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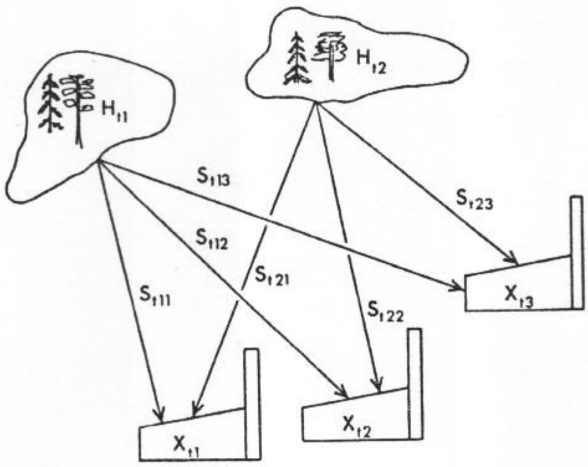
STOCHASTIC DYNAMIC PROGRAMMING WITH A LINEAR PROGRAMMING SUBROUTINE:

APPLICATION TO ADAPTIVE PLANNING AND COORDINATION IN THE FOREST INDUSTRY ENTERPRISE



by

Peter Lohmander

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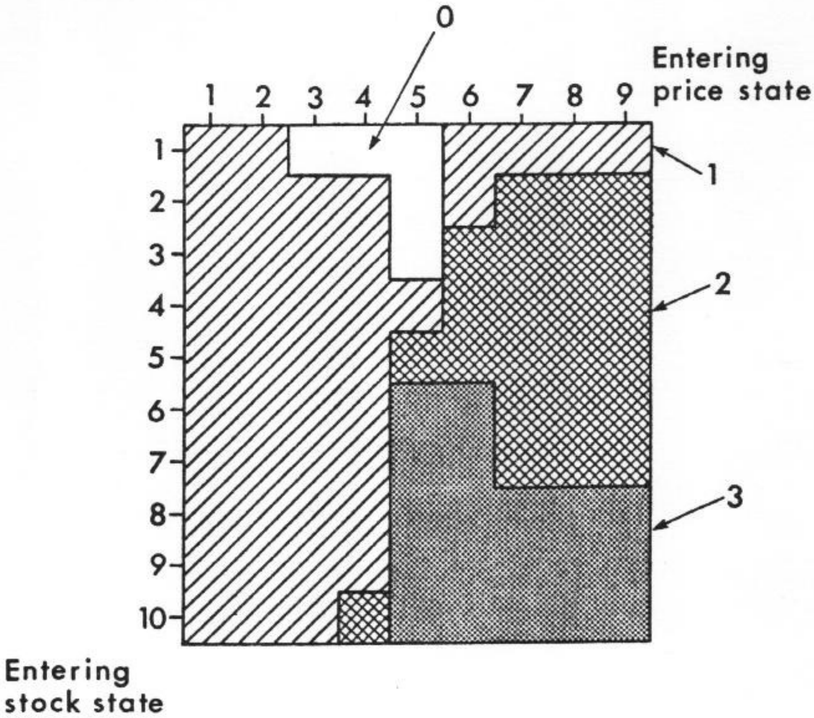

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Stochastic dynamic programming with a linear programming subroutine: Application to adaptive planning and coordination in the forest industry enterprise

Abstract

Most economic planning situations in complex enterprises have the following properties: (1) Many activities must be spatially coordinated, (2) decisions at time t affect the possible activities in later periods and (3) important information, particularly concerning future product prices, is not available in advance. Sequential and spatially coordinated decisions are necessary. This paper includes a review of important economic principles, observations and methodological progress in this area. Particular emphasis is directed towards the method of stochastic optimal control in discrete time with linear programming solutions for each possible stage and state. The intertemporal coordination problem of the integrated forest industry enterprise in a stochastic product market is defined.

Analytical and numerical optimization solutions are derived. Variable reduction is used to simplify the simplex subproblem and the main results of a typical solution are discussed.

When the future prices of the final products are stochastic;

a. The expected present value of the profit is higher when the suggested method is used instead of deterministic multi period linear programming.

b. The expected dual variables (shadow prices) associated with the industrial capacity restrictions are underestimated via deterministic multi period linear programming.

c. Industrial flexibility is valuable and the optimal levels of maximum production capacity are higher than in a deterministic world.

A computer program for stochastic dynamic programming with a linear programming subroutine is designed and included.

1. Introduction

1.1. The issue

This study covers several topics. A combination of these will in this paper be shown to be very useful in the economic planning of integrated forest industry enterprises.
The ambition is to accept and explicitly make use of the properties of most real world planning problems: The future can not be perfectly predicted, many activities must be coordinated already in the present time period and present decisions and activities influence the planning and activity options in the future.

Clearly, the structure of this problem is the following: Since the future conditions (parameters) are not yet known, it is generally optimal to have flexibility. Adaptive optimization is relevant. Coordination of the present activities require efficient deterministic planning tools. In this paper, the suggestion is to use stochastic dynamic programming, which serves as the adaptive optimization tool, and linear programming (possibly quadratic programming) which takes care of the deterministic optimization and coordination of the decisions at each point in time and for each possible stochastic state of the system.

1.2. On the relevance of the topic

The methodology created in operations analysis has found many applications in the forest – forest industry sector. Löfgren (1989) writes that the easily available methods of operations analysis, such as linear programming, are based on the assumption of perfect information concerning future prices. He also claims that adaptive optimization may be useful if the price probability distribution is known but that the common sence of the forest owner most likely works as well as explicit optimization. Löfgren states that the forest economic optimization may be performed on the stand level in case the optimization problem is truely linear (exogenous price and cost parameters in the objective function) and there are no (arbitrary) "even harvest level" (or similar) restrictions on the total harvest program.

In the analysis of this paper, we will consider the comments presented by Löfgren (1989).
- We will not assume that future conditions (prices etc.) are known.
- We will not consider problems of low complexity (which make "common sence solutions" trivial and optimal).
- We will consider economic planning problems where many activities must be coordinated and explicit use of constrained optimization can not be avoided.

1.3. Past efforts and findings

It is suggested that the following classification is used:

a. Deterministic integrated enterprise economics and planning

Koopmans and Beckman (1957), Andersson (1963), Näslund (1965), von Malmborg (1967), Lindgren and Näslund (1968), Lönner (1968), Holvid (1970), Ljungman (1971), Nilsson (1974), Lohmander (1985), Lohmander (1988f)

Koopmans and Beckman (1957) gave the foundations of some of the problems of interest in the present analysis, particularly those of economic location of activities. Within the natural resource enterprise, the transportation problem is typically a subproblem of great importance. However, it is dangerous if the transportation problem becomes the only area of interest. All possible activities within the firm are of relevance in the planning situation and suboptimization should be avoided. Andersson (1963) looks at the district management problem in forestry from a less mathematical but more operational view. Some effort is directed towards the collection of planning data and data reliabilty within the enterprise. Näslund (1965) suggests relevant methods from management theory to be used in the forest sector and von Malmborg (1967) shows how to use linear programming in the integrated forest - farm enterprise. Lindgren and Näslund (1968) give a survey of possible planning approaches in the forest sector based on mathematical optimization, particularly linear programming. They also report from past application experiences. The option of decomposition and decentralized decision making is stressed through a detailed example from a forest enterprise with many departments. Lönner (1968) presents a system for short term planning of forest district management including harvesting, storage and transportation. Clearly, one of the key reasons why short term planning is applied is the presence of unpredictable situations. Nevertheless, deterministic methods were used. Also the studies by Holvid (1970) and Ljungman (1971) are based on deterministic linear programming. Nilsson (1974) attacks one of the very unpredictable planning situations, namely the windthrow area management. Lohmander (1985) contains an introduction to the use of deterministic linear programming in the integrated forest enterprise and Lohmander (1988f) shows how to derive simple harvest activity selection rules from a formal linear programming model.

b. Stochastic integrated enterprise economics and planning

Bellman and Zadeh (1970), Hakansson (1971), Nilsson (1979), Hof, Robinson and Betters (1988)

It is difficult to find a representative set of publications in this class. However, some interesting ideas have been presented by the suggested authors. Note that approaches consistent with stochastic optimal control theory seldom have been used in the past applications.

c. Deterministic forest stand economics and planning

Näslund (1969), Heaps (1984), Johansson and Löfgren (1985), Heaps (1986), Lohmander (1988d), Magnusson (1988), Valsta (1988), Lohmander (1989)

These references are relevant in this paper mainly to show that the forest stand management problem has gained much attention in the literature. Clearly, in most cases, the assumptions of "stand separable management" are seldom satisfied. Usually, there are economies of scale in harvesting operations, technical restrictions etc. that make separability vanish. Coordination and constrained optimization becomes important.

d. Stochastic forest stand economics and planning

Norstrom (1975), Risvand (1976), Lohmander (1987a), (1988b), (1988c), (1988e)

Since it has recently been realized that the future is far from predictable, the explicit treatment of stochastic future events and adaptive optimization have become natural ingredients in forest economics. This new interest is largely a result of the discoveries in catastrophe and chaos theory and of course the associated sciences biology and chemistry. Two recent publications in that area are Puu (1987) and Puu (1989).

e. Deterministic optimization

Bellman (1953), Hållsten (1958), Wolfe (1959), Wolfe and Dantzig (1962), van de Panne and Whinston (1964), Balinski (1965), van de Panne and Whinston (1966), Denardo (1970), McCormick (1970), Florian and Robillard (1971), Lemke, Salkin and Spielberg (1971), Geoffrion and Marsten (1972)

f. Stochastic optimization and estimation

Manne (1960), Breiman (1964), Cocks (1968), Derman and Veinott (1972), Garstka and Rutenberg (1973), Satia (1973), Smallwood (1973), Kumar and Varaiya (1986), Lohmander and Helles (1987b), Kurzhanski (1988), Williams (1988)

g. Stochastic optimization with deterministic constrained optimization subproblems

Simon (1956), Breiman (1964), Rockafellar and Wets (1978), Rockafellar and Wets (1986), Rockafellar (1987), Rockafellar and Wets (1987), Byrnes and Kurzhanski (1988), Varaiya and Wets (1988)

This field is the one which makes the solution of the problem approached in this paper possible. The author is convinced that most real world problems in principle should be solved via methods in this class. The analytical and numerical development of these methods is of great importance to future decision making in a complex and unpredictable world.

h. Recent contributions to the general stochastic investment theory

Baldwin (1982), Brennan and Schwartz (1985), McDonald and Siegel (1985), McDonald and Siegel (1986), Pindyck (1988)

McDonald and Siegel (1985) observe that the profit is a kinked convex function of price when there is an option to shut down production during low price periods. Hence, it can be shown that mean preserving increases in price risk (nonstrictly) increase the expected profit. This paper contains a relaxation of their assumptions: A natural resource (forest) which can be extracted (harvested) over time in different ways and alternative processing plants (pulp and paper mills, saw mills etc.) are explicitly introduced. Hence, it will turn out that product price risk is valuable to the enterprise since: 1/ There is an option to shut down (or just decrease) total production during low price periods and 2/ The natural resource may be distributed between the different processing plants in the most profitable way, depending on the levels of the different product prices.

Pindyck (1988) presents a continuous time model with irreversible investments and stochastic prices. He finds, as in this paper, that the marginal capacity values (shadow prices) are increasing functions of price variability. Pindyck makes the special assumption that there is an option to add capital continuously and incrementally. Hence, there is little reason to invest before the high prices have been observed. The conclusion is that firms should hold less capacity than if prices were predictable and investments were reversible. Finally, Pindyck stresses that the result partly is a function of the particular assumptions and that more generalized models should be studied. He suggests that numerical methods should be used to solve these more complex problems. The authour of this paper is convinced that the main result (less capacity) derived by Pindyck would have been different (more capacity) in a model where investments take time. It generally takes many years to construct a pulp mill, not to mention the time it takes in Sweden to get a building and capacity expansion permission ! Hence, if demand rapidly increases, there is no time available to build a new factory. It is already too late.

1.4. The deterministic problem

A simplified "standard" formulation of the integrated forest industry enterprise problem, different versions of which can be found in Lohmander (1985) and in many of the earlier publications, is the following:

$$\max_{H,S,x} \Pi = \sum_t e^{-rt} \left[-\sum_i c_{ti} H_{ti} - \sum_i \sum_j g_{tij} S_{tij} + \sum_j \bar{p}_{tj} x_{tj} \right] \qquad (1)$$

s.t.

$$\sum_t H_{ti} \le A_i \qquad \text{(for each i)} \qquad (2)$$

$$x_{tj} \le \bar{I}_{tj} \qquad \text{(for each t, j)} \qquad (3)$$

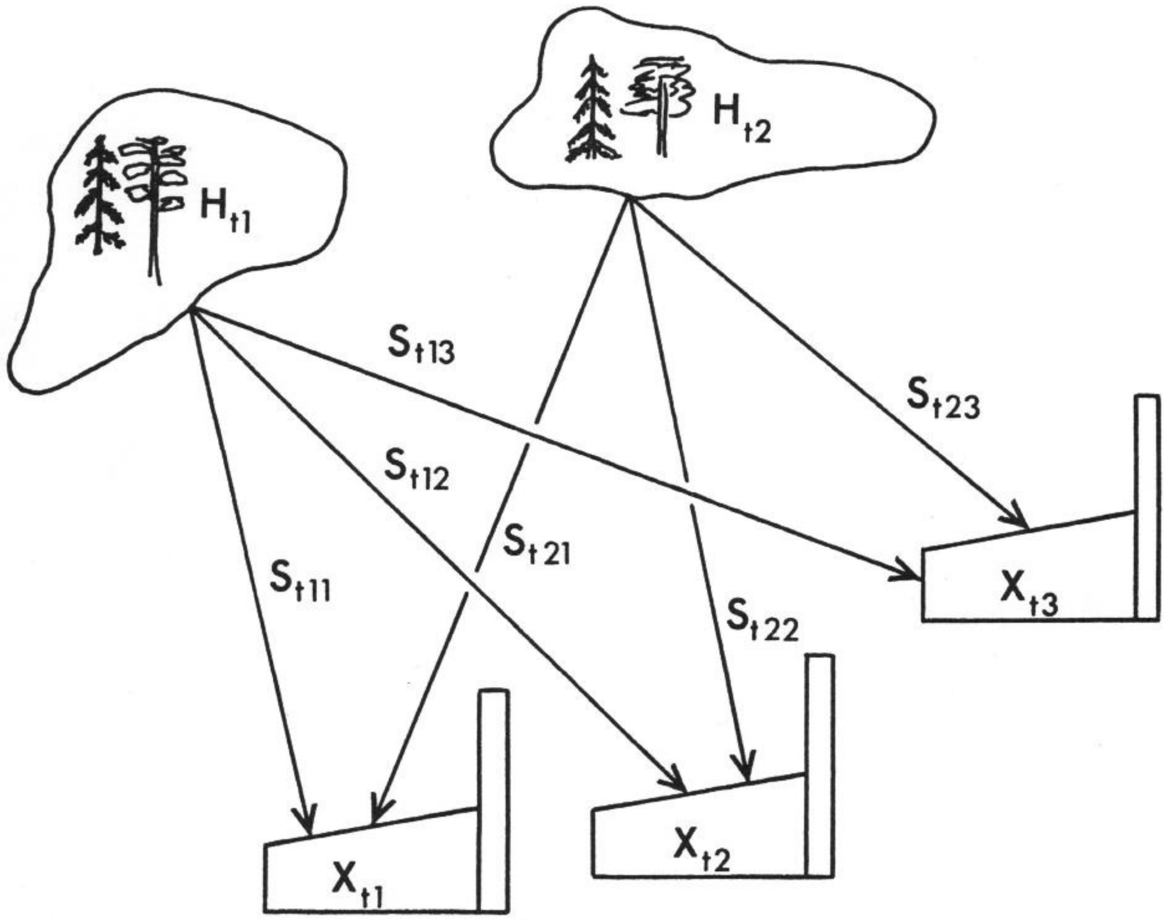
$$\varphi_{ti} H_{ti} - \sum_j S_{tij} = 0 \qquad \text{(for each t, i)} \qquad (4)$$

$$\sum_i S_{tij} - \beta_{tj} x_{tj} = 0 \qquad \text{(for each t, j)} \qquad (5)$$

The objective function (1) is the total present value of all future activities, Π.

Π is a function of the harvest area levels, H, the transport levels, S, and the industrial production levels, x. t, r, i and j denote time period, rate of interest, stand index and production plant index. A_i denotes the initial area in stand class i, $\bar{I}_{tj}$ is the production capacity in plant j at time t, φ_{ti} is the stand density (volume per area unit) in stand class i at time t and β_{tj} is the input coefficient in plant j at time t.

Figure 1.
The "traditional" integrated forest industry enterprise problem.



c, g and $\overline{p}$ are the harvest cost per area unit, the transportation cost per volume unit and the final product net price (price minus variable costs). $\overline{p}$ (and the transformed price p, compare the mathematical appendix) are in the rest of the analysis called "product price". (2) represents the intertemporal harvest area restriction for each stand class, (3) is the production capacity restriction for each plant and time period, (4) are the transport balance equations in the different forest areas and (5) expresses the transport balances in the different plant nodes.

There exist practical problems in the application of the model (1) – (5). Some of these are:

a. The size of the simplex matrix increases rapidly with the number of periods, the number of stand classes, the number of plants etc.. Hence, at least the number of periods has to be restricted and the solution in the final periods can not be regarded as relevant to the real world decision problem. (In the final model period, there is no reason to save any forest resources for future periods.)

b. Nonlinearities may exist in the real world decision problem. Some of these may be taken care of by quadratic programming (as long as the objective function is concave and the feasible region is a convex set).

c. Particularly if we make the plant capacity investments (and hence the plant capacities in future periods) endogenous, economies of scale may exist that make the objective function strictly convex. Then, linear programming becomes irrelevant.

d. Integer programming may be necessary when investment activities are endogenous.

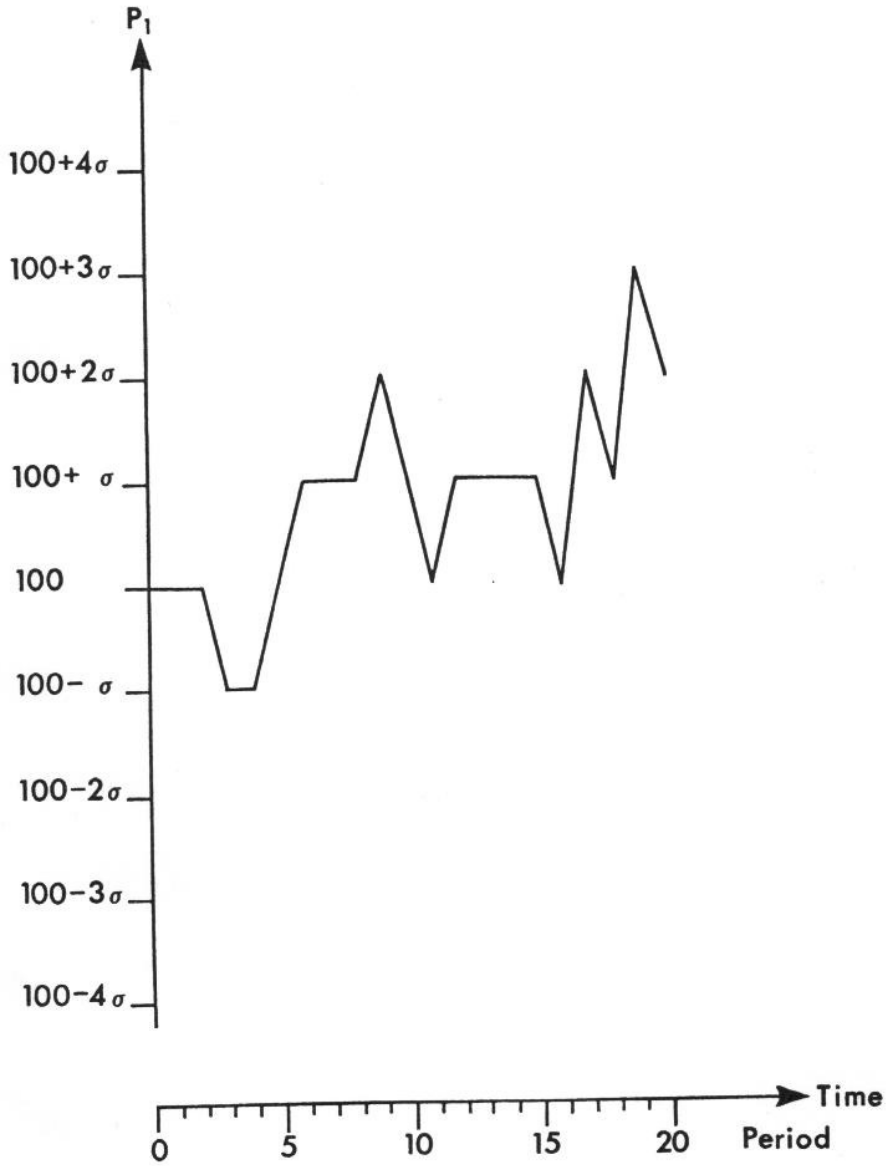
e. The coefficients of the "real world" objective function are stochastic (or at least very expensive to predict perfectly for $t > 1$).

Figure 2.

A typical sample path of a product price process according to the numerical analysis in this paper. The net price is represented by a first order discrete time autoregressive process defined via the transition probability matrix given in the numerical appendix. The discrete state space contains 9 states, which makes the transition probability matrix contain only (9 X 9 =) 81 elements. This makes it possible to use the dynamic programming method with limited execution time. The discrete state process "approximates" a stationary continuous state discrete time process:

$P_{t+1} = A + BP_t + \epsilon_t$ such that $A > 0, 0 < B < 1, \epsilon \in N(0,\sigma)$

In the illustration, $A = 50$, $B = .5$, $\sigma = 10$.



f. Clearly, all coefficients in the simplex problem are difficult (or expensive) to predict for $t > 1$.

The ambition of the analysis presented in this paper is mainly to treat the problem e.: stochastic objective function coefficients. The reason is that the other simplex problem coefficients frequently are much easier, much less expensive, to predict. More specifically, the main effort will be directed towards the problem where the product prices, $\bar{p}_{tj}$, can be described as a stochastic Markov process. The decisions are taken sequentially when the prices $\bar{p}_{tj}$ have been observed (without noise).

The essential questions that should be possible to answer via the presented approach are the following:
- How should the activities within the integrated forest industry enterprise be coordinated in order to satisfy the balance equations ?
- How sensitive are the optimal activity levels in different parts of the enterprise to the states of the product markets ?
- What is the total effect of stochastic product prices on the expected profitability of the integrated enterprise ?
- Are the optimal investment program and the optimal plant capacities functions of the stochastic properties of product prices ?
- What is the value of flexibility in the stochastic environment ?

2. Analysis
2.1. Analytical investigation

This section contains the definition of a simplified linear programming problem which is further analysed in the mathematical appendix. This formulation may be interpreted as the optimization problem in a future period. All coefficients are known except for some objective function coefficients, prices, which are stochastic in the future period. The future decisions are adaptive: First the coefficient outcome is observed without noise and then the optimal activity decisions are taken. In a multi period problem, the decisions in earlier and later periods are connected via the forest resource constraint. If more is harvested in period one, then less can be harvested in the future. Clearly, the solution to the "complete" global problem (when the objective function is the expected present value of the profits in all periods), must give consistent expected marginal valuation of the (common) forest resource. In other words, the expected dual variable associated with the forest resource constraint must have the same value in the present period and in the future. This is easily shown via stochastic dynamic programming. Hence, the main effort in this section will be to investigate how the expected dual variable vector is affected by increasing risk in a future period. The results may be used in the shape of parameter changes in the optimization problems of the earlier periods. Hence, some important properties of the optimal primal and dual solutions under the influence of stochastic objective function coefficients are derived. The general results of relevance to optimal economic decisions in the integrated forest – industry enterprise are discussed in connection to presented figures.

The simplified problem is :

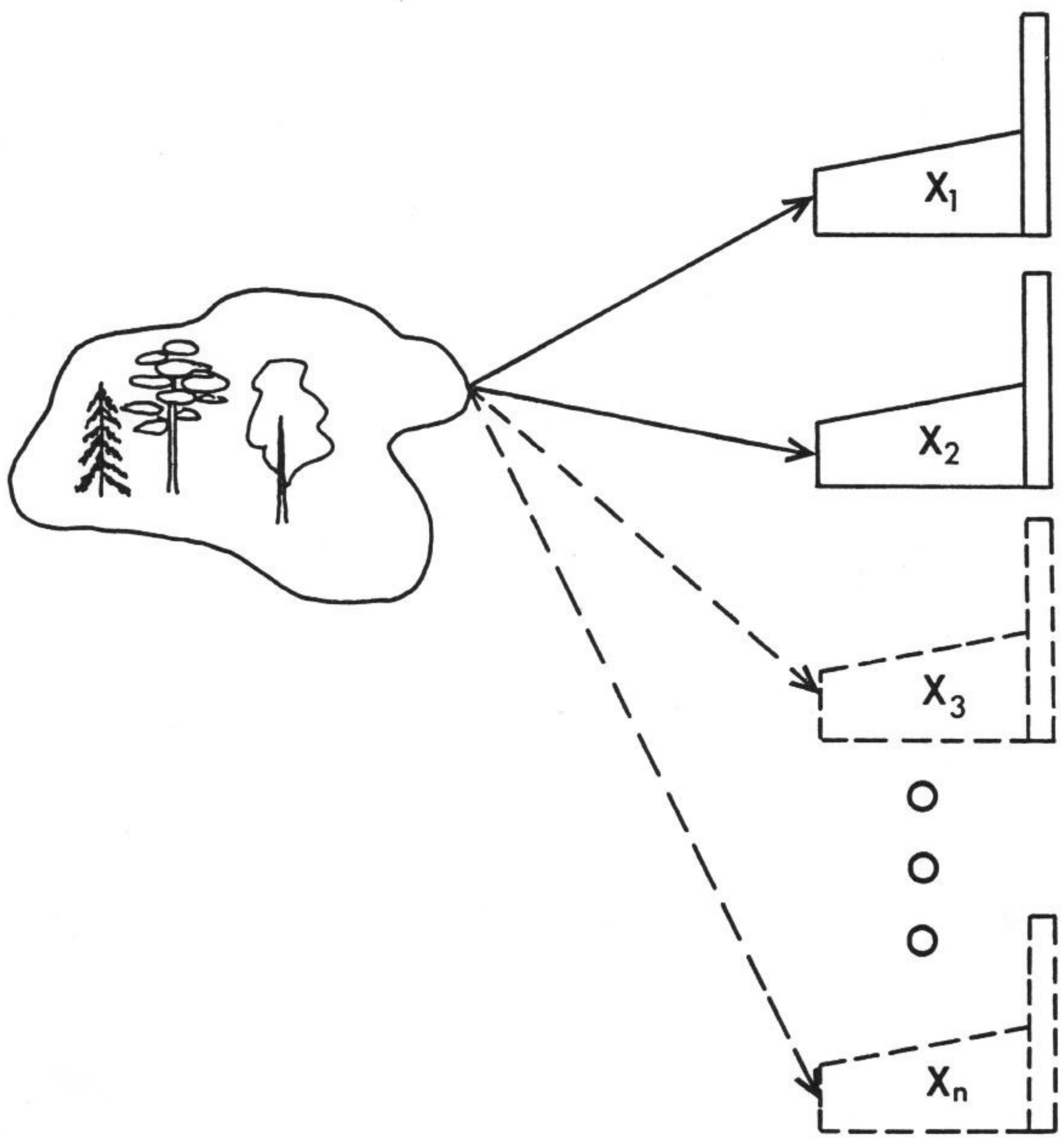
$$\max \Pi = \sum_{j=1}^{n} p_j x_j \tag{6}$$

$$\text{s.t.} \quad \sum_{j=1}^{n} \alpha x_j \leq R \tag{7}$$

$$x_j \leq I_j \text{ for } j \in (1,\dots,n) \tag{8}$$

Π denotes total profit and R is the total forest resource stock to be used in the period under consideration. Each x_j is expressed in a unit such that the forest resource input required in the production of one x_j is equal to α. I_j is the maximum factory capacity expressed in the relevant x_j unit and p_j denotes the unit net price. A more general version of the same problem, which is the first presented version in the mathematical appendix, can easily be constructed.

Figure 3.
The simplified integrated forest industry problem. The transportation activities are not treated as primal variables. The objective function coefficients of the industrial production activities x_j are reduced by the unit costs that derive from harvesting and transportation. The activities in the different factories are connected via the forest resource constraint.



In figure 3, the physical meaning of the simplified problem is shown. The set of possible (unique) optimal solutions under different assumptions are highlighted in the figures 4, 6, 7 and 8. The expected profit of the firm is strictly higher in the presence of price variability than in the deterministic case. This is shown in figure 5. The mathematical appendix contains Kuhn–Tucker analysis of the constrained optimization problem and some comparative statics analysis essential to the later analysis. In particular, the dual variables (the marginal values of the resources: the forest and the factory capacities) are determined as functions of the product prices. Since each corner solution in the linear programming problem is defined by a specific equation system, the relevant equation system and hence the partial derivatives of the dual variables with respect to the parameters are functions of the parameters.

In figure 9, the key results from the mathematical appendix are shown in graphical form. It is found that the dual varables (shadow prices) have interesting properties with respect to risk effects: The shadow prices associated with the factory capacities are kinked convex functions of every product price. According to the Jensen inequality, this means, which is also shown in figure 9(b) and 9(c), that the expected shadow prices (marginal values) of the factory capacities are (nonstrictly) higher in case prices are stochastic than if they are known constants (and the expected price in the stochastic case is equal to the price in the constant case). In figure 9(a), it is found that the shadow price of the forest resource is a kinked concave function of every product price (for positive prices) and a kinked convex function (for prices in the neigbourhood of origo).

Hence, if prices may take positive and negative values (which is possible since prices here are defined as net prices), the expected marginal forest resource value is ambiguously affected by increasing risk in the prices. If, on the other hand, prices are always positive, then the Jensen inequality makes the expected marginal forest resource value (nonstrictly) lower in the presence of risk than in the case of constant prices (when the constant price is equal to the expected price in the stochastic case).

Figure 4.

The linear programming problem when there are two possible products, x_1 and x_2. Depending on the price vector, two different solutions are optimal. Of course, if $p_1 = p_2$, then the optimal solution is not unique. In the analysis, it will everywhere be assumed that the forest resource constraint and all of the industrial capacities ($\bar{I}$ is transformed to I in the mathematical appendix.) I_j, $j \in (1,...,n)$ except for one of them restrict the optimal solution. This means that the solution is determined by n linear equations. Hence, we may assume that the number of strictly positive x_j is also equal to n. In the figure, n = 2 and the number of strictly positive x_j is 2.

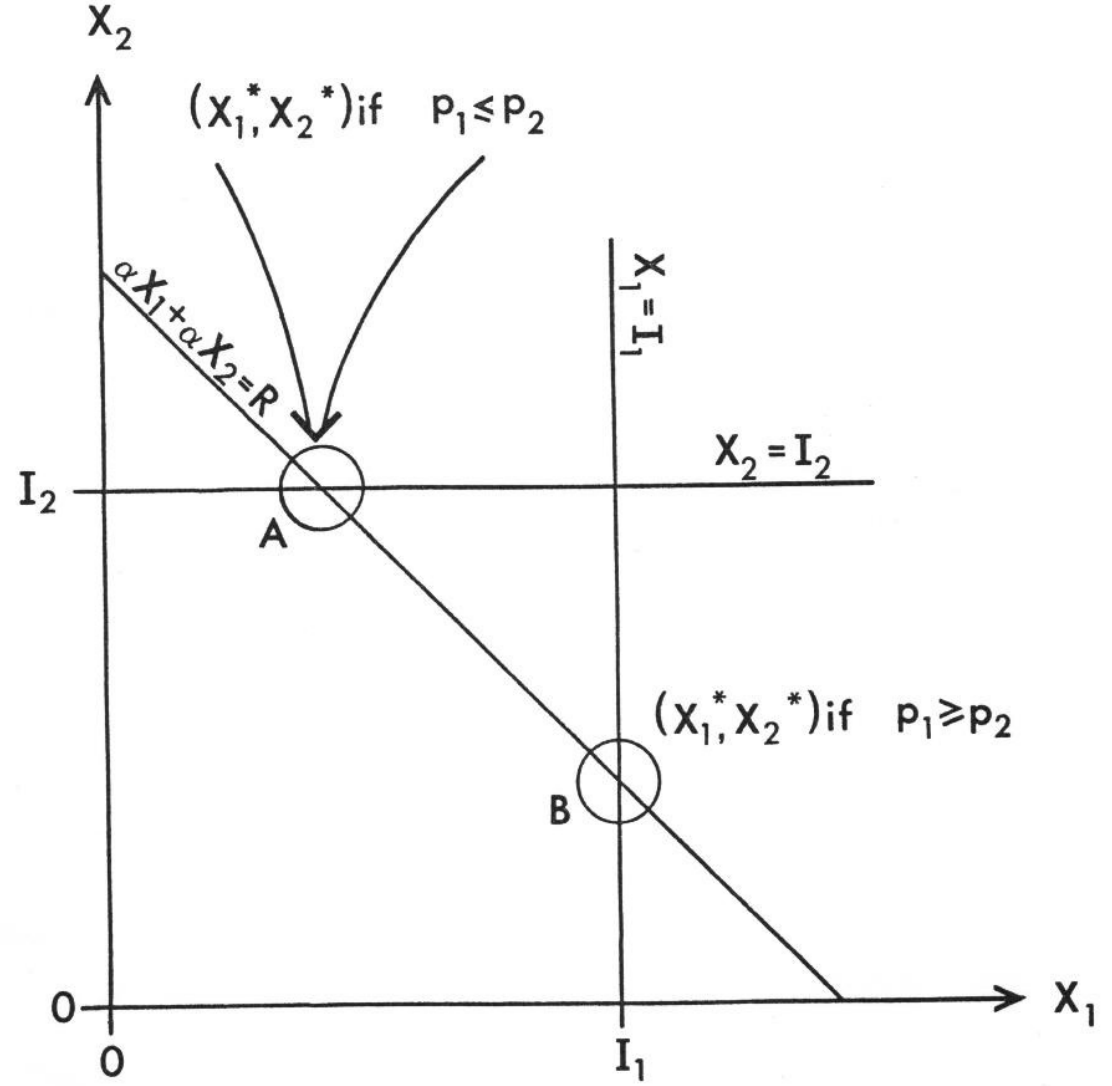


Figure 5.

The graph shows Π^*, the optimal value of the objective function in the two variable problem when the industrial capacities are very high and never restrict the optimal solution. If $p_1 < p_2$, it is optimal not to produce x_1 at all. Hence, Π^* is not affected by changes in p_1 as long as $p_1 \leq p_2$. If $p_1 > p_2$, then all of the forest resource should be used in the production of x_1 and x_2 must be zero. Then, Π^* is proportional to p_1. We assume that p_1 is stochastic. $\text{Prob}(p_1 = p_2^-) = \text{Prob}(p_1 = p_2^+) = 1/2$. Note that Π^* is a kinked convex function of p_1. Clearly, from the Jensen inequality, $E(\Pi^*(p_1)) \geq \Pi^*(E(p_1))$. Thus, stochastic product prices improve the expected profit when adaptive optimization is used. Note that Π^* is a kinked convex function of p_1 also when the industrial capacities are restricting the optimal solutions as in figure 4.

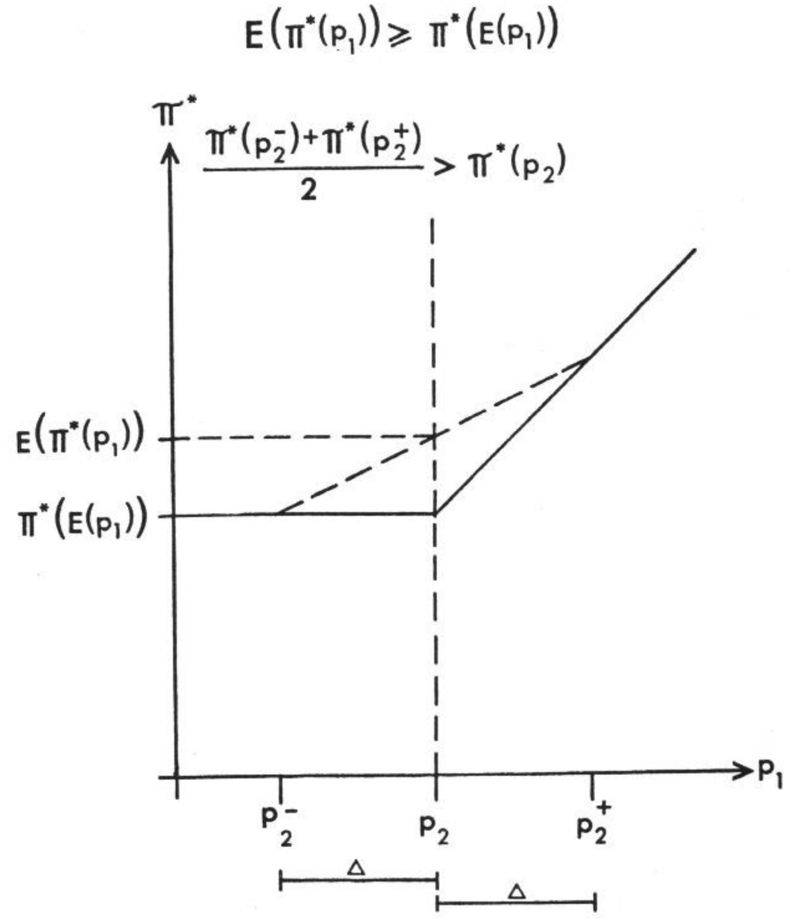


Figure 6.

The graph shows the primal problem in the case of 3 products. In the illustrated case, all potential unique optimal solutions (feasible corners) are restricted by the forest resource plane and one of the industrial capacity planes. Hence, six possible unique solutions exist. Since every solution is determined by two equations, only two x_j's take strictly positive values in the solutions. One of the x_j's is always zero.

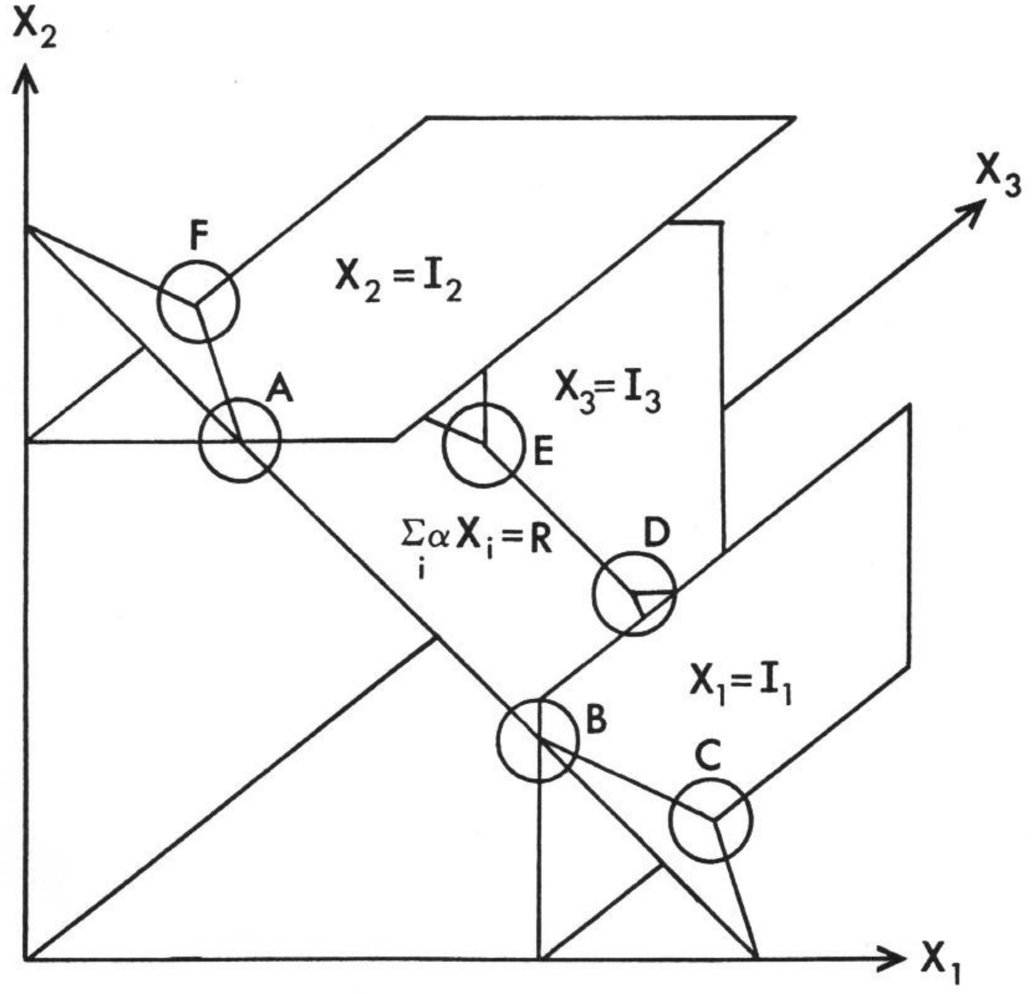


Figure 7.
The graph shows the forest resource restriction plane seen from "above". The feasible region is the area restricted by A – B – C – D – E – F – A . The six possible unique optimal solutions correspond to those in figure 6.

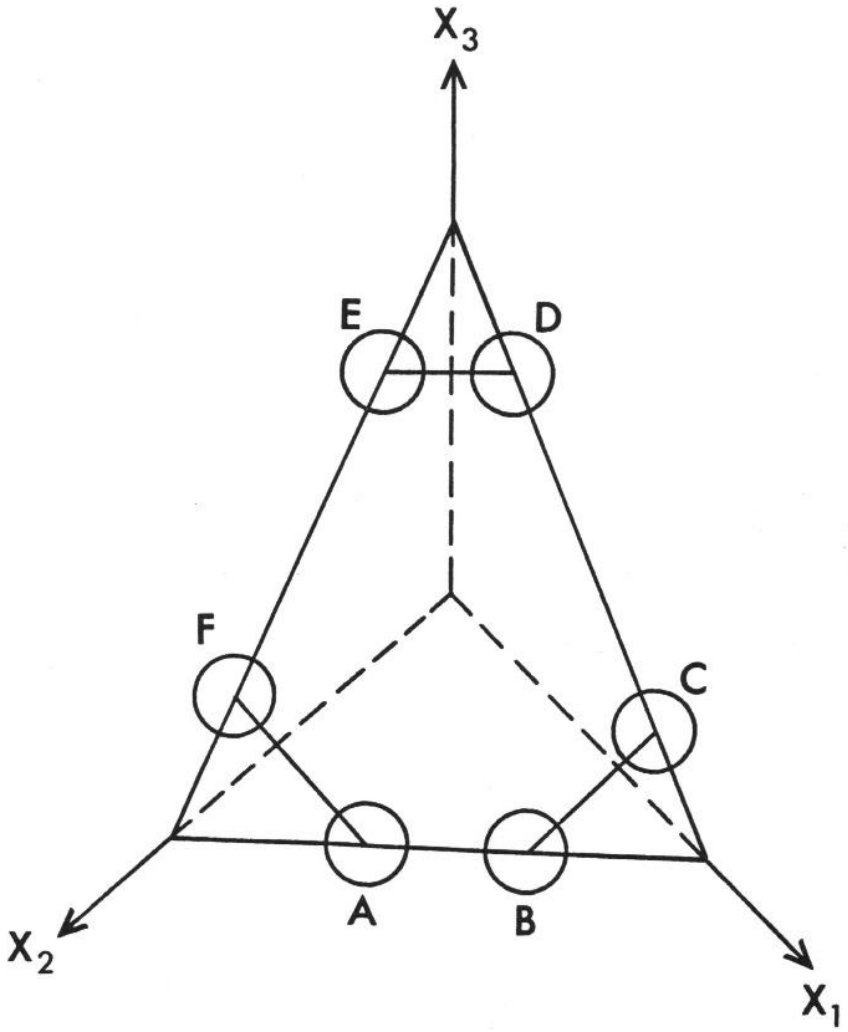


Figure 8.

If the industrial capacities decrease as represented by the shifts marked by arrows, then the feasible region becomes the area restricted by $Z_1 - Z_2 - Z_3 - Z_1$. Hence, only three possible unique optima exist. Each unique (feasible corner) optimum is restricted by the forest resource restriction plane and two of the industrial capacity planes. Hence, three equations give three strictly positive x_j's. The problem investigated in this paper is the n dimensional version of the illustrated situation : The forest resource and n–1 industrial capacities restrict the optimal solution. Every x_i, $i \in (1,...,n)$ is strictly positive.

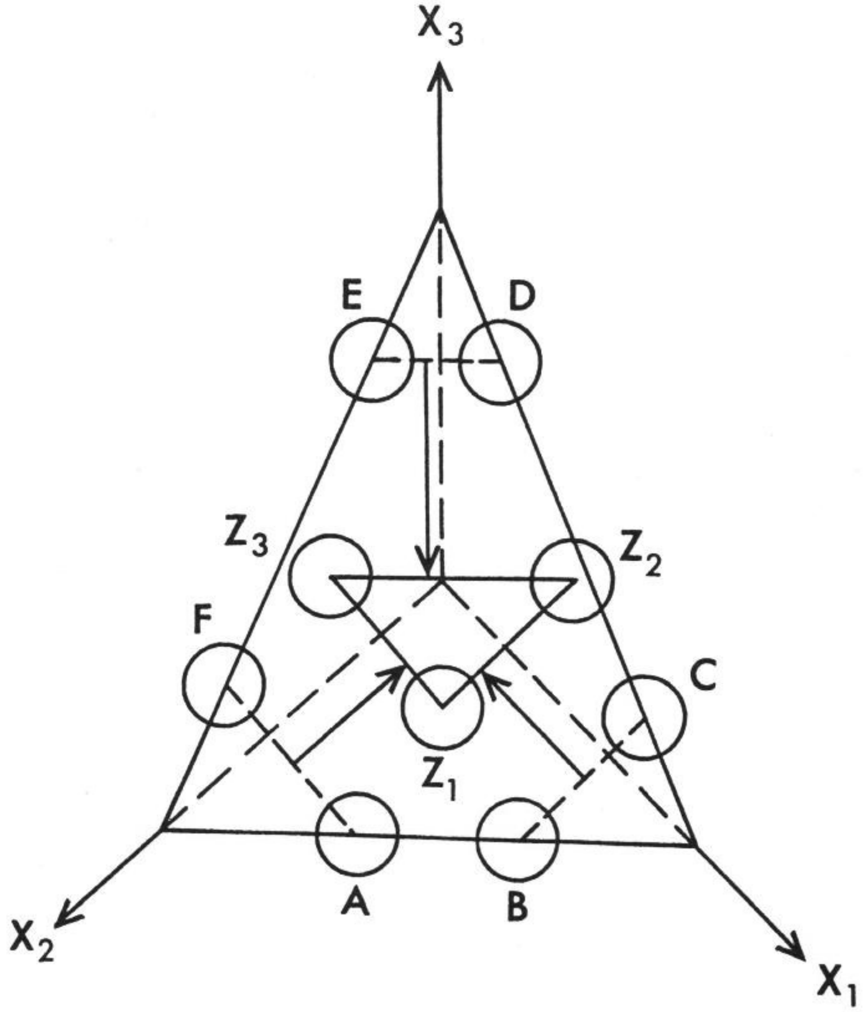
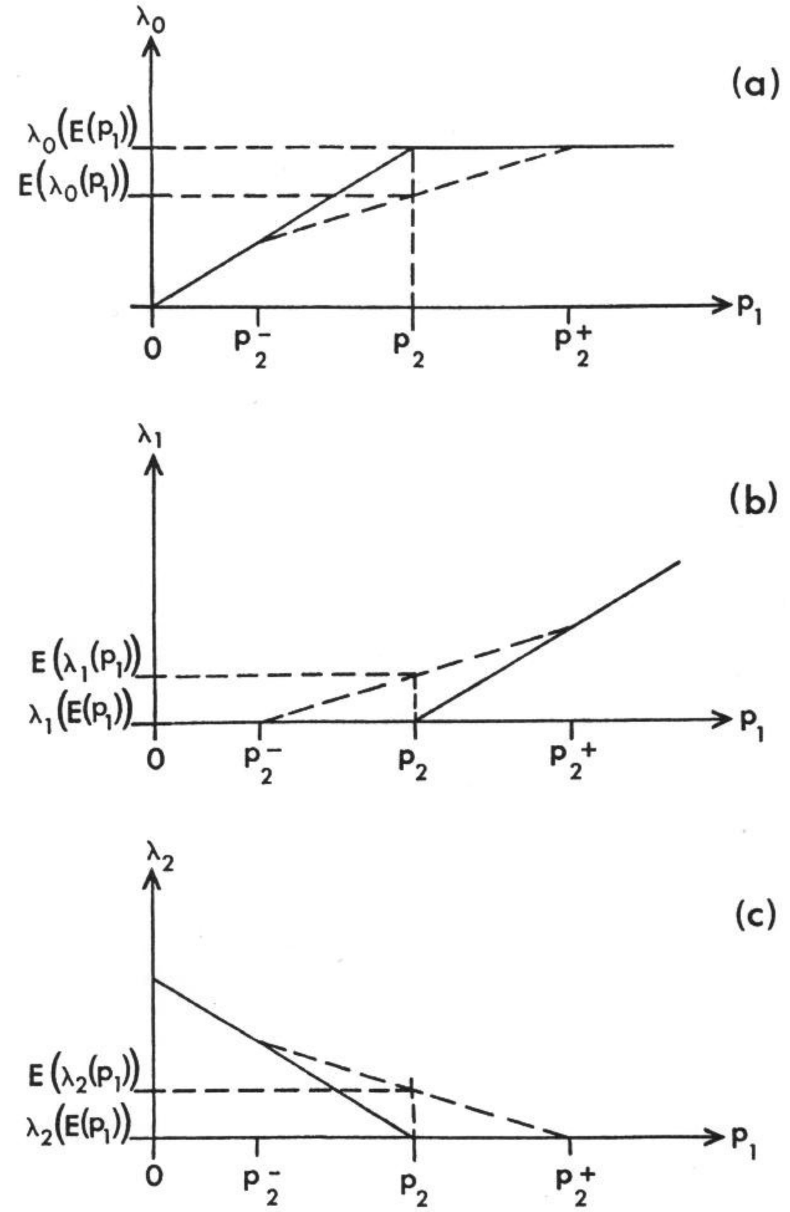


Figure 9.

The graphs (a), (b) and (c) show how the shadow prices (dual variables) of the forest resource (λ_0), of the industrial capacity I_1 (λ_1) and of the industrial capacity I_2 (λ_2) are affected by the price p_1. In the mathematical appendix it is proved that the qualitative results illustrated hold also in the n dimensional case. Hence, λ_0 is a kinked concave function of every product price (for positive prices). λ_j, $j \in (1,...,n)$ is a kinked convex function of every product price. Hence, $E(\lambda_0(p_i)) \leq \lambda_0(E(p_i))$ (if p_i is always positive !) and $E(\lambda_j(p_i)) \geq \lambda_j(E(p_i))$ for $j \in (1,...,n)$.



2.2. Numerical investigation

In the analytical section it was found that the expected value of the enterprise and the expected marginal factory capacity values are higher in the presence of price risk than in a corresponding (the same expected values of the prices) deterministic environment. It was also found that the expected marginal value of the forest resource is higher, the same or lower under risk than in a corresponding deterministic situation. Now, in order to obtain more detailed answers to our questions, it is necessary to consult a numerically specified optimization model.

The optimization problem in the numerical analysis is the following:

$$\max_{\substack{x_t \in \underline{X}(h_t,\phi) \\ h_t \in (0,1,\ldots,C)}} W\left[h_t, x_t; t, \phi, C, i\right] = e^{-rt}\Pi\left[x_t; \phi, K_t(h_t), t, i\right] + \sum_{j=1}^{J} V_{tij} W^{*}\left[h^{*}_{t+1}, x^{*}_{t+1}; (t+1), \phi, C(h_t, G_t), j\right] \qquad (9)$$

correction 90-01-19: ~~S_{tij}~~ → V_{tij}

W is the expected present value, h and x are the harvest level and the industrial production vector and t denotes time period. ϕ is the vector of known exogenous coefficients in the linear programming subproblem, C is the state of the forest resource and i denotes the state of the stochastic price vector (the price level index in the specific application). $\underline{X}$ is the feasible industrial production set, which is a function of the harvest level and other coefficients of the optimization problem. Π denotes the linear programming subproblem which is solved for every possible combination of the states i and C and in every period t. (This LP subproblem, $\Pi(.)$, is in the numerical optimizations defined as a quantified version of the "simplified problem" presented in the equations (6) – (8). The simplex method algorithm with the big M method, as presented by Wagner(1975), is used in the program. The particular coefficients used are found in the simplex matrix data set included in the end of the LP subroutine.) K, which is a function of the harvest level, is the forest resource restriction of the LP subproblem. r denotes the rate of interest and β_{tij} is the probability of transition from state i in period t to state j in period t+1. Of course, the elements of the transition probability matrix are derived from the stochastic process parameters. The details may be found in the included computer program. In case the available forest resource increases with time (new stands become sufficiently old to be harvested etc.), this may be handled via the parameter G. In the present analysis, this option has not been used. Stars (*) indicate optimal values. The initial forest resource state is C_0. The number of stochastic price vector states is I and TTOT denotes the number of periods in the optimization problem.

The solution is found via recursive calculations (from TTOT and backwards). In order to start the recursions, W^* is given the value zero for every possible state in period TTOT+1.

In the computer program and outputs (compare the numerical appendix), the following definitions are valid:

HMAT(C,i) — The optimal harvest (extraction) strategy matrix as a function of the entering forest resource state C and the price state i.

WMAT(C,i) — The optimal present value matrix ($= W^*(.)$) as a function of the entering forest resource state C and the price state i.

Figure 10.

The optimal harvesting strategy map in the integrated forest – industry enterprise (with one forest and two factories) as a function of the product price state (the net price level, p_1, of one of the products, which is represented by a stochastic process of the form shown in figure 2) and the entering stock state (the number of not yet harvested forest area units). The price of the other product, p_2, is assumed constant. The illustrated strategy map is relevant and optimal only in period 1 in the 5 period example which is defined in more details in the numerical appendix. Note in particular that the optimal harvest level is a decreasing function of price in one price interval and an increasing function of price in an other interval ! Explanation: If $p_1 << p_2$ (the entering price state is 1 or 2 and the entering stock state is 1), then the probability is very low that $p_1 > p_2$ in the next period (since the autocorrelation of the stationary p_1 process is rather high). Hence, we expect it to be better to harvest now than in the next period. If $p_1 \leq p_2$ and the price difference is small (the entering price state is 3, 4 or 5), then the probability is rather high that $p_1 > p_2$ in the next period. We select to speculate and save the forest at least one more period.

Assumptions; $r = 5\%$, $\sigma = 40$, $t = 1$.

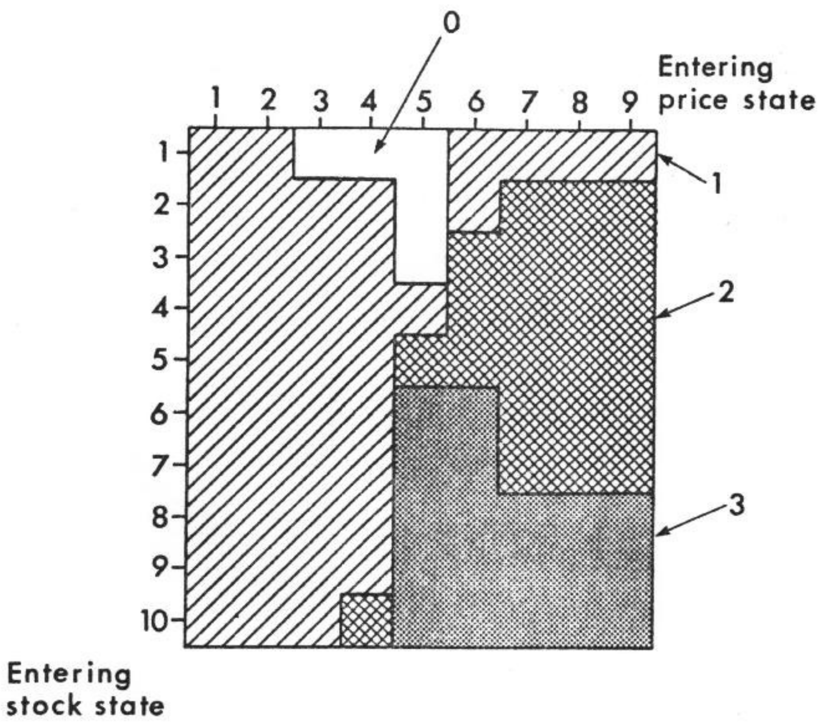


Figure 11.

The optimal harvesting strategy map. Compare figure 10.
Because of lower price variability, less i gained by "speculation" than in the case shown in figure 10. In this case, the optimal harvest level in period 1 is higher than or equal to the optimal level in figure 10. Assumptions; r = 5%, $\sigma = 10$, t = 1.

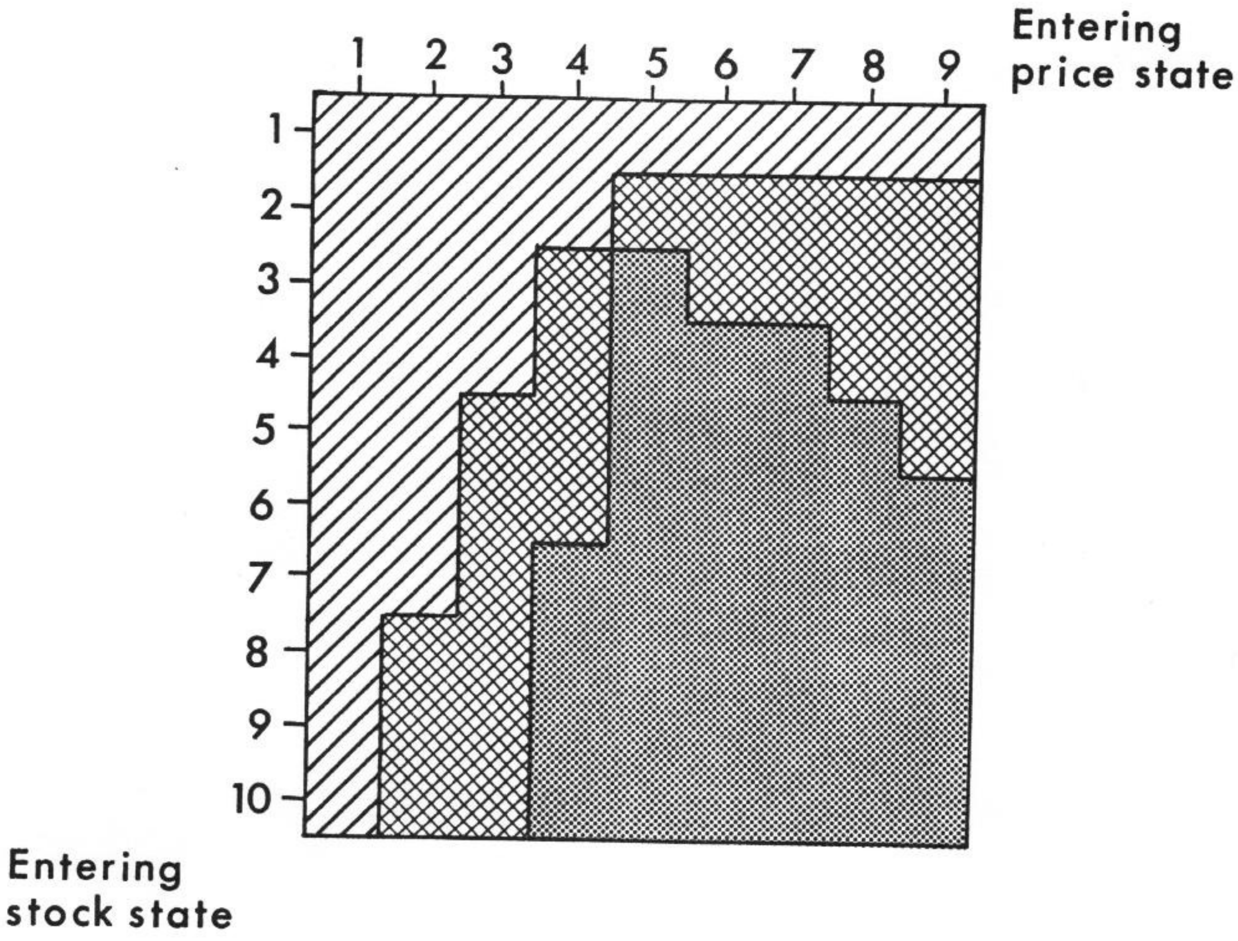


Figure 12.

The optimal harvesting strategy map. Compare figures 10 and 11.
Because of a higher rate of interest than in the case shown in figure 10, the present value of instant harvesting is much higher than the expected present value of later harvesting (ceteres paribus). Hence, the optimal harvest level is higher than or equal to the optimal level in figure 10. Assumptions; $r = 20\%$, $\sigma = 40$, $t = 1$.

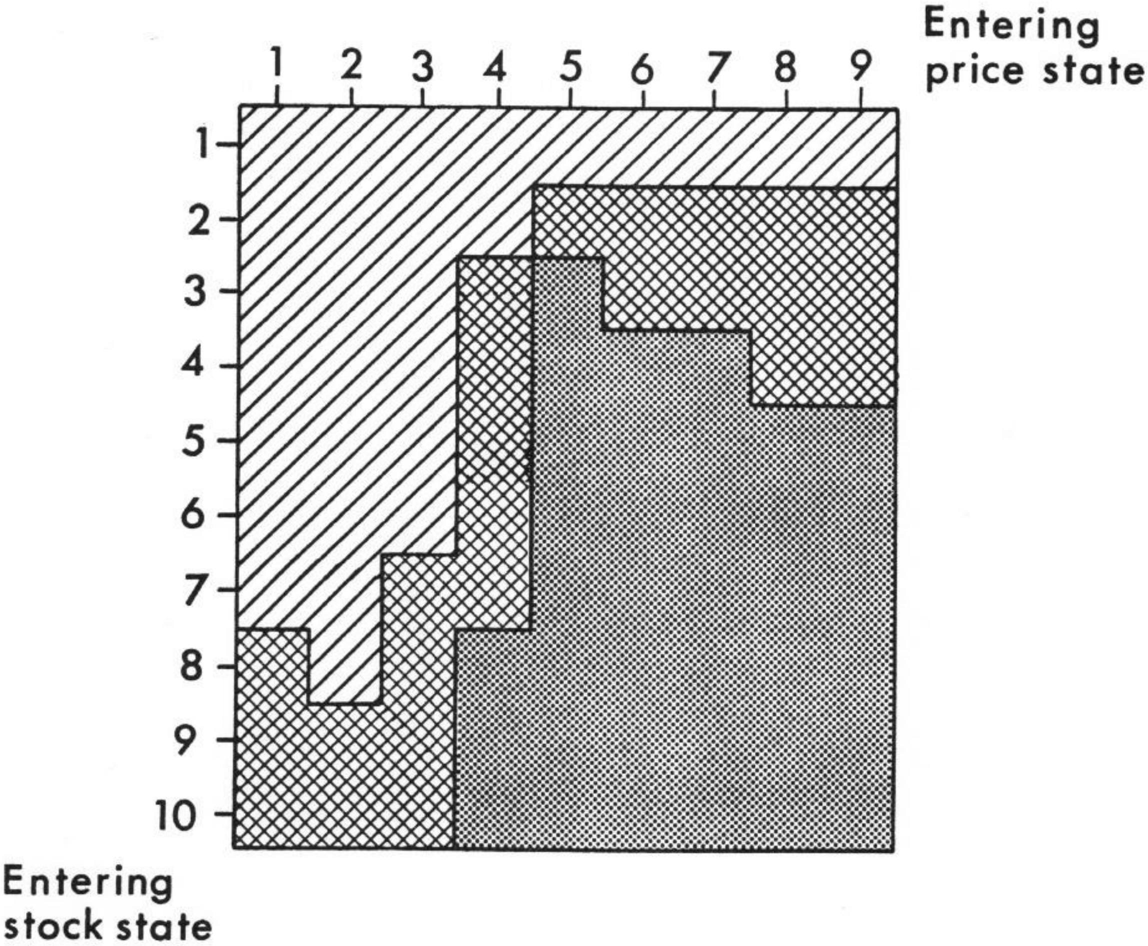


Figure 13.

The expected present value of the firm, W^*, in period 1 (in a five period analysis) as a function of the price of product 1 (p_1) in period 1 and the standard deviation of the stochastic component of the process p_1, σ. If p_1 is held constant, W^* is an increasing function of σ. The reason is that adaptive optimization is used: Production increases during good years and decreases during worse years. Compare the argument based on the Jensen inequality shown in figure 5.

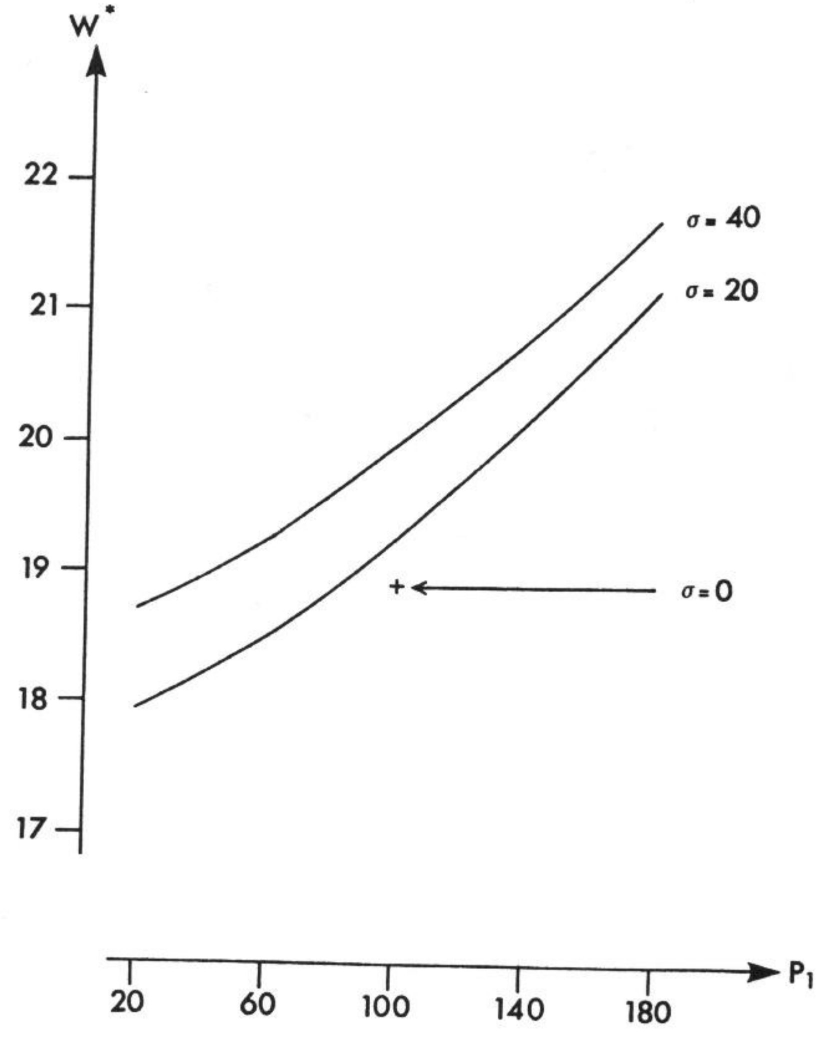


Figure 14.

The expected present value of the firm, W^*, as a function of time, t, the standard deviation of the stochastic component in the price process p_1, σ, and the size of the remaining forest resource, C. W^* is an increasing function of C and of σ. As t increases, the number of good price options decreases. Furthermore, as t increases, the number of years between which the forest resource may be distributed decreases. (The profit function is concave in the forest input each year (for positive input volumes) and distribution over many years is generally optimal.) Hence, W^* is a decreasing function of time.

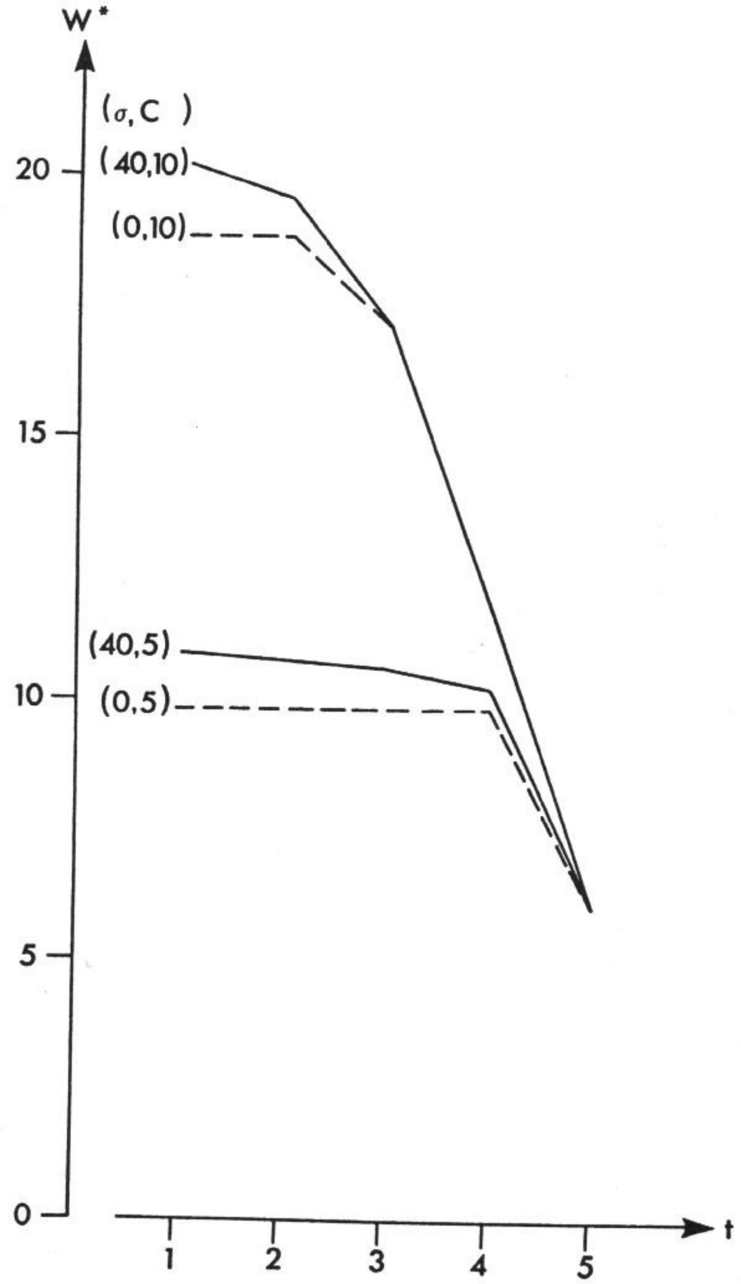


Figure 15.

The expected present value W^* is an increasing function of the size of the remaining forest resource (entering stock state) and the standard deviation of the stochastic component in the price process p_1, σ.

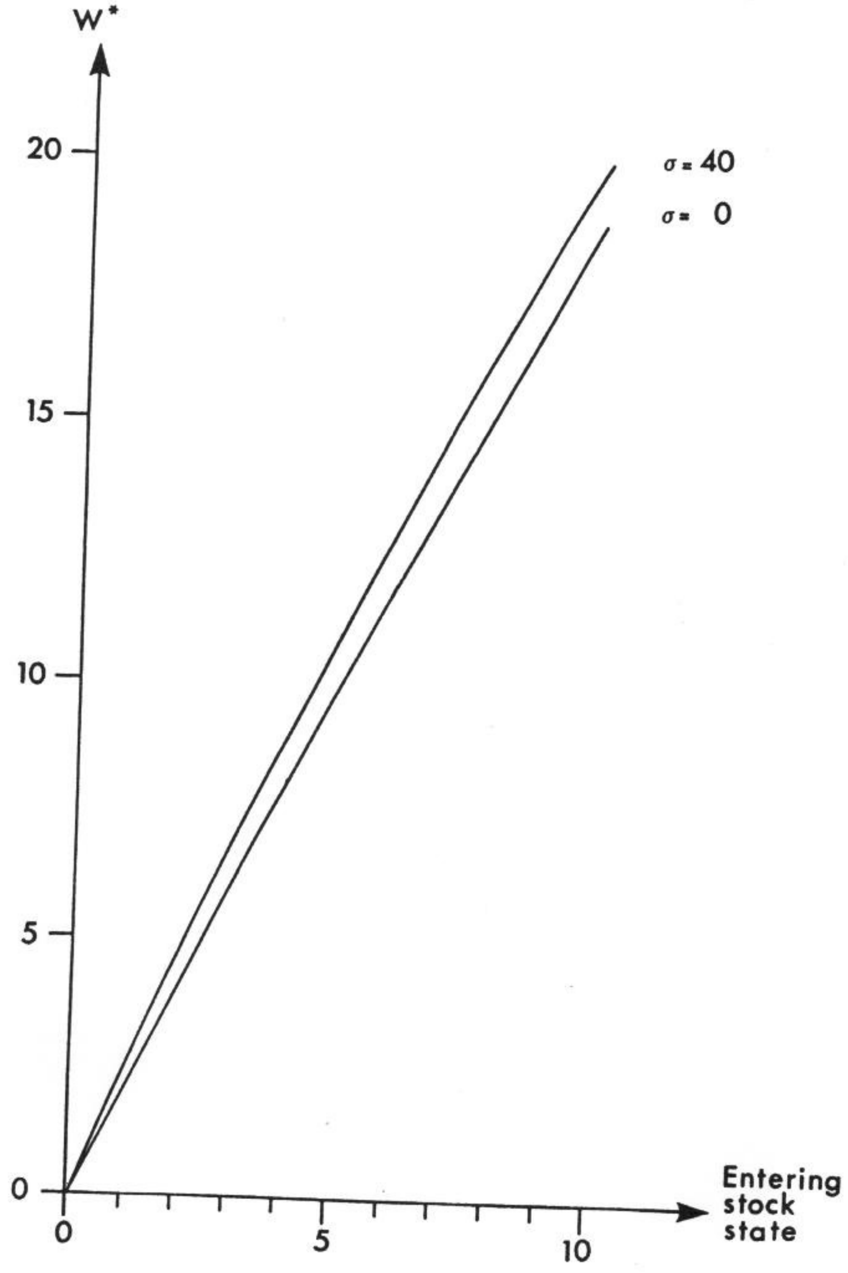


Figure 16.

The expected "marginal" value (calculated via solutions based on discrete capacity alternatives) of 10 capacity units in factory 1 (where product 1 is produced), "$E(\lambda_1)$", as a function of the entering stock state, σ and p_1. Clearly, the expected marginal capacity value increases very much with the product price variability. This is consistent with the results presented in the mathematical appendix and in figure 9(b). Assumptions: r = 5%, t = 1.

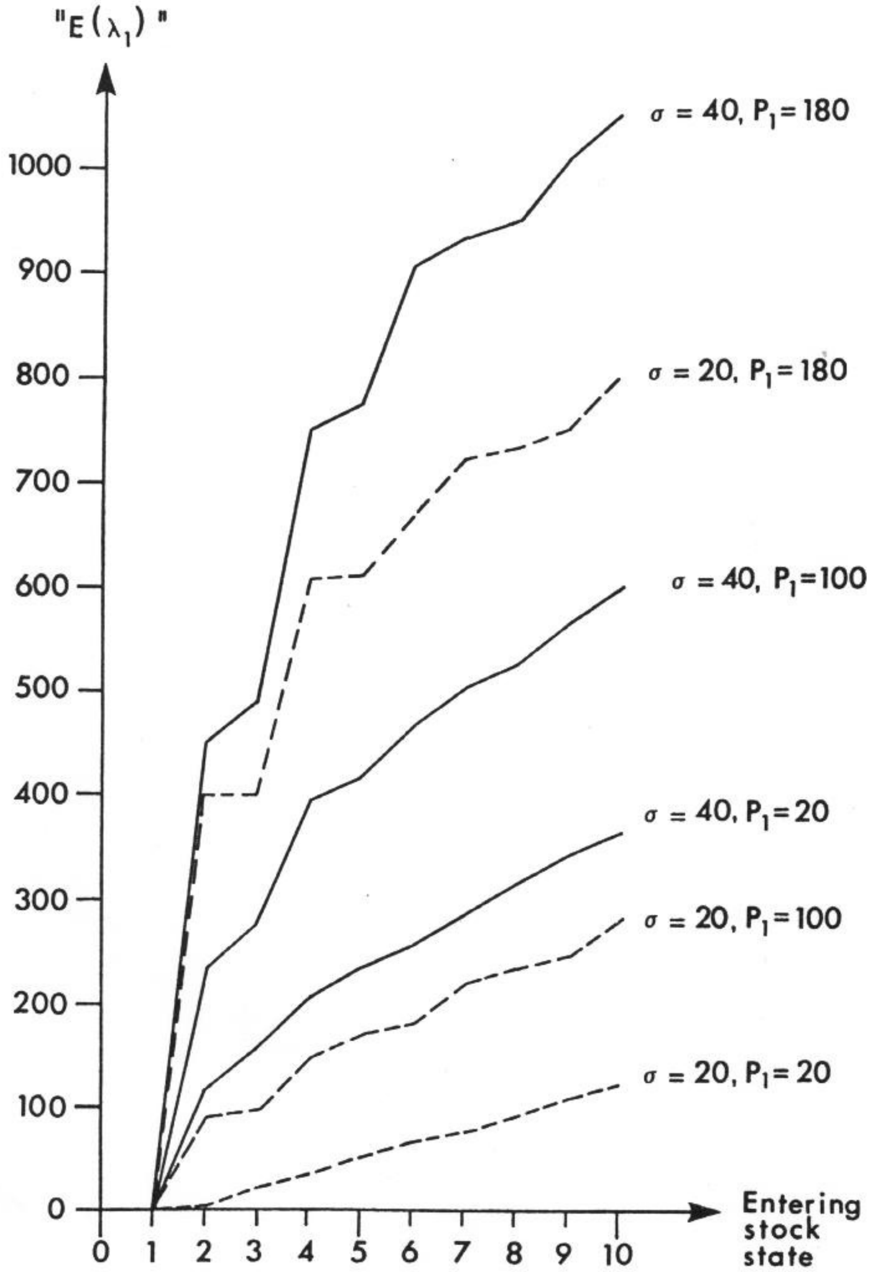
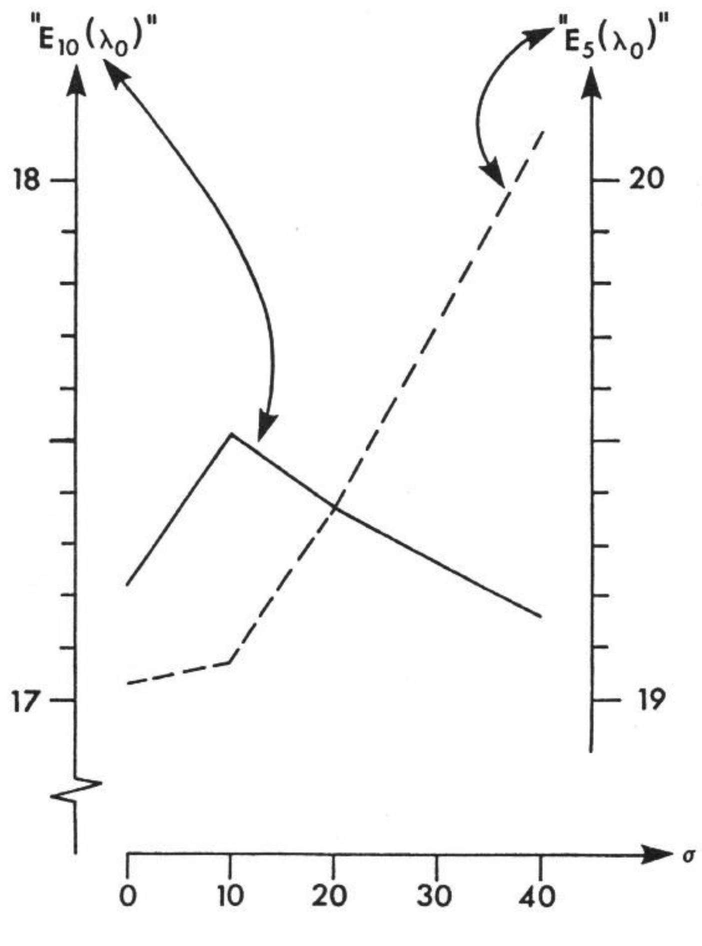


Figure 17.

The expected "marginal" value (calculated via solutions based on discrete resource alternatives) of the forest resource, "$E_{C_0}(\lambda_0)$", as a function of σ. C_0 is the initial state (size) of the forest resource. Assumptions: Compare figure 16.

For low values of C_0, the expected marginal forest value is positively affected by σ. The optimal harvesting stops completely during low price years. Hence, the marginal unit of the forest is used during a year with better product prices (and hence higher forest input shadow price) than during an average year. The capacity of the factories will never restrict the optimal solution. However, when C_0 is high, the effect of σ on the expected marginal forest resource value is ambiguous. The marginal forest unit will most likely be used during a year when the marginal profitability is zero. The probability is high that the factory capacities will restrict the optimum and make the value of the marginal forest resource equal to zero. These results can also be found via inspection of the analytical appendix and figure 9 a.



3. Obtained results and future options

The new approach has made it possible to model and understand the principles of optimal planning in the integrated natural resource enterprise in the presence of stochastic markets. Clearly, the suggested forest sector is just one possibility. The methodology may also be used in the oil and mineral sectors. The qualitative results discussed in connection to the graphs should hold also in such applications. It is obvious that forests, exactly as oil and mineral resources, are "flexible" resources that can be used in the production of many final products. The different factories or processing plants that typically belong to the enterprise are generally less flexible. They can be used in the production of a specific final product only. Hence, almost exactly the same model should be relevant in those related sectors of the economy.

However, the reader should be aware that modern technology makes it possible to construct more flexible production plants than in the past. Hence, the relevance of any specific model should always be properly investigated. We found in figure 5 that the stochastic market was preferable to a constant market. We could gain from variability. This property of the enterprise was due to the fact that the natural resource was flexible, could be used in the production of many final products. An other reason, which was not found in figure 5 but partly in figure 14, is that most natural resources are flexible over time. They may be used during the best (most profitable) years and production may go down during periods with worse market conditions. Of course, these production changes may have effects that are not desirable in other parts of the economy. Employment will change and social problems may occur. On the other hand, since a large fraction of the modern production plants in the process industry become more and more automatic and less labour intensive, the employment level and variability decrease . The author assumes that temporary production reductions in a particular plant may be profitable even if the employment is held constant. The proper distribution of the natural resource over time and final products should always be a high priority problem.

Finally, the author looks forward to future applications of the presented methods in the natural resource enterprises of reality. Some preliminary steps of action have already been taken in that direction.

References

Andersson, S., District management and work supervision in some forest management districts in central Sweden, Swedish University of Agricultural Sciences, Dept. of Operational Efficiency, No. 21, 1963

Baldwin, C.Y., Optimal sequential investment when capital is not readily reversible, Journal of Finance, Vol. 37, 763–782, June 1982

Balinski, M.L., Integer programming: Methods, uses, computation, Management Science, Vol. 12, No. 3, November, 1965

Bellman, R., An introduction to the theory of dynamic programming, The RAND Corporation, Santa Monica, 1953

Bellman, R.E., Zadeh, L.A., Decision–making in a fuzzy environment, Management Science, Vol. 17, No. 4, December 1970

Breiman, L., Stopping–rule problems, in Applied combinatorial mathematics, Beckenbach, E.F. (editor), New York Wiley cop., Engineering and physical sciences extension series, 99–0432162–0, 1964

Brennan, M.J., Schwartz, E.S., Evaluating natural resource investments, Journal of Business, Vol. 58, No. 2, 1985

Byrnes, C., Kurzhanski, A. (editors), Modelling and adaptive control, Lecture notes on control and information sciences, v. 105, Springer Verlag, 1988

Cocks, K.D., Discrete stochastic programming, Management Science, Vol. 15, No. 1, September 1968

Denardo, E.V., On linear programming in a Markov decision problem, Management Science, Vol. 16, No. 5, January 1970

Derman, C., Veinott, A.F.Jr., Constrained Markov decision chains, Management Science, Vol. 19, No. 4, Part I, December 1972

Florian, M., Robillard, P., An implicit enumeration algorithm for the concave cost network flow problem, Management Science, Vol. 18, No. 3, November 1971

Garstka, S.J., Rutenberg, D.P., Computation in discrete stochastic programs with recourse, Operations Research, Vol. 21, 1973

Geoffrion, A.M., Marsten, R.E., Integer programming algorithms: A framework and state-of-the-art survey, Management Science, Vol. 18, No. 9, May 1972

Hakansson, N.H., Optimal enterpreneurial decisions in a completely stochastic environment, Management Science, Vol. 17, No. 7, March 1971

Heaps, T., The forestry maximum principle, Journal of Economic Dynamics and Control, Vol. 7, 1984

Heaps, T., Cost benefit analysis of growing Douglas fir in British Columbia, Lohmander, P. (editor), Research on economic planning in natural resource sectors, Swedish University of Agricultural Sciences, Dept. of Forest Economics, No. 68, 1986

Hof, J.G., Robinson, K.S., Betters, D.R., Optimization with expected values of random yield coefficients in renawable resource linear programs, Forest Science, Vol. 34, No. 3, 1988

Holvid, A., En modell för integrerad virkestransportplanering, The Royal College of Technology, Dept. of Mathematics, Sweden, (mimeo) Mars 1970

Hållsten, B., Dynamisk programmering och företagsekonomiska problem, Erhvervsøkonomisk Tidsskrift, No. 4, 1958

Johansson, P.O., Löfgren, K.G., The economics of forestry and natural resources, Basil Blackwell Ltd, 1985,

Koopmans, T.C., Beckmann, M., Assignment problems and the location of economic activities, Econometrica, Vol. 25, 1957

Kumar, P.R., Varaiya, P., Stochastic systems. Estimation, identification and adaptive control, Prentice–Hall, Englewood Cliffs, 1986

Kurzhanski, A.B., Identification – A theory of guaranteed estimates, Lohmander, P. (editor), Economic planning of dynamic resource harvesting, Proceedings from the Economic Planning Group conference, Scandinavian Society of Forest Economics, Oslo, May, 1988, Swedish University of Agricultural Sciences, Dept. of Forest Economics, WP–84, 1988

Lemke, C.E., Salkin, H.M., Spielberg, K., Set covering by single–branch enumeration with linear–programming subproblems, Operations research, Vol. 19, 1971

Lindgren, J–E., Näslund, B., Planering inom skogsbruket med användande av matematisk programmering, Norstedts, Stockholm, 1968

Ljungman, L., A model and algorithms for scheduling of logging areas, Swedish University of Agricultural Sciences, Dept. of Operational Efficiency, No. 45, 1971

Lohmander, P., Principles of budgeting within the forest industry enterprise via linear programming (in Swedish), Swedish University of Agricultural Sciences, Dept. of Forest Economics, 2nd edition, 1985

Lohmander, P., The economics of forest management under risk, Swedish University of Agricultural Sciences, Dept. of Forest Economics, No. 79, 1987 (1987a)

Lohmander, P., Helles, F., Windthrow probability as a function of stand characteristics and shelter, Scandinavian Journal of Forest Research, Vol. 2, No, 2, 1987 (1987b)

Lohmander, P., Helles, F., Skovstruktur og Stormfald, en analyse af novemberstormen 1981, Dansk Skovforenings Tidsskrift, No. 3, 1987 (1987c)

Lohmander, P., Continuous extraction under risk, International Institute for Applied Systems Analysis, System and Decision Sciences, WP-86-16, Mars 1986, and Systems Analysis - Modelling - Simulation, Vol. 5, issue 2, 1988 (1988b)

Lohmander, P., Pulse extraction under risk and a numerical forestry application, International Institute for Applied Systems Analysis, System and Decision Sciences, WP-87-49, June 1987, and Systems Analysis - Modelling - Simulation, Vol. 5, issue 3, 1988 (1988c)

Lohmander, P., Optimal resource control in continuous time without Hamiltonian functions, Swedish University of Agricultural Sciences, Dept. of Forest Economics, WP-73, 1988, forthcoming in Systems Analysis - Modelling - Simulation june, 1989 (1988d)

Lohmander, P., A quantitative adaptive optimization model for resource harvesting in a stochastic environment, Swedish University of Agricultural Sciences, Dept. of Forest Economics, WP-74, 1988, forthcoming in Systems Analysis - Modelling - Simulation (1988e)

Lohmander, P., The rotation age, the constrained Faustmann problem and the initial conditions, forthcoming in Systems Analysis - Modelling - Simulation (1988f)

Lohmander, P., Optimal forest harvesting over time in the presence of air pollution and growth reduction, Swedish University of Agricultural Sciences, Dept. of Forest Economics, WP-85, 1989

Löfgren, K.G., Skogshushållning och nationalekonomer – en rättvis betraktelse, Sveriges Skogsvårdsförbunds Tidskrift, No. 1, 1989

Lönner, G., Ett system för kortsiktig planläggning av drivning, lagring och vidaretransport av rundvirke, Forskningsstiftelsen Skogsarbeten, Stockholm, 1968

Magnusson, P-H., Det maximala nuvärdet för tall och gran i mellersta Sverige, en numerisk analys, Swedish University of Agricultural Sciences, Dept. of Forest Economics, WP-76, 1988

von Malmborg, G., Ekonomisk planering av lantbruksföretaget, Jordbrukets Utredningsinstitut, Stockholm, 1967

Manne, A.S., Linear programming and sequential decisions, Management Science, Vol. 6, 1960

McCormick, G.P., The variable reduction method for nonlinear programming, Management Science, Vol. 17, No. 3, November 1970

McDonald, R., Siegel, D.R., Investment and the valuation of firms when there is an option to shut down, International Economic Review, Vol 26, 331–349, June 1985

McDonald, R., Siegel, D.R., The value of waiting to invest, Quarterly Journal of Economics, Vol 101, 707–728, November 1986

Nilsson, S., The planning process at simulated windthrows, Swedish University of Agricultural Sciences, Dept. of Operational Efficiency, No. 71, 1974

Nilsson, S., The economic effects of variations in the flow of wood and in stock levels – A case study, Swedish University of Agricultural Sciences, Dept. of Operational Efficiency, No. 130, 1979

Norstrom, C.J., A stochastic model for the growth period decision in forestry, Swedish Journal of Economics, 1975

Näslund, B., Företagsekonomiska forskningsmetoder med synpunkter på deras tillämpbarhet inom skogsbruket, Svenska Skogsvårdsföreningens Tidskrift, 1965

Näslund, B., Optimal rotation and Thinning, Forest Science, Vol. 15, 1969

van de Panne, C., Whinston, A., Simplicial methods for quadratic programming, Naval Research Logistics Quarterly, Vol. 11, 1964

van de Panne, C., Whinston, A., A comparision of two methods of quadratic programming, Operations Research, Vol. 14, 1966

Pindyck, R.S., Irreversible investment, capacity choice, and the value of the firm, The American Economic Review, Vol. 78, No. 5, December 1988

Puu, T., Complex dynamics in continuous models of the business cycle, Lecture Notes in Economics and Mathematical Systems, 293:227, Springer–Verlag, Berlin, 1987

Puu, T., Essays on nonlinear economic dynamics, Lecture Notes in Economics and Mathematical Systems, Springer–Verlag, Berlin, 1989

Risvand, J., A stochastic model for the cutting policy decision in forestry, Agricultural University of Norway, Dept. of Mathematics and Statistics, Vol. 55, 1976

Rockafellar, R.T., Wets, R., The optimal recourse problem in discrete time: L^1– multipliers for inequality constraints, SIAM Journal on control and optimization Vol. 16, No. 1, January 1978

Rockafellar, R.T., Wets, R., A Lagrangian finite generation technique for solving linear–quadratic problems in stochastic programming, Mathematical Programming Studies, North–Holland, No. 28, 1986

Rockafellar, R.T., Generalized linear–quadratic programming and optimal control, SIAM Journal on Control and Optimization, Vol. 25, No. 3, May 1987

Rockafellar, R.T., Wets, R., Generalized linear–quadratic problems of deterministic and stochastic optimal control in discrete time, International Institute for Applied Systems Analysis, System and Decision Sciences, WP–87–52, June 1987

Satia, J.K., Lave, R.E., Markovian decision processes with probabilistic observation of states, Management Science, Vol. 20, No. 1, September 1973

Simon, H.A., Dynamic programming under uncertainty with a quadratic criterion function, Econometrica, Vol. 24, 1956

Smallwood, R.D., Sondik, E.J., The optimal control of partially observable Markov processes over a finite horizon, Operations Research, Vol. 21, 1973

Valsta, L.T., A comparision of numerical methods to optimize stand treatment, Lohmander (editor), Economic planning of dynamic resource harvesting, Proceedings from the Economic Planning Group conference, Oslo, May, 1988, Scandinavian Society of Forest Economics, Swedish University of Agricultural Sciences, Dept. of Forest Economics, WP–84, 1988

Varaiya, P., Wets, R.J–B., Stochastic dynamic optimization approaches and computation, International Institute for Applied Systems Analysis, System and Decision Sciences, WP–88–87, September 1988,

Wagner, H.M., Principles of operations research with applications to managerial decisions, Prentice Hall, 2nd edition, 1975

Williams, B.K., Markov: A methodology for the solution of infinite time horizon Markov decision processes, Applied Stochastic Models and Data Analysis, Vol. 4, 253–271, 1988

Wolfe, P., The simplex method for quadratic programming, Econometrica, Vol. 27, No. 3, 1959

Wolfe, P., Dantzig, G.B., Linear programming in a Markov chain, Operations Research, Vol. 10, 1962

M. Mathematical appendix

M.1. The stochastic LP forest enterprise problem in one stage

Consider the following problem:

$$\max \Pi = \sum_{j=1}^{n} \bar{p}_j x_j$$

$$\text{s.t.} \quad \sum_{j=1}^{n} \alpha_j x_j \leq R$$

$$x_j \leq \bar{I}_j \text{ for } j \in (1,\dots,n)$$

The variables and parameters are defined in the main text.
Clearly, we may reformulate the problem in order to obtain a more convenient analysis: We can get rid of the j index placed on α_j if, for each j, proper values are chosen for p_j and I_j and these parameters replace $\bar{p}_j$ and $\bar{I}_j$. (Compare the main text and figure 3. for more details.) The more convenient problem, which now will be analysed is :

$$\max \Pi = \sum_{j=1}^{n} p_j x_j$$

$$\text{s.t.} \quad \sum_{j=1}^{n} \alpha x_j \leq R$$

$$x_j \leq I_j \text{ for } j \in (1,\dots,n)$$

M.2. The Kuhn–Tucker conditions

The Lagrange function is :

$$L = \sum_j p_j x_j + \lambda_0 (R - \alpha \sum_j x_j) + \sum_j \lambda_j (I_j - x_j)$$

The Kuhn–Tucker conditions are :

$$\frac{\delta L}{\delta \lambda_0} = R - \alpha \Sigma x_j \qquad \geq 0$$

$$\frac{\delta L}{\delta \lambda_i} = I_i - x_i \qquad \geq 0 \text{ for } i \in (1,\ldots,n)$$

$$\frac{\delta L}{\delta x_j} = p_j - \alpha\lambda_0 - \lambda_j \qquad \leq 0 \text{ for } j \in (1,\ldots,n)$$

$$\lambda_i \geq 0 \text{ for } i \in (0,\ldots,n)$$

$$x_j \geq 0 \text{ for } j \in (1,\ldots,n)$$

$$\lambda_i \frac{\delta L}{\delta \lambda_i} = 0 \text{ for } i \in (0,\ldots,n)$$

$$x_j \frac{\delta L}{\delta x_j} = 0 \text{ for } j \in (1,\ldots,n)$$

M.3. One important dual equation system

From M.2., $\frac{\delta L}{\delta x_j} \leq 0$, for $j \in (1,\ldots,n)$, one dual equation system will be defined.
Clearly, we have n weak inequalities (potential linear equations) and n+1 variables $(\lambda_0 - \lambda_n)$ to be determined.

Definition

The forest resource and the industrial capacities $I_2,...,I_n$ restrict the optimal solution. I_1 does not restrict the optimal solution (if we introduce a slack variable, this is strictly positive in that restriction).

Observation

$\lambda_1 = 0$. λ_0 and $\lambda_2,...,\lambda_n$ should be determined from the following equation system:

$$\begin{bmatrix} \alpha & 0 & 0 & . & 0 \\ \alpha & 1 & 0 & . & 0 \\ \alpha & 0 & 1 & . & 0 \\ . & . & . & . & . \\ \alpha & 0 & 0 & . & 1 \end{bmatrix} \begin{bmatrix} \lambda_0 \\ \lambda_2 \\ \lambda_3 \\ . \\ \lambda_n \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ . \\ p_n \end{bmatrix}$$

Simplify notation and let us write the system as :

$$\begin{bmatrix} D \end{bmatrix} \begin{bmatrix} \lambda \end{bmatrix} = \begin{bmatrix} p \end{bmatrix}$$

Application of Cramers rule gives λ_0, the shadow price (marginal value) of the forest resource :

$$\lambda_0 = \frac{\begin{vmatrix} p_1 & 0 & 0 & . & 0 \\ p_2 & 1 & 0 & . & 0 \\ p_3 & 0 & 1 & . & 0 \\ . & . & . & . & . \\ p_n & 0 & 0 & . & 1 \end{vmatrix}}{\begin{vmatrix} D \end{vmatrix}}$$

Laplace expansion of the involved determinants gives us the explicit result :

$\lambda_0 = \frac{p_1}{\alpha}$. Hence, the shadow price of the forest resource is equal to the net price divided by the input requirement in "marginal" production (the product x_1, the production of which is not restricted by the capacity limit I_1 in the optimal solution). Note particularly that λ_0 is not a function of any other price than p_1.

Clearly, we can derive the result:

$$\frac{\delta\lambda_0}{\delta p_1} = \frac{1}{\alpha}$$

The marginal values, the shadow prices, of the limiting production capacities, can also be determined via Cramers rule. We consider only λ_2 explicitly. However, since all λ_i, $i \in (2,...,n)$ enter the problem in the same manner, the result similar to the result derived for λ_2 can easily be shown to hold for λ_i, $i \in (2,...,n)$.

$$\lambda_2 = \frac{\begin{vmatrix} \alpha & p_1 & 0 & . & 0 \\ \alpha & p_2 & 0 & . & 0 \\ \alpha & p_3 & 1 & . & 0 \\ . & . & . & . & . \\ \alpha & p_n & . & . & 1 \end{vmatrix}}{|\,D\,|}$$

Laplace expansion gives the result :

$$\lambda_2 = p_2 - p_1$$

and more generally, since all λ_i, $i \in (2,...,n)$ enter the problem in a symmetric way, we have :

$$\lambda_i = p_i - p_1 \qquad \text{for } i \in (2,...,n)$$

Clearly, we may extract the following derivatives :

$$\frac{\delta\lambda_i}{\delta p_1} = -1 \qquad \text{for } i \in (2,...,n)$$

$$\frac{\delta\lambda_i}{\delta p_i} = +1 \qquad \text{for } i \in (2,...,n)$$

We should note that λ_i, $i \in (2,...,n)$, the shadow price (the marginal value) of the limiting production capacity I_i, is an increasing function of the price of product i and a decreasing function of the price of the "marginal product" (the product x_1, which is not restricted by I_1 in the optimal solution). λ_i, $i \in (2,...,n)$ is not a function of the other prices p_j, $j \neq i$.

To make the exposition complete, we note that by definition :

$$\lambda_1 = 0$$

$$\frac{\delta\lambda_1}{\delta p_i} = 0 \qquad \text{for } i \in (1,...,n)$$

N. Numerical Appendix

N.1. Illustrative example

A typical solution to the stochastic dynamic optimization problem with a linear programming subroutine is shown below.

```
TTOT, R, CO, L =                5                  .05              10
 5000

PRPAR1, PRPAR2, PRCOEF, PRTREND, PRSTDEV
 2.5   .5   40   0   1

TRANSITION PROBABILITY MATRIX OF THE STOCHASTIC STATE INDEX
ROW = STATE(t+1), COLUMN = STATE(t)
  0.054  0.007  0.004  0.000  0.000  0.000  0.000  0.000  0.000
  0.243  0.090  0.054  0.007  0.004  0.000  0.000  0.000  0.000
  0.401  0.403  0.242  0.090  0.054  0.007  0.004  0.000  0.000
  0.243  0.403  0.399  0.403  0.242  0.090  0.054  0.007  0.004
  0.054  0.090  0.242  0.403  0.399  0.403  0.242  0.090  0.054
  0.004  0.007  0.054  0.090  0.242  0.403  0.399  0.403  0.243
  0.000  0.000  0.004  0.007  0.054  0.090  0.242  0.403  0.401
  0.000  0.000  0.000  0.000  0.004  0.007  0.054  0.090  0.243
  0.000  0.000  0.000  0.000  0.000  0.000  0.004  0.007  0.054

THE PERIOD IS =  5


THE OPTIMAL EXTRACTION STRATEGY MATRIX HMAT(.) IN PERIOD  5
row = entering stock, column = stochastic state index (1 - 9)

  1  1  1  1  1  1  1  1  1
  2  2  2  2  2  2  2  2  2
  2  2  3  3  3  3  3  3  3
  2  2  3  3  3  3  3  3  3
  2  2  3  3  3  3  3  3  3
  2  2  3  3  3  3  3  3  3
  2  2  3  3  3  3  3  3  3
  2  2  3  3  3  3  3  3  3
  2  2  3  3  3  3  3  3  3
  2  2  3  3  3  3  3  3  3

THE EXPECTED PRESENT VALUE MATRIX WMAT(.) IN PERIOD  5
row = entering stock, column = stochastic state index

  2004.  2006.  2051.  2084.  2291.  2484.  2850.  3206.  3593.
  3291.  3429.  3664.  3886.  4292.  4684.  5250.  5805.  6387.
  3860.  4268.  4841.  5405.  6003.  6600.  7199.  7799.  8383.
  3860.  4268.  4841.  5405.  6003.  6600.  7199.  7799.  8383.
  3860.  4268.  4841.  5405.  6003.  6600.  7199.  7799.  8383.
  3860.  4268.  4841.  5405.  6003.  6600.  7199.  7799.  8383.
  3860.  4268.  4841.  5405.  6003.  6600.  7199.  7799.  8383.
  3860.  4268.  4841.  5405.  6003.  6600.  7199.  7799.  8383.
  3860.  4268.  4841.  5405.  6003.  6600.  7199.  7799.  8383.
  3860.  4268.  4841.  5405.  6003.  6600.  7199.  7799.  8383.
```

THE PERIOD IS = 4

THE OPTIMAL EXTRACTION STRATEGY MATRIX HMAT(.) IN PERIOD 4
row = entering stock, column = stochastic state index (1 - 9)

1	1	1	1	0	1	1	1	1
1	1	1	1	1	2	2	2	2
1	1	1	1	2	2	2	2	2
2	2	2	2	3	2	2	2	2
2	2	2	2	3	3	3	3	3
2	2	3	3	3	3	3	3	3
2	2	3	3	3	3	3	3	3
2	2	3	3	3	3	3	3	3
2	2	3	3	3	3	3	3	3
2	2	3	3	3	3	3	3	3

THE EXPECTED PRESENT VALUE MATRIX WMAT(.) IN PERIOD 4
row = entering stock, column = stochastic state index

2014.	2022.	2094.	2156.	2363.	2556.	2894.	3222.	3602.
3964.	3990.	4095.	4186.	4501.	4797.	5317.	5824.	6399.
5510.	5626.	5853.	6062.	6535.	6985.	7680.	8359.	9103.
6796.	7046.	7448.	7832.	8430.	9010.	9788.	10555.	11387.
7931.	8341.	8918.	9482.	10204.	10914.	11731.	12542.	13379.
8480.	9145.	9995.	10834.	11715.	12595.	13478.	14362.	15223.
8480.	9145.	9995.	10834.	11715.	12595.	13478.	14362.	15223.
8480.	9145.	9995.	10834.	11715.	12595.	13478.	14362.	15223.
8480.	9145.	9995.	10834.	11715.	12595.	13478.	14362.	15223.
8480.	9145.	9995.	10834.	11715.	12595.	13478.	14362.	15223.

THE PERIOD IS = 3

THE OPTIMAL EXTRACTION STRATEGY MATRIX HMAT(.) IN PERIOD 3
row = entering stock, column = stochastic state index (1 - 9)

1	1	1	0	0	1	1	1	1
1	1	1	1	0	1	2	2	2
1	1	1	1	1	2	2	2	2
1	1	1	1	2	2	2	2	2
1	1	1	1	3	2	2	2	2
1	1	1	2	3	3	2	2	2
2	2	2	2	3	3	3	3	3
2	2	2	2	3	3	3	3	3
2	2	3	3	3	3	3	3	3
2	2	3	3	3	3	3	3	3

THE EXPECTED PRESENT VALUE MATRIX WMAT(.) IN PERIOD 3
row = entering stock, column = stochastic state index

2030.	2049.	2131.	2204.	2402.	2588.	2913.	3228.	3606.
4007.	4045.	4160.	4262.	4568.	4857.	5359.	5847.	6412.
5914.	5969.	6126.	6266.	6688.	7087.	7760.	8417.	9147.
7595.	7707.	7943.	8161.	8666.	9149.	9902.	10638.	11452.
9125.	9322.	9667.	9994.	10596.	11175.	12000.	12810.	13700.
10546.	10837.	11301.	11747.	12438.	13109.	13966.	14812.	15728.
11813.	12226.	12825.	13408.	14189.	14955.	15862.	16761.	17697.
12850.	13394.	14144.	14880.	15779.	16667.	17625.	18580.	19538.
13387.	14180.	15162.	16133.	17149.	18163.	19181.	20199.	21192.
13387.	14180.	15162.	16133.	17149.	18163.	19181.	20199.	21192.

THE PERIOD IS = 2

THE OPTIMAL EXTRACTION STRATEGY MATRIX HMAT(.) IN PERIOD 2
row = entering stock, column = stochastic state index (1 - 9)

1	1	0	0	0	1	1	1	1
1	1	1	1	0	1	2	2	2
1	1	1	1	0	2	2	2	2
1	1	1	1	1	2	2	2	2
1	1	1	1	2	2	2	2	2
1	1	1	1	3	3	2	2	2
1	1	1	1	3	3	3	2	2
1	1	1	2	3	3	3	3	3
2	1	1	2	3	3	3	3	3
2	2	2	2	3	3	3	3	3

THE EXPECTED PRESENT VALUE MATRIX WMAT(.) IN PERIOD 2
row = entering stock, column = stochastic state index

2054.	2082.	2165.	2240.	2430.	2608.	2924.	3232.	3609.
4043.	4086.	4205.	4312.	4614.	4899.	5389.	5865.	6423.
5976.	6035.	6194.	6336.	6749.	7140.	7801.	8445.	9168.
7851.	7930.	8125.	8302.	8777.	9227.	9966.	10688.	11492.
9588.	9707.	9968.	10208.	10759.	11285.	12098.	12894.	13775.
11240.	11414.	11752.	12070.	12679.	13263.	14099.	14919.	15827.
12802.	13034.	13447.	13841.	14515.	15168.	16039.	16897.	17833.
14267.	14569.	15071.	15552.	16302.	17032.	17945.	18848.	19803.
15582.	15990.	16601.	17194.	18011.	18812.	19758.	20697.	21670.
16798.	17297.	18014.	18715.	19615.	20500.	21488.	22471.	23469.

THE PERIOD IS = 1

THE OPTIMAL EXTRACTION STRATEGY MATRIX HMAT(.) IN PERIOD 1
row = entering stock, column = stochastic state index (1 - 9)

1	1	0	0	0	1	1	1	1
1	1	1	1	0	1	2	2	2
1	1	1	1	0	2	2	2	2
1	1	1	1	1	2	2	2	2
1	1	1	1	2	2	2	2	2
1	1	1	1	3	3	2	2	2
1	1	1	1	3	3	2	2	2
1	1	1	1	3	3	3	3	3
1	1	1	1	3	3	3	3	3
1	1	1	2	3	3	3	3	3

THE EXPECTED PRESENT VALUE MATRIX WMAT(.) IN PERIOD 1
row = entering stock, column = stochastic state index

2076.	2111.	2193.	2268.	2450.	2622.	2933.	3235.	3610.
4075.	4119.	4240.	4348.	4646.	4928.	5409.	5877.	6430.
6019.	6080.	6241.	6385.	6792.	7176.	7828.	8462.	9181.
7915.	7994.	8188.	8363.	8831.	9276.	10008.	10723.	11522.
9759.	9858.	10093.	10308.	10840.	11346.	12151.	12939.	13815.
11520.	11653.	11941.	12209.	12784.	13334.	14163.	14975.	15882.
13224.	13395.	13736.	14057.	14681.	15280.	16139.	16982.	17918.
14847.	15056.	15459.	15842.	16525.	17185.	18076.	18953.	19900.
16399.	16653.	17120.	17565.	18304.	19021.	19936.	20840.	21800.
17871.	18185.	18718.	19231.	20020.	20789.	21732.	22665.	23643.

```
10 DEFDBL A-H
20 REM program STDYNLP
30 REM STOCHASTIC DYNAMIC PROGRAMMING WITH A LINEAR PROGRAMMING SUBROUTINE
40 REM BASED ON THE SIMPLEX ALGORITHM WITH AND THE BIG-M METHOD
50 REM LOHMANDER PETER 89-03-08
60 LPRINT CHR$(27);"E"
70 LPRINT CHR$(27);"G"
80 REM *********************************************************************
90 DIM SKU(50),A(50,50),NXIEX(50),VX(50)
100 DIM WMAT(9,100),TRMAT(9,9),FIMAT(9,100),HMAT(9,100)
110 DIM FDEV(11),WMAT2(9,100),X(20)
120 REM *********************************************************************
130 REM GENERAL PARAMETERS (INPUT IN A LATER STAGE)
140 REM *********************************************************************
150 OUTO= 0
160 OUT2 = 0
170 OUT3 = 1
180 TTOT = 5
190 R = .05
200 CO = 10
210 L = 5000
220 LPRINT"TTOT, R, CO, L = ",TTOT,R,CO,L
230 REM *********************************************************************
240 REM PARAMETERS OF THE STOCHASTIC STATE PROCESS (INPUT IN A LATER STAGE)
250 REM *********************************************************************
260 PRPAR2 = .5
270 PRPAR1 = 5*(1-PRPAR2)
280 PRSTDEV=1
290 PRTREND = 0
300 PRCOEF = 40
310 LPRINT" "
320 LPRINT"PRPAR1, PRPAR2, PRCOEF, PRTREND, PRSTDEV"
330 LPRINT PRPAR1; PRPAR2; PRCOEF; PRTREND; PRSTDEV
340 LPRINT" "
350 REM *********************************************************************
360 REM CALCULATION OF THE STOCHASTIC STATE INDEX TRANSITION PROBABILITY MATRIX
370 REM *********************************************************************
380 FOR IDI = 1 TO 11
390 DEV = IDI - 1
400 FDEV(IDI) = 1/(2*3.141593*PRSTDEV^2)^.5*EXP(-DEV^2/2/PRSTDEV^2)
410 NEXT IDI
420 FOR PO = 1 TO 9
430 EP = PRPAR1 + PRPAR2*PO
440 FOR P1 = 1 TO 9
450 DEV = ABS(P1-EP)
460 IDI = DEV + 1
470 TRMAT(P1,PO) = FDEV(IDI)
480 NEXT P1
490 REM *********************************************************************
500 REM CORRECTION FOR TRUNCATION, REFLECTING BARRIERS AND DISCRETE SPACE
510 REM *********************************************************************
520 PROBTOT = 0
530 FOR P1 = 1 TO 9
540 PROBTOT = PROBTOT + TRMAT(P1,PO)
550 NEXT P1
560 FOR P1 = 1 TO 9
570 TRMAT(P1,PO) = TRMAT(P1,PO)/PROBTOT
580 NEXT P1
590 NEXT PO
600 LPRINT"TRANSITION PROBABILITY MATRIX OF THE STOCHASTIC STATE INDEX"
610 LPRINT"ROW = STATE(t+1), COLUMN = STATE(t)"
620 FOR P1 = 1 TO 9
630 FOR PO = 1 TO 9
640 X(PO) = TRMAT(P1,PO)
650 NEXT PO
660 LPRINT USING"###.###";X(1);X(2);X(3);X(4);X(5);X(6);X(7);X(8);X(9)
670 NEXT P1
680 REM *********************************************************************
690 REM DEFINITION OF WMAT2(P,V) (= 0 FOR ALL P,V) IN THE FINAL PERIOD
700 REM *********************************************************************
710 FOR P = 1 TO 9
720 FOR V = 1 TO CO
730 WMAT2(P,V) = 0
740 NEXT V
750 NEXT P
```

```
760 REM ************************************************************************
770 REM STOCHASTIC DYNAMIC PROGRAMMING VIA THE BACKWARD ALGORITHM
780 REM ************************************************************************
790 FOR S = 1 TO TTOT
800 T = TTOT - S + 1
810 LPRINT" "
820 LPRINT"THE PERIOD IS = ";T
830 LPRINT" "
840 REM ************************************************************************
850 REM OPTIMIZATION OF THE DECISION VECTOR AT TIME t VIA LINEAR PROGRAMMING
860 REM FOR EACH POSSIBLE STOCHASTIC INDEX STATE AND FOR EACH POSSIBLE STATE
870 REM OF THE RESOURCE AVAILABILITY
880 REM ************************************************************************
890 REM
900 FOR PIND = 1 TO 9
910 FOR VOLIN = 1 TO CO
920 PRMAX = 0
930 HOPT = 0
940 FOR IH = 0 TO VOLIN
950 REM ************************************************************************
960 REM EQUATION OF MOTION
970 REM ************************************************************************
980 INGROWTH = 0
990 VOLOUT = VOLIN - IH + INGROWTH
1000 W = WMAT2(PIND,VOLOUT)
1010 REM ************************************************************************
1020 REM THE LINEAR PROGRAMMING SUBROUTINE IS CALLED AND THE OPTIMUM IS FOUND
1030 REM ************************************************************************
1040 GOSUB 1680
1050 PR = OBJLP + EXP(-R)*W
1060 IF PR > PRMAX THEN HOPT = IH
1070 IF PR > PRMAX THEN PRMAX = PR
1080 NEXT IH
1090 FIMAT(PIND,VOLIN) = PRMAX
1100 HMAT(PIND,VOLIN) = HOPT
1110 NEXT VOLIN
1120 NEXT PIND
1130 LPRINT" "
1140 LPRINT"THE OPTIMAL EXTRACTION STRATEGY MATRIX HMAT(.) IN PERIOD ";T
1150 LPRINT"row = entering stock, column = stochastic state index (1 - 9)"
1160 LPRINT" "
1170 FOR V = 1 TO CO
1180 FOR P = 1 TO 9
1190 X(P) = HMAT(P,V)
1200 NEXT P
1210 LPRINT USING"###";X(1);X(2);X(3);X(4);X(5);X(6);X(7);X(8);X(9)
1220 NEXT V
1230 REM ************************************************************************
1240 REM DETERMINATION OF THE EXPECTED VALUE MATRIX, NAMELY WMAT(P,V)
1250 REM ************************************************************************
1260 FOR PO = 1 TO 9
1270 FOR VOLIN = 1 TO CO
1280 WMAT(PO,VOLIN) = 0
1290 FOR P1 = 1 TO 9
1300 WMAT(PO,VOLIN) = WMAT(PO,VOLIN) + TRMAT(P1,PO)*FIMAT(P1,VOLIN)
1310 NEXT P1
1320 NEXT VOLIN
1330 NEXT PO
1340 REM ************************************************************************
1350 REM PRINTOUT OF THE EXPECTED PRESENT VALUE MATRIX, NAMELY WMAT(P,V)
1360 REM ************************************************************************
1370 LPRINT" "
1380 LPRINT"THE EXPECTED PRESENT VALUE MATRIX WMAT(.) IN PERIOD ";T
1390 LPRINT"row = entering stock, column = stochastic state index"
1400 LPRINT" "
1410 FOR V = 1 TO CO
1420 FOR P = 1 TO 9
1430 X(P) = WMAT(P,V)
1440 NEXT P
1450 LPRINT USING"######.";X(1);X(2);X(3);X(4);X(5);X(6);X(7);X(8);X(9)
1460 NEXT V
1470 REM ************************************************************************
1480 REM BEFORE A CHANGE IN PERIOD, WE LET WMAT(P,V) REPLACE WMAT2(P,V) !
1490 REM ************************************************************************
1500 REM INPUT"GO ON = 1 ?",G
1510 REM IF G<1 THEN STOP
1520 FOR P = 1 TO 9
1530 FOR V = 1 TO CO
1540 WMAT2(P,V) = WMAT(P,V)
1550 NEXT V
1560 NEXT P
1570 NEXT S
1580 STOP
1590 REM
1600 REM
1610 END
```

```
1680 REM ***********************************************************************
1690 REM SUBROUTINE FOR LINEAR PROGRAMMING VIA THE SIMPLEX METHOD WITH BIG-M.
1700 REM ***********************************************************************
1710 READ MO,NO
1720 FOR I = 1 TO MO
1730 FOR J = 1 TO NO
1740 A(I,J) = 0
1750 NEXT J
1760 NEXT I
1770 READ M,N,AVALUE
1780 IF M=0 THEN GOTO 1810
1790 A(M,N) = AVALUE
1800 GOTO 1770
1810 RESTORE
1820 REM ***********************************************************************
1830 REM NEW PARAMETERS ENTER FROM THE MAIN PROGRAM TO THE SIMPLEX TABLE
1840 REM ***********************************************************************
1850 A(1,1) = - 100 - (PIND-5)*PRCOEF + T*PRTREND
1860 A(2,6) =IH*20
1870 IF OUT3 = 1 THEN PRINT"A(1,1) = ";A(1,1);" A(2,6) = ";A(2,6)
1880 IF OUTO= 0 THEN GOTO 1930
1890 PRINT"THE INITIAL SIMPLEX TABLE IS:"
1900 FOR M = 1 TO MO
1910 PRINT USING"####.###";A(M,1);A(M,2);A(M,3);A(M,4);A(M,5);A(M,6)
1920 NEXT M
1930 FOR I = 1 TO 50
1940 NXIEX(I) = 0
1950 NEXT I
1960 DIFP=.000001
1970 DIFM=-.000001
1980 REM ***********************************************************************
1990 REM FORCE THE ARTIFICIAL VARIABLES INTO THE SYSTEM SOLUTION
2000 REM ***********************************************************************
2010 FOR NH = 1 TO NO
2020 IB=0
2030 AKOST=A(1,NH)
2040 IF(AKOST>999.9 AND AKOST<1000.1) THEN IB=1
2050 IF(IB=0)THEN 2150
2060 M=0
2070 FOR MX=1 TO MO
2080 AB=A(MX,NH)
2090 IF(AB>.999 AND AB<1.001) THEN M=MX
2100 NEXT MX
2110 IF(M=0) THEN PRINT"ART.VAR. MISSING IN EQUATION ";NH
2120 FOR N = 1 TO NO
2130 A(1,N) = A(1,N) - A(M,N)*1000
2140 NEXT N
2150 NEXT NH
2160 REM ***********************************************************************
2170 REM LOOK FOR THE MOST NEGATIVE COEF. IN THE OBJECTIVE FUNCTION
2180 REM ***********************************************************************
2190 NMAX = NO - 1
2200 IF OUTO= 0 THEN GOTO 2240
2210 FOR M = 1 TO MO
2220 PRINT USING"#####.###";A(M,1);A(M,2);A(M,3);A(M,4);A(M,5);A(M,6)
2230 NEXT M
2240 NA=0
2250 AK = -.000001
2260 FOR N = 1 TO NMAX
2270 AEV = A(1,N)
2280 IF AEV>AK THEN 2310
2290 NA=N
2300 AK=AEV
2310 NEXT N
2320 IF NA=0  THEN 2720
2330 REM ***********************************************************************
2340 REM WHAT IS THE MAXIMUM VALUE WHICH XI CAN TAKE ?
2350 REM ***********************************************************************
2360 AKOEF = 0
2370 AKVOT = 100000
2380 MA = 0
2390 FOR KKM = 2 TO MO
2400 EVKOEF = A(KKM,NA)
2410 AVOL = A(KKM,NO)
2420 IF (EVKOEF>DIFM AND EVKOEF<DIFP) THEN 2490
2430 IF (EVKOEF<0) THEN 2490
2440 EVKVOT = AVOL/EVKOEF
2450 IF(EVKVOT>AKVOT)THEN 2490
2460 AKVOT = EVKVOT
2470 AKOEF = EVKOEF
2480 MA = KKM
2490 XXX=1
2500 NEXT KKM
2510 NXIEX(NA) = MA
```

```
2520 REM *********************************************************************
2530 REM ROW DIVISION WHICH IMPLIES THAT Xi BECOMES Xi = 1
2540 REM *********************************************************************
2550 FOR N = 1 TO NO
2560 A(MA,N) = A(MA,N)/AKOEF
2570 NEXT N
2580 REM *********************************************************************
2590 REM REDUCTION OF EACH ROW
2600 REM *********************************************************************
2610 FOR M = 1 TO MO
2620 FAKTOR = A(M,NA)
2630 IF(M=MA)THEN FAKTOR = 0
2640 FOR N = 1 TO NO
2650 A(M,N) = A(M,N) - FAKTOR * A(MA,N)
2660 NEXT N
2670 NEXT M
2680 GOTO 2190
2690 REM *********************************************************************
2700 REM FEED THE RESULTS INTO THE PRIMAL RESULT VECTOR FOR X, VX(NX)
2710 REM *********************************************************************
2720 FOR NX = 1 TO NMAX
2730 IKO = 0
2740 AKOLL = 0
2750 FOR MEV = 1 TO MO
2760 TAL = A(MEV,NX)
2770 AKOLL = AKOLL + TAL*TAL
2780 IF(TAL>.9999 AND TAL<1.0001) THEN IKO = MEV
2790 NEXT MEV
2800 IF (AKOLL>1.0001) THEN IKO = 0
2810 IF (IKO > 1)THEN VX(NX) = A(IKO,NO)
2820 NEXT NX
2830 REM *********************************************************************
2840 REM FEED THE RESULTS INTO THE DUAL RESULT VECTOR (SHADOW PRICES) SKU(NX)
2850 REM *********************************************************************
2860 FOR NX = 1 TO NMAX
2870 SKU(NX) = A(1,NX)
2880 NEXT NX
2890 IF(OUTO= 0 AND OUT2 = 0) THEN GOTO 2920
2900 PRINT"PRIMAL RESULT VECTOR";VX(1);VX(2);VX(3);VX(4);VX(5)
2910 PRINT"DUAL   RESULT VECTOR";SKU(1);SKU(2);SKU(3);SKU(4);SKU(5)
2920 IF OUT3 = 1 THEN PRINT"THE OPTIMAL OBJECTIVE FUNCTION VALUE = ";A(1,NO)
2930 OBJLP = A(1,NO)
2940 RETURN
2950 REM *********************************************************************
2960 REM END OF THE SIMPLEX SUBROUTINE
2970 REM *********************************************************************
2980 REM
2990 REM
3000 REM
3010 REM *********************************************************************
3020 REM SIMPLEX MATRIX DATA SET
3030 REM *********************************************************************
3040 DATA 004,006
3050 DATA 001,001,-100,001,002,-100
3060 DATA 002,001,001,002,002,001,002,003,001,002,006,050
3070 DATA 003,001,001,003,004,001,003,006,030
3080 DATA 004,002,001,004,005,001,004,006,030
3090 DATA 0,0,0
3100 REM *********************************************************************
3110 REM END OF SIMPLEX MATRIX DATA SET
3120 REM *********************************************************************
```