

# Optimization in forestry

- Motivation, principles, methods and results

*Peter Lohmander*

*Professor of forest management and economic optimization  
Faculty of Forest Sciences, SLU, Umea, Sweden*

[www.Lohmander.com](http://www.Lohmander.com)

Forest Ecology and Management  
seminar series at SLU

14:30 – 15:30, December 13, 2011

# Optimization in forestry

- **Motivation** from the point of view of **Forest Ecology & Management**

Citation from "Forest Ecology & Management", "Silviculture", <http://www.seksko.se/> (2011-12-05):

"The aim of silvicultural research is to evaluate the effects of forestry practices, like regeneration, pre-commercial thinning, thinning, fertilization, and felling, on the future development of forests. The research is based on long-term field experiments, survey-studies, laboratory experiments, and simulations.

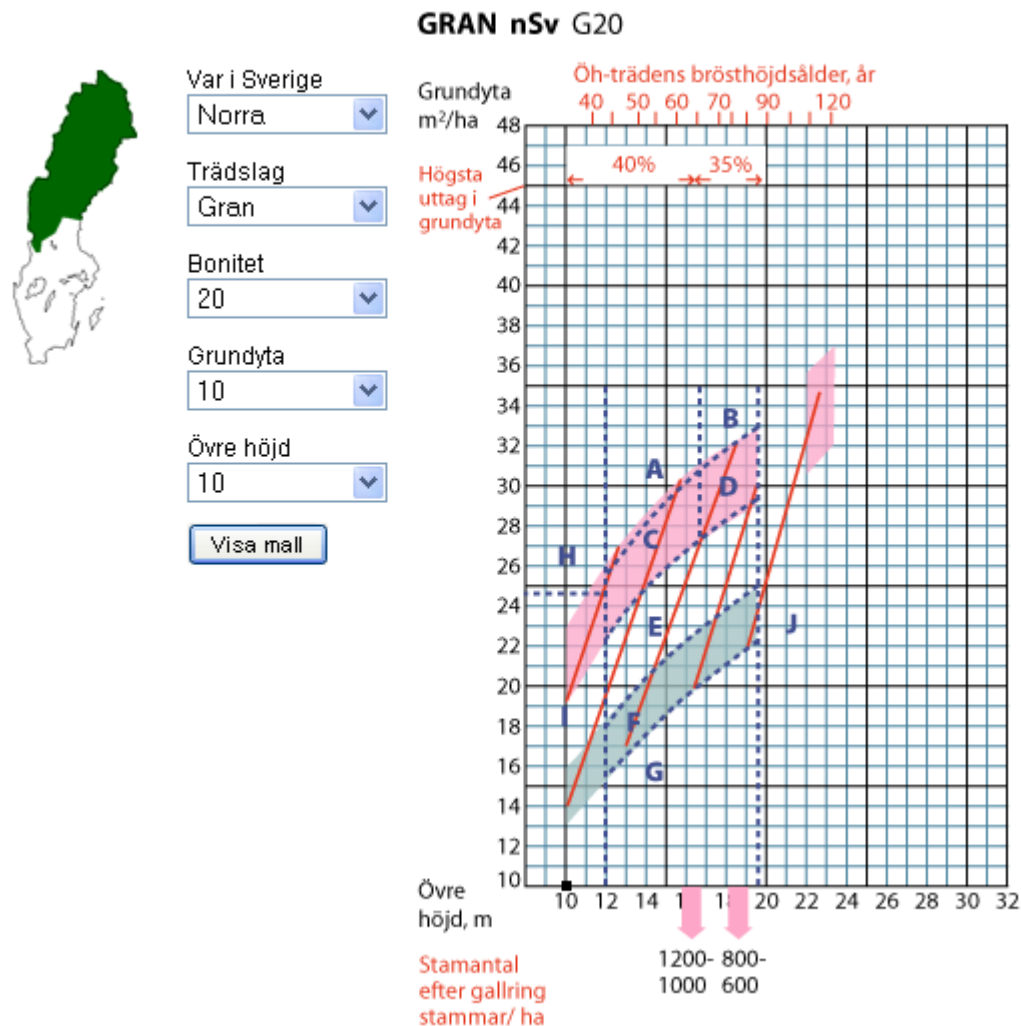
The department is responsible for the development of knowledge for forming silvicultural practices and systems, which enable a sustainable use of the forest resource. The topic is central to the department's education, at both undergraduate and graduate level." (End of citation)

## OBSERVATION:

In order to develop "the best" methods in forestry, we need well defined objective functions and explicit optimization of activities in forestry.

*Peter Lohmander*

## Gallringsmall - tall och gran



A classical tool that is interesting to use and that makes it possible to "simulate" different combinations of thinning times, thinning volumes and clearfellings.

### **OBSERVATION!**

There is an infinite number of such possible combinations!

The tool does not optimize the combined Decisions.

**If you want the best plan,**  
**you should optimize**  
**the decisions!**

# A simple but typical deterministic final felling problem

## Example:

Stand volume right now = 100 m<sup>3</sup> per hectare

The stand grows 5 m<sup>3</sup> per hectare and year.

Price – harvest cost = 200 SEK / m<sup>3</sup>

Today,  $t=0$ , we have a forest stand with the following properties:

$B(t)$  = Value of the stand if we harvest  $t$  years from now. (SEK/ha)

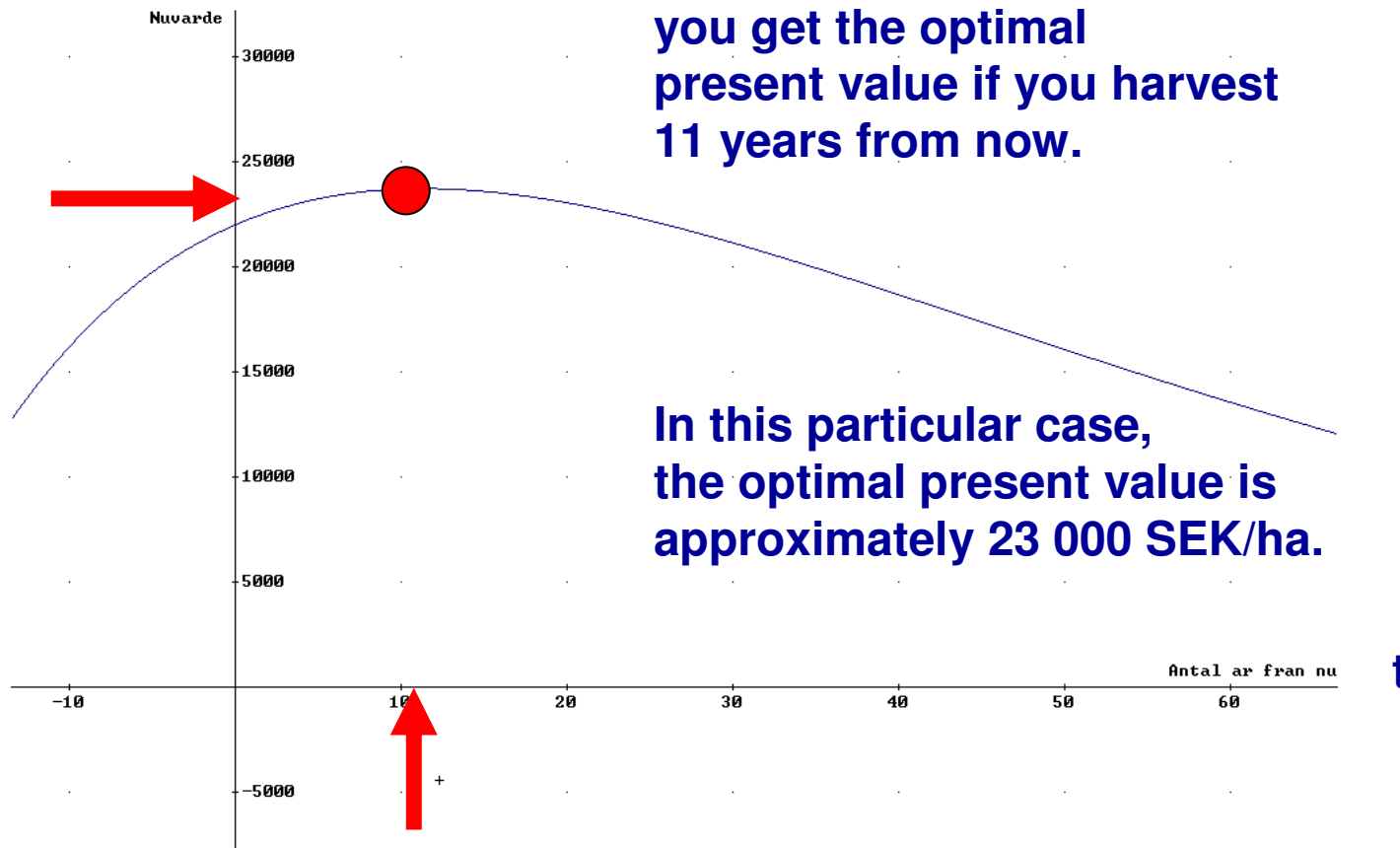
$$B(t) = a + bt \quad a = 20000, b = 1000$$

$L$  = Value of the land (bare land) after harvesting ( = 2000 SEK/ha).

$r$  = Rate of interest in the capital market (in continuous time).



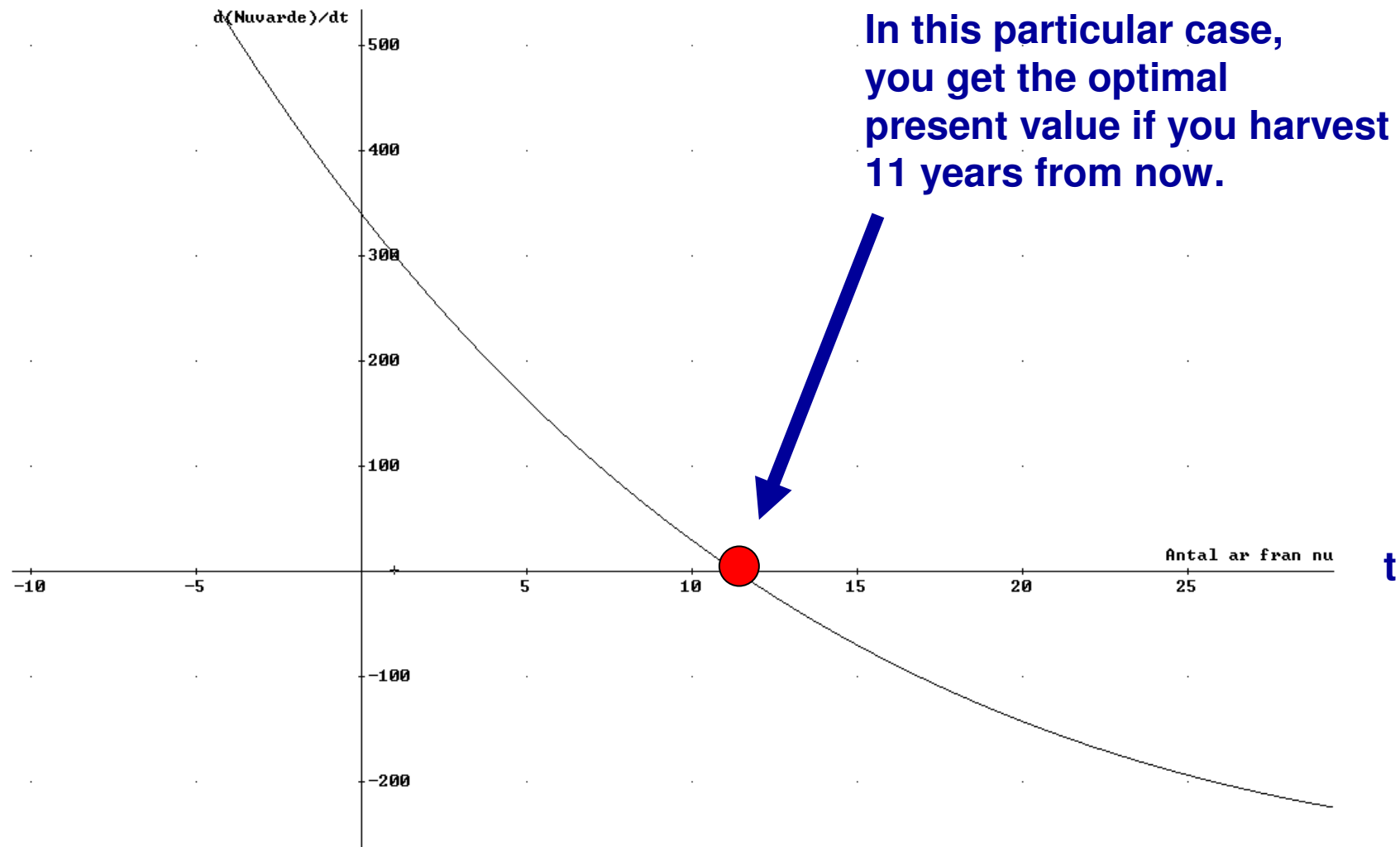
## Present value



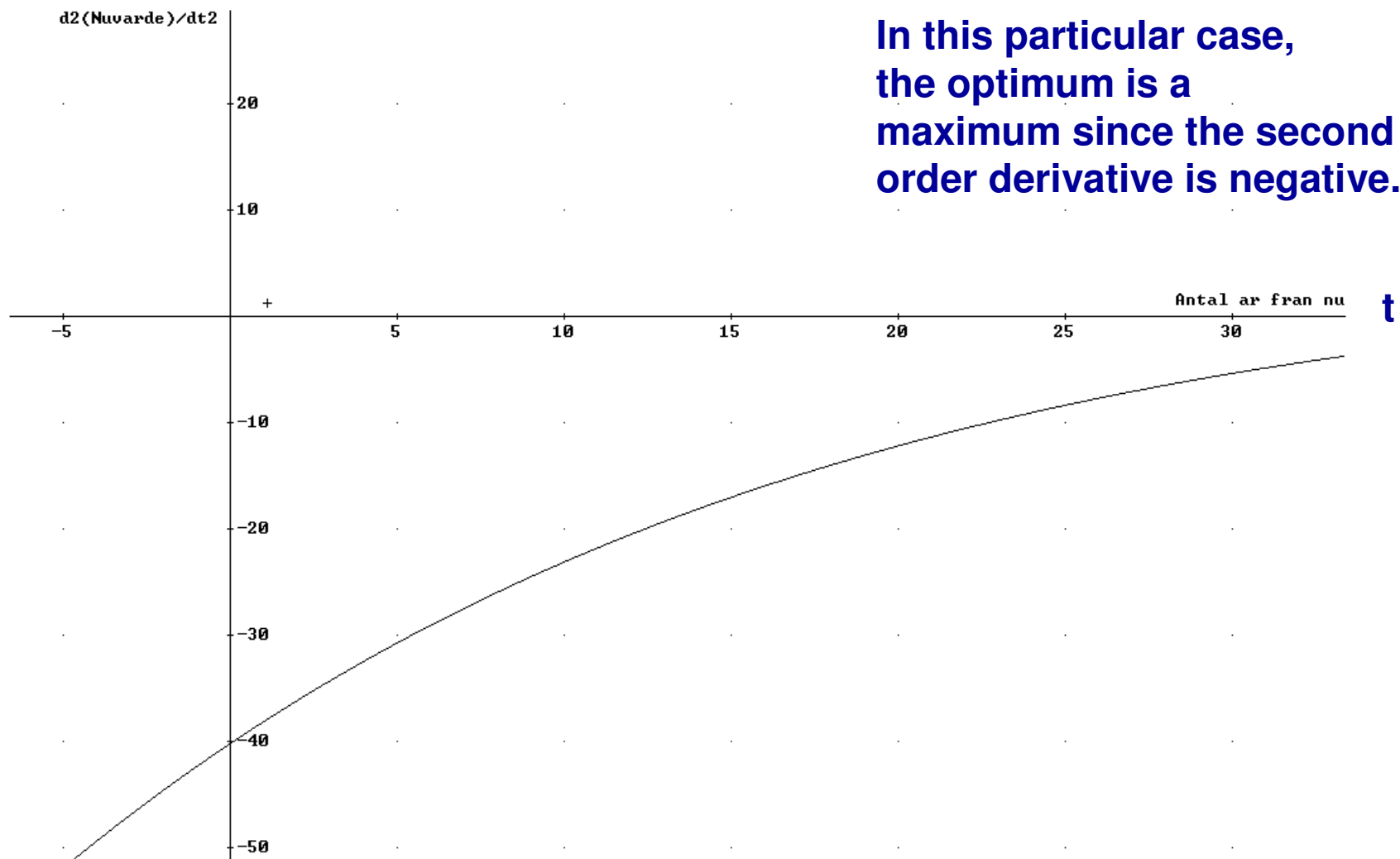
With one (or two) decision(s), you can plot the objective function and easily find the optimum for particular parameter values.

$$\text{Present value} = \text{EXP}(-0.03 \cdot t) \cdot (20000 + 1000 \cdot t + 2000)$$

$d(\text{Present value}) / dt$



$d^2(\text{Present value}) / dt^2$



**It is better to derive the optimal decision as a function of the parameters!**

$$\Pi(t) = e^{-rt} (B(t) + L)$$

**Objective function**

$$\Pi'(t) = -re^{-rt} (B(t) + L) + e^{-rt} B'(t)$$

**First derivative w.r.t. t**

$$\Pi'(t) = e^{-rt} (-r(B(t) + L) + B'(t)) = 0$$

**First derivative w.r.t. t = 0**

↑  
 $e^{-rt} > 0$

$$-r(B(t) + L) + B'(t) = 0$$

First derivative w.r.t.  $t = 0$



$$-r(B(t) + L) + B'(t) = 0$$

$$B(t) = a + bt$$

$$-r(a + bt + L) + b = 0$$

$$-ra - rbt - rL + b = 0$$

$$-rbt = -b + ra + rL$$

$$t = \frac{-b + r(a + L)}{-rb}$$

$$t = \frac{1}{r} - \frac{(a + L)}{b}$$



Optimal harvest time,  $t$ ,  
as a function of the  
parameters.

$$t^* = \frac{1}{r} - \frac{(a + L)}{b}$$

Optimal harvest time,  $t$ ,  
as a function of the  
parameters.

a	b	L	r	$t^*$
20000	1000	2000	3% →	11
20000	1000	2000	4% →	3

# Optimal present value

$$\Pi^* = e^{-rt^*} (B(t^*) + L)$$

$$\Pi^* = e^{-rt^*} (a + bt^* + L)$$



# ***Maximum or minimum?***

## ***Unique maximum or minimum?***

The second order maximum condition is important and should be investigated.

More details:

<http://www.lohmander.com/Kurser/SkogEkIntro/SEIntroWeb.html>

# A simple but typical deterministic **plantation** problem

More details:

<http://www.lohmander.com/Kurser/SkogEkIntro/SEIntroWeb.html>

# A simple but typical deterministic **thinning** problem

More details:

<http://www.lohmander.com/Kurser/SkogEkIntro/SEIntroWeb.html>

# ***We should optimize:***

**Seedlings or seeds?**

**Numbers of seedlings (or seeds)?**

**Species mix of seedlings (or seeds)?**

**When to harvest?**

**How much to harvest?**

**What species to harvest?**

**Quality, dimension or other selection criteria?**

# One important decision:

- Continuous cover forestry with trees of different sizes and ages?

or

- Forestry with clear fellings and trees of the same age and size?

*Read about these things and look at excursions:*

<http://www.lohmander.com/Kont11/Kont11.htm>

[http://www.lohmander.com/Arjeplog\\_2011/Arjeplog\\_2011.htm](http://www.lohmander.com/Arjeplog_2011/Arjeplog_2011.htm)





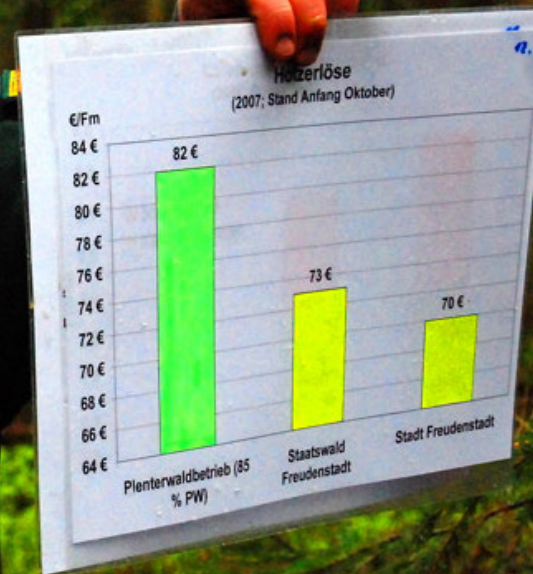
**Arjeplog, Sweden, 2011**





**Schwartzwald, Germany, 2009**

















**Neuchatel, Switzerland, 2009**





























































**SkogsNolia, Umea, Sweden, 2011**











# ***We should *also optimize:****

**Types of harvesters, forwarders and other machines?**

**Numbers of such machines?**

**Sizes of harvest areas?**

**Infrastructure investments and maintenance?**

**Road and/or railroad network properties such as distances  
between roads, qualities of roads,...**

**Investments in labour, education,...**



**SCA Nordliner, Umeå, Sweden, 2008**





**Baden- Wurttemberg, Germany, 2009**





**Dåva 2, Umea, Sweden, 2010**













**Skelleftekraft, Lycksele, Sweden, 2010**





***Roundwood stored at Dåva 2 CHP, Umeå,  
Sweden, 2010***







OPTIMAL RESULTS FROM DHINV  
Software by  
Peter Lohmander 2010

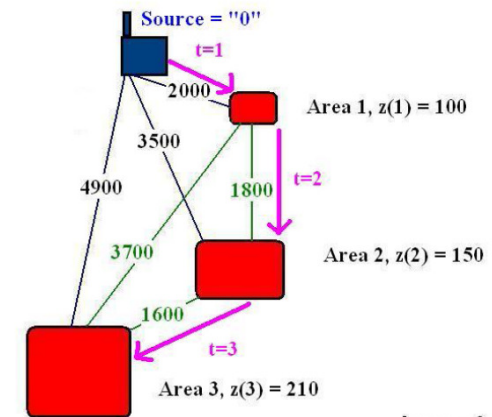
OPTIMAL TIME AND STATE DEPENDENT DECISIONS AND EXPECTED PRESENT VALUES

t = 1	i(t)	E(PV)	i(t+1)	DEC	CVIA	Entering Partial States
1	34830.	5	1	0	0 0 0	

t = 2	i(t)	E(PV)	i(t+1)	DEC	CVIA	Entering Partial States
5	40538.	7	2	1	1 0 0	

t = 3	i(t)	E(PV)	i(t+1)	DEC	CVIA	Entering Partial States
7	45062.	8	3	2	1 1 0	

t = 4	i(t)	E(PV)	i(t+1)	DEC	CVIA	Entering Partial States
8	51517.	8			1 1 1	



kmax = 3  
(Areas)







***Electrical generator at Dåva 2 CHP, close to Umeå, Sweden, 2010.***



***We should *also optimize:****

**Forest industry investments and maintenance  
(sawmills, pulp mills, combined heat and power  
stations, ...)**

**...**

***When?***

***Where?***

***Capacities?***

***Properties?***

**...**





***Continuous cover forestry excursion, Bräcke, Sweden, 2011.***



# Optimization in forestry

## - Motivation

*Person A says:*

*"This forest management activity is the best".*

*You **should** say:*

*"It is possible that A is right. With some combination of objective function, constraints and information structure, A may be right."*

*"However, with most other combinations of objective function, constraints and information structure, A is probably wrong."*

*"Please describe your objective function, constraints and information structure! Otherwise, I can not determine if your suggested forest management activity is the best ( = optimal) !"*



# Optimization in forestry

## - Principles

*Always start with the  
problem definition!*

*Objective function,  
Constraints,  
Information structure*



# Optimization in forestry

## - **Methods**

*We should determine the activities in forestry that optimize the objective function with consideration of the relevant constraints and the information structure.*

*The optimal method is a function of the objective function, constraints and information structure.*

*Sometimes we may desire solutions in the form of general functions.*

*Sometimes we may need a concrete figure, a numerical solution.*



$$\max \Pi = p_1 q_1 + p_2 q_2 + \dots + p_n q_n$$

*s.t.*

$$a_{1,1} q_1 + a_{1,2} q_2 + \dots + a_{1,n} q_n \leq c_1$$

$$a_{2,1} q_1 + a_{2,2} q_2 + \dots + a_{2,n} q_n \leq c_2$$

.....

$$a_{m,1} q_1 + a_{m,2} q_2 + \dots + a_{m,n} q_n \leq c_m$$

**A classical linear programming problem**





**Baden- Wurttemberg, Germany, 2009**









**Schwarzwald, Germany, 2009**





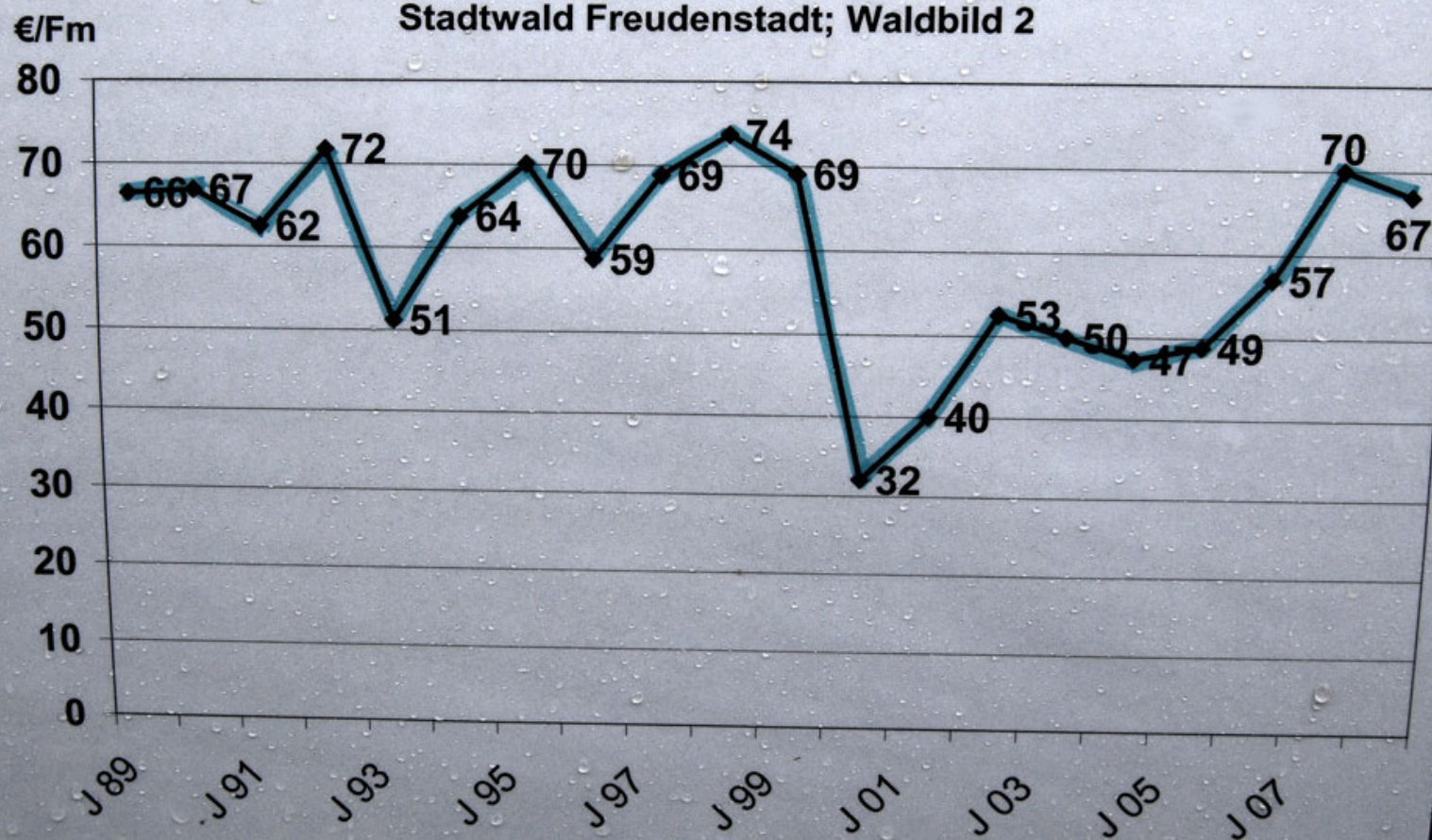






③

# Holzpreise je Fm verkaufte Masse Stadtwald Freudenstadt; Waldbild 2





# Optimization in forestry

## - Methods

*Do we have perfect information about future events?*

*The answer is generally **NO**. Most people pretend that perfect information exists, because it is easier!*

**Reality:**

*Stochastic prices of forest products,*

*Stochastic insect damages,*

*Stochastic fungi damages,*

*Stochastic storms and windthrows,*

*Stochastic forest fires,*

*...*



# Optimization in forestry

## - Methods

***MOSTLY (!!!), we do not have perfect information concerning all future events. Then, this method is necessary if we want to optimize activities in forestry:***

***Stochastic dynamic programming (in discrete time and with discrete state space)***

***Special version:***

***Stochastic optimal control in continuous time (and with continuous state space).***

***Important observation:***

***Continuous time is not always more relevant. Furthermore, in continuous time and space formulations, the functional forms are often simplified to irrelevant functions.***



$$W^*(i, t) = \max_{\substack{h \\ h \in H(i)}} \left( R(i, t, h) + e^{-r} \sum_{j=1}^J \tau(j | i, t, h) W^*(j, t + 1) \right)$$

***Stochastic dynamic programming (in discrete time and with discrete state space)***



$$\text{Min } Z = \sum_i w_i$$

s.t.

$$w_i - \beta \sum_j \tau(j|i, u) w_j \geq R_{i,u} \quad \forall i, u \Big|_{u \in U(i)}$$

Sometimes, solutions to **Stochastic dynamic programming problems (in discrete time and with discrete state space)** can be calculated via linear programming. Then, commercial software can be used.



***Stochastic optimal control in continuous time  
(and with continuous state space):***

$$\int_t^{\infty} e^{-rt} (ku - pu^2 + fx - gx^2) dt$$

$$dx = (u - mx - n) dt + sx dz$$



# Optimal CCS, Carbon Capture and Storage, Under Risk

*The objective function is the total present value of CO2 storage minus CCS costs.*

$$\int_t^{\infty} e^{-rt} (ku - pu^2 + fx - gx^2) dt$$

Discounting  
factor

u =  
control =  
CCS  
level

x = The total  
storage level  
of CO2



# The controlled storage

*A stochastic differential equation:*

$$dx = (u - mx - n) dt + sx dz$$

Change of the  
CO2 storage level.

Control =  
CCS level.

Expected CO2 leakage.

The CO2 storage level is to some  
extent affected by stochastic leakage  
and other stochastic events.  
Z = standard Wiener process.



# Optimization in forestry

## - Methods

*Sometimes we may desire solutions in the form of  
**general functions:***

*Then, we need **general analytical methods** to determine  
optimal objective function values and activities, as more  
or less general functions of different parameters.*



# Optimization in forestry

## - Methods

*Sometimes we may **need a concrete figure**, a numerical solution.*

*Then, we can use **numerical methods** to determine the optimal objective function values and activities.*

*Sometimes, commercially **available software** with mathematical programming subroutines can be used to solve the problems.*

*Sometimes, the easiest way is to **create a new software**.*



# Optimization in forestry

## - Results

Now, different examples will follow!

**Make sure that you know the:**

*The objective function*

*The relevant constraints*

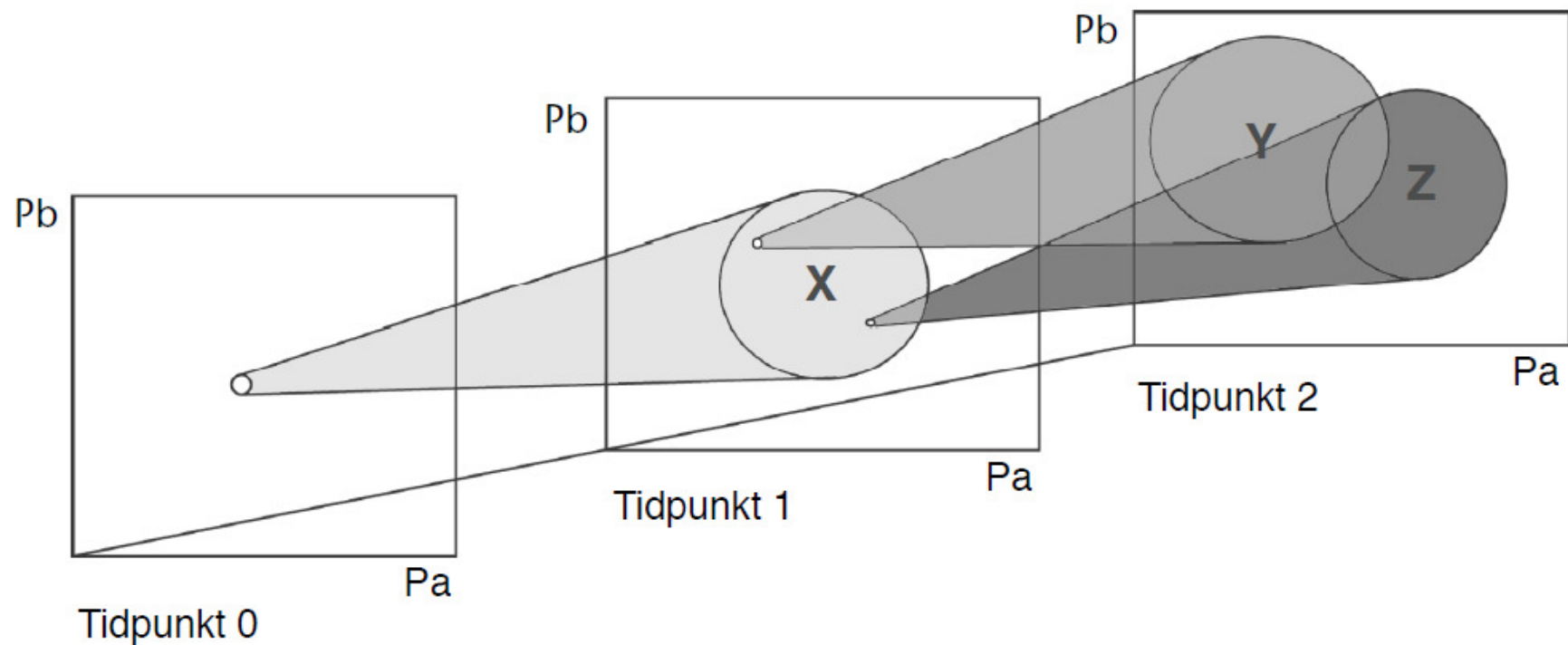
*The information structure*

*And the optimal forestry decisions!*



**Arjeplog, Sweden, 2011**





**Lohmander, P., Optimala beslut inför osäker framtid, FAKTA SKOG, SUAS, Nr 10, 2001**

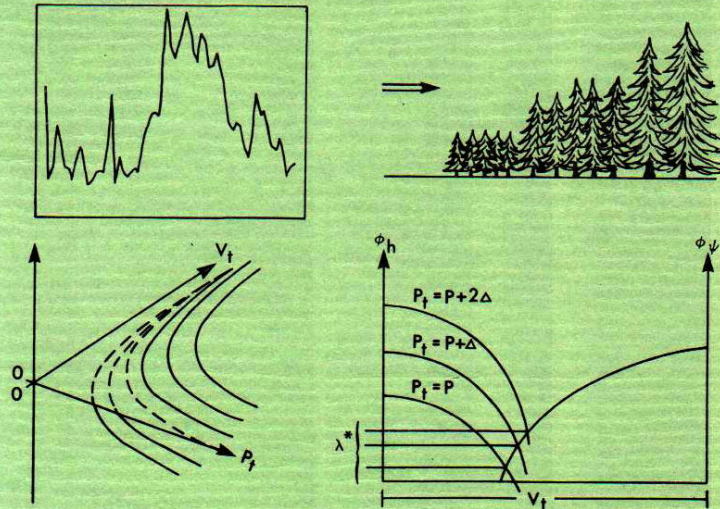
- <http://www.slu.se/PageFiles/33707/2001/FS01-10.pdf>



SVERIGES LANTBRUKSUNIVERSITET  
INSTITUTIONEN FÖR SKOGSEKONOMI

## THE ECONOMICS OF FOREST MANAGEMENT UNDER RISK

By Peter Lohmander



INSTITUTIONEN FÖR SKOGSEKONOMI

SWEDISH UNIVERSITY OF AGRICULTURAL SCIENCES  
DEPARTMENT OF FOREST ECONOMICS

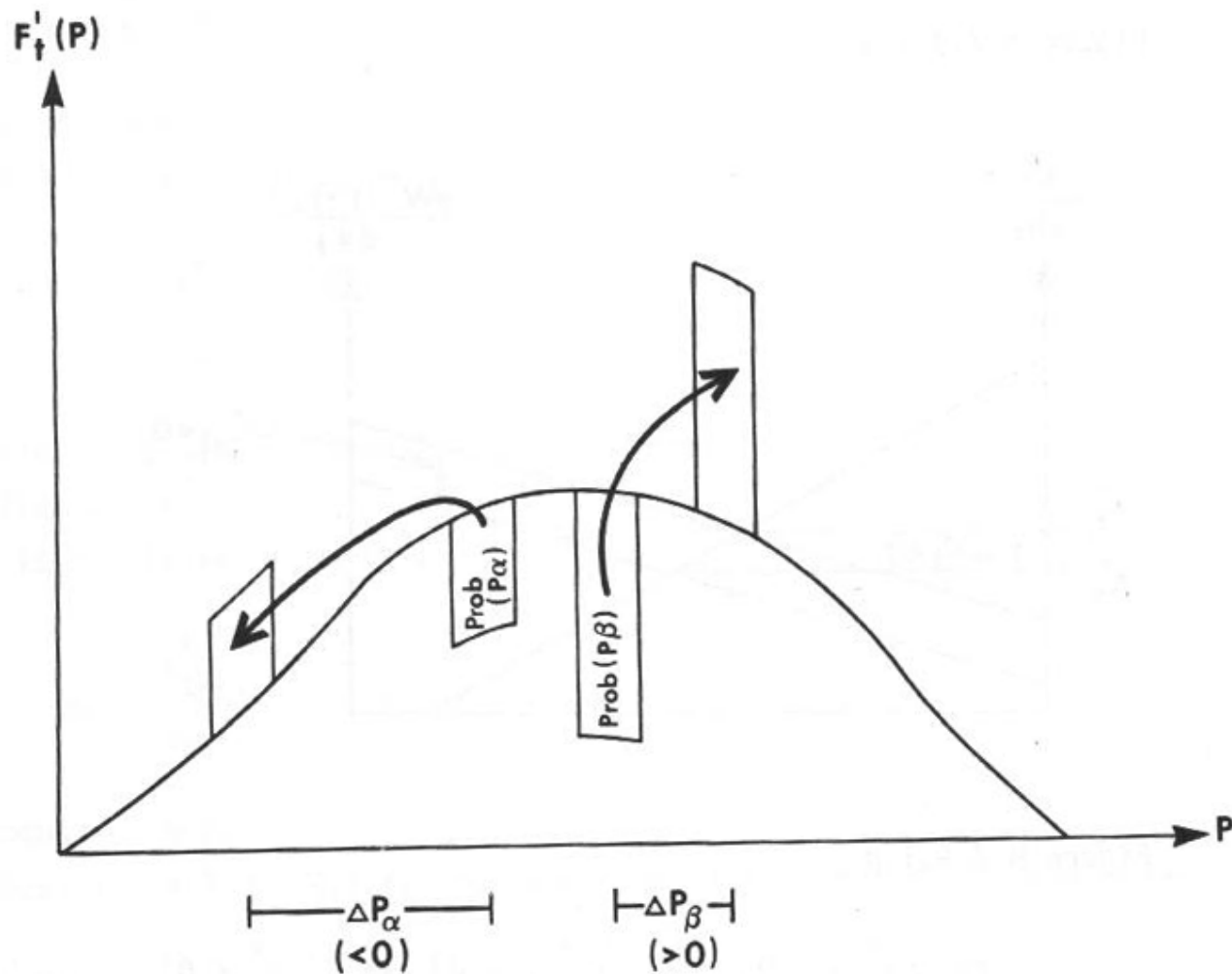
RAPPORT 79  
REPORT

UMEÅ 1987  
ISSN 0348-2049  
ISBN 91-576-3229-4

Lohmander, P., The economics of forest management under risk, Swedish University of Agricultural Sciences, Dept. of Forest Economics, Report 79, 1987

(Doctoral dissertation) (Doktorsavhandling), 311p76





A mean preserving spread (MPS) according to Rothschild and Stiglitz.

"Probability density" is moved from the center of the distribution to the tails in such a way that the expected value of the stochastic variable is not changed.

# Optimal continuous harvesting with stochastic prices and growth (Early)

Lohmander, P., Continuous extraction under risk, SYSTEMS ANALYSIS - MODELLING - SIMULATION, Vol. 5, No. 2, 131-151, 1988

Lohmander, P., Continuous extraction under risk, IIASA, International Institute for Applied Systems Analysis, Systems and Decisions Sciences, WP-86-16, March 1986

<http://www.iiasa.ac.at/Admin/PUB/Documents/WP-86-016.pdf>

<http://www.lohmander.com/WP-86-016.pdf>

Lohmander, P., The economics of forest management under risk, Swedish University of Agricultural Sciences, Dept. of Forest Economics, Report 79, 1987 (Doctoral dissertation) (Doktorsavhandling), 311p



# **Optimal pulse harvesting with stochastic prices and growth** (Early)

**Lohmander, P., Pulse extraction under risk and a numerical forestry application, SYSTEMS ANALYSIS -MODELLING - SIMULATION, Vol. 5, No. 4, 339-354, 1988**

**Lohmander, P., Pulse extraction under risk and a numerical forestry application, IIASA, International Institute for Applied Systems Analysis, Systems and Decisions Sciences, WP-87-49, June 1987**

**<http://www.iiasa.ac.at/Admin/PUB/Documents/WP-87-049.pdf>**

**<http://www.lohmander.com/WP-87-049.pdf>**

**Lohmander, P., The economics of forest management under risk, Swedish University of Agricultural Sciences, Dept. of Forest Economics, Report 79, 1987 (Doctoral dissertation) (Doktorsavhandling), 311p**

# **Optimal spatial dynamic harvesting with stochastic windthrows** (Early)

**Lohmander, P., Helles, F., Windthrow probability as a function of stand characteristics and shelter, SCANDINAVIAN JOURNAL OF FOREST RESEARCH, Vol. 2, No. 2, 227-238, 1987**

**Lohmander, P., Helles F., Skovstruktur og stormfald, En analyse af novemberstormen 1981, Dansk Skovforenings Tidskrift, No. 3, 1987**

**Lohmander, P., The economics of forest management under risk, Swedish University of Agricultural Sciences, Dept. of Forest Economics, Report 79, 1987 (Doctoral dissertation) (Doktorsavhandling), 311p**

**Lohmander, P., Ekonomiskt optimal avverkning med hansyn till stormfallningar, SKOGSBRUKETS EKONOMI, Skogsakta Konferens, No. 11, 32-37, 1988**



# MORE OPTIMIZATION IN FORESTRY?

Welcome to PhD Courses in these areas!

Optimization in Dynamic and Stochastic  
Decision Problems (2011, ..?..)

Forest Economics (2012)

Applied Problem Solving via Computer  
Programming (2009,..?..)

# Forest Economics

**7,5 credits**

PFS0057

**Date:**

2012-01-23 -- 2012-02-24

**Language:**

English

**Prerequisites:**

Suggested background: Most importantly, the course participants should be able to follow and understand the lectures. A general background in some quantitative area is suggested according to the three alternatives found below:

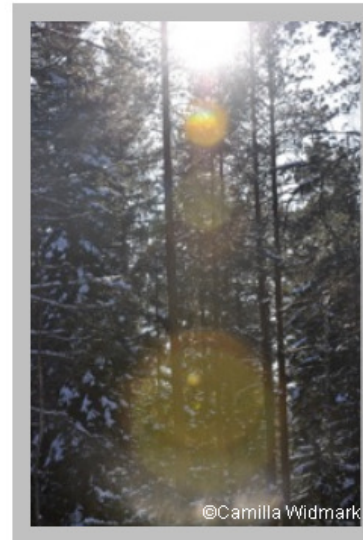
Alternative 1: Doctoral student and MSc och BSc in forestry (or jägmästare) including higher level courses in mathematical statistics and/or forest economics.

Alternative 2: Doctoral student and MSc or BSc in economics, management or business administration including higher level courses in quantitative analysis.

Alternative 3: Doctoral student with other background (such as engineering) including quantitative analysis and some economics and/or management.

**Objectives:**

After the course, the student should understand the fundamental principles of economic forestry, including the derivation of optimal decision rules in some situations. The student should be well aware of the underlying assumptions of the most typical decision rules, based on deterministic representations of reality. The student should be able to determine and understand in what ways the optimal decisions and decision rules change in case the underlying assumptions change within deterministic, single decision variable, forestry problems. The student should understand how optimal combinations of decisions can be determined in the presence of constraints caused by technology and laws. The student should also have some understanding of econometrics applied to forest economics, the consequences of information assumptions and different kinds of physical and economic disturbances, fundamental examples and applications of optimal stopping theory and more general stochastic multi period control in problems of forest sector relevance.



<http://www-sekon.slu.se/~cala/forskarutbildning/forskarutbildning/PFS0057.html>



**Optimization in Dynamic and Stochastic Decision Problems 2011**  
**SLU, Umea, Sweden**



[http://www.lohmander.com/ODSDP\\_2011/ODSDP\\_2011.pdf](http://www.lohmander.com/ODSDP_2011/ODSDP_2011.pdf)

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# Optimization in forestry

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<http://www.Lohmander.com>

