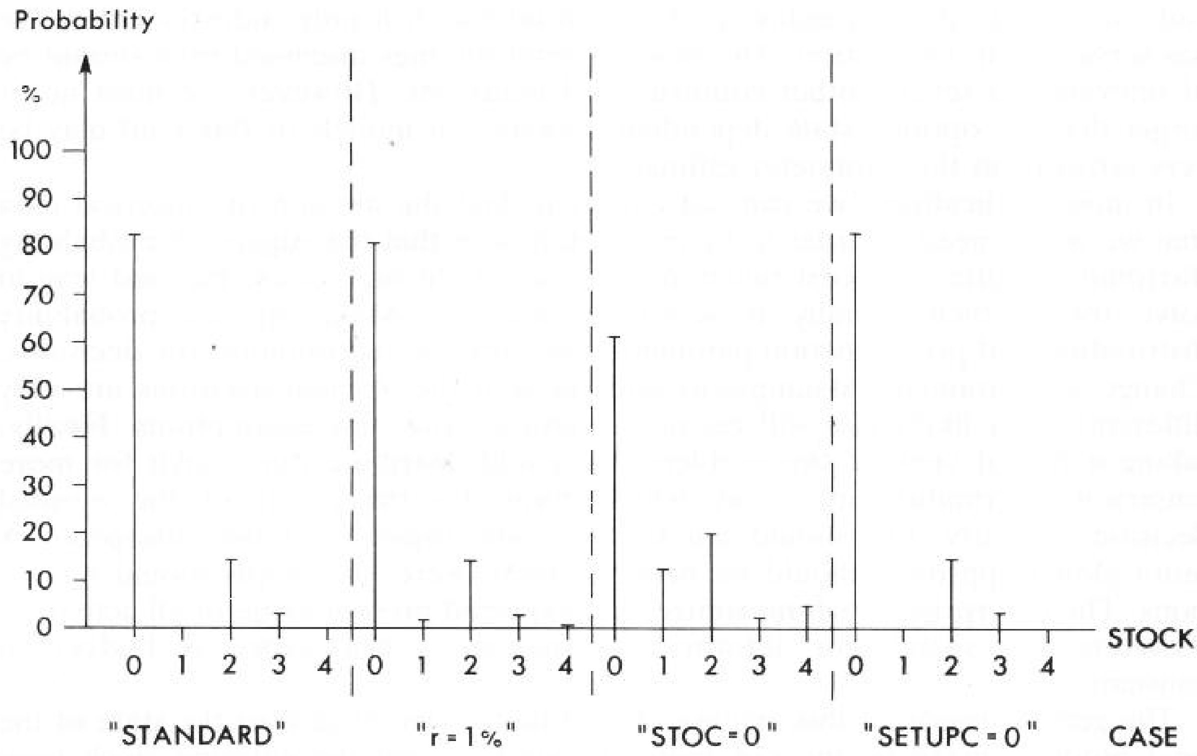

Figure 14 The stock probabilities will approach the values shown in this diagram as time approaches infinity if the optimal state dependent decisions are always made. The stock is given in the unit "*6 month consumption at full production capacity utilization". The illustrated cases correspond to the cases and definitions given in connection to Figure 13.

that the stock level is in the three intercommunicating states. We need three linear equations to determine these probabilities. One equation says that the sum of the probabilities is 1. Two equations contain relations between state probabilities through transition probabilities. We could construct one more equation containing transition probability information but then we get to many equations. The resulting stock state probabilities are shown in Figure 14. Several more general definitions, theorems and proofs concerning the highly interesting Markov chains and state probability distributions can be found in Grimmett and Stirzaker (1985).

In Figure 14, we find that the probability that a stock level corresponding to the industrial consumption of 6 month or more (at full production capacity utilization) is optimal, is only about 20% in three of the tested cases. In the low storage cost case, this probability is about 40%. Compare these results to the latest real world stock levels shown in Figure 3. The similarity is obvious even if the stock state resolution is low! It should not be difficult to increase the stock state resolution in a purely applied and normative investigation. In any case, the model gives "reasonable" results!

5. DISCUSSION

The analysis in this paper represents one step towards a multi market treatment of the simultaneous stochastic dynamic optimization of production, stock level and sales in the process industry. An artificial Swedish pulp industry enterprise has served as an illustration. The more general findings discussed here should be of relevance in several other countries and industries. However, we must never forget that the optimal state dependent decisions in models of this kind may be very sensitive to the parameter estimates.

In most applications, we can not expect to find the amount of empirical data that we would need in order to be completely sure that the suggested probability distributions, price and cost functions are correct. In such cases, the best way to solve the problem usually is sensitivity analysis: Make specific probability distribution and price function parameter assumptions and optimize the decisions. Change the parameter assumptions and check if the optimal decisions are very different. Most likely, we will get new decisions with new assumptions. Finally, taking a partial view of the problem, we could distribute our search for more empirical information in a way which minimizes the errors of the optimal decisions. Clearly, this should not be the main objective of the enterprise. A more global approach should be better if there were no computational restrictions. The enterprise which maximizes the expected present value of all activities, including the search for information, maximizes the expected budget of consumption.

The general lesson of this analysis should be that we must take the state of the final products markets, the raw materials markets and the entering stock level into account when the production, stock and sales level decisions are taken. Typical variations in such factors frequently mean much more to the economically optimal decisions than typical variations in the factors that several simple and well known inventory formulae take into account.

Not to forget: It is important from several points of view to build bridges between different partial theories in economics, business and management.

Hopefully, this analysis will contribute to a more complete and interesting theory of management, including production theory, inventory management and market economics, in which we do not have to pretend that the future is already known.

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NUMERICAL APPENDIX

This appendix contains the computer program which solves the stochastic dynamic infinite horizon production and inventory Markov chain optimization problem. The resulting linear programming problem is solved via policy iteration which makes it possible to run the program on a standard 386 personal computer. The language is QUICK BASIC but the code can easily be translated to other languages and/or dialects. The program is based on seven different subroutines and contains detailed remarks that hopefully make the program self instructive.

Table N.1 contains the indata file INDYNSTO.DAT with the parameters suggested by the user. Table N.2 shows the result file UTDYNSTO.DAT which will appear if the indata file in Table N.1 is used. Note that the result file also contains the list of used parameter assumptions.

Table N.1 An example of the parameter indata file INDYNSTO.DAT. The file should contain 8 lines. Explanations are given in the text.

```
910912 0947 20
1 1 1
05
0
1 10 2
14 -.2 3
1
.08 .08 .08 .08 .36 .08 .08 .08 .08
```

Indata File Instruction

The indata file INDYNSTO.DAT should contain the following information in the following order (compare Table N.1):

On row 1: Data, time, maximum number of main iterations (20 was satisfactory in all tested cases.)

On row 2: Profit weights (the weights of the revenue function, the production cost function and the storage cost function in the profit function.)

On row 3: The real rate of interest, r (%).

On row 4: The production independent fix cost per period, FIXC.

On row 5: The cost function parameters COSTA, COSTB and COSTC. Compare the explanations in Figure 8.

On row 6: The price function parameters PRICEA, PRICEB and PRICEC. Compare the explanations in Figure 7.

On row 7: The marginal cost of storing one unit of stock from one period to the next, STOC.

On row 8: The probabilities of the 9 different combinations of the price function state (i) and the cost function state (j). The probabilities of the state combinations (i, j) are given in the following order: (1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3).

Table N.2 An example of the result file UTDYNSTO.DAT. Explanations are given in the text.

```
INFINITE HORIZON MARKET DEPENDENT STOCK OPTIMIZATION
Optimal solution:

The optimal values of Y(i) are:
  138.2  134.2  132.3  142.5  138.5  134.5  148.5  144.5  140.5
  149.1  145.1  143.9  155.5  151.5  147.5  164.5  160.5  156.5
  159.9  155.9  155.2  168.1  164.1  160.1  180.1  176.1  172.1
  170.3  166.3  166.1  180.3  176.3  172.5  195.3  191.3  187.3
  180.3  176.9  176.9  191.9  187.9  185.1  206.9  202.9  199.3

The optimal state dependent decisions are:
 2 0 ,  2 0 ,  0 0 ,  2 2 ,  2 2 ,  2 2 ,  2 2 ,  2 2 ,  2 2 ,
 2 0 ,  2 0 ,  0 0 ,  2 3 ,  2 3 ,  2 3 ,  2 3 ,  2 3 ,  2 3 ,
 2 1 ,  2 1 ,  0 0 ,  2 4 ,  2 4 ,  2 4 ,  2 4 ,  2 4 ,  2 4 ,
 2 2 ,  2 2 ,  0 0 ,  2 5 ,  2 5 ,  0 3 ,  2 5 ,  2 5 ,  2 5 ,
 2 3 ,  0 1 ,  0 1 ,  2 5 ,  2 5 ,  0 4 ,  2 5 ,  2 5 ,  1 5 ,

Date =  910912 . Time =  947
Rate of interest =  5 . Profit weights =  1  1  1 . Fix cost =  0
Production cost parameters =  1  10  2 . Price parameters =  14 -.2  3
Storage cost parameter =  1
Probability vector =  0.080 0.080 0.080 0.080 0.360 0.080 0.080 0.080 0.080
```

The Result File Interpretation

The result file UTDYNSTO.DAT contains the following information (Compare Table N.2): First, the expected optimal present values are listed for the 45 different possible initial conditions. The first list contains 5 rows and 9 columns. The entering stock is equal to the row number—1. Hence, the entering stock is 0 on row 1 and 4 on row 5. Each column represents one combination of the price function state (i) and cost function state (j). The columns represent the combinations (i, j) in the following order: Col 1 = (1, 1), Col 2 = (1, 2), Col 3 = (1, 3), Col 4 = (2, 1), Col 5 = (2, 2), Col 6 = (2, 3), Col 7 = (3, 1), Col 8 = (3, 2) and Col 9 = (3, 3). Hence, as an example, in the middle of the table we find the expected optimal present value if the initial stock level is 2, the initial price level is 2 and the initial cost level is 2.

Next, the optimal state dependent decisions are listed. The order is the same as in the value list above. For each state, the optimal decision combination, two numbers, production and sales, is given. Finally, the parameter list is printed. We may compare Table N.1.

The optimization program DYNSTO.BAS

```
REM ************************************************************************
REM Program DYNSTO.BAS, stochastic dynamic optimization of
REM production and sales where the stock level, the production
REM cost level and the product price level are state space dimensions.
REM Lohmander Peter 91-09-12 kl 08.22
REM ************************************************************************

REM +++++++++++++++++++++++++++++++++++++++
REM SECTION 1, DEFINITIONS AND DIMENSIONS
REM +++++++++++++++++++++++++++++++++++++++
DEFDBL A-Y
DIM SHARED A(45, 46), C(45), Y(45), ZD(45, 18), ZDEC(45)
DIM SHARED DAKT(45), BAKT(45, 46), YBEST(45), XLEV(45), SLEV(45)
DIM QLEV(45), PLEV(45), CLEV(45), PRMAT(18, 18), OPTY(45)
DIM BLOC(45)
OPEN "INDYNSTO.DAT" FOR INPUT AS #1
REM IXMAX is the maximum production quantity and ISMAX is the
REM maximum sales quantity. Production and sales may take
REM the value zero. Hence, there are (IXMAX + 1) possible production
REM levels and (ISMAX + 1) possible sales levels. DECMAX is the
REM total number of possible decisions.
IXMAX = 2: ISMAX = 5
DECMAX = (IXMAX + 1) * (ISMAX + 1)
REM QMAX, PMAX and CMAX are the maximum values of the state
REM variables quantity, price and production cost. Note that
REM the quantity may be zero and the number of quantity states hence
REM is (QMAX + 1). IYMAX is the total number of states.
QMAX = 4: PMAX = 3: CMAX = 3: IYMAX = (QMAX + 1) * PMAX * CMAX
REM IMAX and JMAX denote the number of rows and columns in the
REM linear equation systems.
IMAX = IYMAX: JMAX = IMAX + 1
CLS




REM ++++++++++++++++++++
REM SECTION INFORMATION.
REM ++++++++++++++++++++
PRINT "INFINITE HORIZON STATE DEPENDENT STOCK OPTIMIZATION"
PRINT "*****************************************************"
PRINT ""
PRINT "by Peter Lohmander    ver.  91-09-12 kl. 08.22"
PRINT ""
FOR I = 1 TO 10: SOUND (100 + 200 * I), (11 - I): NEXT I


REM +++++++++++++++++++++++
REM SECTION PARAMETER INPUT
REM +++++++++++++++++++++++
GOSUB 600
DISCF = EXP(-RATEINT / 100)
GOSUB 100
GOSUB 150

REM ++++++++++++++++++++++++++++++++++++++++++++++++++++++
REM SECTION 2, CALCULATION OF PRMAT(.,.), XLEV(.), SLEV(.),
REM QLEV(.), PLEV(.), CLEV(.) AND ZD(.,.)
REM ++++++++++++++++++++++++++++++++++++++++++++++++++++++
REM The price and cost state transition probability matrix is defined.
REM ************************************************************************
FOR P1 = 1 TO PMAX: FOR C1 = 1 TO CMAX
I = (P1 - 1) * CMAX + C1
```

```
PRMAT(I, 1) = PR11: PRMAT(I, 2) = PR12: PRMAT(I, 3) = PR13
PRMAT(I, 4) = PR21: PRMAT(I, 5) = PR22: PRMAT(I, 6) = PR23
PRMAT(I, 7) = PR31: PRMAT(I, 8) = PR32: PRMAT(I, 9) = PR33

NEXT C1: NEXT P1
FOR I = 1 TO (PMAX * CMAX)
PROBTOT = 0
FOR J = 1 TO (PMAX * CMAX)
PROBTOT = PROBTOT + PRMAT(I, J)
NEXT J
FOR J = 1 TO (PMAX * CMAX)
PRMAT(I, J) = PRMAT(I, J) / PROBTOT
NEXT J
NEXT I
REM First, we have to define XLEV(KDEC) and SLEV(KDEC). They are two
REM vectors that give the production level and the sales level
REM respectively, as functions of the decision index KDEC.
REM ***************************************************************
FOR X = 0 TO IXMAX: FOR S = 0 TO ISMAX: KDEC = 1 + X * (ISMAX + 1) + S
XLEV(KDEC) = X: SLEV(KDEC) = S
NEXT S: NEXT X


REM Now, for each state, i, we want to know the stock level, QLEV(i),
REM the price level, PLEV(i), and the production cost level, CLEV(i).
REM ****************************************************************
FOR Q = 0 TO QMAX: FOR P = 1 TO PMAX: FOR C = 1 TO CMAX
I = Q * PMAX * CMAX + (P - 1) * CMAX + C
QLEV(I) = Q: PLEV(I) = P: CLEV(I) = C
NEXT C: NEXT P: NEXT Q




REM Here, the values of the instant economic net
REM profits are determined.
REM ********************************************
FOR I = 1 TO IYMAX
P = PLEV(I): C = CLEV(I): Q = QLEV(I)
  FOR DEC = 1 TO DECMAX
  PROD = XLEV(DEC): SALES = SLEV(DEC)

  PCOST = (COSTB + COSTC * (C - 2)) * PROD
  IF PROD > 0 THEN PCOST = PCOST + COSTA
  IF PROD = 0 THEN PCOST = 0
  IF PCOST < 0 THEN PCOST = 0
  PRICE = PRICEA + PRICEB * SALES + PRICEC * (P - 2)
  REVENUE = PRICE * SALES
  STOCKOUT = Q + PROD - SALES
  CSTOCK = CSTO * STOCKOUT
  IF STOCKOUT < 0 THEN CSTOCK = 0
  PROFIT = PROFR * REVENUE - PROFPC * PCOST - PROFCS * CSTOCK - FIXCOST
  REM *** We can not sell more than stock + production. ***
  IF SALES > (Q + PROD) THEN PROFIT = -1000
  REM *** We can not store more than max capacity until next period. ***
  IF (Q + PROD - SALES) > (QMAX + 1) THEN PROFIT = -1000
  ZD(I, DEC) = PROFIT
  NEXT DEC
NEXT I

REM ++++++++++++++++++++++++++++++++++++++++++++++++++++++
REM SECTION 3, THE INITIAL GUESS IS THAT THE DECISIONS
REM THAT ARE OPTIMAL IN A ONE PERIOD PROBLEM ARE OPTIMAL
REM IN THE INFINITE HORIZON PROBLEM.
REM ++++++++++++++++++++++++++++++++++++++++++++++++++++++
FOR I = 1 TO IYMAX
OPTY(I) = -1
  FOR J = 1 TO DECMAX
  EVE = ZD(I, J)
  IF EVE > OPTY(I) THEN ZDEC(I) = J
  IF EVE > OPTY(I) THEN OPTY(I) = EVE
  NEXT J
Y(I) = OPTY(I)
NEXT I
GOSUB 100
GOSUB 150
```

```
REM +++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
REM SECTION 4, THE EXPECTED OPTIMAL PRESENT VALUES IN THE DIFFERENT
REM STATES ARE CALCULATED UNDER THE ASSUMPTION THAT THE DECISIONS
REM ACCORDING TO THE INITIAL GUESS ARE MADE.
REM +++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++

REM The equations are determined and placed in the equation system.
GOSUB 200
REM The equation system is solved.
GOSUB 400
REM If there are any errors, these may be found.
GOSUB 300
SOUND 1200, 10
IF YTEST > 0 THEN SOUND 3000, 20
IF YTEST > 0 THEN STOP
REM The new solution is placed in the value vector Y(.)
FOR I = 1 TO IYMAX: Y(I) = C(I): NEXT I
REM The solution is shown on the screen.
GOSUB 100
GOSUB 150


REM ++++++++++++++++++++++++++++++++++++
REM SECTION 5, THE MAIN LOOP STARTS HERE.
REM ++++++++++++++++++++++++++++++++++++
TOTSTOP = 0
FOR IMAIN = 1 TO IMAINMAX

REM ++++++++++++++++++++++++++++++++++++++++++++++++++++++
REM SECTION 6, THE MAIN LOOP STOPPING CRITERION IS TESTED.
REM MAYBE THE ITERATION STOPS AND THE RESULTS ARE PRINTED.
REM ++++++++++++++++++++++++++++++++++++++++++++++++++++++
EVSTOP = 0
FOR I = 1 TO IYMAX: EVSTOP = EVSTOP + Y(I): NEXT I
REM POSSIBLY, IT IS TIME TO BE SATISFIED WITH THE SOLUTION.
IF EVSTOP = TOTSTOP THEN GOSUB 500
IF EVSTOP = TOTSTOP THEN END
TOTSTOP = EVSTOP



REM +++++++++++++++++++++++++++++++++++++
REM SECTION 7, THE STATE LOOP STARTS HERE.
REM +++++++++++++++++++++++++++++++++++++
FOR IY = 1 TO IYMAX
DECOLD = ZDEC(IY)

REM +++++++++++++++++++++++++++++++++++++++++
REM SECTION 8, THE DECISION LOOP STARTS HERE.
REM +++++++++++++++++++++++++++++++++++++++++



YEVOPT = -1
FOR IDEC = 1 TO DECMAX
REM CALCULATION OF TRANSITION PROBABILITIES FROM STATE IY.
FOR J = 1 TO IYMAX: BLOC(J) = 0: NEXT J
REM THE STOCK IN THE NEXT PERIOD, STOCK2, IS CALCULATED.
STOCKERR = 0
STOCK1 = QLEV(IY): STOCKDEV = XLEV(IDEC) - SLEV(IDEC)
STOCK2 = QLEV(IY) + STOCKDEV
REM IF THE ENDING STOCK IS TOO SMALL OR TOO BIG, THEN STOCKERR = 1.
IF STOCK2 < 0 THEN STOCKERR = 1
IF STOCK2 > QMAX THEN STOCKERR = 1
IF STOCKERR = 1 THEN GOTO 5
REM THE RELEVANT INSTANT PROFIT IS PLACED IN DAKT(.).
DAKT(IY) = ZD(IY, IDEC)
REM THE PROBABILITIES OF PRICE AND COST CHANGES ARE CALCULATED AND
REM DISCOUNTING IS PERFORMED.
P1 = PLEV(IY): C1 = CLEV(IY): PCSTATE1 = (P1 - 1) * CMAX + C1
FOR PCSTATE2 = 1 TO (PMAX * CMAX)
STATE2 = STOCK2 * PMAX * CMAX + PCSTATE2
PROB2 = PRMAT(PCSTATE1, PCSTATE2)
BLOC(STATE2) = PROB2 * DISCF
NEXT PCSTATE2
```

```
REM CALCULATION OF THE TEST VALUE, YEV, OF Y(IY) IF THE DECISION IDEC IS
REM USED NOW AND THE OLD DECISIONS SET IS USED IN THE FUTURE.
YEV = DAKT(IY)
FOR J = 1 TO IYMAX
YEV = YEV + BLOC(J) * Y(J)
NEXT J
IF YEV < YEVOPT THEN GOTO 5
DECOPT = IDEC
YEVOPT = YEV
5 REM
NEXT IDEC
IF DECOLD = DECOPT THEN GOTO 6
IF (Y(IY) + .00001) > YEVOPT THEN GOTO 6
ZDEC(IY) = DECOPT
REM THE VERY BEST EQUATION IS PLACED IN THE MATRIXES BAKT(.,.) AND DAKT(.).
FOR J = 1 TO IYMAX
BAKT(IY, J) = BLOC(J)
NEXT J
DAKT(IY) = ZD(IY, DECOPT)
PRINT "Y("; IY; ") may become "; YEVOPT;
PRINT "if decision "; XLEV(DECOPT); SLEV(DECOPT); " is made."
SOUND 600, 3




REM ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
REM SECTION 9, THE EQUATION SYSTEM SUBROUTINE IS CALLED FOR AND THE
REM POSSIBLY NEW VALUES OF Y(I), C(I), ARE CALCULATED.
REM ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++

REM THE EQUATION SYSTEM MATRIX IS DETERMINED.
GOSUB 200
REM THE EQUATION SYSTEM IS SOLVED.
GOSUB 400
REM POSSIBLE ERRORS MAY BE DETECTED.
GOSUB 300
SOUND 1200, 7
REM IF YTEST = 0, THEN NEW SOLUTION IS PLACED IN THE VECTOR Y(I).
ZDEC(IY) = DECOLD
IF YTEST > 0 THEN GOTO 6
ZDEC(IY) = DECOPT
FOR I = 1 TO IYMAX: Y(I) = C(I): NEXT I
6 REM

REM THE SOLUTION IS SHOWN ON THE SCREEN.
GOSUB 100

NEXT IY
NEXT IMAIN
END
```

```
100 REM
REM SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
REM SUBROUTINE 100, OUTPUT OF PRELIMINARY RESULTS.
REM SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
CLS
PRINT "INFINITE HORIZON MARKET DEPENDENT STOCK OPTIMIZATION"
PRINT ""
PRINT "The optimal values of Y(i) are:"
FOR I = 1 TO IYMAX
J = I - INT(I / 9) * 9
PRINT USING "#####.#"; Y(I);
IF J = 0 THEN PRINT ""
NEXT I
PRINT ""
PRINT "The optimal state dependent decisions are:"
FOR I = 1 TO IYMAX
J = I - INT(I / 9) * 9
PRODU = XLEV(ZDEC(I)): SALES = SLEV(ZDEC(I))
PRINT PRODU; SALES; ", ";
IF J = 0 THEN PRINT ""
NEXT I
PRINT ""
IF IY < 46 THEN PRINT "Main loop = "; IMAIN; ". State "; IY; " optimized."
PRINT "Date = "; DATE; ". Time = "; TIME
PRINT "Rate of interest = "; RATEINT;
PRINT ". Profit weights = "; PROFR; PROFPC; PROFCS;
PRINT ". Fix cost = "; FIXCOST
PRINT "Production cost parameters = "; COSTA; COSTB; COSTC;
PRINT ". Price parameters = "; PRICEA; PRICEB; PRICEC
PRINT "Storage cost parameter = "; CSTO
PRINT "Probability vector = ";
PRINT USING "##.###"; PR11; PR12; PR13; PR21; PR22; PR23; PR31; PR32; PR33
RETURN
```

```
150
REM SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
REM SUBROUTINE 150, OUTPUT OF FINAL RESULTS TO FILE UTDYNSTO.DAT
REM SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
OPEN "UTDYNSTO.DAT" FOR OUTPUT AS #2
PRINT #2, "INFINITE HORIZON MARKET DEPENDENT STOCK OPTIMIZATION"
PRINT #2, "Optimal solution:"
PRINT #2, ""
PRINT #2, "The optimal values of Y(i) are:"
FOR I = 1 TO IYMAX
J = I - INT(I / 9) * 9
PRINT #2, USING "#####.#"; Y(I);
IF J = 0 THEN PRINT #2, ""
NEXT I
PRINT #2, ""
PRINT #2, "The optimal state dependent decisions are:"
FOR I = 1 TO IYMAX
J = I - INT(I / 9) * 9
PRODU = XLEV(ZDEC(I)): SALES = SLEV(ZDEC(I))
PRINT #2, PRODU; SALES; ", ";
IF J = 0 THEN PRINT #2, ""
NEXT I
PRINT #2, ""
PRINT #2, "Date = "; DATE; ". Time = "; TIME
PRINT #2, "Rate of interest = "; RATEINT;
PRINT #2, ". Profit weights = "; PROFR; PROFPC; PROFCS;
PRINT #2, ". Fix cost = "; FIXCOST
PRINT #2, "Production cost parameters = "; COSTA; COSTB; COSTC;
PRINT #2, ". Price parameters = "; PRICEA; PRICEB; PRICEC
PRINT #2, "Storage cost parameter = "; CSTO
PRINT #2, "Probability vector = ";
PRINT #2, USING "##.###"; PR11; PR12; PR13; PR21; PR22; PR23; PR31; PR32; PR33
CLOSE #2
RETURN
```

```
200 REM
REM SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
REM SUBROUTINE 200, THE RELEVANT EQUATIONS (WITH RESPECT TO THE
REM INITIALLY SUGGESTED DECISIONS) ARE DETERMINED AND PLACED
REM IN THE EQUATION SYSTEM MATRIX A(I,J).
REM SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
REM The decision dependent coefficients are
  REM placed in the DAKT and BAKT matrixes.
  REM First, BAKT(i,j) = 0 for all i, j
  REM ****************************************
  REM ****************************************
  FOR I = 1 TO IMAX: FOR J = 1 TO JMAX: BAKT(I, J) = 0: NEXT J: NEXT I
  REM The stock level in the next period, STOCK2, is calculated.
  REM ************************************************************
  FOR I = 1 TO IYMAX
  LOCDEC = ZDEC(I)
  STOCK1 = QLEV(I): STOCKDEV = XLEV(LOCDEC) - SLEV(LOCDEC)
  STOCK2 = QLEV(I) + STOCKDEV
  IF STOCK2 < 0 THEN GOTO 3
  IF STOCK2 > QMAX THEN GOTO 3
  REM The relevant instant profit vector is placed in DAKT(i).
  REM **********************************************************
  DAKT(I) = ZD(I, ZDEC(I))
  REM The probabilities of different changes in the cost and price levels
  REM are calculated. Discounting is performed.
  REM ******************************************************************
  P1 = PLEV(I): C1 = CLEV(I): PCSTATE1 = (P1 - 1) * CMAX + C1
      FOR PCSTATE2 = 1 TO (PMAX * CMAX)
      STATE2 = STOCK2 * PMAX * CMAX + PCSTATE2
      PROB2 = PRMAT(PCSTATE1, PCSTATE2)
      BAKT(I, STATE2) = PROB2 * DISCF
      NEXT PCSTATE2
3 REM
  NEXT I
REM The relevant linear equation system is placed in the matrix A(i,j).
REM ********************************************************************
FOR I = 1 TO IMAX: FOR J = 1 TO JMAX: A(I, J) = 0: NEXT J: NEXT I
FOR I = 1 TO IMAX: A(I, I) = 1: NEXT I
FOR I = 1 TO IMAX: A(I, JMAX) = DAKT(I): NEXT I
FOR I = 1 TO IMAX: FOR J = 1 TO IMAX
A(I, J) = A(I, J) - BAKT(I, J)
NEXT J: NEXT I
RETURN
```

```
300 REM
REM SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
REM SUBROUTINE 300, TESTS OF THE SOLUTION OF THE LINEAR EQUATION
REM SYSTEM ARE PERFORMED.
REM SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
REM Is the obtained solution reasonable? We know from theory that
REM Y(i) (for every i) should increase or be unchanged after each
REM iteration in the optimal direction. It is possible that there
REM are numerical problems in the linear equation system subroutine
REM in some iterations. Maybe, there is linear dependence in the
REM automatically calculated equation system. Such cases may give all
REM kinds of large errors in the results. Hence, here is a test.
REM If (1) every Y(i) value increases or is unchanged and if (2) the
REM sum of relative increases in the Y(i) values is less than 10, then the
REM new solution is accepted. Otherwise, the old solution is still assumed
REM to be the best solution among the tested solutions.
YTEST1 = 0
FOR I = 1 TO IMAX
IF C(I) < Y(I) THEN YTEST1 = 1
NEXT I
YTEST2 = 0
YRELTOT = 0
FOR I = 1 TO IMAX
YRELINC = C(I) / (Y(I) + 1)
YRELTOT = YRELTOT + YRELINC
NEXT I
IF (YRELTOT / IMAX) > 1000 THEN YTEST2 = 1
REM The first time the equation system is solved, the Y values increase very
REM much. Hence, the test value is adjusted in this case.
IF IMAIN = 1 THEN YTEST2 = 0
YTEST = YTEST1 + YTEST2
IF YTEST > 0 THEN SOUND 1000, 2
PRINT "(NEG. DEVIATION, REL. INCREASE) = ("; YTEST1; ", "; YTEST2; ")"
RETURN
```

```
400 REM
REM SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
REM SUBROUTINE 400, THE LINEAR EQUATION SYSTEM IS SOLVED.
REM SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
REM ******************************************************************
REM Here, the linear equation system is solved via the Gauss method,
REM Ref: Wittmeyer-Kock, I., Elden, L., Lrobok i numeriska metoder,
REM University of Linkping, Dept. of Math., 581 83 Linkping, Sweden,
REM 1986
REM ******************************************************************
FOR I = 1 TO IMAX
ROWCOEFMAX = I
     FOR K = (1 + I) TO IMAX
     IF ABS(A(K, I)) > ABS(A(ROWCOEFMAX, I)) THEN ROWCOEFMAX = K
     NEXT K
     FOR L = 1 TO JMAX
     C1 = A(I, L)
     C2 = A(ROWCOEFMAX, L)
     A(I, L) = C2
     A(ROWCOEFMAX, L) = C1
     NEXT L
     FOR II = (I + 1) TO IMAX
     COEF = A(II, I) / A(I, I)
        FOR J = I TO JMAX
        A(II, J) = A(II, J) - COEF * A(I, J)
        NEXT J
     NEXT II
NEXT I
FOR IBACK = 1 TO IMAX
I = IMAX - IBACK + 1
   FOR II = 1 TO (I - 1)
   COEF = A(II, I) / A(I, I)
     FOR J = 1 TO JMAX
     A(II, J) = A(II, J) - COEF * A(I, J)
     NEXT J
   NEXT II
NEXT IBACK
FOR I = 1 TO IMAX
A(I, JMAX) = A(I, JMAX) / A(I, I)
C(I) = A(I, JMAX)
NEXT I
RETURN
```

```
500 REM
REM SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
REM SUBROUTINE 500, FINAL RESULTS AND SOUNDS.
REM SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
GOSUB 150
GOSUB 100
FOR I = 1 TO 20: SOUND (1000 + I * 100), 5
SOUND (3000 - I * 100), 5: NEXT I
RETURN
```

```
600 REM
REM SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
REM SUBROUTINE 600, THE USER DEFINED PARAMETERS ARE LOADED FROM
REM THE FILE INDYNSTO.DAT.
REM SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
INPUT #1, DATE, TIME, IMAINMAX
INPUT #1, PROFR, PROFPC, PROFCS
INPUT #1, RATEINT
INPUT #1, FIXCOST
INPUT #1, COSTA, COSTB, COSTC
INPUT #1, PRICEA, PRICEB, PRICEC
INPUT #1, CSTO
INPUT #1, PR11, PR12, PR13, PR21, PR22, PR23, PR31, PR32, PR33
RETURN
```