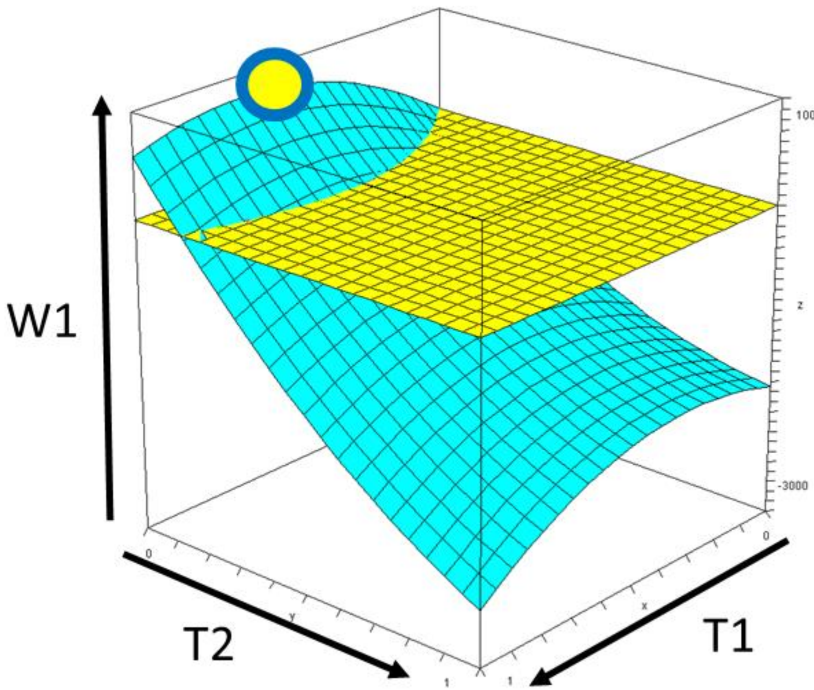

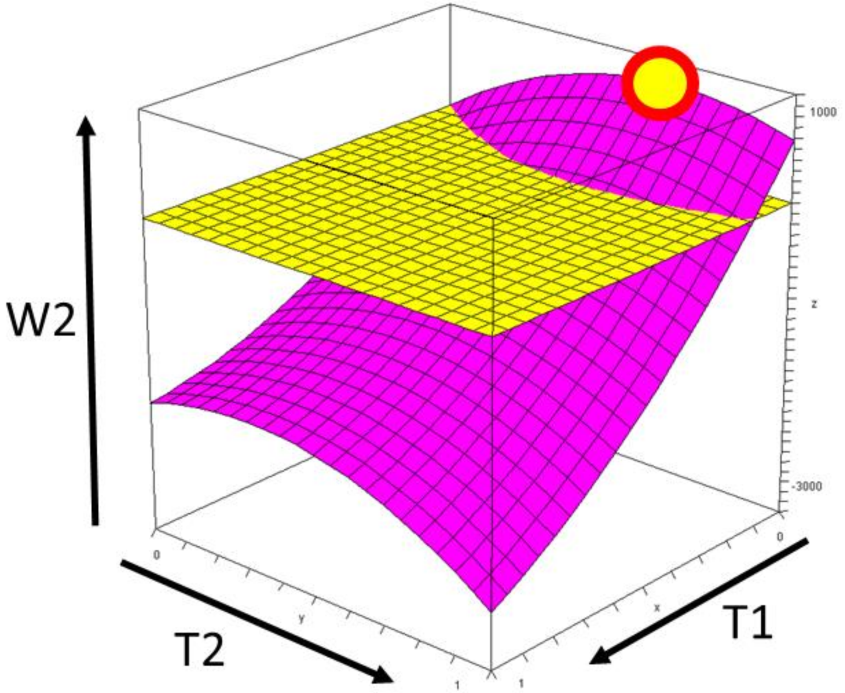

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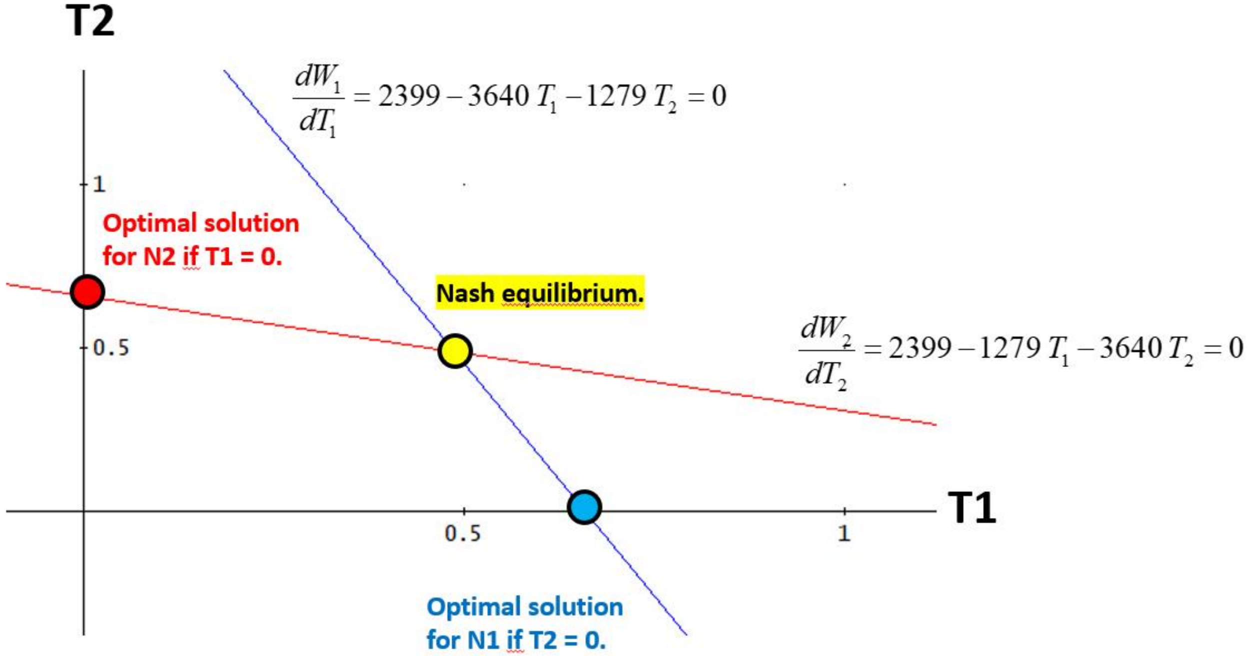


OPTIMAL TARIFFS

AND INTERNATIONAL TRADE

Eleventh International Conference on Statistics for the Twenty-First Century ICSTC 2025

OPTIMAL TARIFFS AND INTERNATIONAL TRADE,
Invited presentation, Eleventh International Conference on Statistics for the Twenty-First Century, ICSTC 2025, December 2025.

Peter Lohmander

Abstract

Optimal tariff strategies, with and without optimal responses, are derived, based on a linear partial equilibrium trade model, A, and a nonlinear general equilibrium trade model with two nations, B.

Under free trade, models A and B are applied to prove that it is always profitable for one nation, N1, to introduce a strictly positive tariff on imports, T1, as long as other nations do not introduce tariffs.

With model B, it is proved that it is rational also for nation N2, to introduce a tariff, T2, if N1, introduces a tariff, T1.

The Nash equilibrium tariff combination is unique and stable. The economic results, in both nations, are however higher with free trade, than in the Nash equilibrium tariff solution.

If both nations know that the other nations will respond to increasing tariffs, and select the optimal tariff, then it is not rational for any nation to leave free trade and introduce tariffs. Free trade agreements are rational for the participants.

It is also shown that it may be rational for a dictator, to introduce a tariff, even if this is not rational for the consumers in the same nation. The dictator may keep the tariff revenues and use these for other purposes. In such a case, the tariff reduces the consumer surpluses in both nations.

Section 1: **Partial equilibrium analysis:**

The free trade solution,

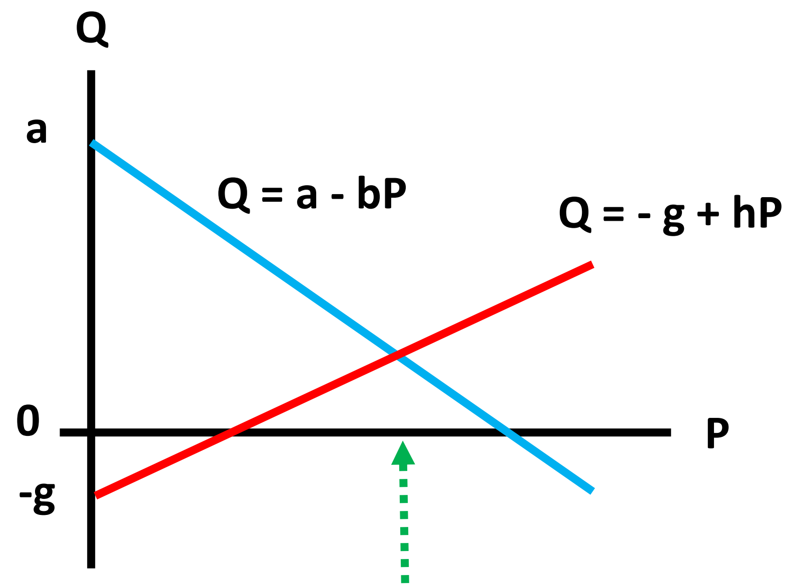
Consumer surplus maximization,

Total surplus (tariff revenue + consumer surplus) maximization,

The optimal deviation from the free trade solution,

The optimal tariff

Linear approximations of demand and supply



$$Q = a - bP$$ **Demand function in N1**

$$Q = -g + hP$$ **Export supply from N2 (or from the "World")**

$$-g + hP = a - bP$$ **Market equilibrium condition**

$$\left(b+h\right)P = a + g$$

$$P = \frac{a+g}{b+h}$$ **Equilibrium price**

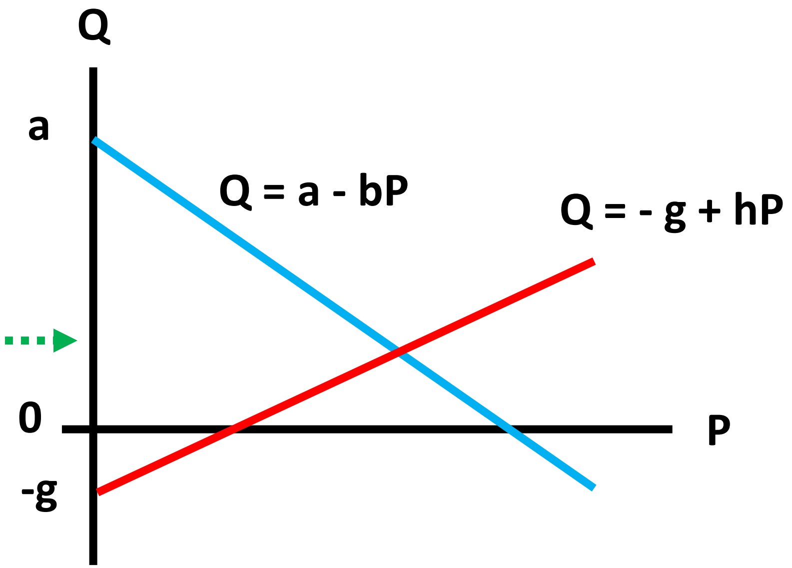
$$P^F = \frac{a+g}{b+h}$$ **Free trade equilibrium price**

$$Q_D^F = a - b\left(\frac{a+g}{b+h}\right)$$ **Demand in N1 in free trade equilibrium**

$$Q_D^F = \frac{a(b+h) - b(a+g)}{b+h}$$

$$\boxed{Q_D^F = \frac{ah - bg}{b+h}}$$

Demand in N1 in free trade equilibrium



$$P^F = \frac{a+g}{b+h}$$

Free trade equilibrium price

$$Q_S^F = -g + h\left(\frac{a+g}{b+h}\right)$$

Export supply from N2 (or ”the from World”) in free trade equilibrium

$$Q_S^F = \frac{-g(b+h)+h(a+g)}{b+h}$$

$$\boxed{Q_S^F = \frac{ah-bg}{b+h} \quad \left(= Q_D^F\right)}$$

Export supply from N2 (World) in free trade equilibrium. This equals the demand in N1.

$$Q = a - bP$$ **Demand function in N1**

$$bP = a - Q$$

$$P = \frac{a}{b} - \frac{1}{b}Q$$ $\boxed{\text{Inverse Demand function in N1}}$

$$Q = -g + hP$$ **Supply function from N2**

$$hP = g + Q$$

$$P = \frac{g}{h} + \frac{1}{h}Q$$ $\boxed{\text{Inverse Supply function from N2}}$

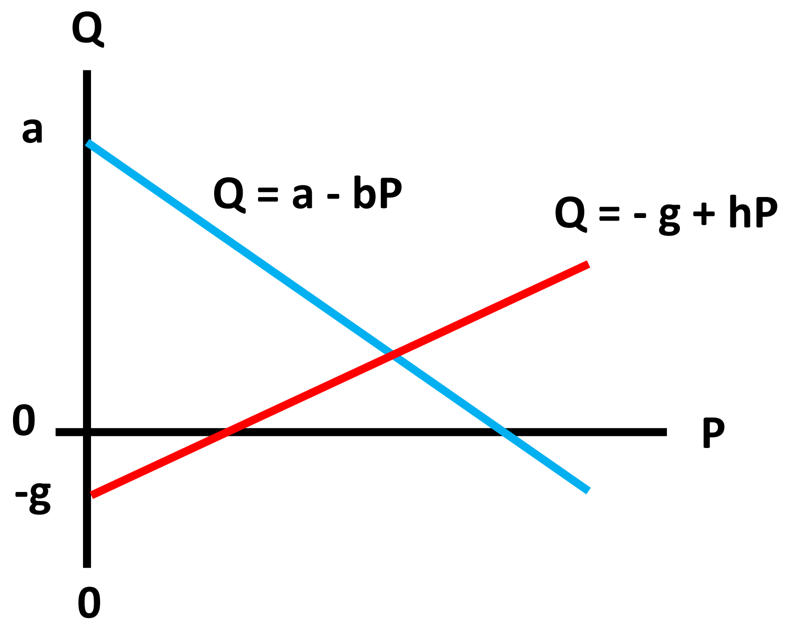
$$Q = a - bP \quad \textbf{Demand}$$

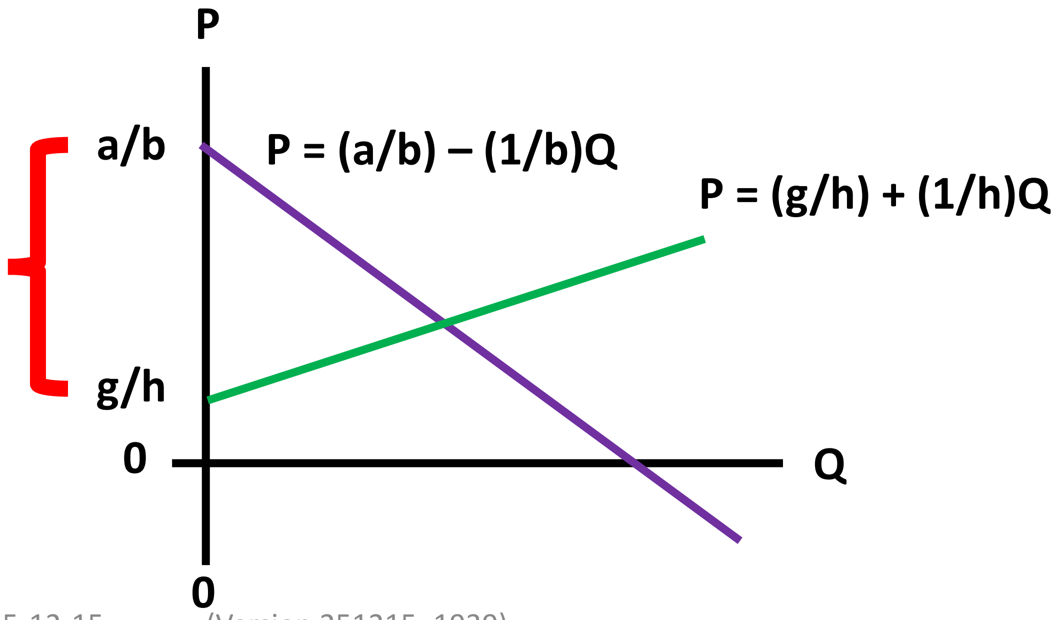
$$P = \frac{a}{b} - \frac{1}{b}Q \quad \textbf{Inverse Demand}$$

$$Q = -g + hP \quad \textbf{Supply}$$

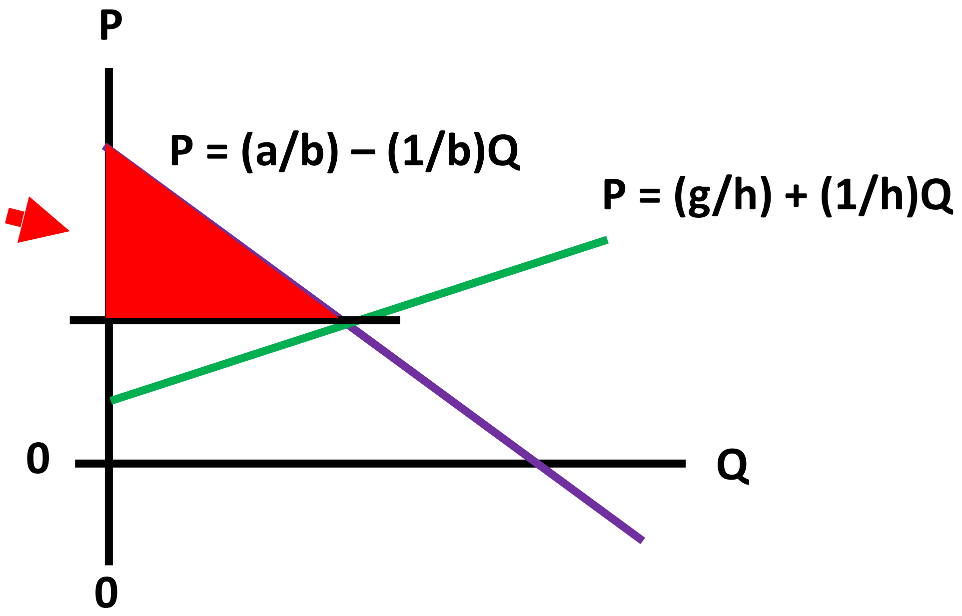
$$P = \frac{g}{h} + \frac{1}{h}Q \quad \textbf{Inverse Supply}$$

$$\boxed{\frac{a}{b} > \frac{g}{h}}$$





Consumer surplus in N1 in free trade equilibrium



$$S = \int_0^Q \left(\frac{a}{b} - \frac{1}{b} q \right) dq - \left(\frac{g}{h} + \frac{1}{h} Q \right) Q \quad , Q = Q_D^F = \frac{ah - bg}{b + h}$$

$$S = \int_0^Q \left(\frac{a}{b} - \frac{1}{b} q \right) dq - \left(\frac{g}{h} + \frac{1}{h} Q \right) Q$$

$$S = \left[\frac{a}{b} q - \frac{1}{2b} q^2 \right]_0^Q - \frac{g}{h} Q - \frac{1}{h} Q^2$$

$$S = \frac{a}{b} Q - \frac{1}{2b} Q^2 - \frac{g}{h} Q - \frac{1}{h} Q^2$$

$$S = \left(\frac{a}{b} - \frac{g}{h} \right) Q - \left(\frac{1}{2b} + \frac{1}{h} \right) Q^2$$

$$S = \left(\frac{a}{b} - \frac{g}{h}\right)Q - \left(\frac{1}{2b} + \frac{1}{h}\right)Q^2$$

Consumer surplus in N1 in free trade equilibrium:

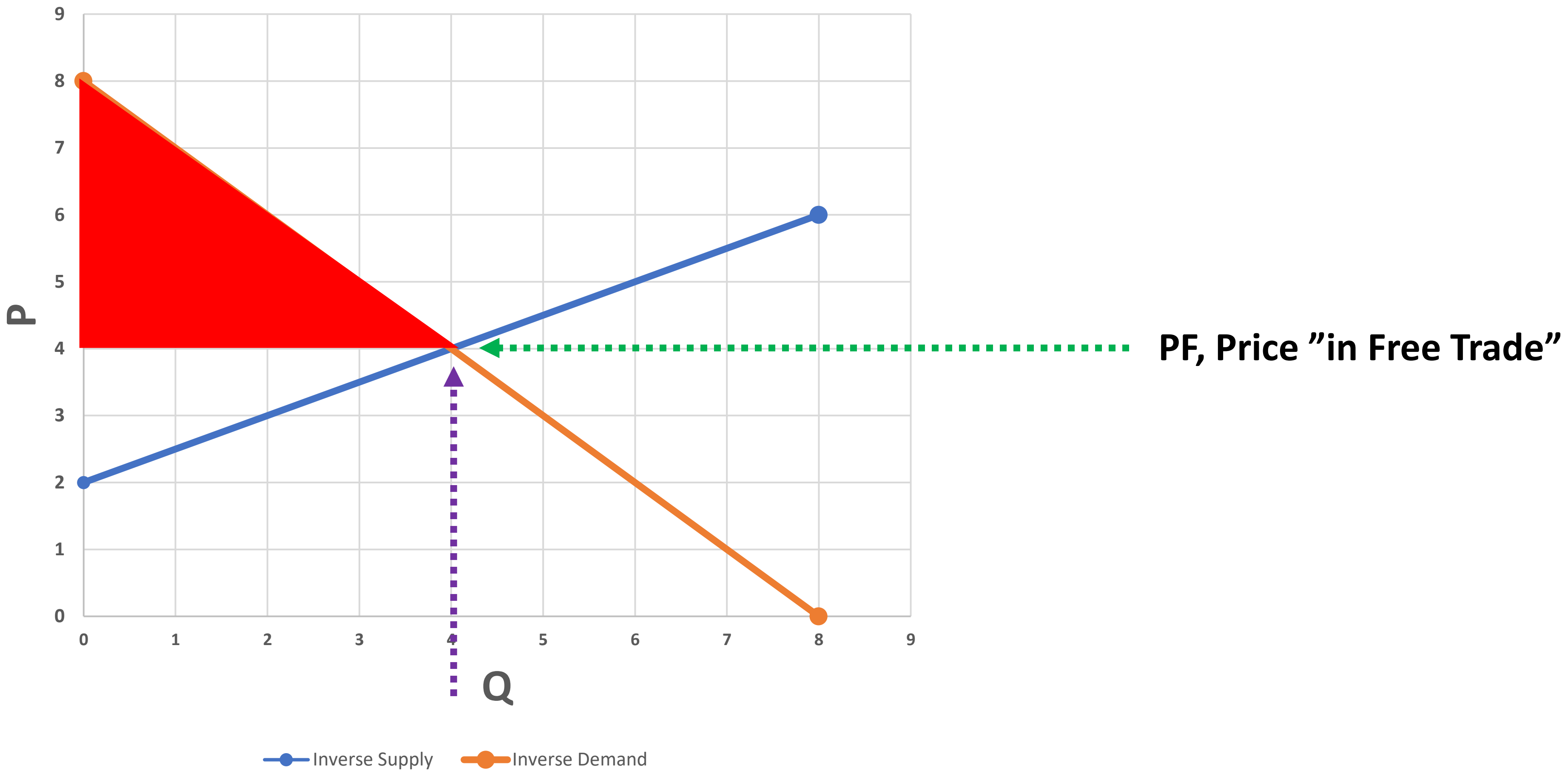
$$S^F = \left(\frac{a}{b} - \frac{g}{h}\right)\left(\frac{ah-bg}{b+h}\right) - \left(\frac{1}{2b} + \frac{1}{h}\right)\left(\frac{ah-bg}{b+h}\right)^2$$

$$\boxed{Q_D^F = \frac{ah-bg}{b+h}}$$

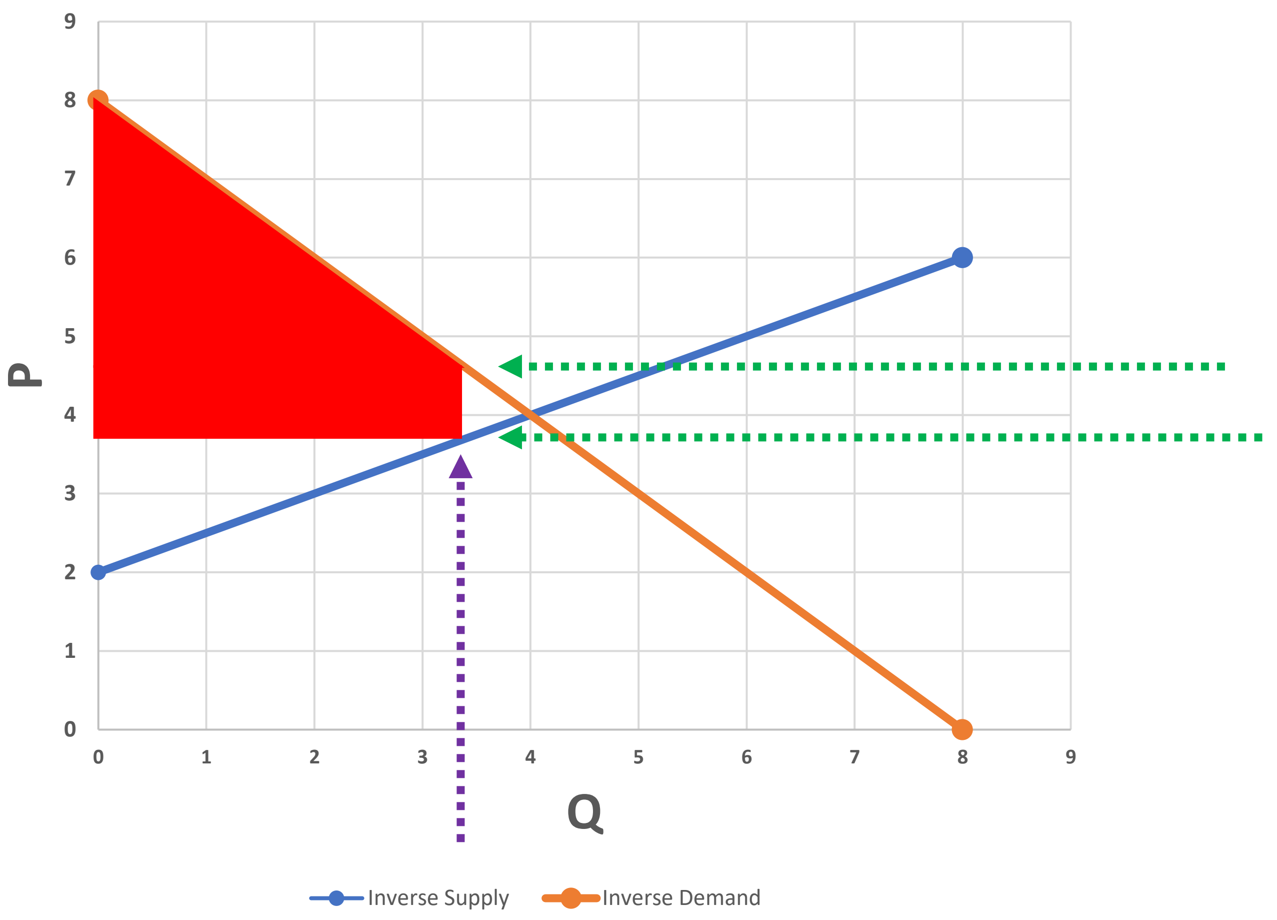
Maybe, it is possible to find an import volume, Q,

that gives a larger consumer surplus

than the free trade equilibrium?

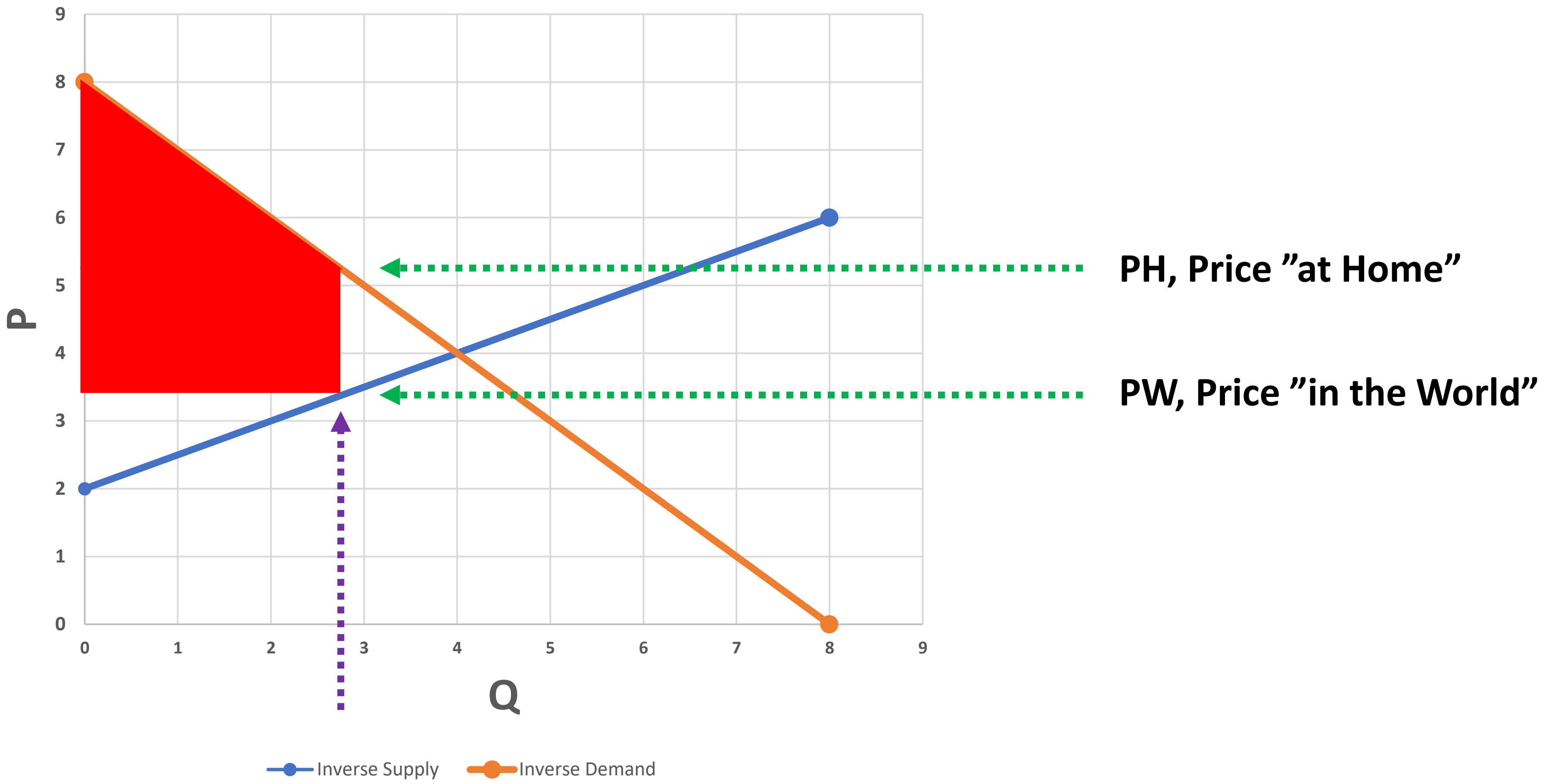


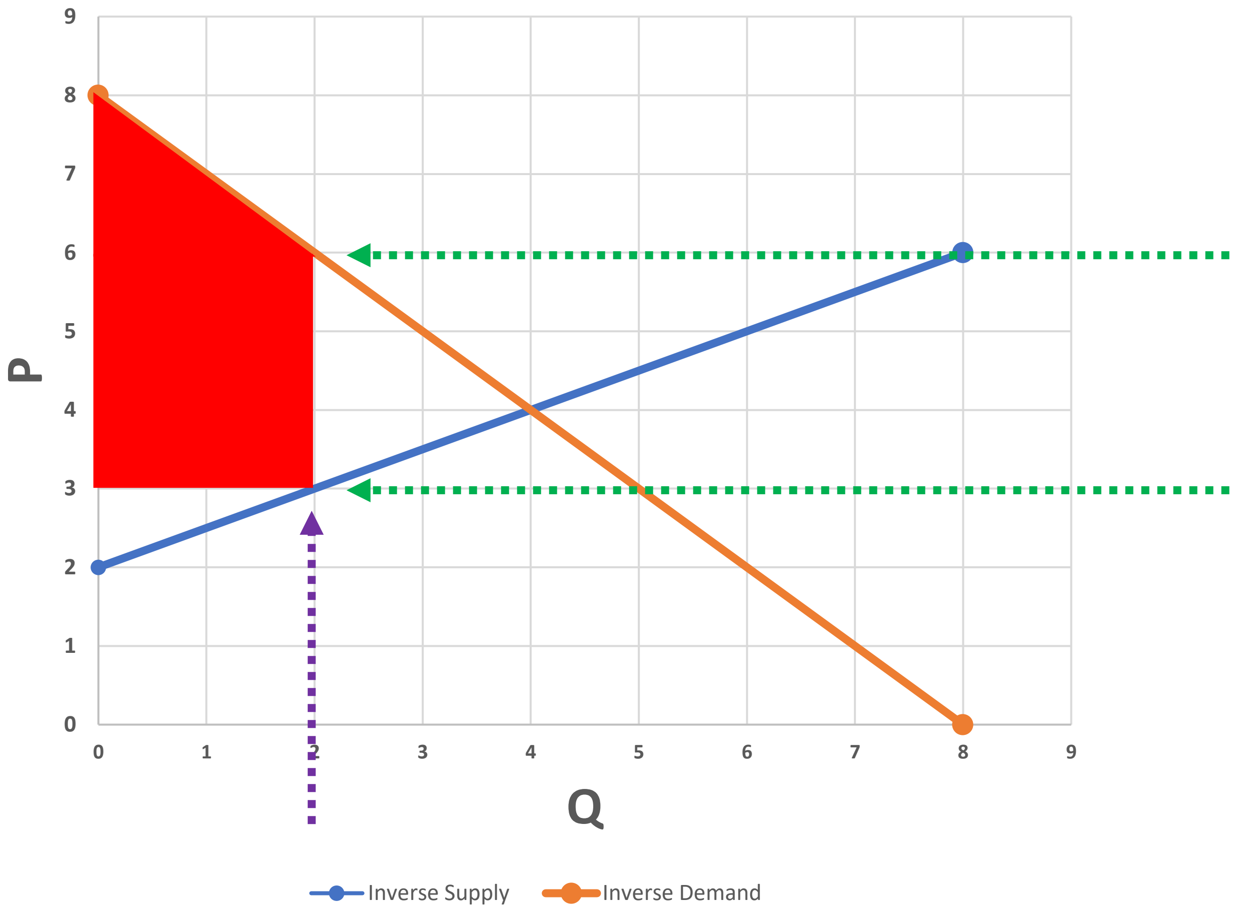
PF, Price ”in Free Trade”
P
Q
Inverse Supply
Inverse Demand



PH, Price ”at Home”

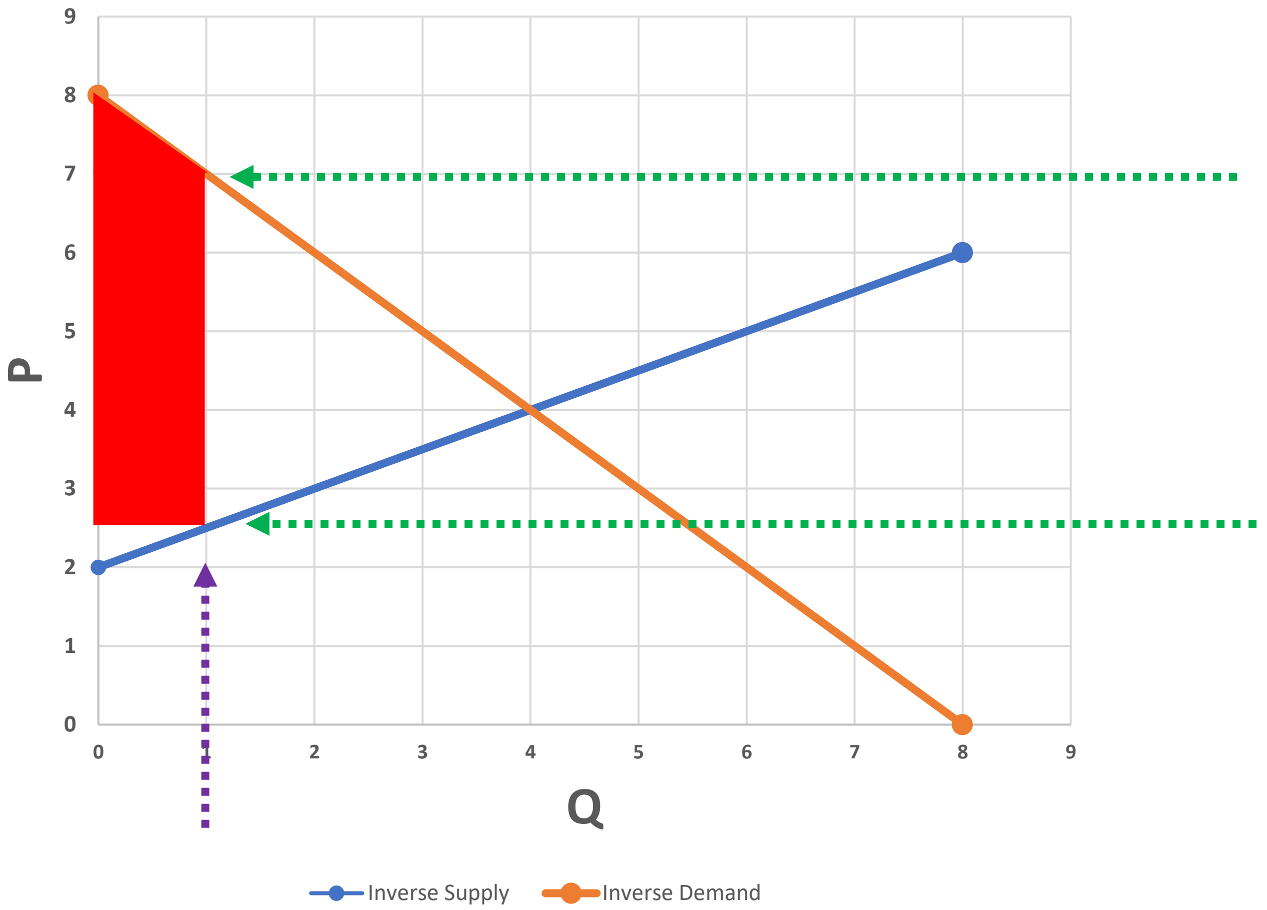
PW, Price ”in the World”


PH, Price ”at Home”
PW, Price ”in the World”
P
Q
Inverse Supply
Inverse Demand



PH, Price ”at Home”

PW, Price ”in the World”



PH, Price ”at Home”

PW, Price ”in the World”

Maximization of consumer surplus in N1, via control of the total import, Q.

$$S = \left(\frac{a}{b} - \frac{g}{h}\right)Q - \left(\frac{1}{2b} + \frac{1}{h}\right)Q^2$$

$$\frac{dS}{dQ} = \left(\frac{a}{b} - \frac{g}{h}\right) - \left(\frac{1}{b} + \frac{2}{h}\right)Q = 0$$

First order optimum condition.

$$\frac{d^2S}{dQ^2} = -\left(\frac{1}{b} + \frac{2}{h}\right) < 0$$

Second order maximum condition.
This is always satisfied,
since b and h are strictly positive.

$$\frac{dS}{dQ} = \left(\frac{a}{b} - \frac{g}{h}\right) - \left(\frac{1}{b} + \frac{2}{h}\right)Q = 0$$

$$\frac{dS}{dQ} = \left(\frac{ah - bg}{bh}\right) - \left(\frac{h + 2b}{bh}\right)Q = 0$$

$$Q = \frac{\left(\frac{ah - bg}{bh}\right)}{\left(\frac{h + 2b}{bh}\right)} = \frac{ah - bg}{2b + h} = Q^*$$

= Optimal import level.

$$Q^* = \frac{ah - bg}{2b + h}$$ = *Optimal import level.*

$$Q^F = \frac{ah - bg}{h + b}$$ = *Free trade import level.*

$$0 < \frac{Q^*}{Q^F} = \frac{\left(\dfrac{ah - bg}{2b + h}\right)}{\left(\dfrac{ah - bg}{b + h}\right)} = \frac{b + h}{2b + h} < 1$$

Important Observation:

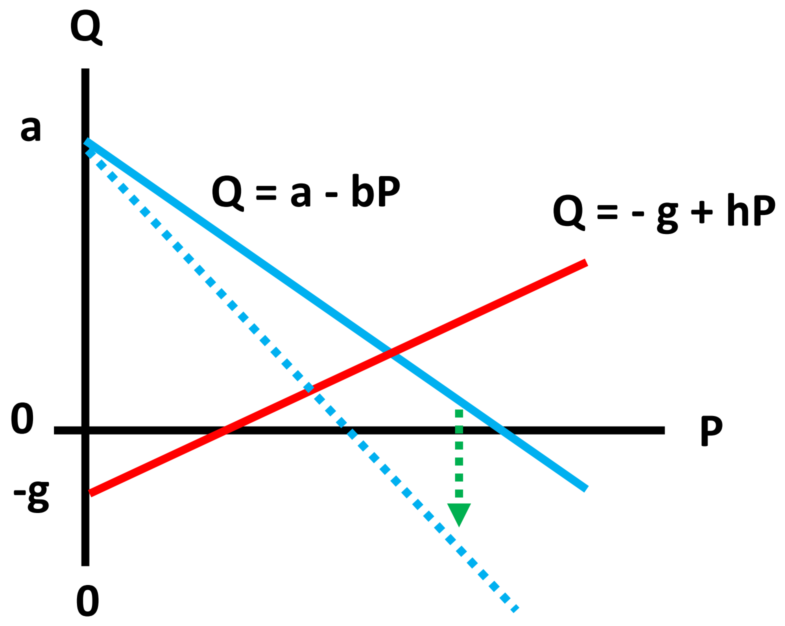
It is always optimal to import less, than according to the free trade solution, as long as the export supply function from other nations is not changed.

$$0 < z = \frac{Q^*}{Q^F} = \frac{b+h}{2b+h} < 1$$

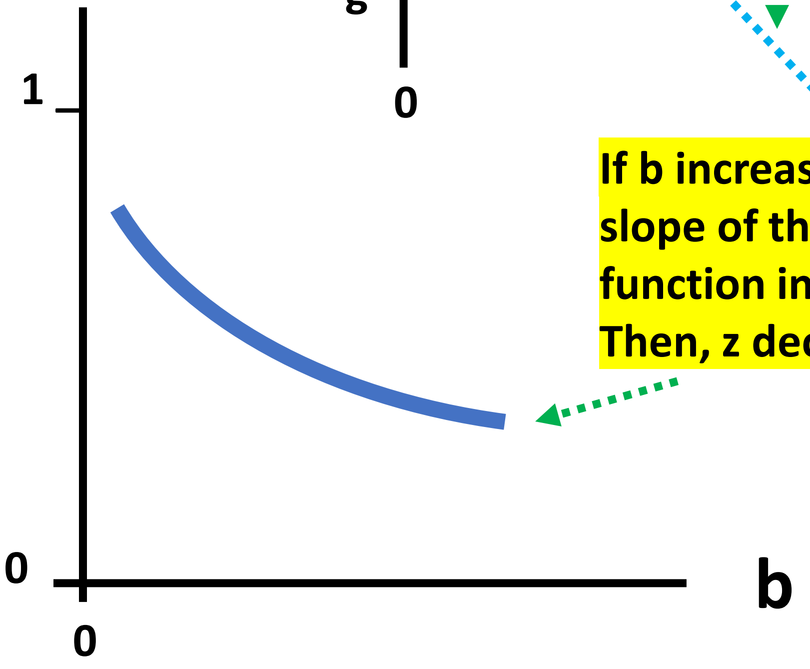
$$\frac{dz}{db} = \frac{(1)(2b+h)-(b+h)(2)}{(2b+h)^2}$$

$$\frac{dz}{db} = \frac{2b+h-2b-2h}{(2b+h)^2}$$

$$\frac{dz}{db} = \frac{-h}{(2b+h)^2} < 0$$



$$z = \frac{Q^*}{Q^F}$$



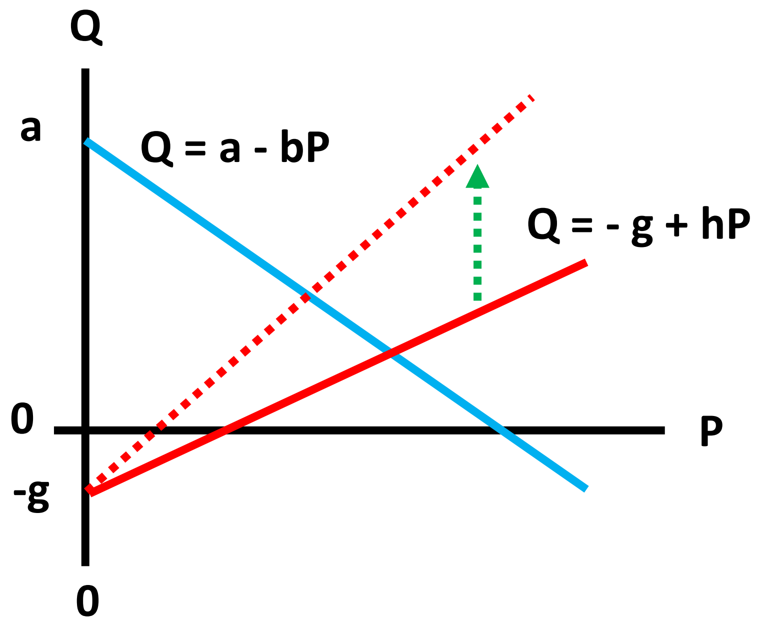
If b increases, the negative slope of the demand function increases.
Then, z decreases.

$$0 < z = \frac{Q^*}{Q^F} = \frac{b+h}{2b+h} < 1$$

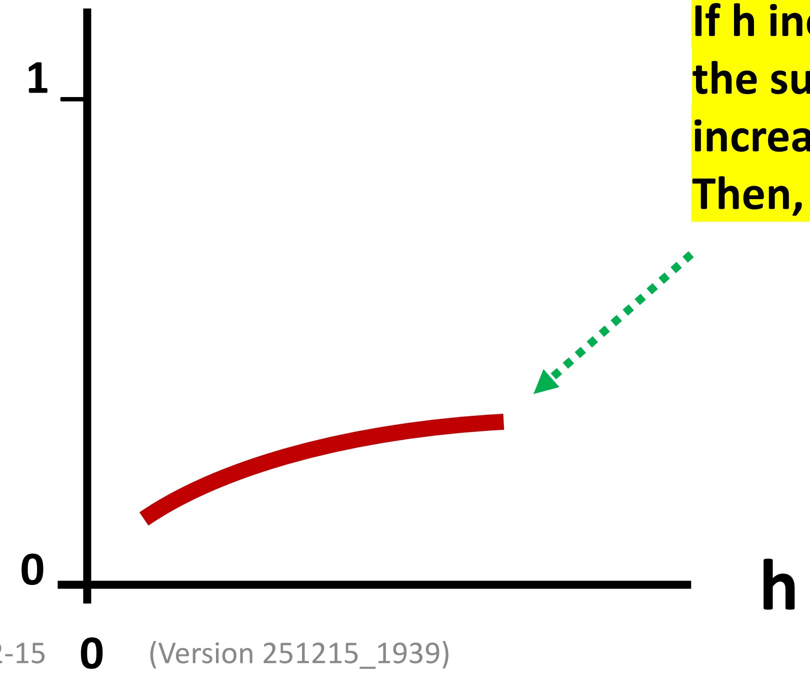
$$\frac{dz}{dh} = \frac{(1)(2b+h)-(b+h)(1)}{\left(2b+h\right)^2}$$

$$\frac{dz}{dh} = \frac{2b+h-b-h}{\left(2b+h\right)^2}$$

$$\frac{dz}{dh} = \frac{b}{\left(2b+h\right)^2} > 0$$



$$z = \frac{Q^*}{Q^F}$$



If h increases, the slope of the supply function increases.
Then, z increases.

$$Q^* = \frac{ah - bg}{2b + h}$$

= *Optimal import level.*

$$P_H^* = \frac{a}{b} - \frac{1}{b} Q^*$$

= *Optimal price for the customers at home (H).*

(This is determined from the demand function.)

$$P_W^* = \frac{g}{h} + \frac{1}{h} Q^*$$

= *Optimal price for the exporters from the outside World (W).*

(This is determined from the supply function.)

$$T^* = P_H^* - P_W^*$$

= *Optimal Tariff.*

$$T^* = \left(\frac{a}{b} - \frac{1}{b}Q^*\right) - \left(\frac{g}{h} + \frac{1}{h}Q^*\right)$$ **= Optimal Tariff.**

$$T^* = \frac{a}{b} - \frac{1}{b}Q^* - \frac{g}{h} - \frac{1}{h}Q^*$$

$$T^* = \left(\frac{a}{b} - \frac{g}{h}\right) - \left(\frac{1}{b} + \frac{1}{h}\right)Q^*$$

$$T^* = \left(\frac{ah - bg}{bh}\right) - \left(\frac{b+h}{bh}\right)\frac{(ah - bg)}{(2b+h)}$$

$$T^* = \left(\frac{ah - bg}{bh}\right) - \left(\frac{b+h}{bh}\right)\frac{(ah-bg)}{(2b+h)}$$

$$T^* = \left(\frac{ah - bg}{bh}\right)\left(1 - \frac{(b+h)}{(2b+h)}\right)$$

$$T^* = \left(\frac{ah - bg}{bh}\right)\left(\frac{(2b+h)}{(2b+h)} - \frac{(b+h)}{(2b+h)}\right)$$

$$T^* = \left(\frac{ah - bg}{bh}\right)\left(\frac{2b+h-b-h}{2b+h}\right)$$

$$T^* = \left(\frac{ah - bg}{bh}\right)\left(\frac{b}{2b+h}\right)$$

The Optimal Tariff

$$\boxed{T^* = \frac{ah - bg}{2bh + h^2}}$$

$$\left(b > 0 \wedge h > 0 \wedge ah - bg > 0\right) \Rightarrow T^{*} > 0$$

The Optimal Tariff is <u>ALWAYS</u> strictly positive as long as other nations do not have tariffs.

$$\boxed{T^{*} = \frac{ah - bg}{2bh + h^{2}}}$$

The optimal tariff and the effects of parameter changes:

$$T^* = \frac{ah - bg}{2bh + h^2}$$

$$\boxed{\frac{dT^*}{da} = \frac{1}{2b + h} > 0}$$

$$\frac{dT^*}{db} = \frac{(-g)\left(2bh + h^2\right) - (ah - bg)(2h)}{\left(2bh + h^2\right)^2}$$

$$\frac{dT^*}{db} = \frac{-2bgh - gh^2 - 2ah^2 + 2bgh}{\left(2bh + h^2\right)^2}$$

$$\boxed{\frac{dT^*}{db} = \frac{-(2a + g)h^2}{\left(2bh + h^2\right)^2} < 0}$$

$$\frac{dT^*}{dg} = \frac{(-b)\left(2bh + h^2\right) - \left(2bh + h^2\right) \times 0}{\left(2bh + h^2\right)^2}$$

$$\boxed{\frac{dT^*}{dg} = \frac{-b}{2bh + h^2} < 0}$$

$$\frac{dT^*}{dh} = \frac{(a)\left(2bh + h^2\right) - (ah - bg)(2b + 2h)}{\left(2bh + h^2\right)^2}$$

$$\frac{dT^*}{dh} = \frac{2abh + ah^2 - 2abh - 2ah^2 + 2b^2 g + 2bgh}{\left(2bh + h^2\right)^2}$$

$$\boxed{\frac{dT^*}{dh} = \frac{-ah^2 + 2b^2 g + 2bgh}{\left(2bh + h^2\right)^2}}$$

?

$$\frac{dT^*}{dh} = \frac{-ah^2 + 2b^2 g + 2bgh}{\left(2bh + h^2\right)^2}$$

(h/b) very large: $$\frac{dT^*}{dh} > 0$$

(h/b) very small: $$\frac{dT^*}{dh} < 0$$

Concrete example:

a=8, b=1, g=4, h=2

Inverse demand function:

P = (a/b) – (1/b)Q

P = 8 - Q

Inverse supply function:

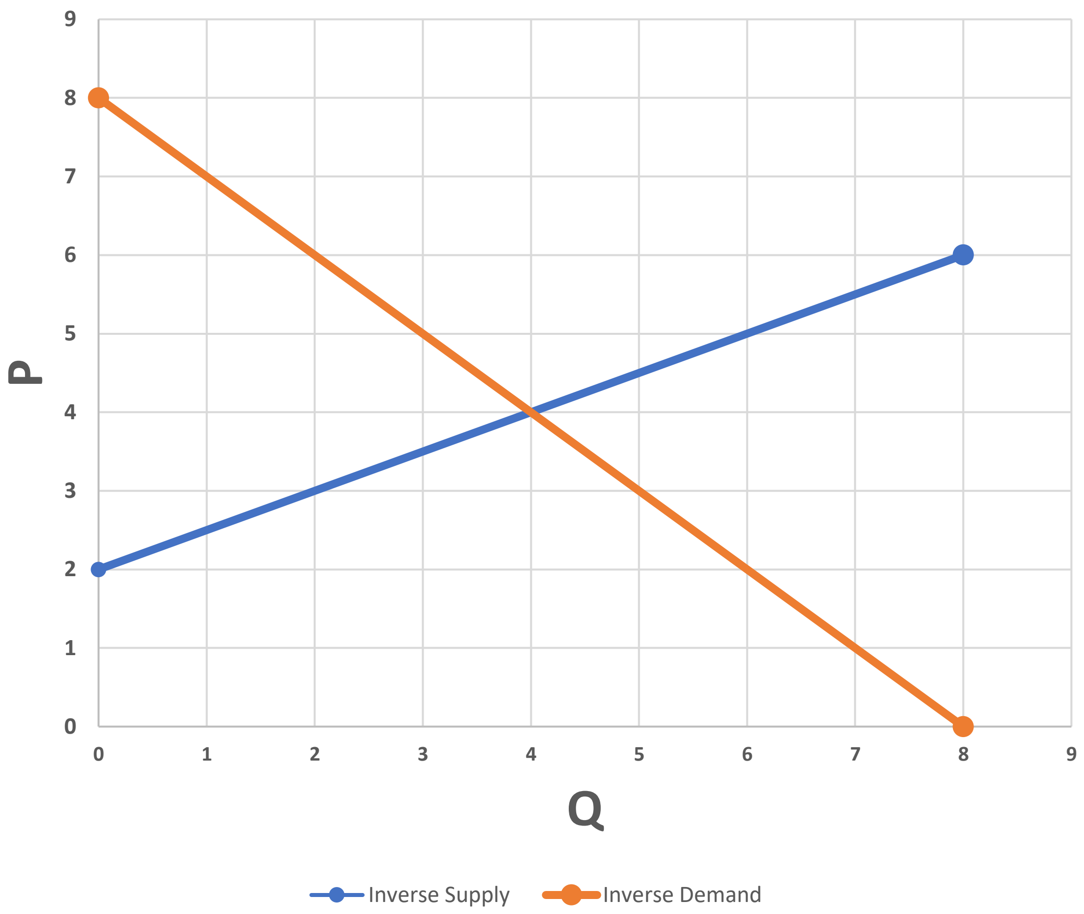
P = (g/h) – (1/h)Q

P = 2 + (1/2)Q

$$Q_D^F = \frac{ah - bg}{b + h} = \frac{16 - 4}{3} = 4$$

$$Q_S^F = \frac{ah - bg}{b + h} = 4$$

Free Trade Solution:



Concrete example:

a=8, b=1, g=4, h=2

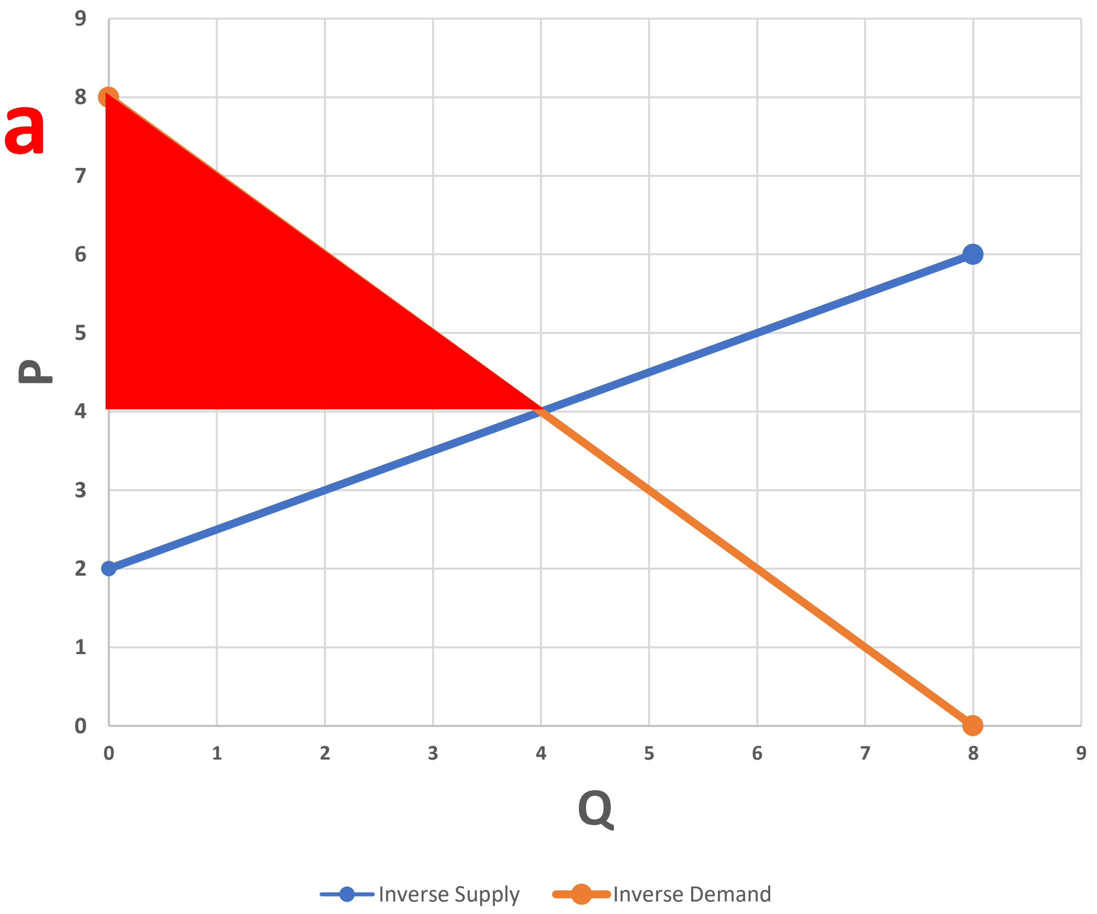
Free Trade Solution:

Consumer Surplus = Red Area = (4*4)/2 = 8

$$S=\left(\frac{a}{b}-\frac{g}{h}\right)Q-\left(\frac{1}{2b}+\frac{1}{h}\right)Q^2$$

$$S=6Q-Q^2$$

$$S=6\times 4-\left(4\right)^2=8$$



Concrete example:

a=8, b=1, g=4, h=2

The optimal import level:

$$Q^* = \frac{ah - bg}{2b + h}$$

$$Q^* = \frac{16 - 4}{2 + 2} = \frac{12}{4} = 3$$

Optimization of the import level:

Here, we optimize the import volume.

A tariff is used to reduce the import volume, which also creates a tariff revenue.

The objective function is the sum of the consumer surplus and the tariff revenue.

This maximizes the total surplus of the consumers in the importing country N1 in case the tariff revenue later is redistributed to the consumers.

Observation:** **In the real world, it is quite possible that the tariff revenue is not completely redistributed to the consumers. In such cases, it is quite possible that the consumers do not benefit from the tariff, even if the total surplus is maximized via the tariff.

Concrete example:

a=8, b=1, g=4, h=2

The optimal price "at home" in N1:

$$P_H^* = \frac{a}{b} - \frac{1}{b}Q^* = 8 - 3 = 5$$

The optimal price "in the World market":

$$P_W^* = \frac{g}{h} + \frac{1}{h}Q^* = 2 + \left(\frac{1}{2}\right)3 = 3.5$$

Concrete example:

a=8, b=1, g=4, h=2

The optimal tariff:

$$T^{*} = \frac{ah - bg}{2bh + h^{2}}$$

$$T^{*} = \frac{16 - 4}{4 + 2^{2}} = \frac{12}{8} = 1.5$$

Concrete example:

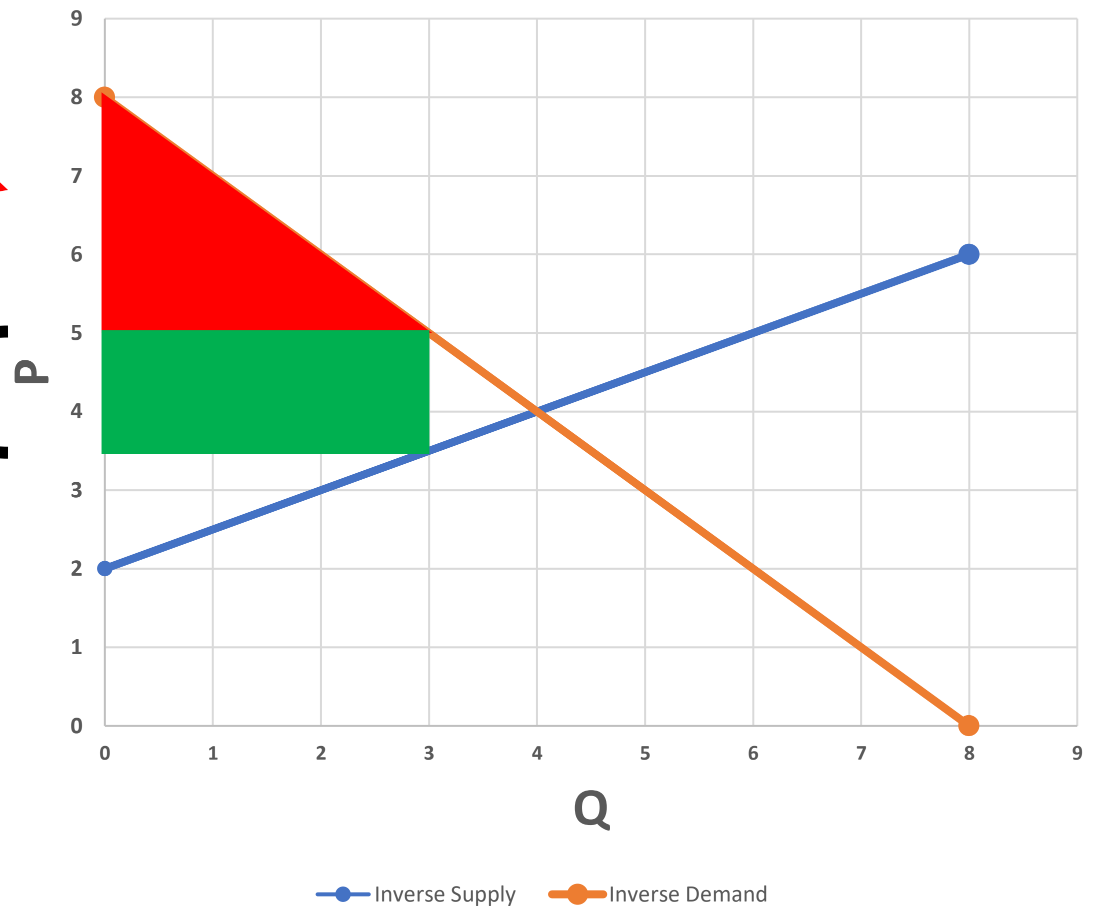
a=8, b=1, g=4, h=2

Optimal Tariff Solution:

Consumer Surplus = Red Area
= (3*3)/2 = 4.5

Tariff Gain = Green Area
= (3*1.5) = 4.5

T^*



Total Surplus = 9.

$$S = \left(\frac{a}{b} - \frac{g}{h}\right)Q - \left(\frac{1}{2b} + \frac{1}{h}\right)Q^2$$

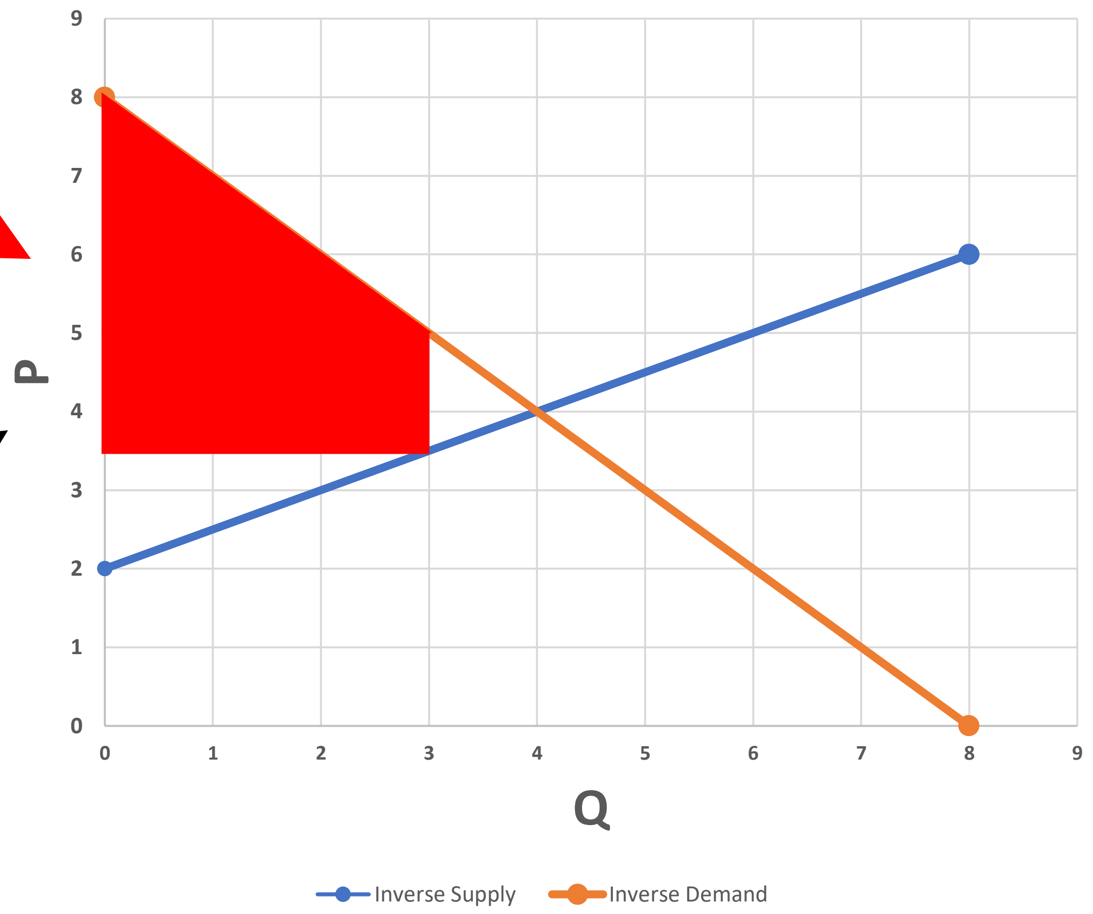
$$S = 6Q - Q^2$$

$$S = 6 \times 3 - (3)^2 = 9$$

Concrete example:
a=8, b=1, g=4, h=2

__Alternative__ solution with the same total surplus, via a Consumer Cartel, *__without__* tariffs:

Consumer Surplus = Red Area
= (3*3)/2 + 1.5*3 = __9__



$$S = \left(\frac{a}{b} - \frac{g}{h}\right)Q - \left(\frac{1}{2b} + \frac{1}{h}\right)Q^2$$

$$S = 6Q - Q^2$$

$$S = 6 \times 3 - (3)^2 = 9$$

Alternative interpretation:

$$P = \frac{C}{Q} = 2 + 0.5Q$$ (Cost per unit)

$$C = 2Q + 0.5Q^2$$ (Cost)

$$\frac{dC}{dQ} = 2 + Q$$ (Marginal cost)

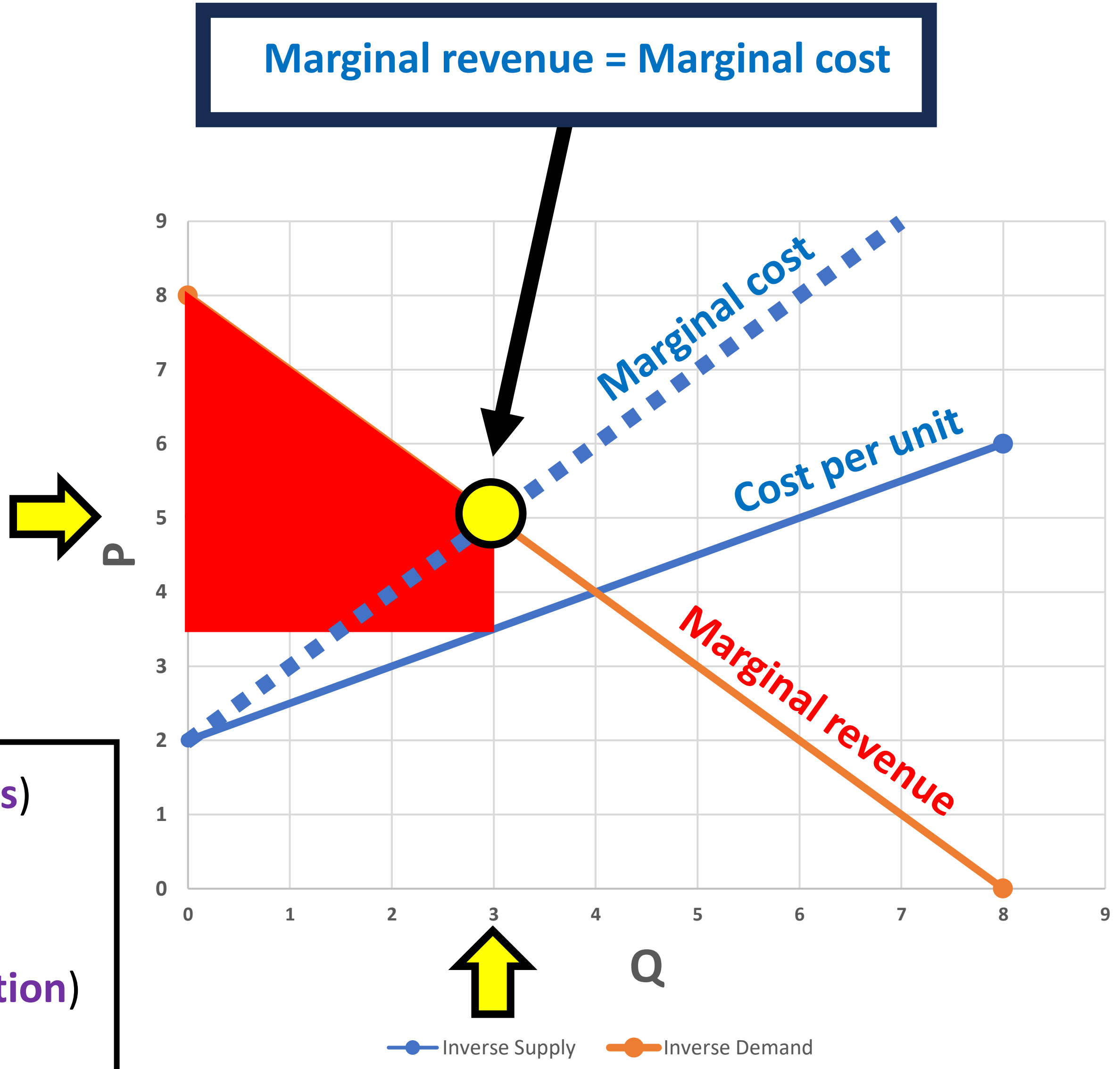
$$\Phi(Q)$$ (Revenue)

$$\max_{Q}\left(\Phi(Q) - C(Q)\right)$$ (Maximization of surplus)

$$\frac{d\Phi}{dQ} - \frac{dC}{dQ} = 0$$ (First order optimum condition)

$$\frac{d\Phi}{dQ} = \frac{dC}{dQ}$$ (Marginal revenue = Marginal cost)

Peter Lohmander 2025-12-15



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Some results from the concrete example:

The total surplus in N1 increases from 8 to 9, when the free trade solution is replaced by the optimized solution. (The consumer surplus is reduced from 8 to 4.5 and the tariff revenue increases from 0 to 4.5.)

Observation:
In the real world, it is quite possible that the tariff revenue is not completely redistributed to the consumers.
In such cases, it is quite possible that the consumers do not benefit from the tariff,
even if the total surplus is maximized via the tariff.

Free Trade Equilibrium Price = 4

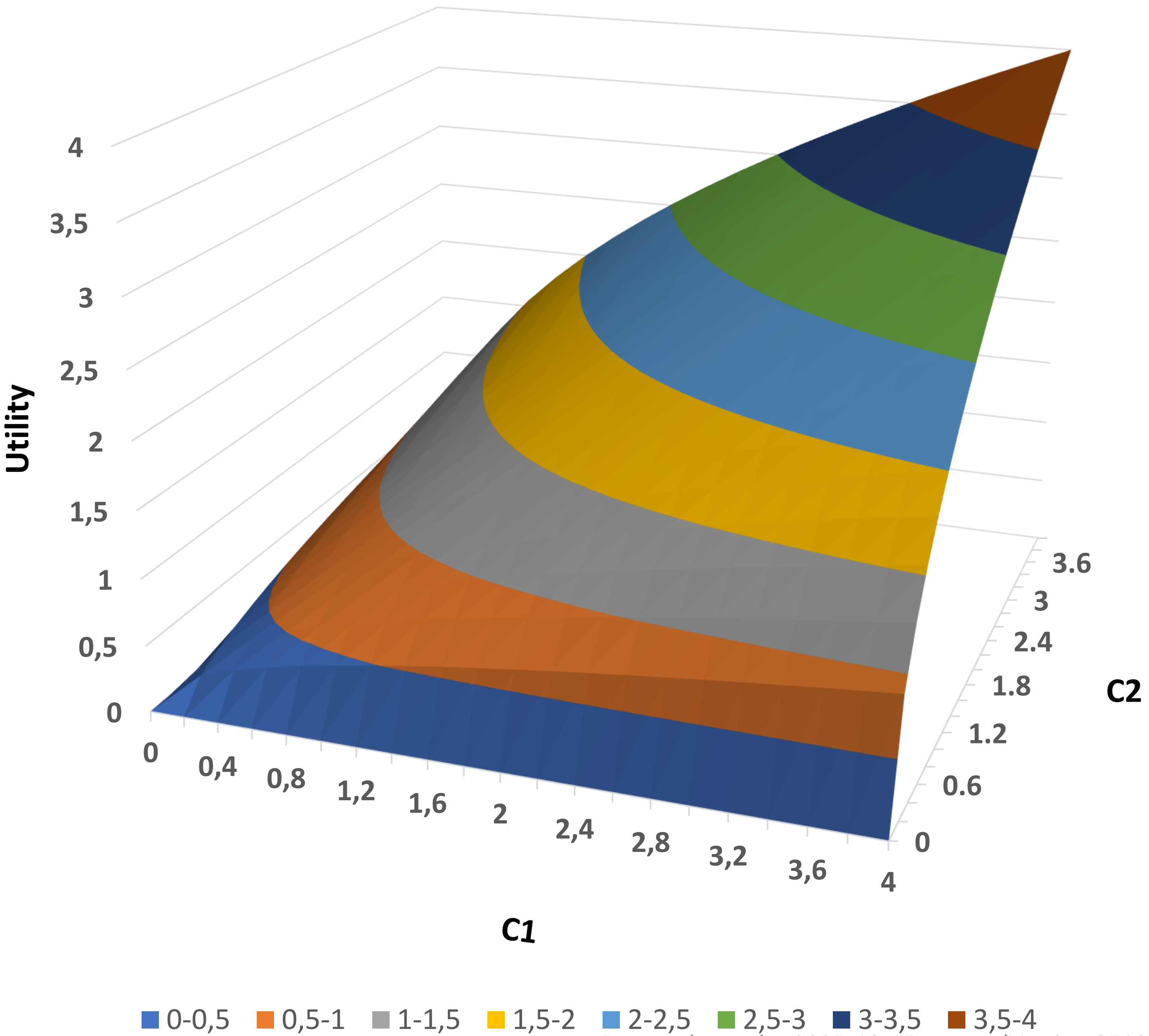
Optimal Tariff = 1.5
World Price in new equilibrium when the optimal tariff is applied = 3.5
Price in N1 when the optimal tariff is applied = 5

The Optimal Tariff = <u>37.5 %</u> of the Free Trade Equilibrium Price
The Optimal Tariff = <u>42.9 %</u> of World Price in new equilibrium when the optimal tariff is applied.

Section 2: **General equilibrium analysis, 2 Nations:**

The free trade solution.

International trade equilibrium based on optimized production and optimized consumption.

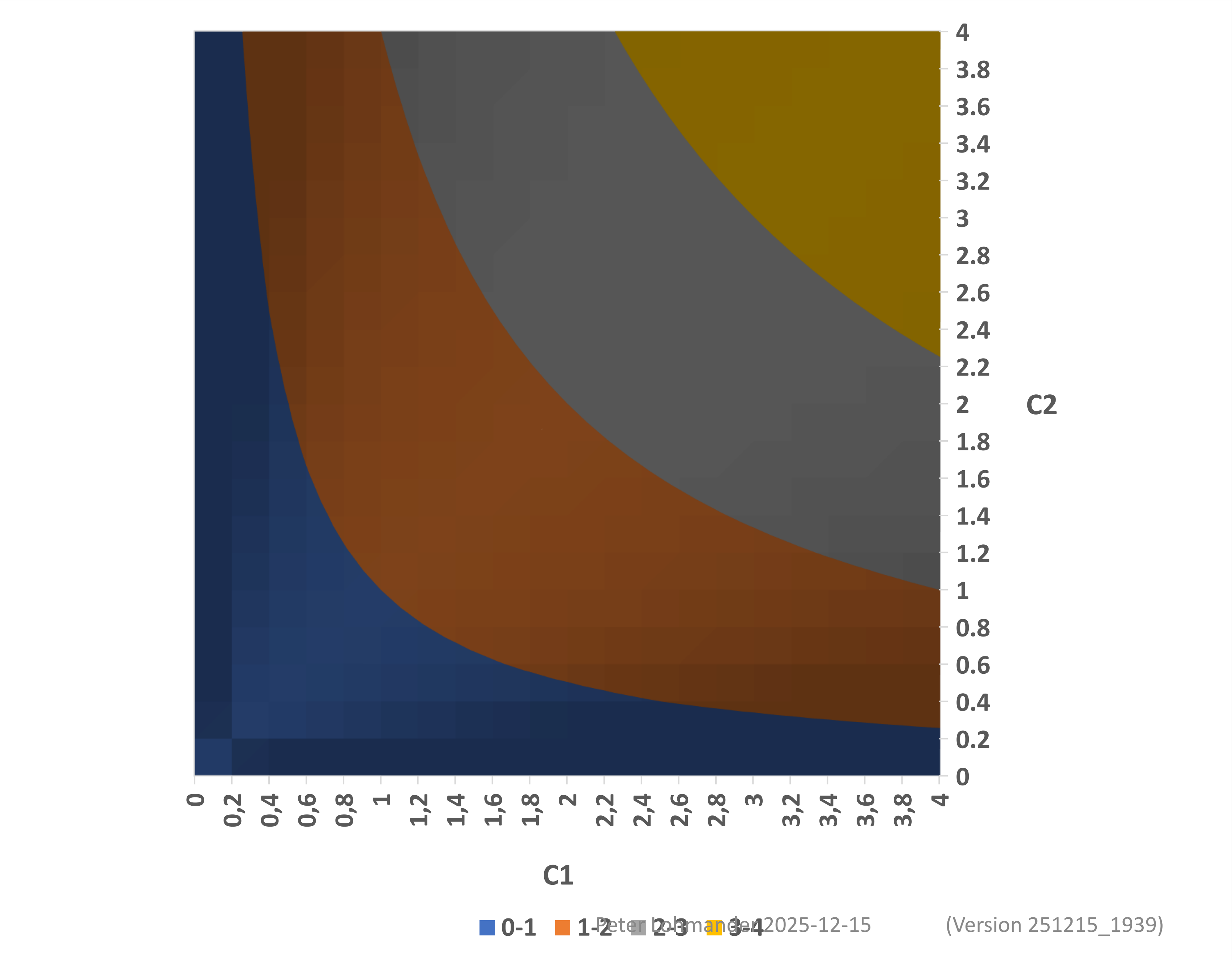


Typical utility function.

$$U = {c_1}^{\alpha_1} {c_2}^{\alpha_2}$$

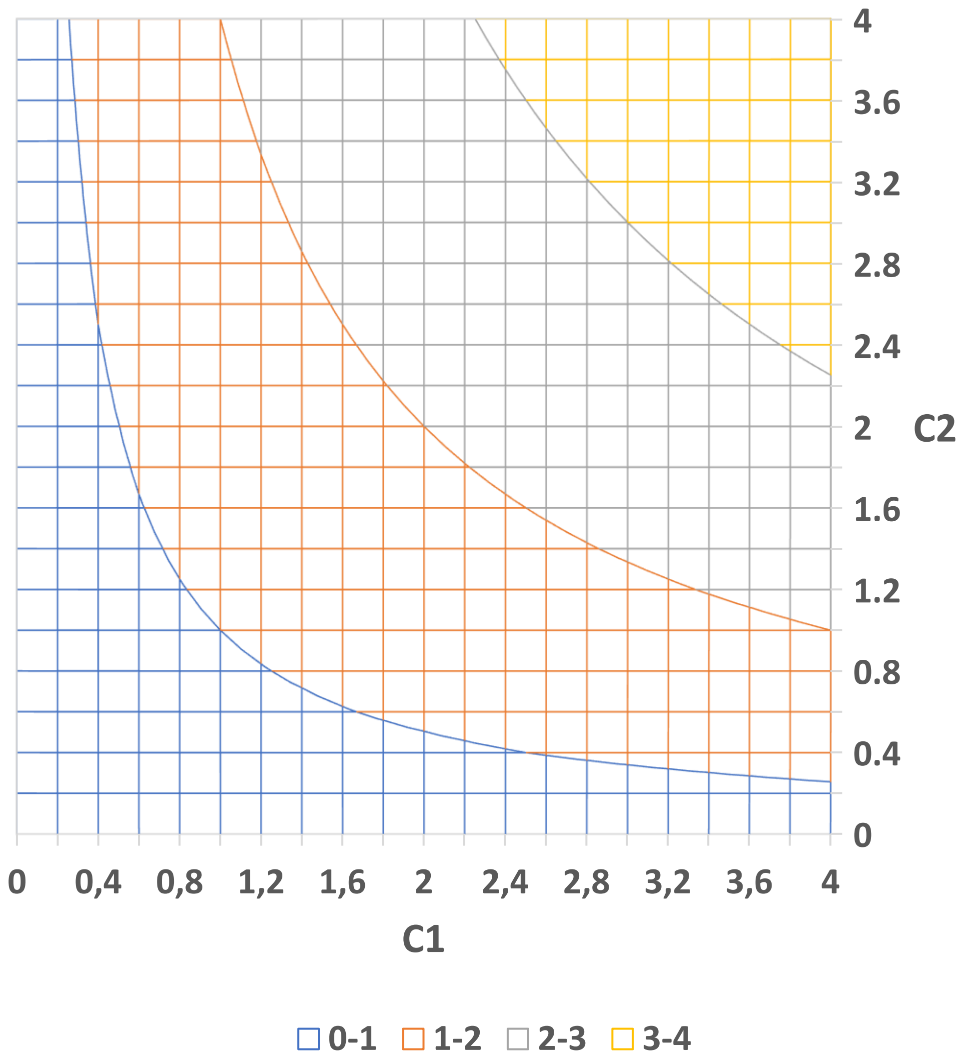
In this illustration,

$$\alpha_1 = \alpha_2 = \frac{1}{2}$$



Level curves of the utility function

(Iso utility curves)



Level curves of the utility function

(Iso utility curves)

Nonlinear analysis

Utility

Consumption levels

$$\max U_i = U_i\left(c_{i,1},..,c_{i,\eta}\right) = \kappa_i c_{i,1}{}^{\alpha_{i,1}} \times \ldots \times c_{i,\eta}{}^{\alpha_{i,\eta}}$$

$$\sum_{j=1}^{\eta} \alpha_{i,j} = 1$$

Consumption budget constraint

$$p_1 c_{i,1} + \ldots + p_\eta c_{i,\eta} \le B_i$$

(In the examples, $\kappa_i = 1$ **.)**

Lagrange function of consumption optimization:

$$L_i = \kappa_i c_{i,1}^{\ \alpha_{i,1}} \times \ldots \times c_{i,\eta}^{\ \alpha_{i,\eta}} + \lambda_i \left(B_i - p_1 c_{i,1} - \ldots - p_\eta c_{i,\eta} \right)$$

$$\left.\begin{aligned}
\frac{dL_i}{d\lambda_i} &= B_i - p_1 c_{i,1} - \ldots - p_\eta c_{i,\eta} \\
\frac{dL_i}{dc_{i,1}} &= \frac{\alpha_{i,1} U_i}{c_{i,1}} - p_1 \lambda_i \\
\frac{dL_i}{dc_{i,\eta}} &= \frac{\alpha_{i,\eta} U_i}{c_{i,\eta}} - p_\eta \lambda_i
\end{aligned}\right\} \text{First order derivatives}$$

$$
\begin{cases}
\dfrac{dL_i}{d\lambda_i} = B_i - p_1 c_{i,1} - \ldots - p_\eta c_{i,\eta} = 0 \\[2ex]
\dfrac{dL_i}{dc_{i,1}} = \dfrac{\alpha_{i,1} U_i}{c_{i,1}} - p_1 \lambda_i = 0 \\[2ex]
\bullet \\[2ex]
\dfrac{dL_i}{dc_{i,\eta}} = \dfrac{\alpha_{i,\eta} U_i}{c_{i,\eta}} - p_\eta \lambda_i = 0
\end{cases}
$$

First order optimum conditions of interior solution

$$\frac{dL_i}{dc_{i,j}} = \frac{\alpha_{i,j} U_i}{c_{i,j}} - p_j \lambda_i \quad \forall i \in \{1,...,I\}, j \in \{1,...,\eta\}$$

$$\left(\frac{dL_i}{dc_{i,j}} = 0 \right) \Rightarrow \left(\frac{\alpha_{i,j} U_i}{c_{i,j}} - p_j \lambda_i = 0 \right)$$

$$\frac{\alpha_{i,j} U_i}{c_{i,j}} = p_j \lambda_i$$

$$c_{i,j} = \frac{\alpha_{i,j} U_i}{p_j \lambda_i}$$

$$B_i = \sum_{j=1}^{\eta} p_j c_{i,j}$$

$$B_i = \sum_{j=1}^{\eta} p_j \left(\frac{\alpha_{i,j} U_i}{p_j \lambda_i} \right)$$

$$B_i = \sum_{j=1}^{\eta} \frac{\alpha_{i,j} U_i}{\lambda_i}$$

$$B_i = \frac{U_i}{\lambda_i} \sum_{j=1}^{\eta} \alpha_{i,j}$$

$$\frac{U_i}{\lambda_i} = B_i \left(\sum_{j=1}^{\eta} \alpha_{i,j} \right)^{-1} = B_i \quad for \quad \sum_{j=1}^{\eta} \alpha_{i,j} = 1$$

$$c_{i,j} = \frac{B_i \alpha_{i,j}}{p_j}, \quad \forall i, j$$

Observation:

The optimal demand (or consumption) function, in nation i, for product j, is proportional to the consumption budget and to the respective utility function exponent.

It is inversely proportional to the product price.

$$c_{i,j} = \frac{B_i \alpha_{i,j}}{p_j}, \quad \forall i, j$$

Let us consider one of the nations and two products.

The notation is simplified:

$$c_i = B \frac{\alpha_i}{p_i}$$

The optimal consumption ratio is a function of the relative prices.

Product 1: $$c_1 = B \frac{\alpha_1}{p_1}$$

Product 2: $$c_2 = B \frac{\alpha_2}{p_2}$$

$$\boxed{\frac{c_2}{c_1} = \left(\frac{\alpha_2}{\alpha_1}\right)\left(\frac{p_1}{p_2}\right)}$$

In a nation, the production possibility frontier is:

$$m_1 (x_1)^2 + m_2 (x_2)^2 \leq 1 \quad , \quad x_1 \geq 0, x_2 \geq 0$$

The parameters m1 and m2 may be different in different nations.

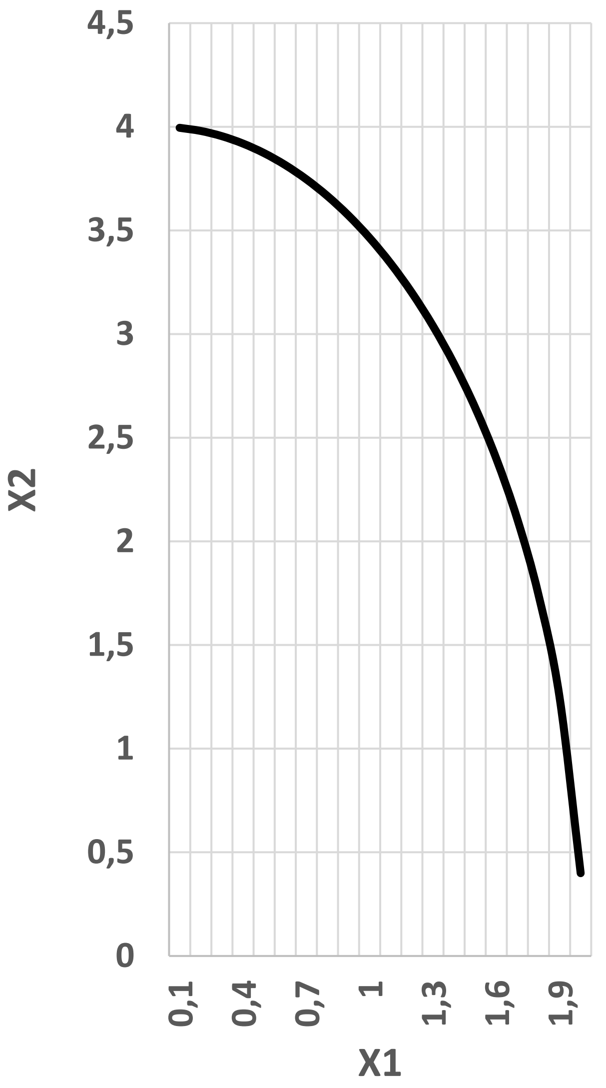
The production volumes of products 1 and 2, produced in N1, are x_{11} and x_{12}.

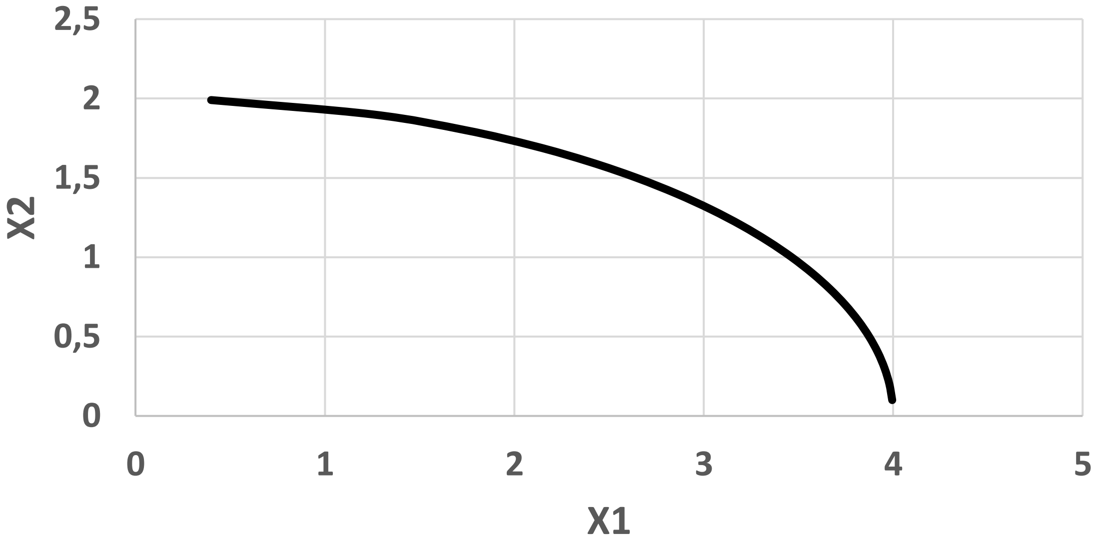
The production volumes of products 1 and 2, produced in N2, are x_{21} and x_{22}.

N1: $$m_{11} (x_{11})^2 + m_{12} (x_{12})^2 \leq 1 \quad , \quad x_{11} \geq 0, x_{12} \geq 0$$

N2: $$m_{21} (x_{21})^2 + m_{22} (x_{22})^2 \leq 1 \quad , \quad x_{21} \geq 0, x_{22} \geq 0$$

Production possibility frontiers





Nation 1

Nation 2

Production optimization:

$$\max_{x_1} \pi = p_1 x_1 + p_2 x_2$$

$$s.t. \quad m_1 \left(x_1 \right)^2 + m_2 \left(x_2 \right)^2 \leq 1 \quad , \quad x_1 \geq 0, x_2 \geq 0$$

Lagrange function of production optimization:

$$L = p_1 x_1 + p_2 x_2 + \lambda \left(1 - m_1 x_1^{\,2} - m_2 x_2^{\,2} \right)$$

The Karush- Kuhn- Tucker conditions are applied.
With full production according to the production possibility frontier, and strictly positive production of all kinds of products, the following equations hold.

$$\begin{cases} \dfrac{dL}{d\lambda} = 1 - m_1 {x_1}^2 - m_2 {x_2}^2 = 0 \\ \dfrac{dL}{dx_1} = p_1 - 2m_1\lambda x_1 \qquad = 0 \\ \dfrac{dL}{dx_2} = p_2 - 2m_2\lambda x_2 \qquad = 0 \end{cases}$$

$$\begin{cases} \dfrac{dL}{d\lambda} = 1 - m_1 x_1{}^2 - m_2 x_2{}^2 = 0 \\ \dfrac{dL}{dx_1} = p_1 - 2m_1\lambda x_1 \qquad = 0 \\ \dfrac{dL}{dx_2} = p_2 - 2m_2\lambda x_2 \qquad = 0 \end{cases}$$

$$\begin{cases} x_1 = \dfrac{p_1}{2m_1\lambda} \\ x_2 = \dfrac{p_2}{2m_2\lambda} \end{cases}$$

$$\frac{x_2}{x_1} = \frac{\left(\dfrac{p_2}{2m_2\lambda}\right)}{\left(\dfrac{p_1}{2m_1\lambda}\right)}$$

$$\frac{x_2}{x_1} = \frac{m_1}{m_2}\left(\frac{p_2}{p_1}\right)$$

$$Let\ R = \frac{p_1}{p_2}$$

The optimal production ratio is a function of the relative prices.

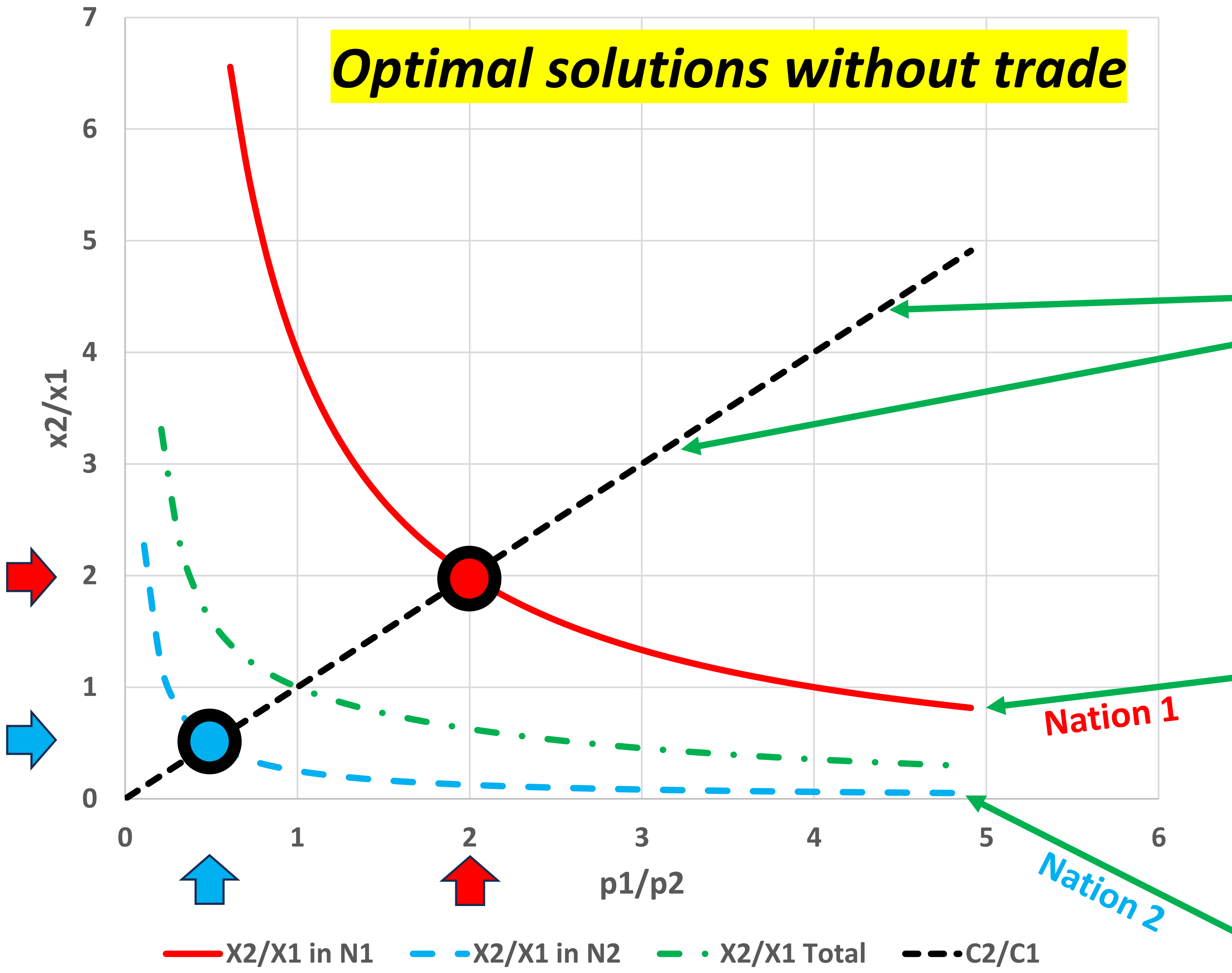
$$\boxed{\frac{x_2}{x_1} = \frac{m_1}{m_2} R^{-1}}$$

$$\frac{x_2}{x_1} = \frac{m_1}{m_2} R^{-1}$$

Optimal production ratios in the two nations:

N1: $\left(\max x_1, \max x_2\right) = (2,4) \Rightarrow \left(m_1, m_2\right) = \left(\frac{1}{4}, \frac{1}{16}\right)$ $\Rightarrow$ $\boxed{\frac{x_2}{x_1} = 4R^{-1}}$

N2: $\left(\max x_1, \max x_2\right) = (4,2) \Rightarrow \left(m_1, m_2\right) = \left(\frac{1}{16}, \frac{1}{4}\right)$ $\Rightarrow$ $\boxed{\frac{x_2}{x_1} = \frac{1}{4} R^{-1}}$

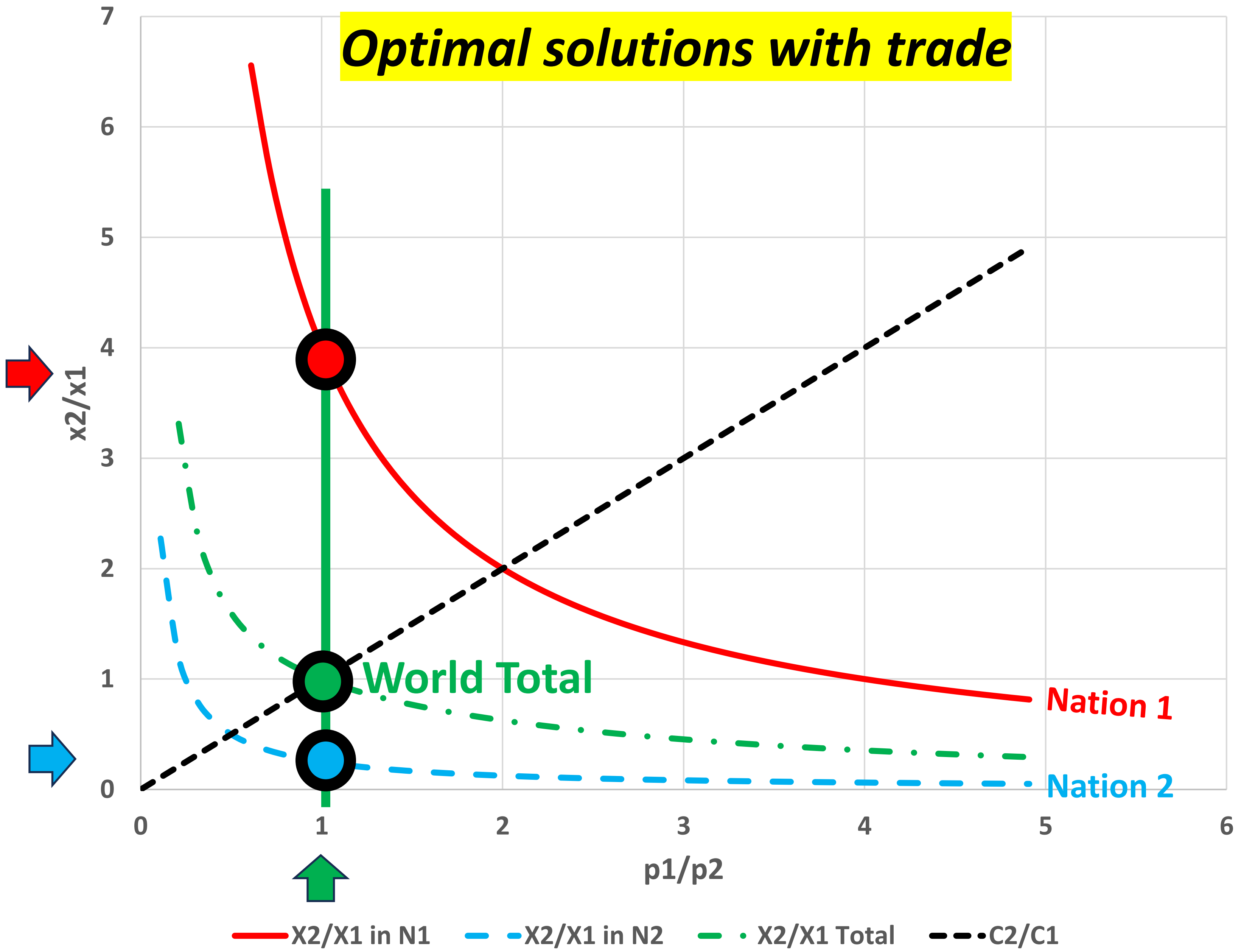


The optimal consumption ratio is a function of the relative prices.

$$\frac{c_2}{c_1} = \left(\frac{\alpha_2}{\alpha_1}\right)\left(\frac{p_1}{p_2}\right)$$

The optimal production ratio is a function of the relative prices.

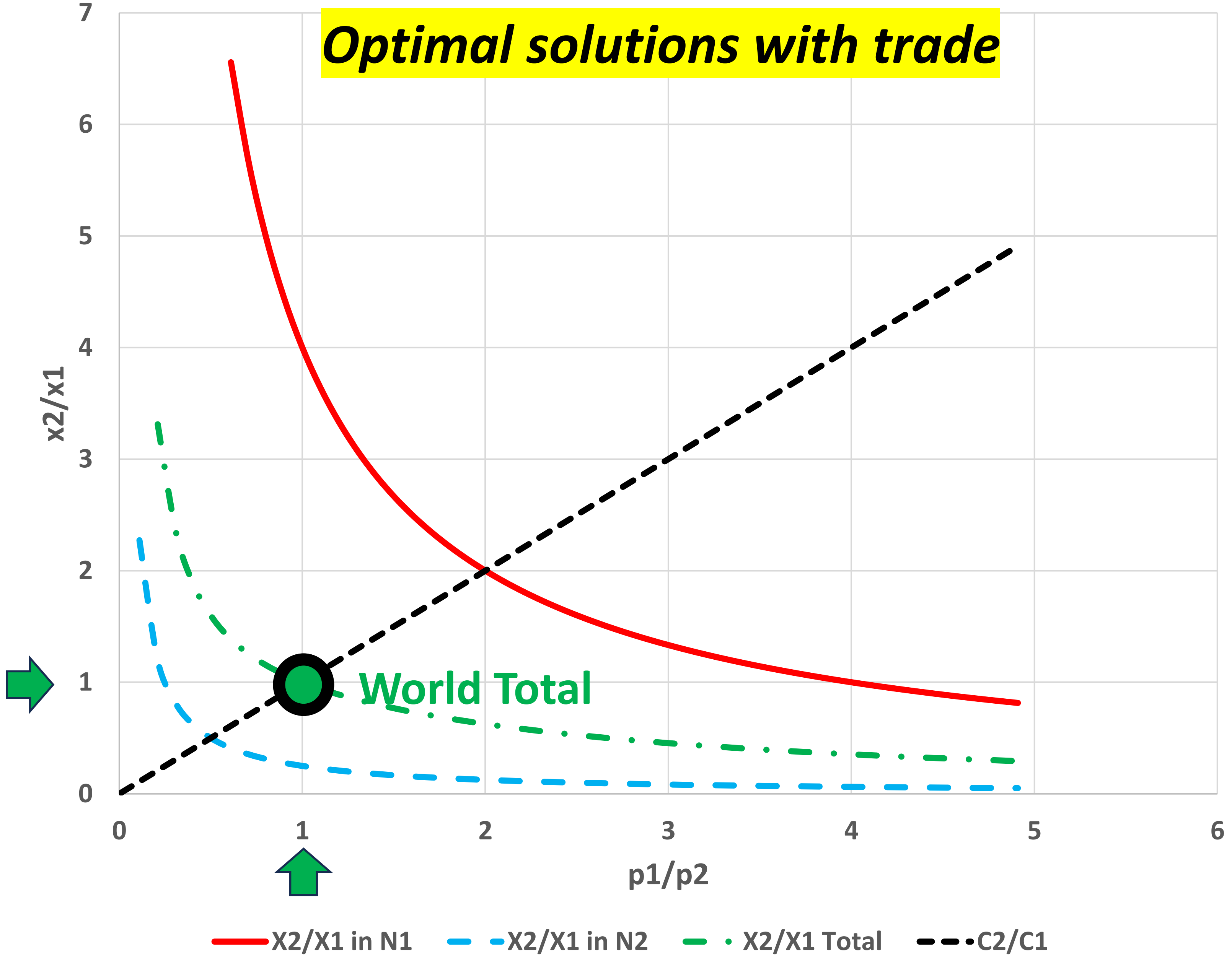
$$\frac{x_2}{x_1} = \frac{m_1}{m_2} R^{-1}$$



The equilibrium relative price in the world, with free trade, is found via the point denoted ”World Total”.

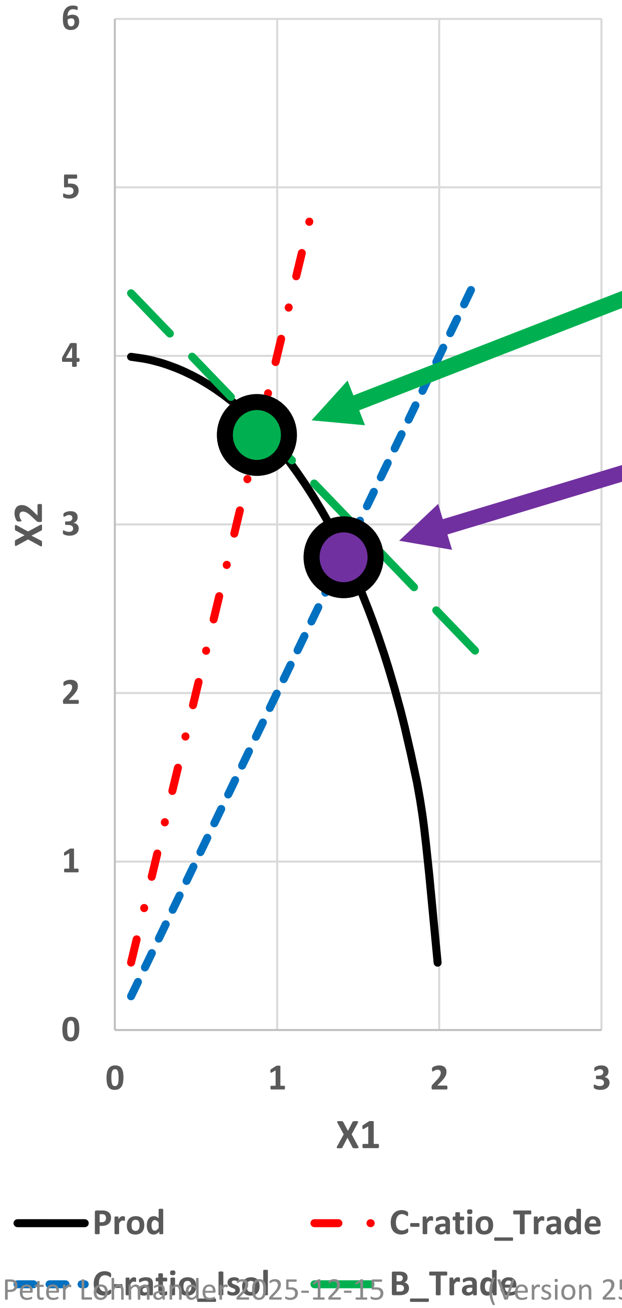
This equilibrium relative price, which is ”1” in the example, determines the optimal relative production levels in the different nations.

”x2/x1” increases in Nation 1 and decreases in Nation 2 when free trade is introduced.



Optimal solutions with trade
World Total
x2/x1
p1/p2
0
1
2
3
4
5
6
7
X2/X1 in N1
X2/X1 in N2
X2/X1 Total
C2/C1

Nation 1

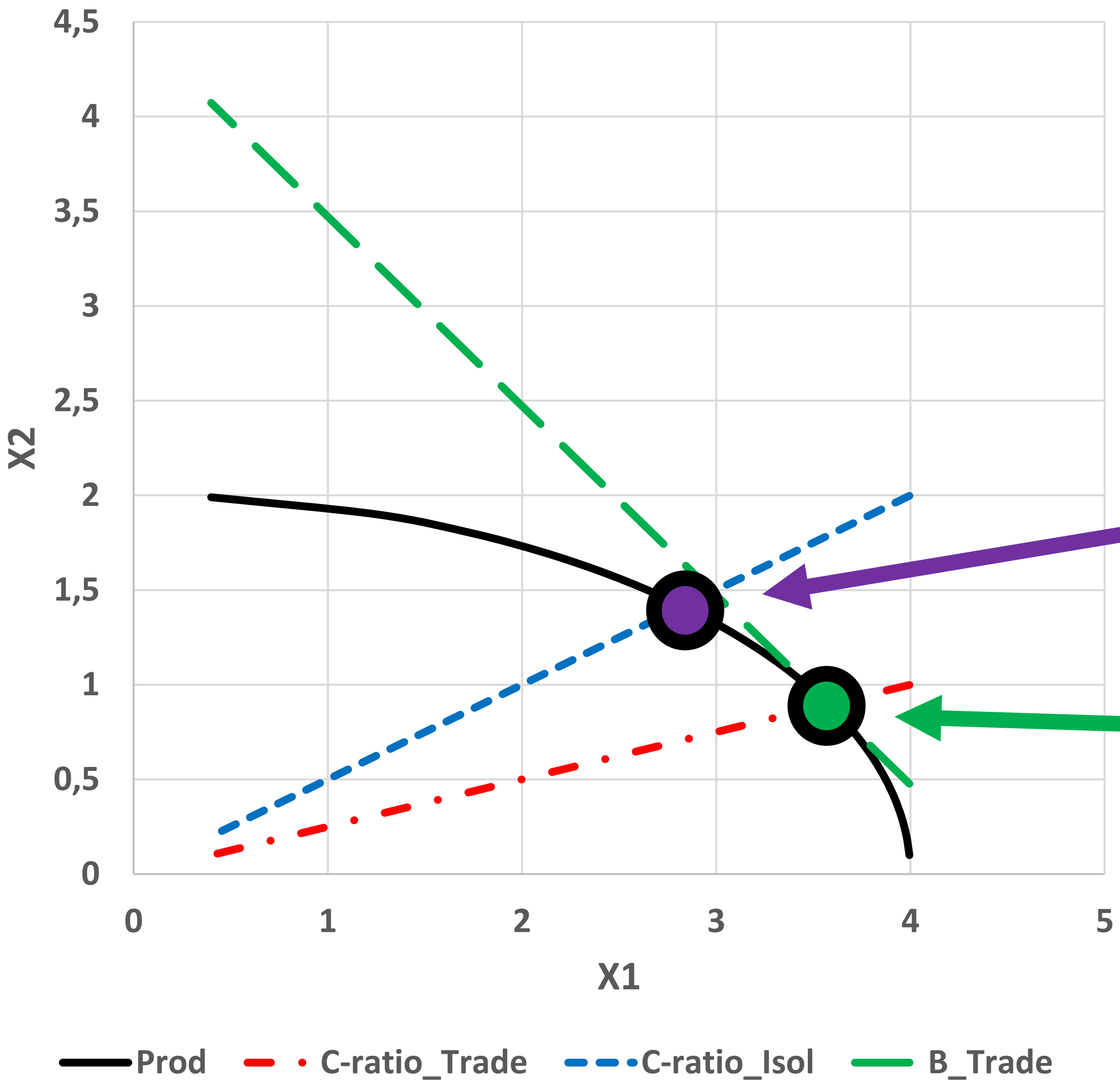


Optimal production with free trade

Optimal production with no trade

Free trade leads to increased "Specialization".

Nation 2



Optimal production with no trade

Optimal production with free trade

Free trade leads to increased "Specialization".

Mathematical determination of the optimal absolute production volumes in the different countries:

$$\boxed{k = \frac{x_2}{x_1} = \frac{m_1}{m_2} R^{-1}} \Rightarrow \boxed{x_2 = k\, x_1}$$

$$m_1 (x_1)^2 + m_2 (x_2)^2 = 1$$

$$m_1 (x_1)^2 + m_2 (k\, x_1)^2 = 1$$

$$\left(m_1 + m_2 k^2\right)(x_1)^2 = 1$$

The production possibility frontier

$$\left(m_1 + m_2 k^2\right)\left(x_1\right)^2 = 1$$

$$x_1^{\ 2} = \frac{1}{\left(m_1 + m_2 k^2\right)}$$

$$x_1 = \sqrt{\frac{1}{\left(m_1 + m_2 k^2\right)}}$$

Optimal absolute production volume

$$\boxed{x_1 = \left(m_1 + m_2 k^2\right)^{-\left(\frac{1}{2}\right)}}$$

$$k = \frac{x_2}{x_1} = \frac{m_1}{m_2} R^{-1}$$

$$x_1 = k^{-1} x_2$$

$$m_1 \left(k^{-1} x_2 \right)^2 + m_2 \left(x_2 \right)^2 = 1$$

$$m_1 k^{-2} \left(x_2 \right)^2 + m_2 \left(x_2 \right)^2 = 1$$

$$\left(m_1 k^{-2} + m_2 \right) \left(x_2 \right)^2 = 1$$

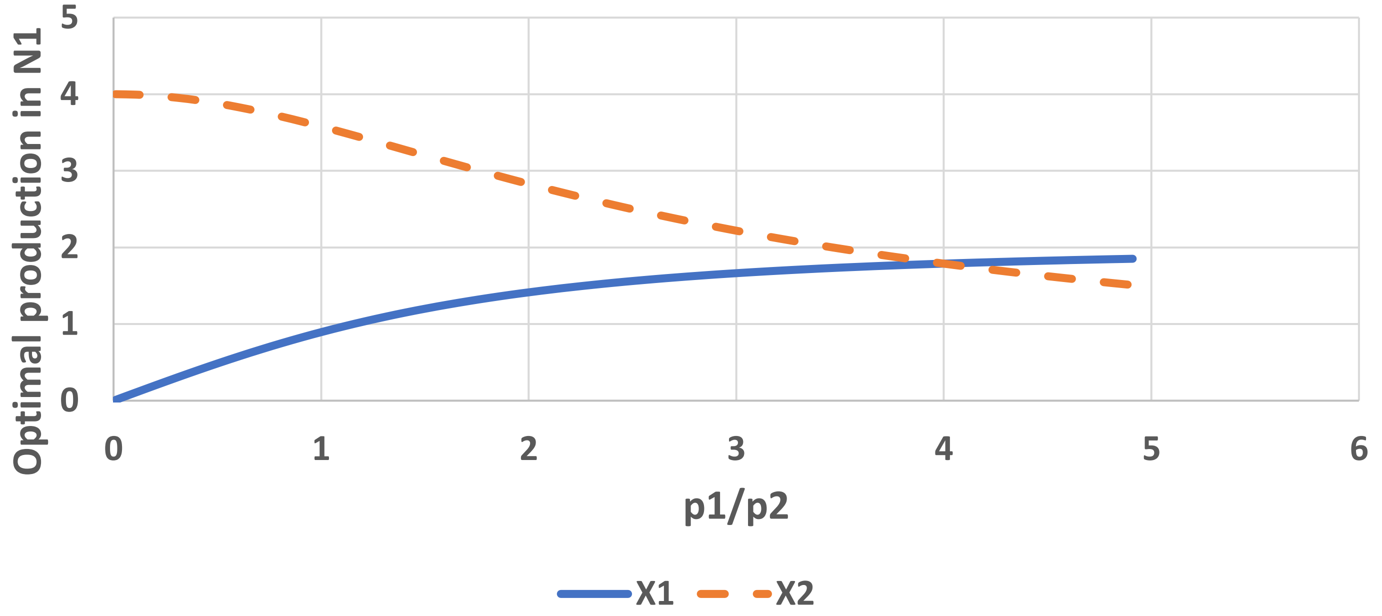
$$\left(m_1 k^{-2} + m_2\right)\left(x_2\right)^2 = 1$$

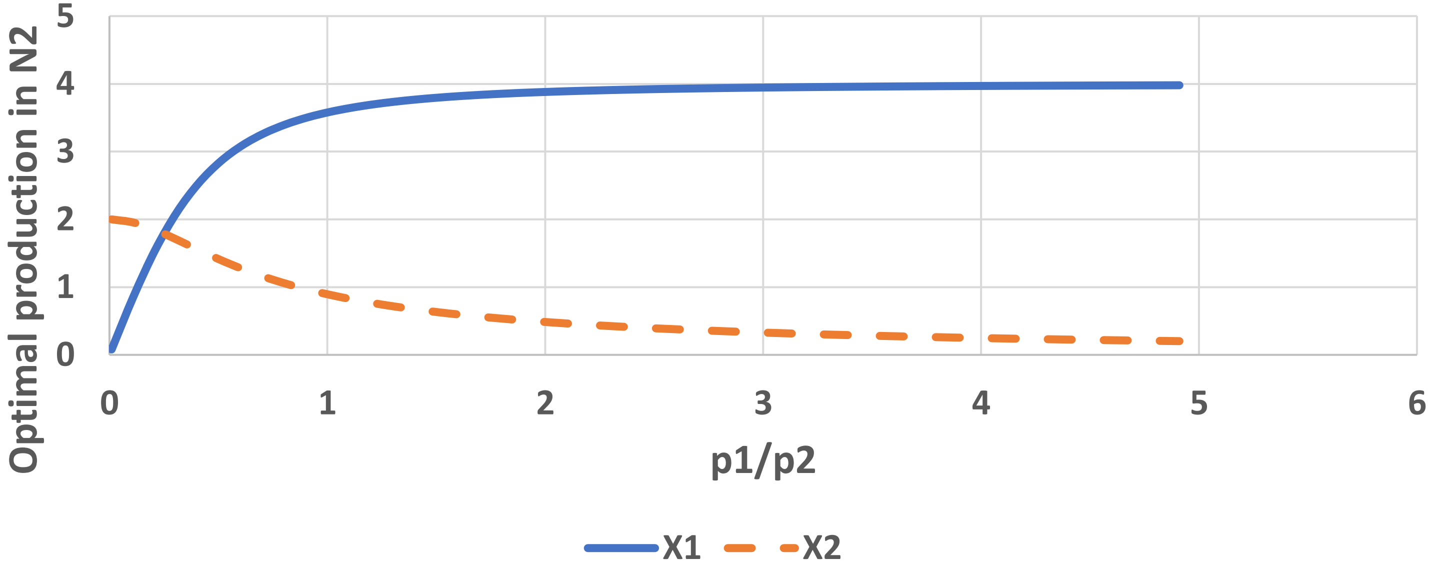
$$x_2^{\ 2} = \frac{1}{\left(m_1 k^{-2} + m_2\right)}$$

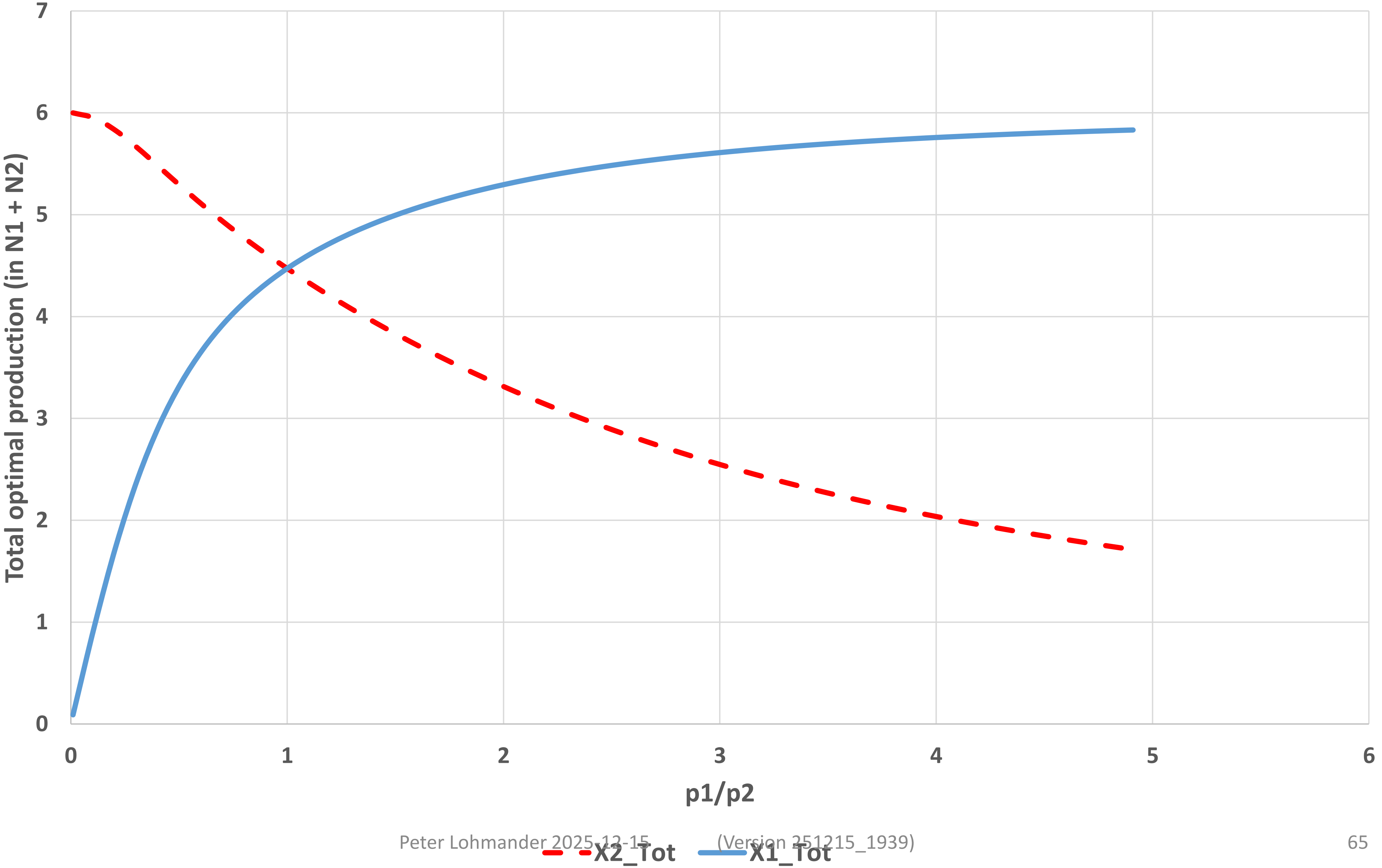
$$x_2 = \sqrt{\frac{1}{\left(m_1 k^{-2} + m_2\right)}}$$

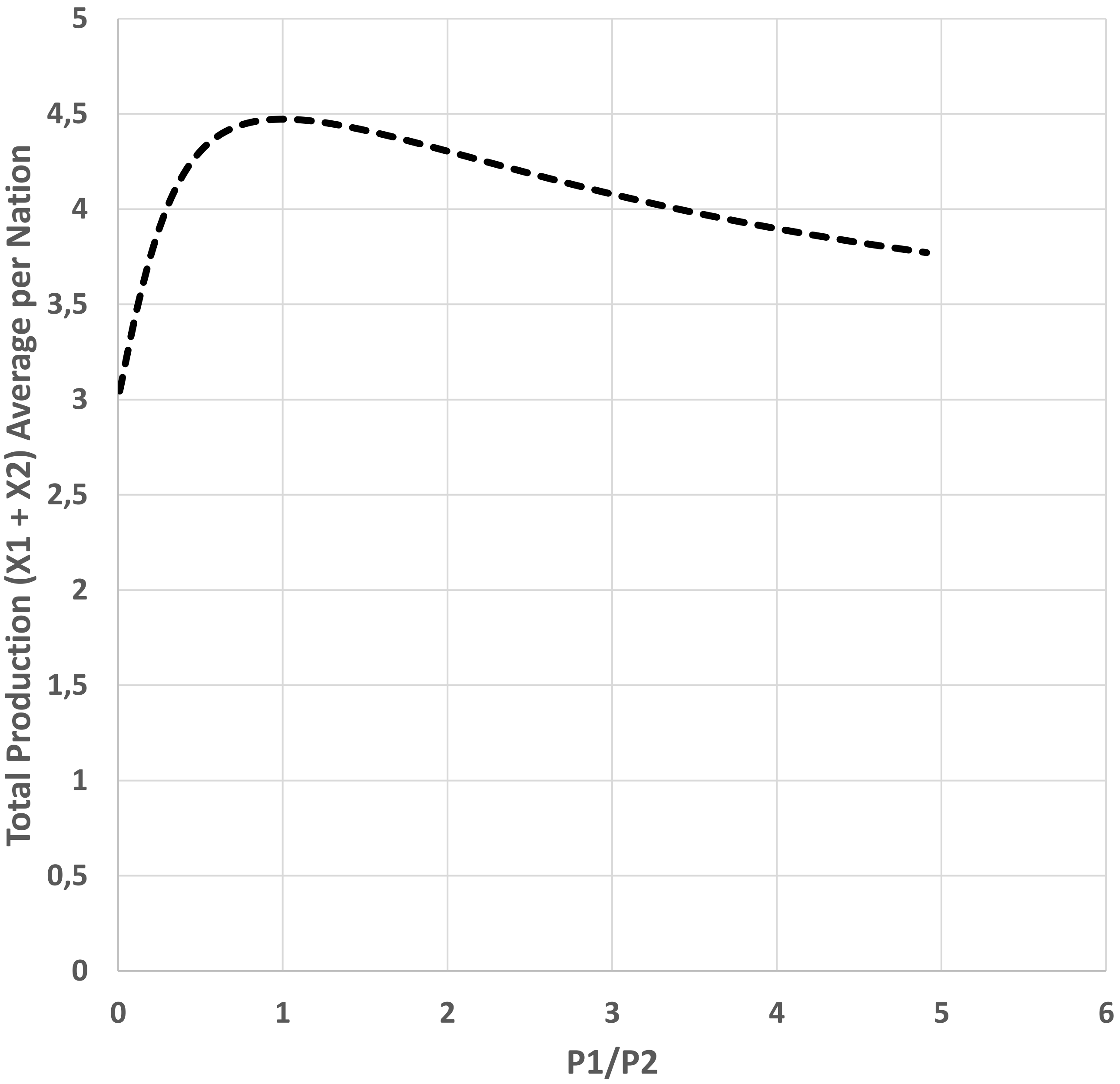
Optimal absolute production volume

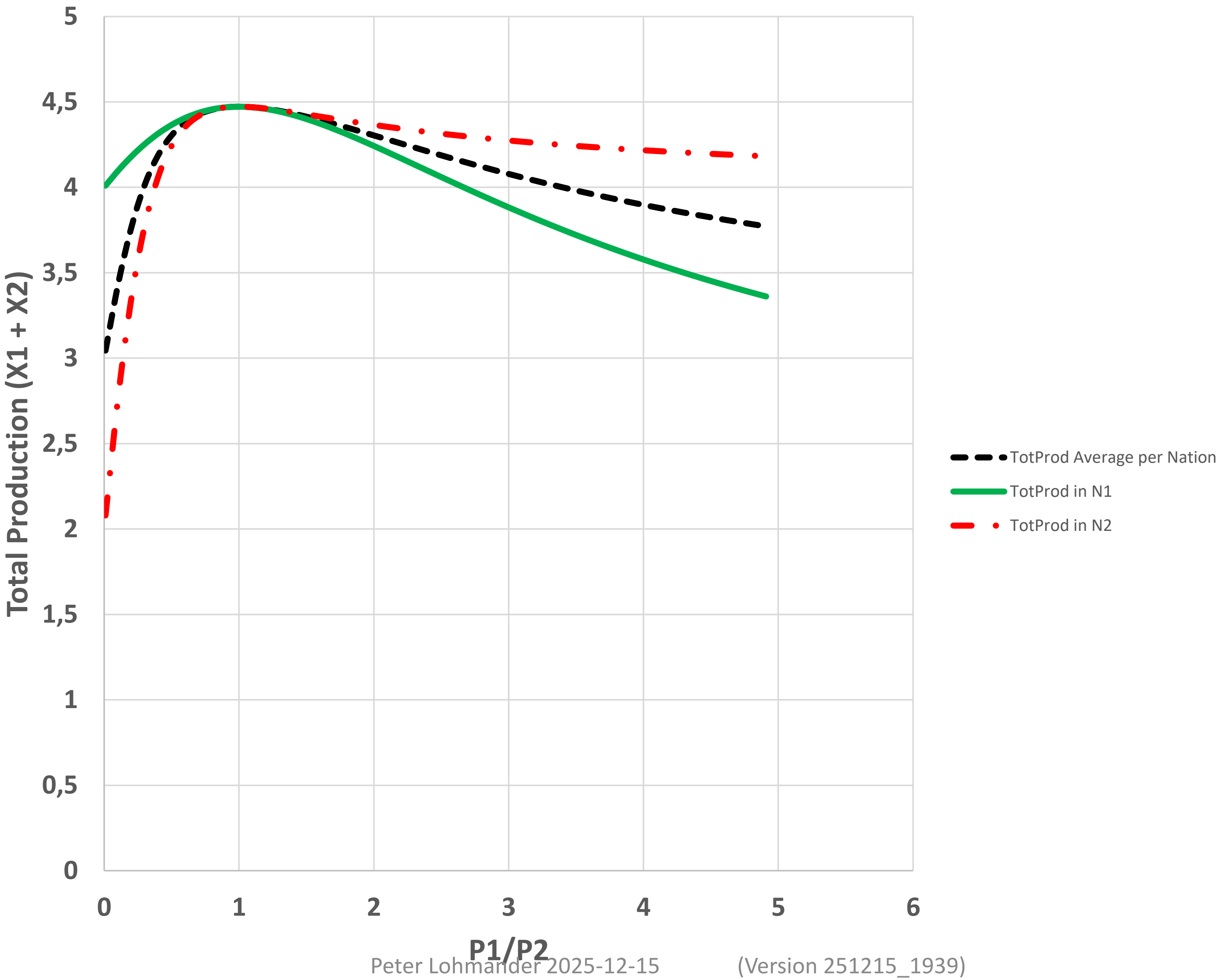
$$\boxed{x_2 = \left(m_1 k^{-2} + m_2\right)^{-\left(\frac{1}{2}\right)}}$$


Optimal production in N1
0
1
2
3
4
5
0
1
2
3
4
5
6
p1/p2
X1
X2


Optimal production in N2
0
1
2
3
4
5
0
1
2
3
4
5
6
p1/p2
X1
X2


Total optimal production (in N1 + N2)
p1/p2
0
1
2
3
4
5
6
7
X2_Tot
X1_Tot


5
4,5
4
3,5
3
2,5
2
1,5
1
0,5
0
0
1
2
3
4
5
6
Total Production (X1 + X2) Average per Nation
P1/P2



5
4,5
4
3,5
3
2,5
2
1,5
1
0,5
0
0
1
2
3
4
5
6
Total Production (X1 + X2)
P1/P2
TotProd Average per Nation
TotProd in N1
TotProd in N2

Consumption levels in the optimized system in equilibrium:

	Nation 1 No trade	Nation 1 Free trade	Nation 2 No trade	Nation 2 Free trade
c1	1.414	2.236	2.828	2.236
c2	2.828	2.236	1.414	2.236

Utility levels in the optimized system in equilibrium:

	Nations 1 and 2, No trade **Utility function exponents = (0.5, 0.5)**	Nations 1 and 2, Free trade **Utility function exponents = (0.5, 0.5)**	Nations 1 and 2, No trade **Utility function exponents = (2.0, 2.0)**	Nations 1 and 2, Free trade **Utility function exponents = (2.0, 2.0)**
u1	2.000	2.236	16	25
u2	2.000	2.236	16	25

Utility levels in the optimized system in equilibrium increase when free trade is introduced.

The relative utility increase is very sensitive to the values of the exponents in the utility functions.

If these exponents are low, (= 0.5), then the utility levels increase by 11.8034 % when free trade is introduced.

If these exponents are high, (= 2.0), then the utility levels increase by 56.25 % when free trade is introduced.

Section 3: **Optimal tariffs without and with coordination, general equilibrium analysis, 2 Nations, with tariffs.**

The trade solution with tariffs.

International trade equilibrium based on optimized production and optimized consumption.

One or both nations introduce tariffs on imports.

Dynamic, nonlinear, non-zero sum, game theory. Nash equilibrium and cooperative solution.

The optimal tariffs and utility changes in both nations, with/without cooperation.

T1 is the relative tariff in Nation 1 on imports of product 2.

T2 is the relative tariff in Nation 2 on imports of product 1.

RELP is the relative price p2/p1, where p1 and p2 are the world market prices of the products 1 and 2.

p3 is the price of product 1 inside Nation 1 as a result of the world market price p1 and Tariff_1 in Nation 1 on imported product 1.

p4 is the price of product 2 inside Nation 2 as a result of the world market price p2 and Tariff_2 in Nation 2 on imported product 2.

Tariff_1 = p1 * T1
p3 = p1 + Tariff_1

Tariff_2 = p2 * T2
p4 = p2 + Tariff_2

Relative prices in N1 and in N2.

RELP_N1 = p2 / p3

RELP_N2 = p4 / p1

x11, x12, x21 and x22 are the optimal production levels in N1 and in N2.

Obs!
These four equations are consistent with the earlier derived equations.

x11 = (1 / (m11 * (m11 / m12 * RELP_N1 ^ (2) + 1))) ^ .5

x12 = (m12 ^ (-1) - 1 / (m11 * RELP_N1 ^ (2) + m12)) ^ .5

x21 = (1 / (m21 * (m21 / m22 * RELP_N2 ^ (2) + 1))) ^ .5

x22 = (m22 ^ (-1) - 1 / (m21 * RELP_N2 ^ (2) + m22)) ^ .5

For each combination (T1, T2):

The consumption levels are functions of the consumption budgets in N1 and N2.

The consumption budgets in N1 and N2 are functions of the tariff revenues.

The tariff revenues are functions of the import volumes.

The import volumes are functions of the consumption levels.

For this reason, fix point iteration is used to determine the equilibrium.

This is done via the i-loop, found below.

The 100 steps give a solution very close to the equilibrium.

For each combination (T1, T2):

In one loop, A, the relative prices are adjusted,
until the total world excess supply is zero (for every product).

In a second loop, B, within A, for each relative price level,
fix point iteration is used to determine approximate solutions to all production levels, consumption levels, all consumption budgets, imports and exports.

RESULTS from the output file

The matrix W1(T1_index, T2_index)

W1(T1,T2) = (U1(T1,T2)-U1(0,0))*10000

OBS! The tariff value is 0.2 x index.

T2_index

		T2 = 0 %			60%							
T1	T1_index	0	1	2	3	4	5	6	7	8	9	10
0 %	0	0	-579	-1017	-1351	-1602	-1797	-1947	-2064	-2153	-2221	-2273
	1	504	-158	-661	-1056	-1358	-1605	-1798	-1953	-2077	-2179	-2260
	2	750	18	-551	-991	-1346	-1631	-1856	-2047	-2202	-2327	-2435
60 %	3	818	22	-599	-1086	-1475	-1793	-2056	-2267	-2446	-2595	-2721
	4	765	-89	-751	-1280	-1704	-2050	-2335	-2567	-2767	-2930	-3069
	5	629	-278	-982	-1542	-1989	-2361	-2663	-2919	-3130	-3310	-3460
	6	427	-522	-1259	-1851	-2320	-2712	-3029	-3294	-3519	-3714	-3872
	7	186	-801	-1573	-2182	-2675	-3079	-3415	-3692	-3924	-4125	-4296
	8	-89	-1112	-1908	-2538	-3046	-3467	-3807	-4094	-4337	-4542	-4718
	9	-392	-1440	-2259	-2907	-3429	-3856	-4205	-4501	-4747	-4956	-5136
	10	-703	-1781	-2614	-3278	-3811	-4245	-4600	-4898	-5152	-5364	-5547

Determination of an approximation W1(T1,T2)

Hypothesis: **The function can be approximated by a multivariate Taylor polynomial of the second order. (A quadratic polynominal can be used.)**

$$W_1(T_2, T_2) = k_1 T_1 + k_{11} T_1{}^2 + k_2 T_2 + k_{12} T_1 T_2 + k_{22} T_2{}^2$$

$$W_1 = k_1 T_1 + k_{11} T_1^2 + k_2 T_2 + k_{12} T_1 T_2 + k_{22} T_2^2$$

Regression analysis based on the numerically determined values of W1(T1,T2)

Regression Statistics	
Multiple R	**0.999251068**
R Square	**0.998502696**
Adjusted R Square	**0.966051431**
Standard Error	**50.31183919**
Observations	**36**

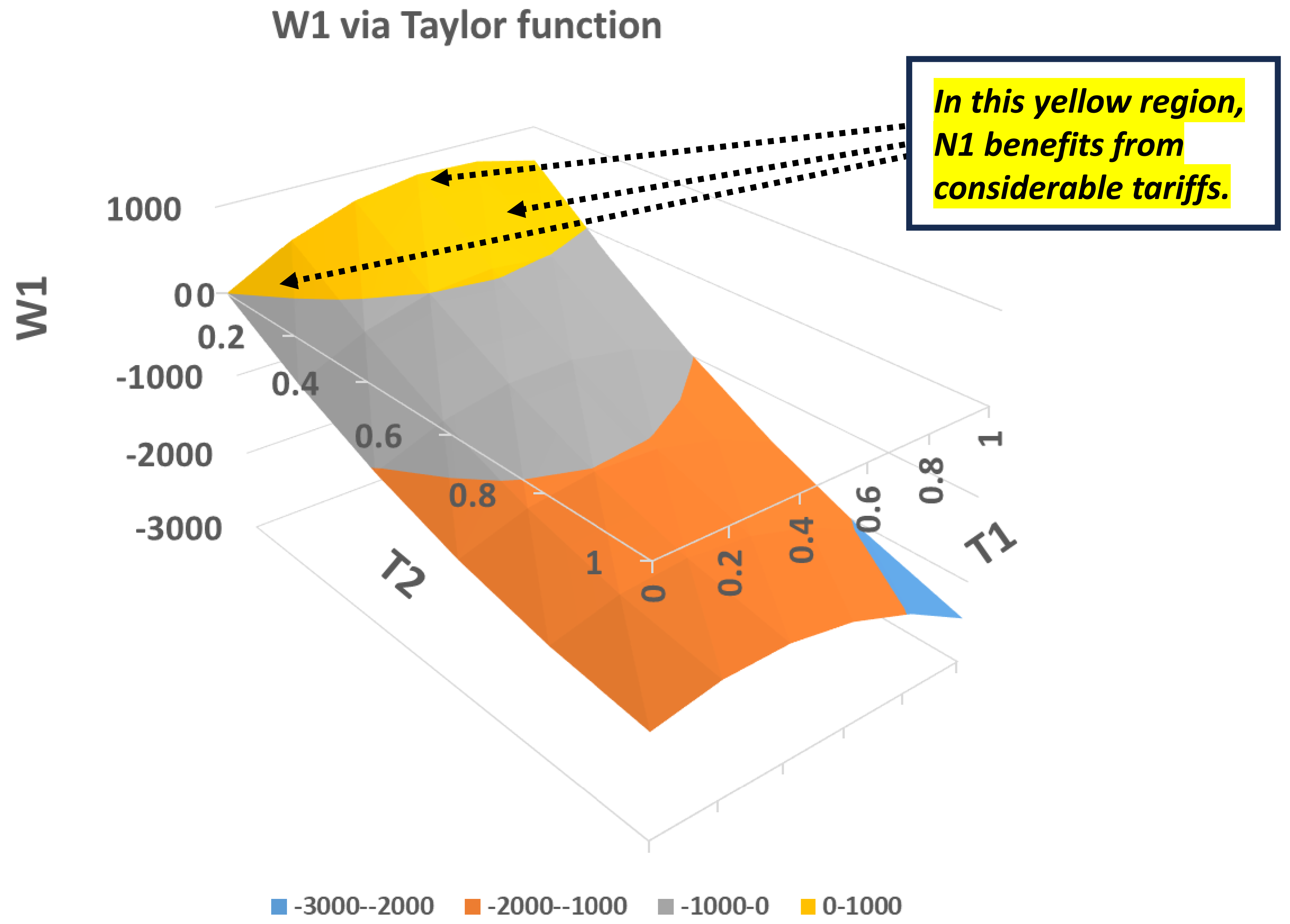
	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0	#N/A	#N/A	#N/A
T1	2399.057995	72.00241976	33.31913015	7.68387E-26
T1_2	-1820.216135	78.26211912	-23.25794593	3.47836E-21
T2	-3101.7426	72.00241976	-43.07831057	3.15284E-29
T1T2	-1279.316822	58.32762742	-21.93329094	1.93636E-20
T2_2	1349.203508	78.26211912	17.23954734	1.94472E-17

Estimated function:

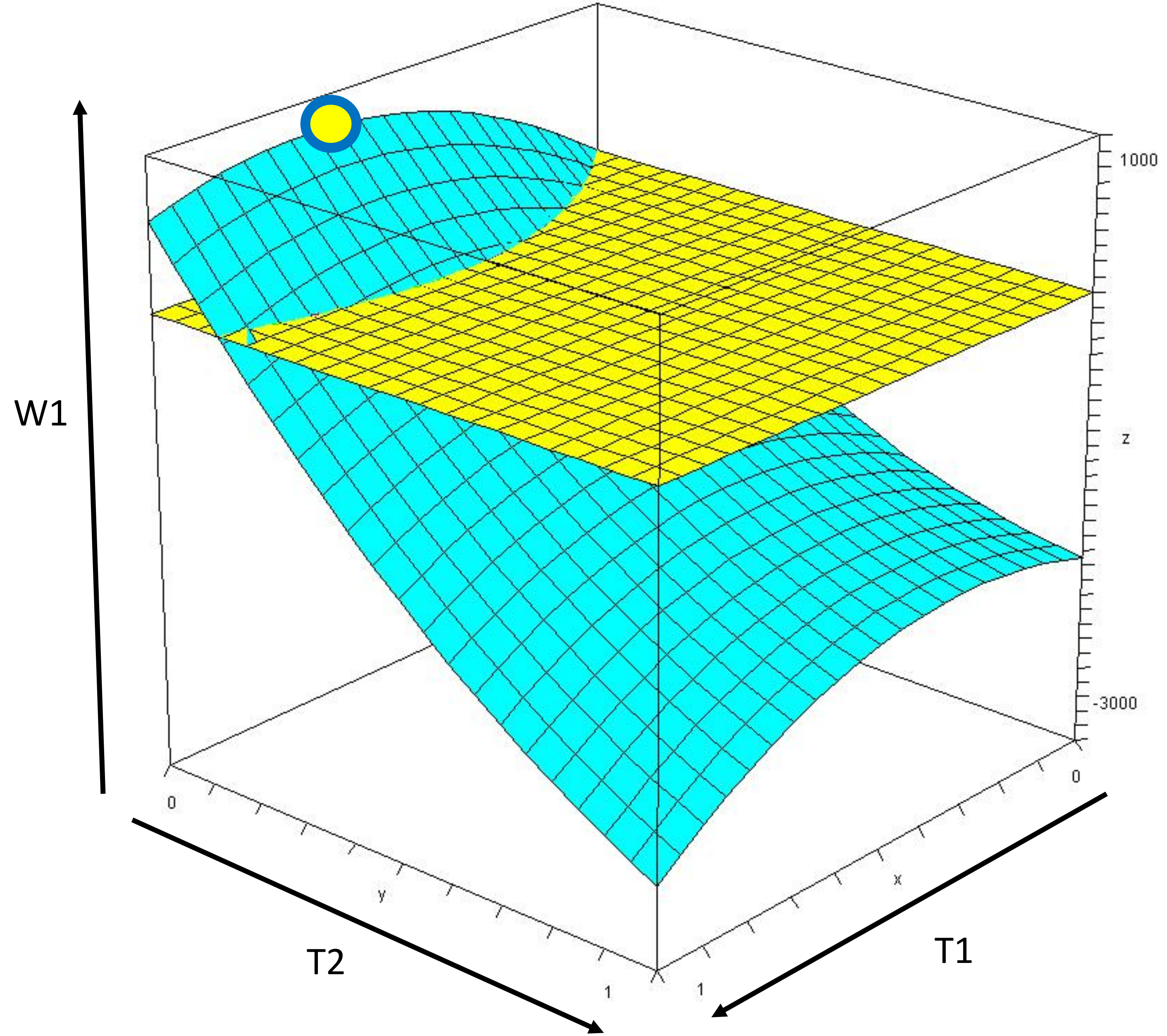
$$W_1 \approx 2399\, T_1 - 1820\, T_1^2 - 3102\, T_2 - 1279\, T_1 T_2 + 1349\, T_2^2$$

With computer characters:

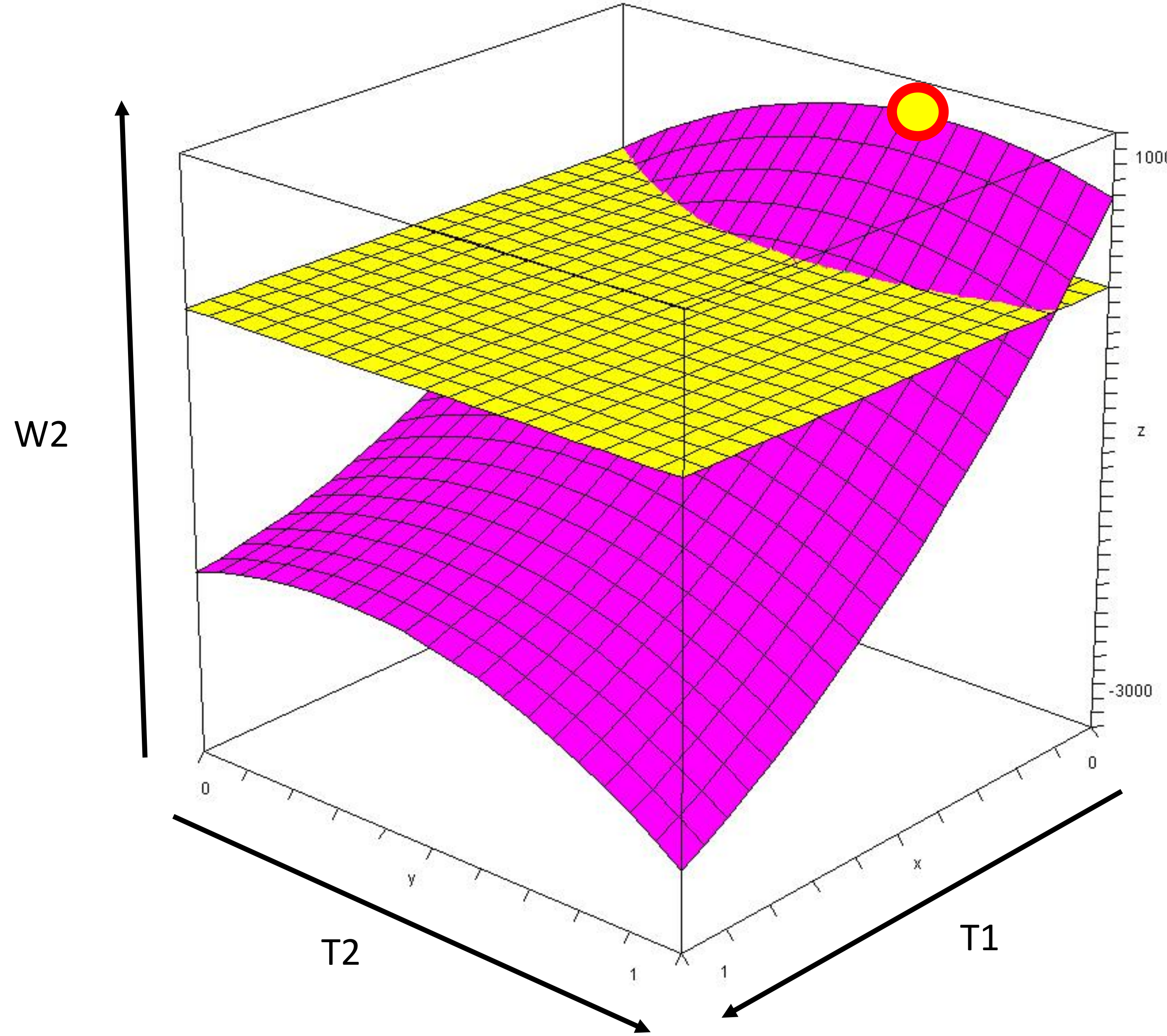
W1 = 2399 * T1 - 1820*T1_2 - 3102*T2 - 1279*T1T2 + 1349*T2_2


W1 via Taylor function
In this yellow region,
N1 benefits from
considerable tariffs.
1000
0
-1000
-2000
-3000
W1
0
0.2
0.4
0.6
0.8
1
T2
0
0.2
0.4
0.6
0.8
1
T1
-3000--2000
-2000--1000
-1000-0
0-1000

Total utility in Nation N1 for different tariff combinations.



Total utility in Nation N2 for different tariff combinations.



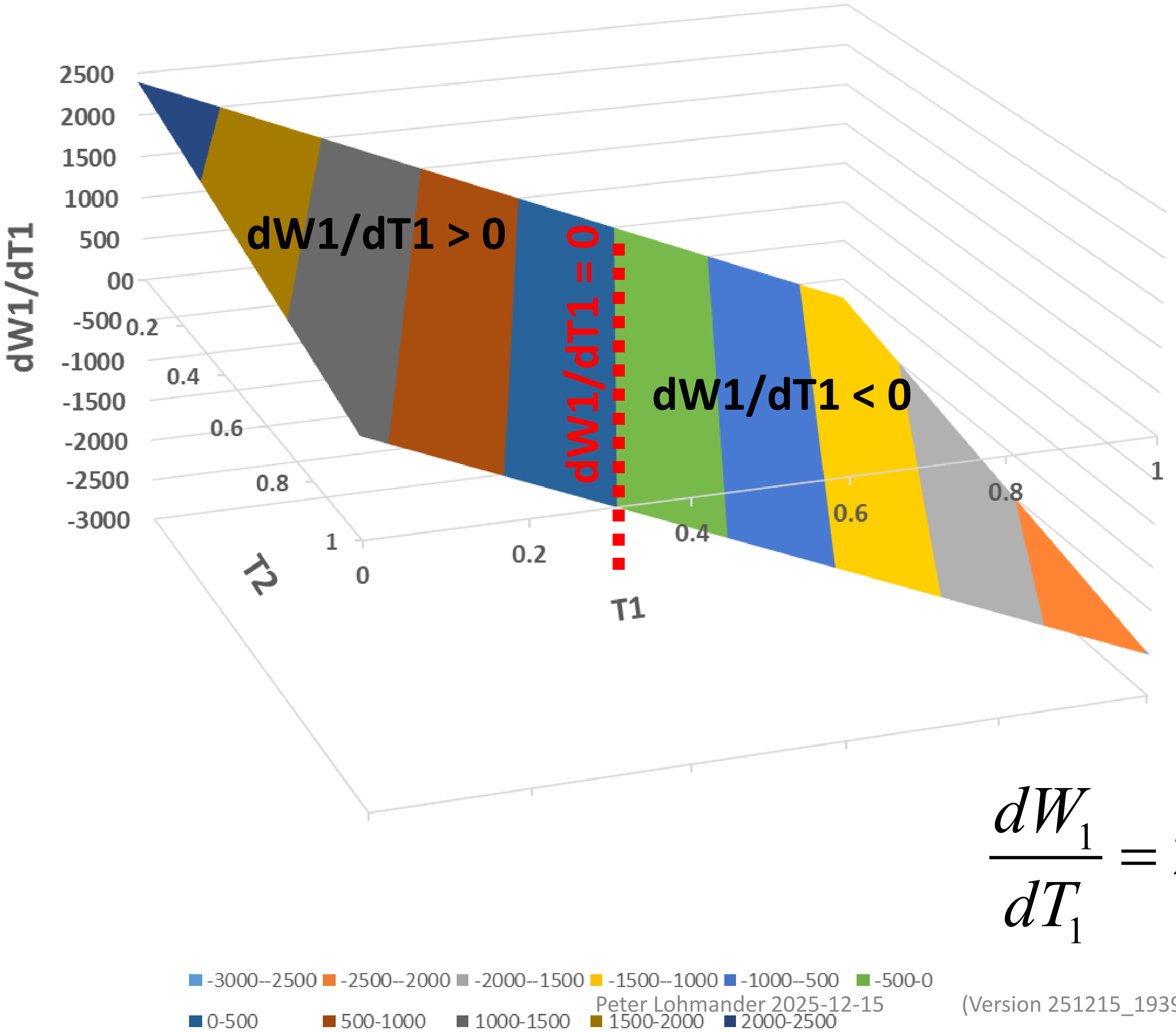
$$W_1 = 2399\,T_1 - 1820\,T_1^{\,2} - 3102\,T_2 - 1279\,T_1 T_2 + 1349\,T_2^{\,2}$$

$$\frac{dW_1}{dT_1} = 2399 - 3640\,T_1 - 1279\,T_2$$

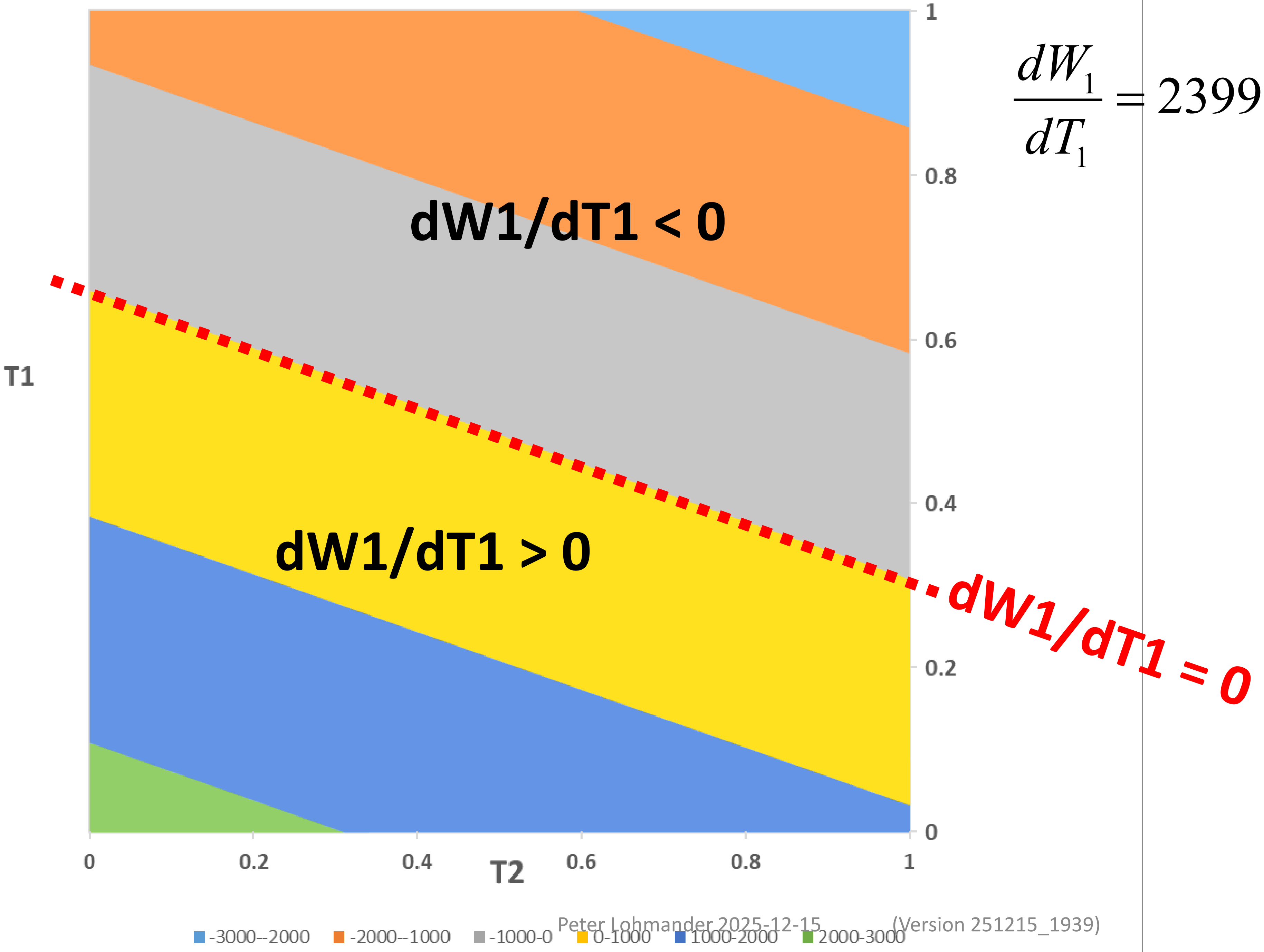
In this particular case, we have a symmetric relationship.

$$W_2 = 2399\,T_2 - 1820\,T_2^{\,2} - 3102\,T_1 - 1279\,T_1 T_2 + 1349\,T_1^{\,2}$$

$$\frac{dW_2}{dT_2} = 2399 - 3640\,T_2 - 1279\,T_1$$



$$\frac{dW_1}{dT_1} = 2399 - 3640\,T_1 - 1279\,T_2$$



$$\frac{dW_1}{dT_1} = 2399 - 3640\,T_1 - 1279\,T_2$$

The matrix W2(T1_index, T2_index)

$$W2(T1,T2) = (U2(T1,T2)-U2(0,0))*10000$$

T2_index

T1_index	0	1	2	3	4	5	6	7	8	9	10
0	0	501	748	825	766	626	422	182	−94	−392	−695
1	−581	−158	13	23	−93	−275	−521	−805	−1115	−1439	−1779
2	−1018	−666	−551	−602	−754	−979	−1266	−1572	−1907	−2263	−2614
3	−1348	−1055	−993	−1086	−1283	−1543	−1844	−2185	−2539	−2906	−3276
4	−1602	−1360	−1348	−1477	−1704	−1990	−2318	−2679	−3046	−3431	−3815
5	−1798	−1603	−1629	−1794	−2051	−2361	−2711	−3078	−3464	−3853	−4248
6	−1948	−1797	−1860	−2052	−2334	−2662	−3029	−3417	−3810	−4203	−4604
7	−2065	−1954	−2047	−2269	−2570	−2919	−3295	−3692	−4098	−4499	−4899
8	−2154	−2078	−2201	−2447	−2766	−3128	−3522	−3927	−4337	−4746	−5150
9	−2221	−2179	−2328	−2594	−2931	−3308	−3712	−4123	−4541	−4956	−5364
10	−2272	−2260	−2435	−2720	−3071	−3462	−3875	−4296	−4717	−5136	−5547

N1 maximizes W1. As long as the derivative

$$\frac{dW_1}{dT_1} = 2399 - 3640\,T_1 - 1279\,T_2$$

exceeds 0, it is rational to increase T1.

Obs: If T2 = 0, it is always rational for N1 to increase T1 from zero.

Then, the optimal value of T1 is determined this way:

$$\left(\frac{dW_1}{dT_1} = 2399 - 3640\,T_1 = 0 \right) \Rightarrow T_1 \approx 66\%$$

Here, N1 finds it optimal to introduce a 66% tariff and deviate from free trade.

N2 maximizes W2. As long as the derivative

$$\frac{dW_2}{dT_2} = 2399 - 3640\,T_2 - 1279\,T_1$$

exceeds 0, it is rational to increase T2.

Obs: If T1 = 0, it is always rational for N2 to increase T2 from zero.

Then, the optimal value of T2 is determined this way:

$$\left(\frac{dW_2}{dT_2} = 2399 - 3640\,T_2 = 0\right) \Rightarrow T_2 \approx 66\%$$

Here, N2 finds it optimal to introduce a 66% tariff and deviate from free trade.

Obs! The total utility decreases if tariffs are introduced.

The matrix Wsum(T1_index, T2_index)

W1(T1,T2) + W2(T1,T2)

T2_index

T1_index	0	1	2	3	4	5	6	7	8	9	10
0	0	−78	−269	−527	−836	−1171	−1525	−1882	−2247	−2613	−2967
1	−77	−316	−648	−1033	−1451	−1880	−2318	−2757	−3193	−3618	−4039
2	−268	−647	−1101	−1593	−2100	−2610	−3123	−3619	−4109	−4590	−5049
3	−530	−1033	−1592	−2173	−2758	−3336	−3900	−4453	−4985	−5500	−5996
4	−837	−1450	−2099	−2757	−3407	−4041	−4653	−5246	−5813	−6360	−6884
5	−1169	−1881	−2612	−3336	−4041	−4721	−5374	−5998	−6594	−7163	−7708
6	−1521	−2319	−3119	−3903	−4654	−5374	−6059	−6711	−7330	−7916	−8476
7	−1879	−2755	−3620	−4451	−5245	−5998	−6710	−7384	−8021	−8624	−9195
8	−2244	−3190	−4110	−4985	−5813	−6595	−7329	−8021	−8673	−9288	−9868
9	−2613	−3619	−4587	−5501	−6359	−7164	−7917	−8624	−9288	−9912	−10500
10	−2975	−4041	−5049	−5998	−6882	−7707	−8475	−9195	−9869	−10500	−11094

In Nash equilibrium:

N1 maximizes W1 via T1, conditional on the fact that N2 maximizes W2 via T2.

N2 maximizes W2 via T2, conditional on the fact that N1 maximizes W1 via T1.

$$\begin{cases} \dfrac{dW_1}{dT_1} = 2399 - 3640\,T_1 - 1279\,T_2 = 0 \\ \dfrac{dW_2}{dT_2} = 2399 - 1279\,T_1 - 3640\,T_2 = 0 \end{cases}$$

In Nash equilibrium:

$$\begin{bmatrix} 3640 & 1279 \\ 1279 & 3640 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} 2399 \\ 2399 \end{bmatrix}$$

$$T_1 = \frac{\begin{vmatrix} 2399 & 1279 \\ 2399 & 3640 \end{vmatrix}}{\begin{vmatrix} 3640 & 1279 \\ 1279 & 3640 \end{vmatrix}} \approx \frac{5664039}{11613759} \approx 0.48770 \qquad \approx 49\%$$

$$T_2 = \frac{\begin{vmatrix} 3640 & 2399 \\ 1279 & 2399 \end{vmatrix}}{\begin{vmatrix} 3640 & 1279 \\ 1279 & 3640 \end{vmatrix}} \approx \frac{5664039}{11613759} \approx 0.48770 \qquad \approx 49\%$$

In Nash equilibrium:

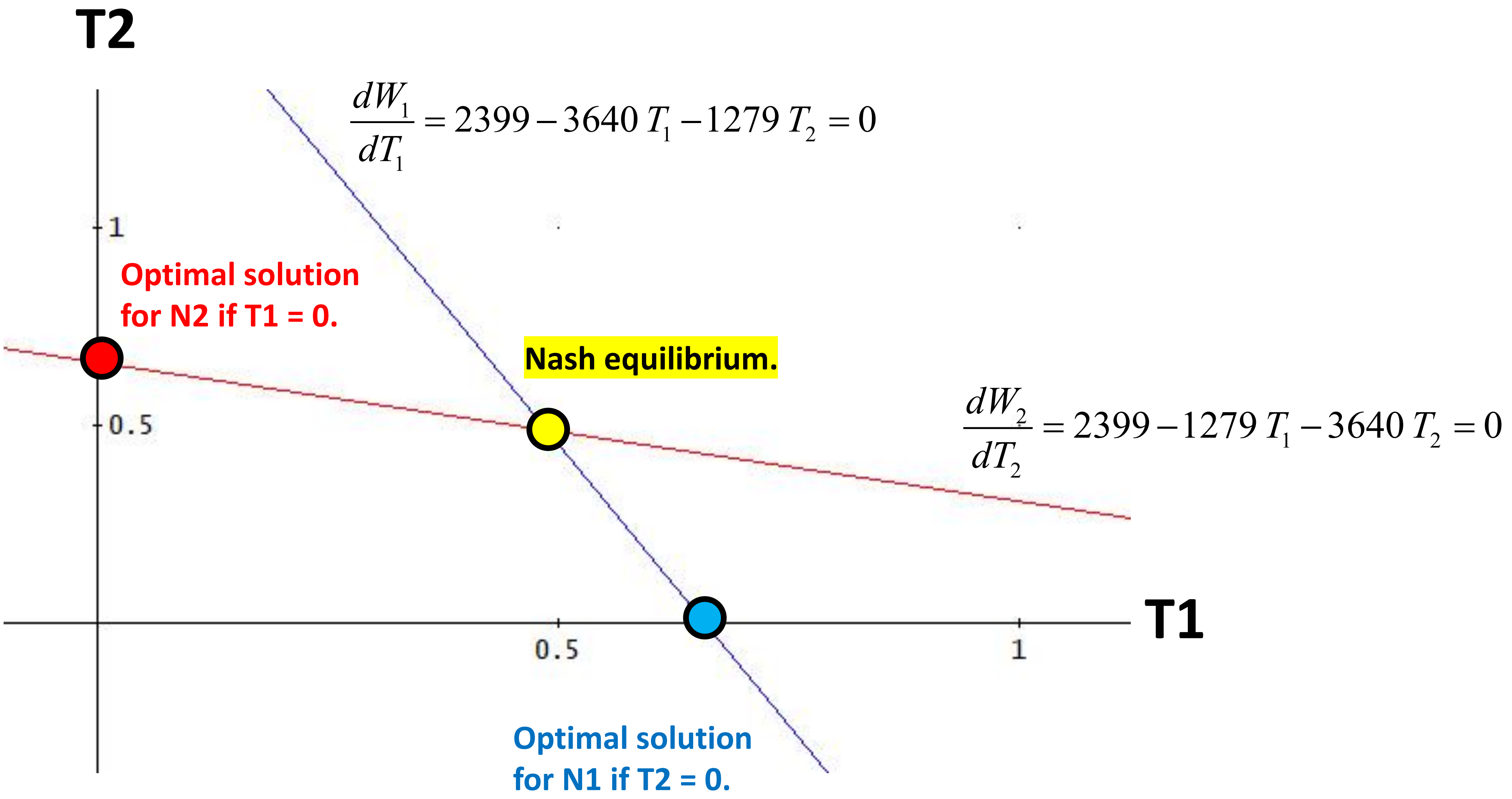
$$(T_1, T_2) = (0.48770, 0.48770) \Rightarrow$$

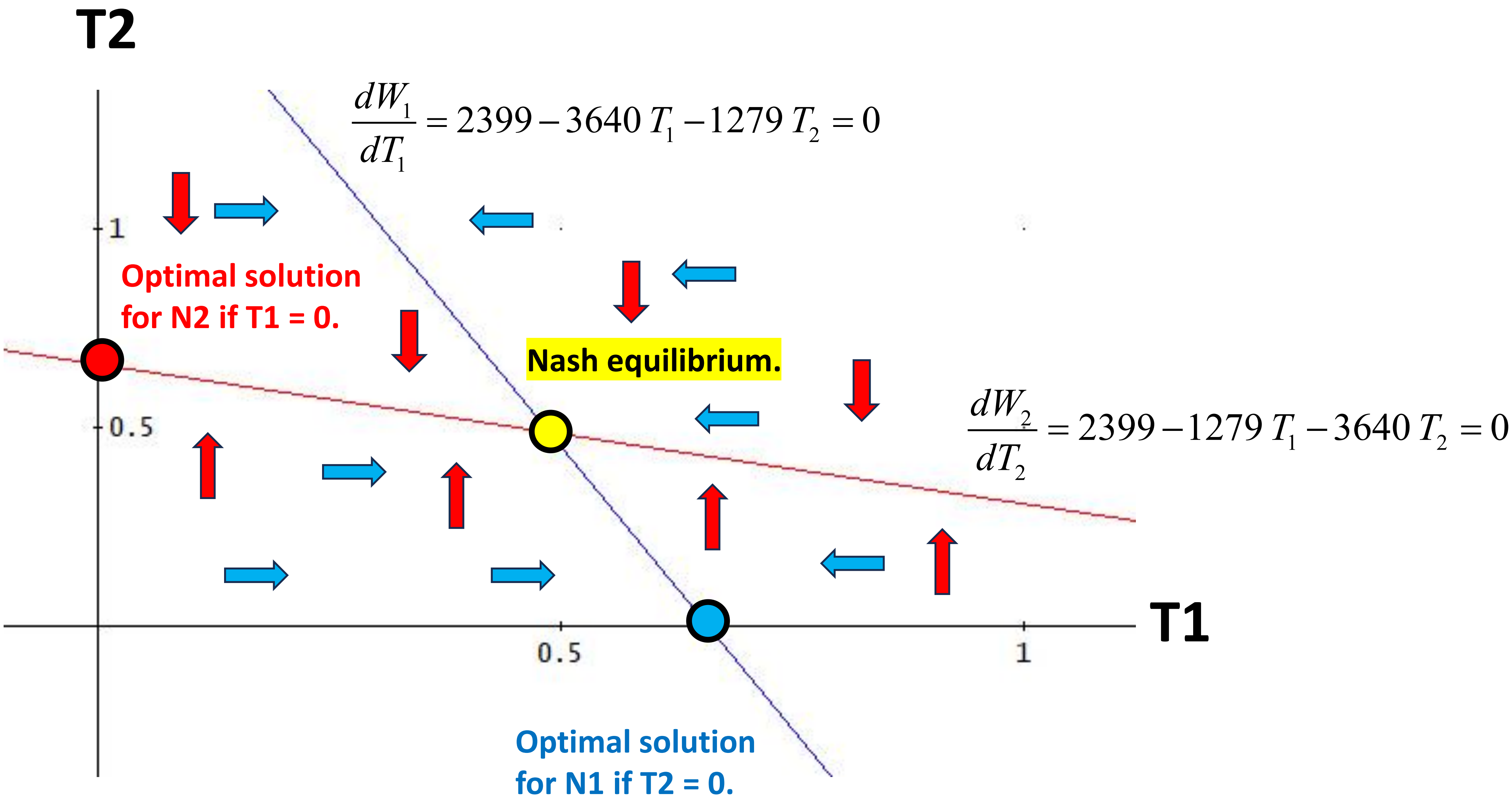
$$W_1 = 2399\, T_1 - 1820\, T_1^2 - 3102\, T_2 - 1279\, T_1 T_2 + 1349\, T_2^2$$

$$W_2 = 2399\, T_2 - 1820\, T_2^2 - 3102\, T_1 - 1279\, T_1 T_2 + 1349\, T_1^2$$

$$W_1 = W_2 \approx -759$$

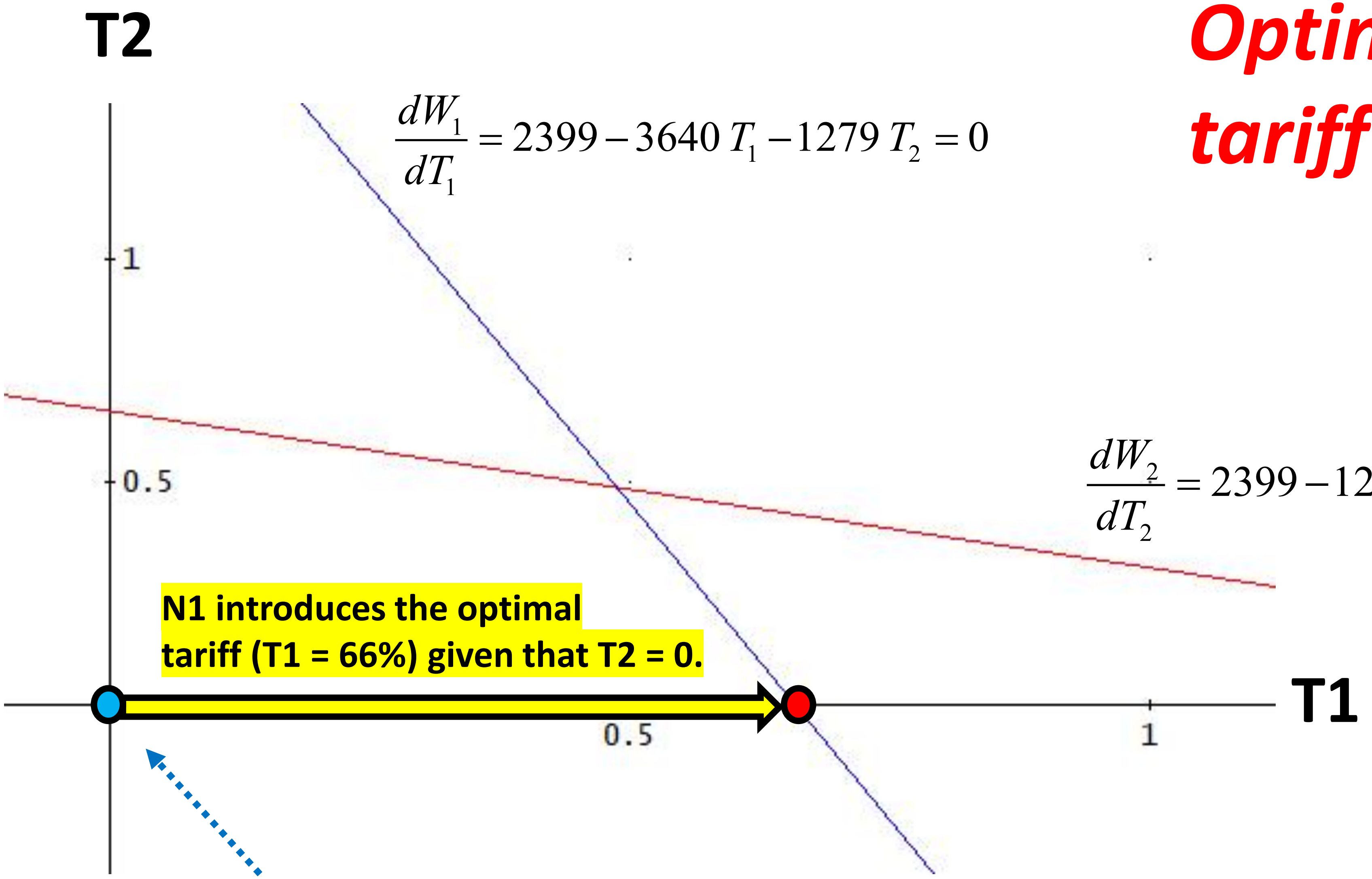
IMPORTANT Observation: ***It is better for N1 and for N2 to select free trade, without any tariffs, than to fight a tariff war, and select the best tariff levels that you can, based on the tariffs in the other nations !!!***


T2
$\frac{dW_1}{dT_1} = 2399 - 3640\,T_1 - 1279\,T_2 = 0$
1
Optimal solution
for N2 if T1 = 0.
Nash equilibrium.
0.5
$\frac{dW_2}{dT_2} = 2399 - 1279\,T_1 - 3640\,T_2 = 0$
T1
0.5
1
Optimal solution
for N1 if T2 = 0.



T2
$\frac{dW_1}{dT_1} = 2399 - 3640\ T_1 - 1279\ T_2 = 0$
1
Optimal solution
for N2 if T1 = 0.
Nash equilibrium.
0.5
$\frac{dW_2}{dT_2} = 2399 - 1279\ T_1 - 3640\ T_2 = 0$
T1
0.5
1
Optimal solution
for N1 if T2 = 0.

Optimal dynamic tariff changes (1)



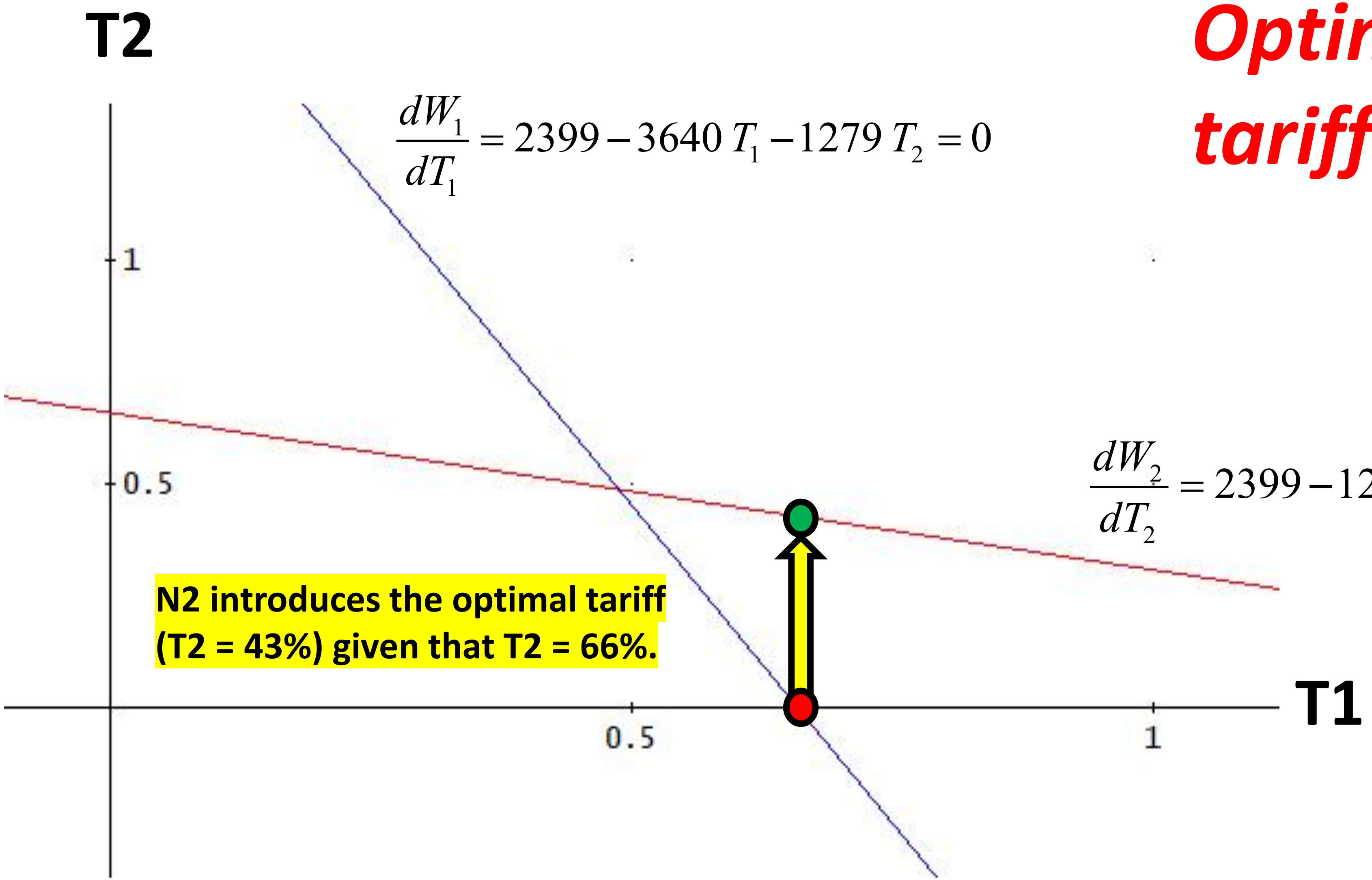
$$\frac{dW_1}{dT_1} = 2399 - 3640\,T_1 - 1279\,T_2 = 0$$

$$\frac{dW_2}{dT_2} = 2399 - 1279\,T_1 - 3640\,T_2 = 0$$

N1 introduces the optimal tariff (T1 = 66%) given that T2 = 0.

We start with free trade (T1,T2) = (0,0).

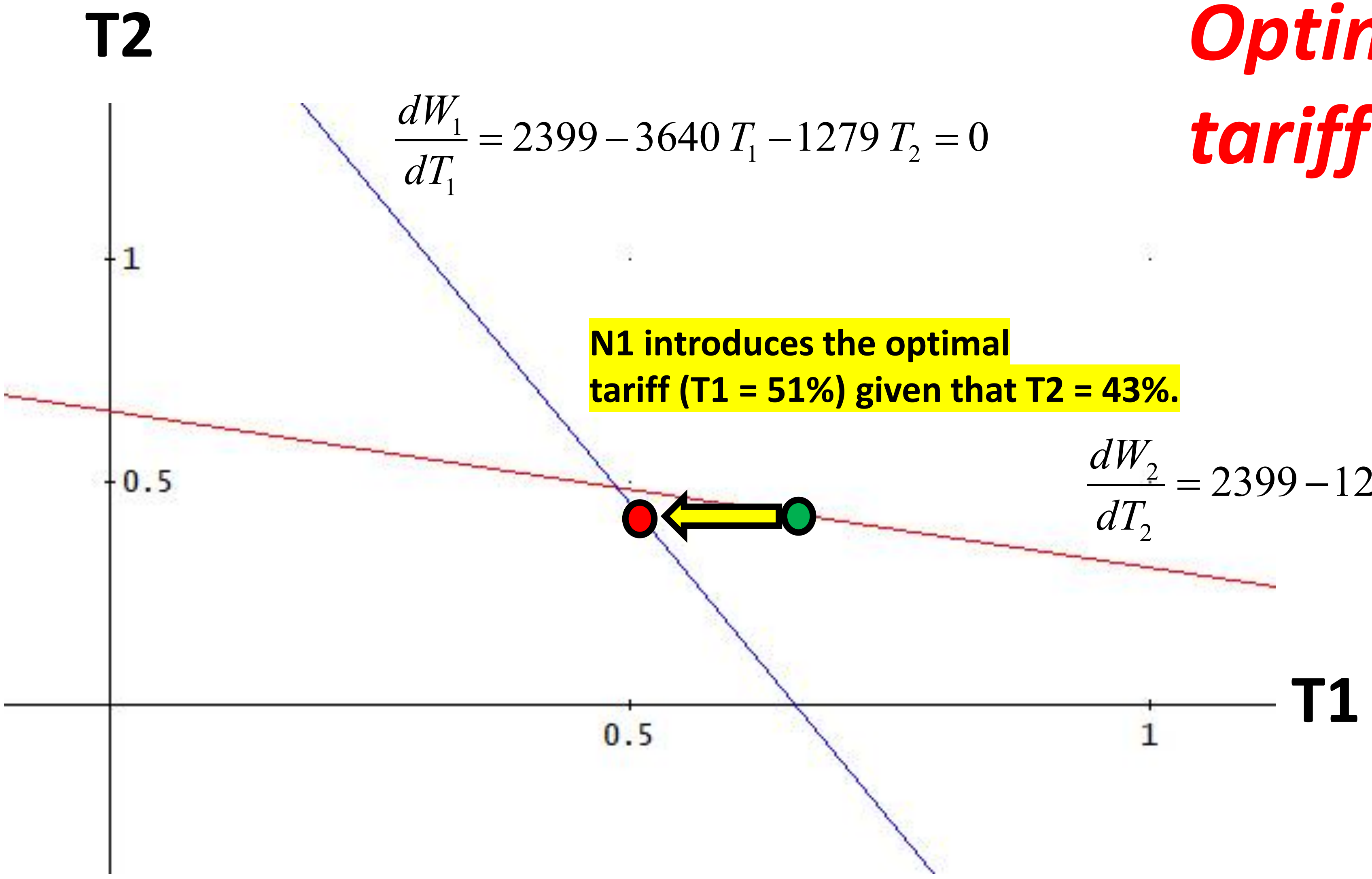
Optimal dynamic tariff changes (2)



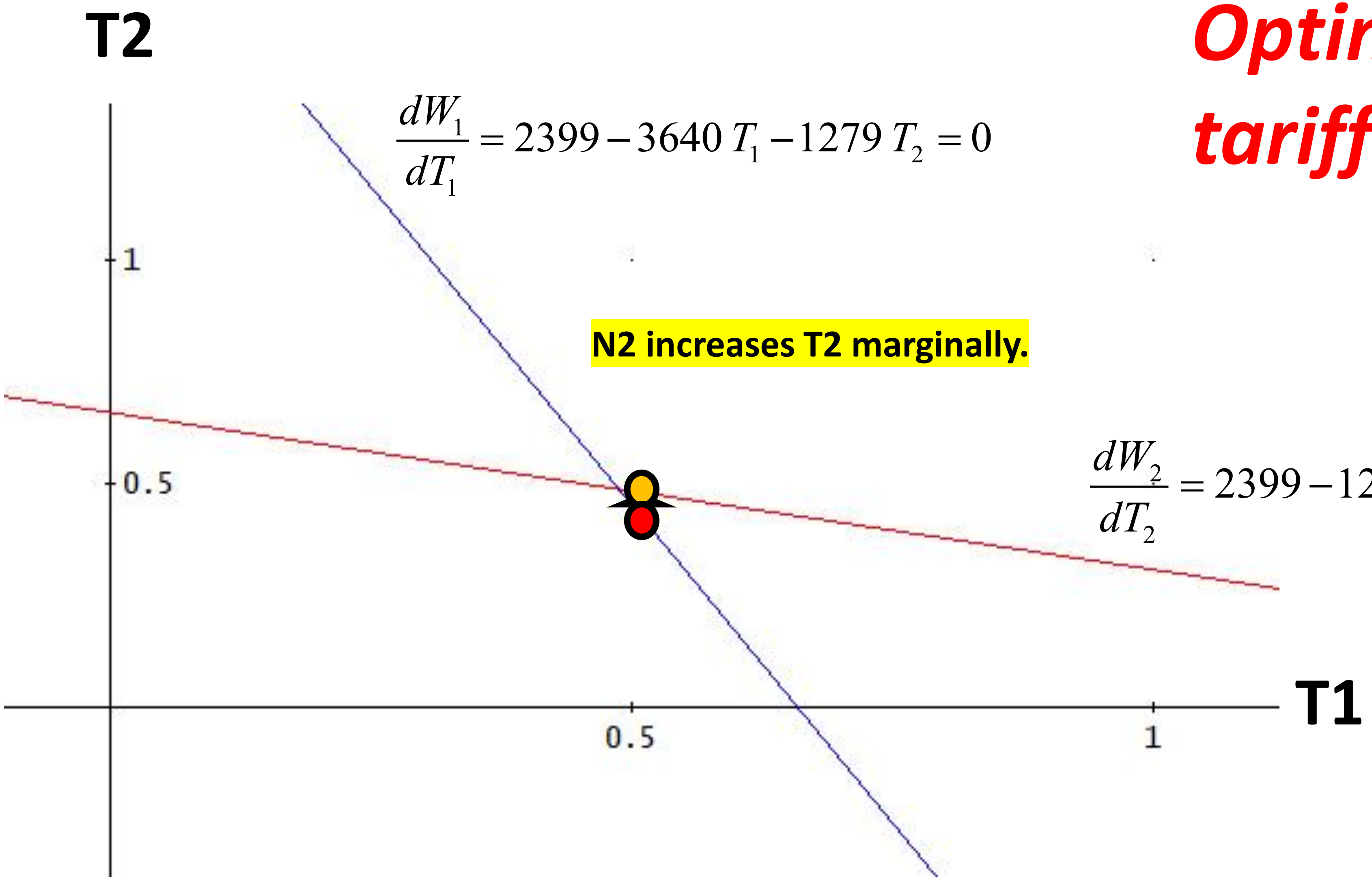
$$\frac{dW_1}{dT_1} = 2399 - 3640\,T_1 - 1279\,T_2 = 0$$

$$\frac{dW_2}{dT_2} = 2399 - 1279\,T_1 - 3640\,T_2 = 0$$

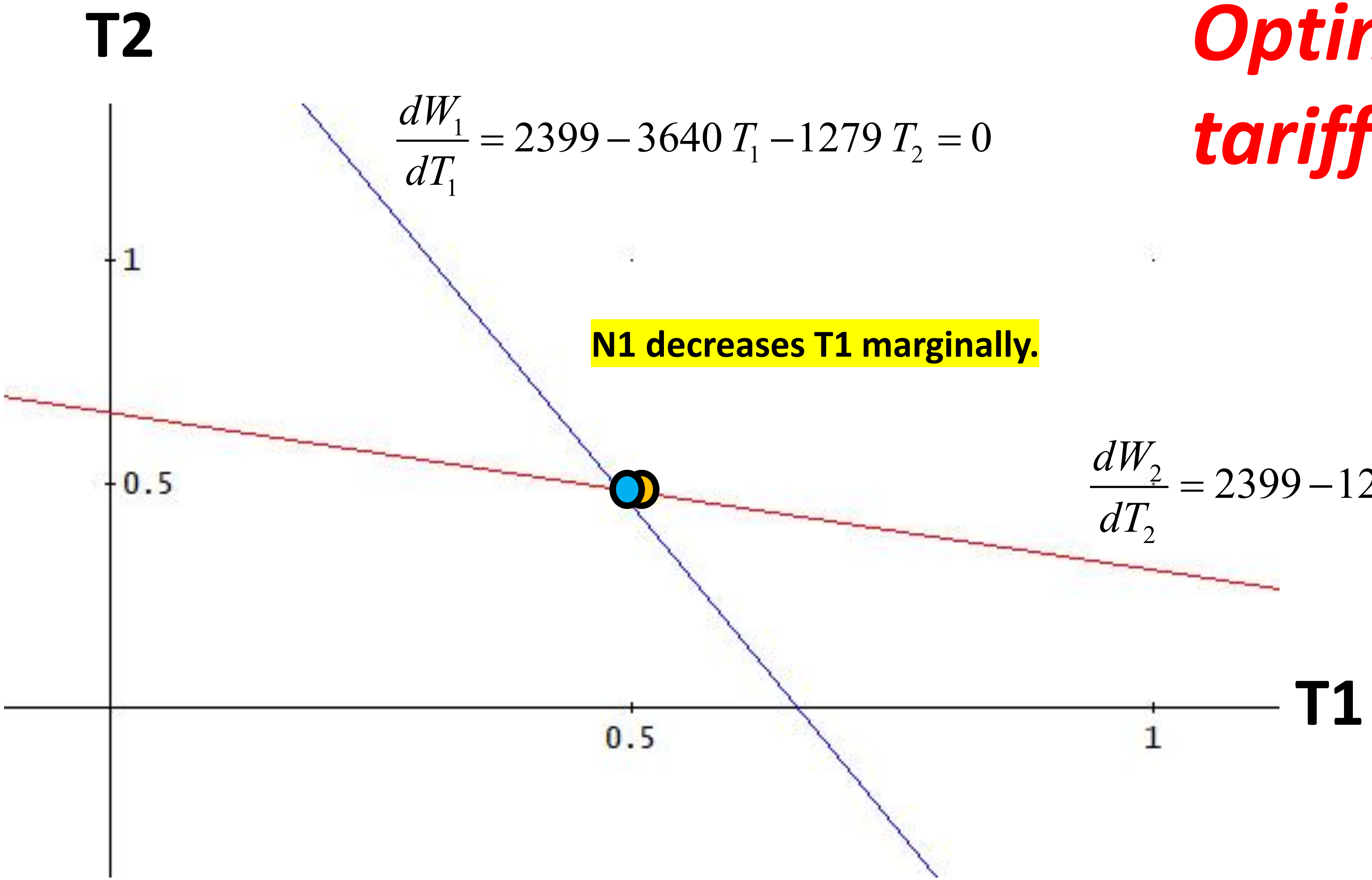
Optimal dynamic tariff changes (3)



Optimal dynamic tariff changes (4)



Optimal dynamic tariff changes (5)

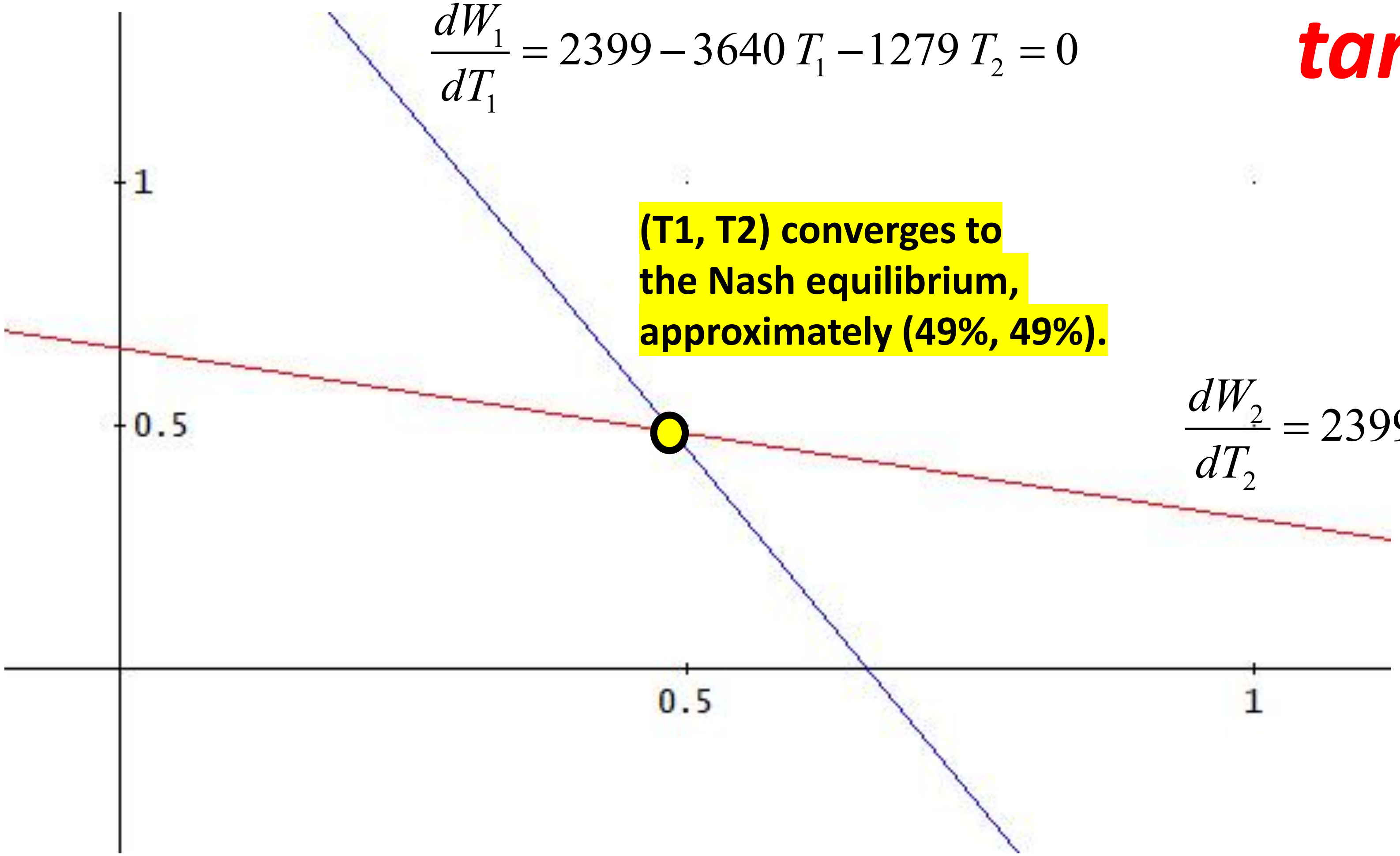


$$\frac{dW_1}{dT_1} = 2399 - 3640\,T_1 - 1279\,T_2 = 0$$

$$\frac{dW_2}{dT_2} = 2399 - 1279\,T_1 - 3640\,T_2 = 0$$

Optimal dynamic tariff changes (6)

$$\frac{dW_1}{dT_1} = 2399 - 3640\,T_1 - 1279\,T_2 = 0$$



$$\frac{dW_2}{dT_2} = 2399 - 1279\,T_1 - 3640\,T_2 = 0$$

In Nash equilibrium:

$$\begin{bmatrix} 3640 & 1279 \\ 1279 & 3640 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} 2399 \\ 2399 \end{bmatrix}$$

All elements in all equations are divided by 2399.
The following system is obtained (with limited precision).

$$\begin{bmatrix} 1.517 & 0.533 \\ 0.533 & 1.517 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Let us generalize the equation system:

$$\begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Nash equilibrium via the general functions and the approximate parameters:

$$T_1 = \frac{\begin{vmatrix} 1 & \alpha_{12} \\ 1 & \alpha_{22} \end{vmatrix}}{\begin{vmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{vmatrix}} = \frac{\alpha_{22} - \alpha_{12}}{\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21}}$$

$$\approx \frac{1.517 - 0.533}{(1.517)^2 - (0.533)^2} \approx 0.488$$

$$T_2 = \frac{\begin{vmatrix} \alpha_{11} & 1 \\ \alpha_{21} & 1 \end{vmatrix}}{\begin{vmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{vmatrix}} = \frac{\alpha_{11} - \alpha_{21}}{\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21}}$$

$$\approx \frac{1.517 - 0.533}{(1.517)^2 - (0.533)^2} \approx 0.488$$

When will the tariff system converge to the Nash equilibrium?

$$\begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Time loop: m(t) is calculated based on n(t) and n(t+1) is calculated based on m(t).

Simplified notation:

$$T_1 = m$$

$$T_2 = n$$

$$\begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{bmatrix} m \\ n \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\left(\alpha_{11} m + \alpha_{12} n = 1\right) \Rightarrow m_t = \frac{1}{\alpha_{11}}\left(1 - \alpha_{12} n_t\right)$$

$$\left(\alpha_{21} m + \alpha_{22} n = 1\right) \Rightarrow n_{t+1} = \frac{1}{\alpha_{22}}\left(1 - \alpha_{21} m_t\right)$$

$$m_t = \frac{1}{\alpha_{11}}\left(1-\alpha_{12}n_t\right)$$

$$n_t = \frac{1}{\alpha_{22}}\left(1-\alpha_{21}m_{t-1}\right)$$

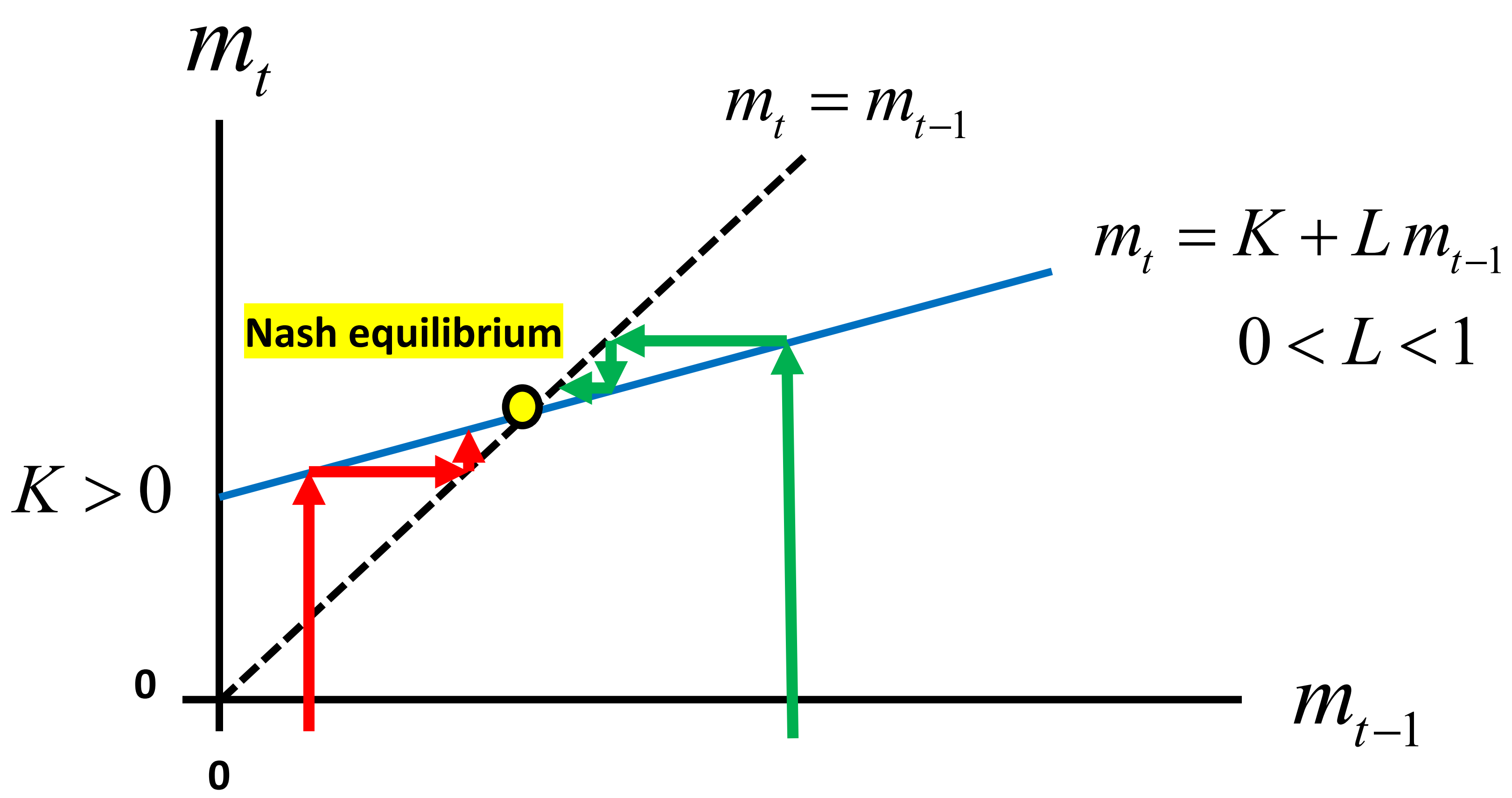
$$m_t = \frac{1}{\alpha_{11}}\left(1-\alpha_{12}\left(\frac{1}{\alpha_{22}}\left(1-\alpha_{21}m_{t-1}\right)\right)\right)$$

$$m_t = \frac{1}{\alpha_{11}}\left(1-\alpha_{12}\left(\frac{1}{\alpha_{22}}-\frac{\alpha_{21}}{\alpha_{22}}m_{t-1}\right)\right)$$

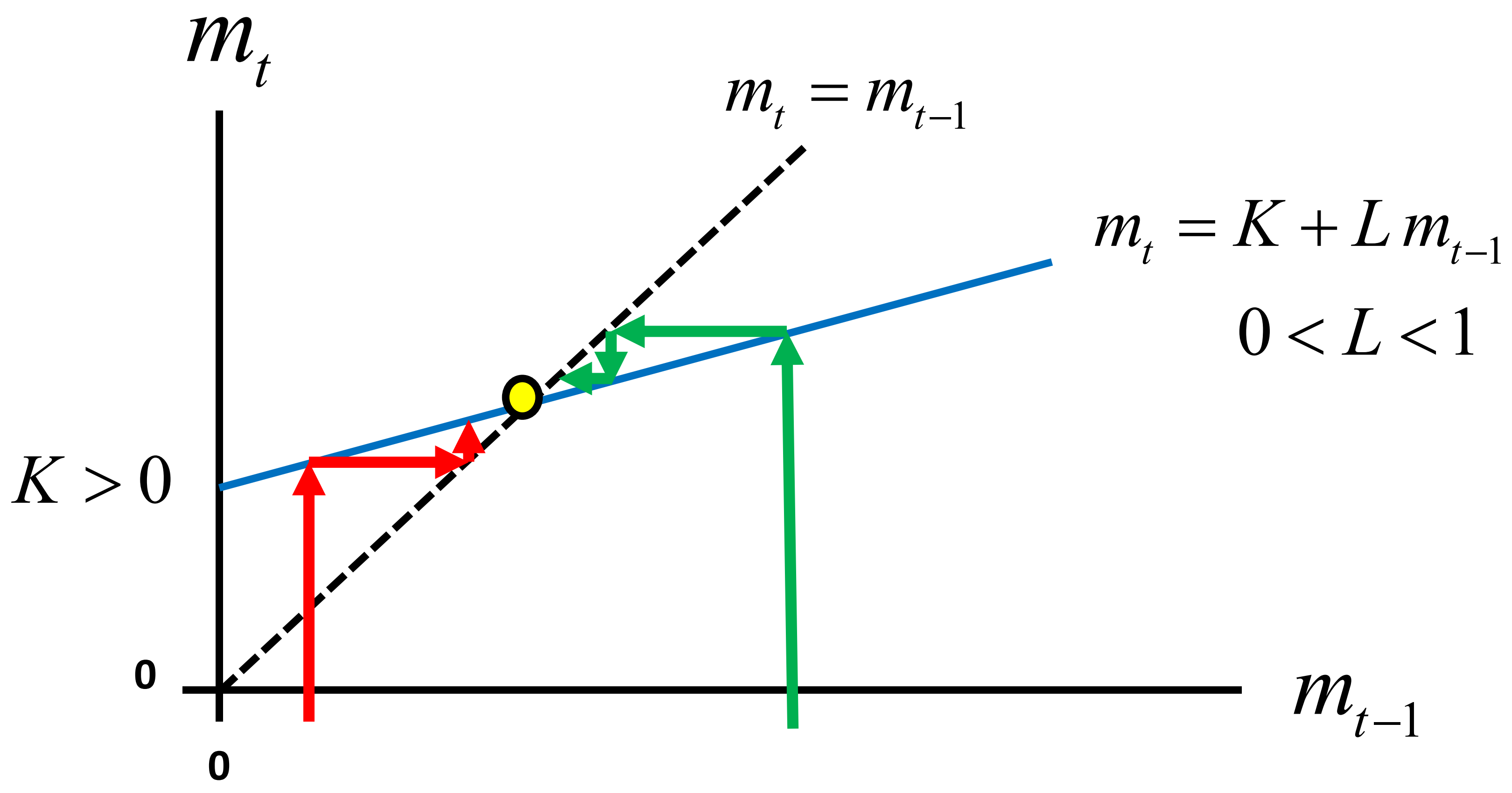
$$m_t = \frac{1}{\alpha_{11}}\left(1-\frac{\alpha_{12}}{\alpha_{22}}+\frac{\alpha_{12}\alpha_{21}}{\alpha_{22}}m_{t-1}\right)$$

$$m_t = \frac{\left(1-\frac{\alpha_{12}}{\alpha_{22}}\right)}{\alpha_{11}}+\frac{\alpha_{12}\alpha_{21}}{\alpha_{11}\alpha_{22}}m_{t-1}$$

$$m_t = \frac{\left(1 - \frac{\alpha_{12}}{\alpha_{22}}\right)}{\alpha_{11}} + \frac{\alpha_{12}\alpha_{21}}{\alpha_{11}\alpha_{22}} m_{t-1} \quad \boxed{m_t = K + L\, m_{t-1}} \quad , \quad K = \frac{\left(1 - \frac{\alpha_{12}}{\alpha_{22}}\right)}{\alpha_{11}}, L = \frac{\alpha_{12}\alpha_{21}}{\alpha_{11}\alpha_{22}}$$



The tariff system converges to the Nash equilibrium if K > 0 and 0 < L < 1.



The tariff system converges to the Nash equilibrium if

$$K = \frac{\left(1 - \dfrac{\alpha_{12}}{\alpha_{22}}\right)}{\alpha_{11}} > 0 \wedge 0 < L = \frac{\alpha_{12}\alpha_{21}}{\alpha_{11}\alpha_{22}} < 0$$

Convergence in the particular case?

$$K = \frac{\left(1 - \dfrac{\alpha_{12}}{\alpha_{22}}\right)}{\alpha_{11}} = \frac{\left(1 - \dfrac{0.533}{1.517}\right)}{1.517} \approx 0.428 > 0$$

$$0 < L = \frac{\alpha_{12}\alpha_{21}}{\alpha_{11}\alpha_{22}} = \frac{0.533 \times 0.533}{1.517 \times 1.517} \approx 0.123 < 1$$

YES!
The particular tariff system converges to the Nash equilibrium!

The Nash equilibrium in general form via the general iteration function:

$$m_t = \frac{\left(1 - \frac{\alpha_{12}}{\alpha_{22}}\right)}{\alpha_{11}} + \frac{\alpha_{12}\alpha_{21}}{\alpha_{11}\alpha_{22}} m_{t-1}$$

$$m^{NE} = \frac{\left(1 - \frac{\alpha_{12}}{\alpha_{22}}\right)}{\alpha_{11}} + \frac{\alpha_{12}\alpha_{21}}{\alpha_{11}\alpha_{22}} m^{NE}$$

$$\left(1 - \frac{\alpha_{12}\alpha_{21}}{\alpha_{11}\alpha_{22}}\right) m^{NE} = \frac{\left(1 - \frac{\alpha_{12}}{\alpha_{22}}\right)}{\alpha_{11}}$$

$$m^{NE} = \frac{\left(\frac{\left(1 - \frac{\alpha_{12}}{\alpha_{22}}\right)}{\alpha_{11}}\right)}{\left(1 - \frac{\alpha_{12}\alpha_{21}}{\alpha_{11}\alpha_{22}}\right)}$$

$$m^{NE} = \frac{\left(1 - \frac{\alpha_{12}}{\alpha_{22}}\right)}{\left(\alpha_{11} - \frac{\alpha_{12}\alpha_{21}}{\alpha_{22}}\right)}$$

$$\boxed{m^{NE} = \frac{(\alpha_{22} - \alpha_{12})}{(\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21})}}$$

This is identical to the equilibrium value of T1 according to the earlier calculation.

To the ***very*** interested reader:

The following topics are included in a separate presentation:

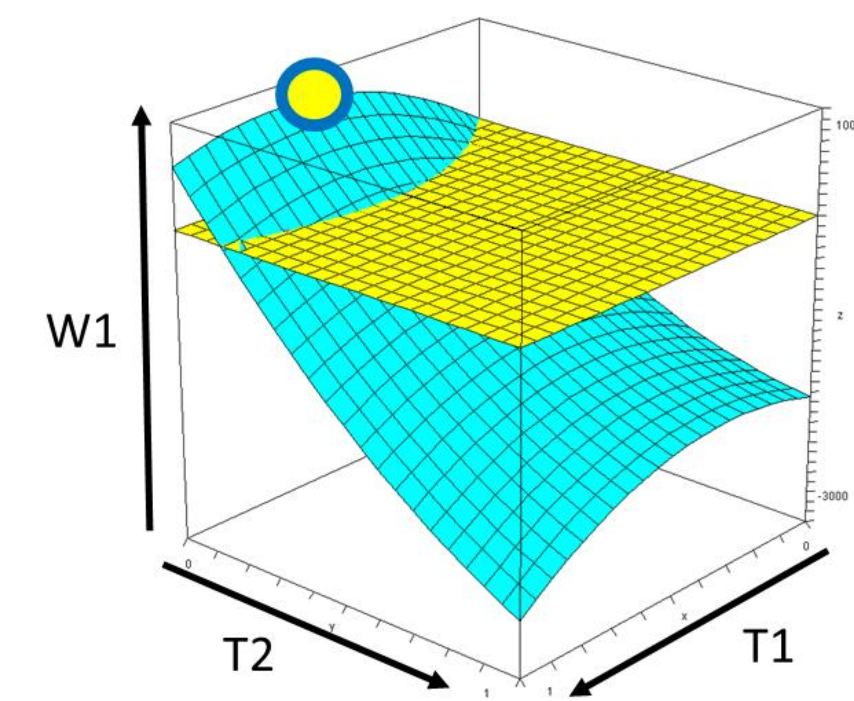
Section 4: **Optimal tariff, general equilibrium analysis, 3 Nations.**

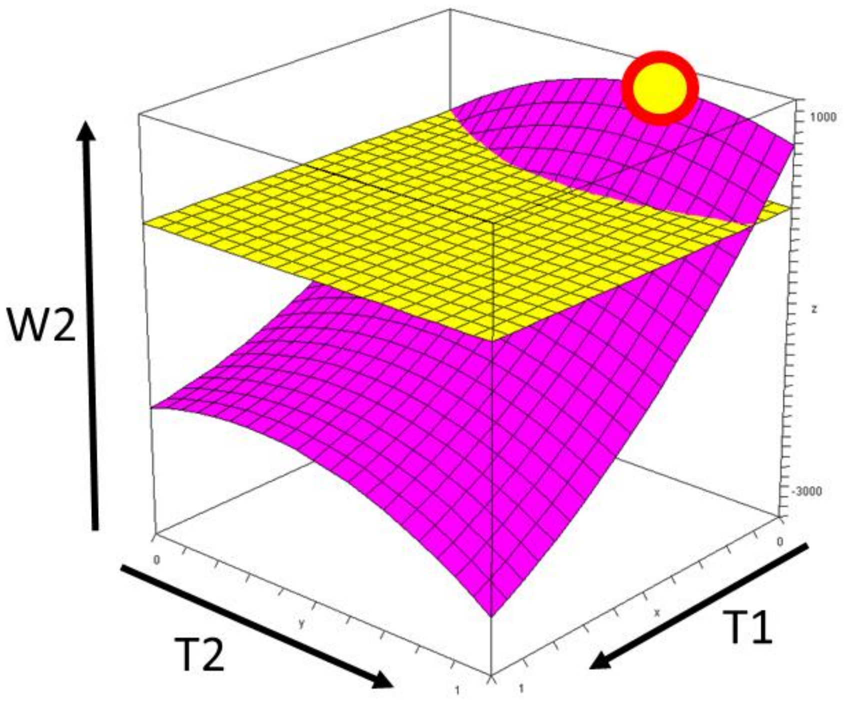
The trade solution with tariff from one nation.

International trade equilibrium based on optimized production and optimized consumption.

One nation, A, introduces tariffs on imports from another nation, B, when B does not import anything from A.

General observations on the optimal tariff and effects on other countries.





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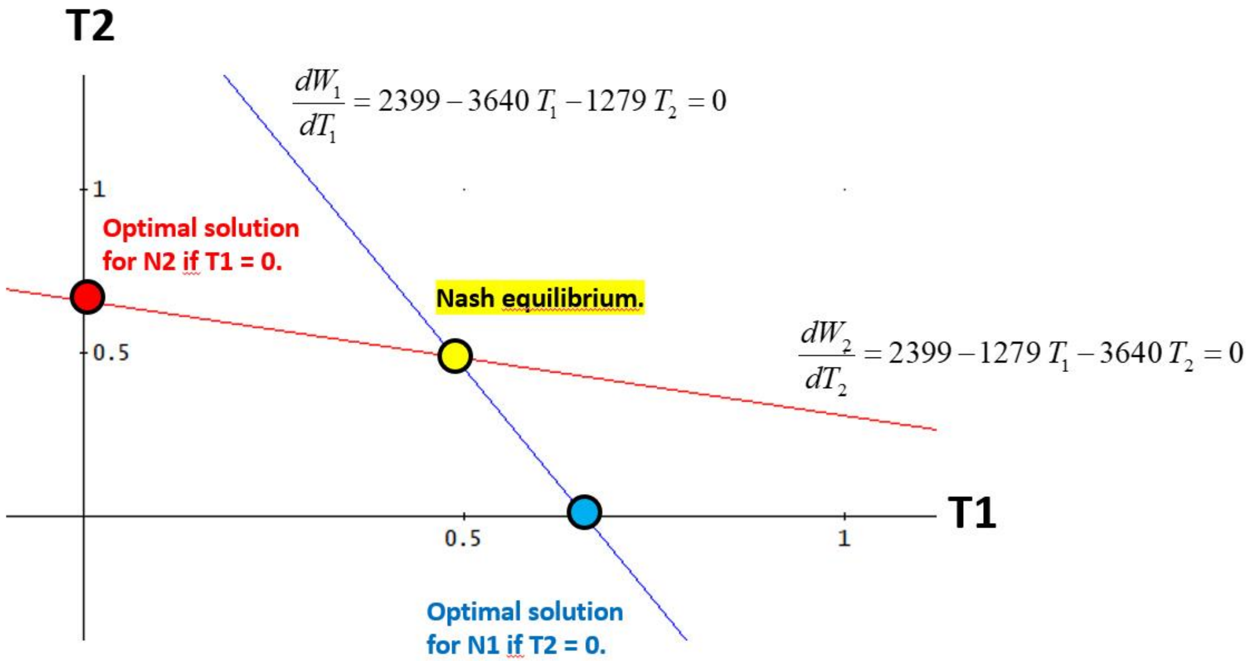

OPTIMAL TARIFFS

AND INTERNATIONAL TRADE

Eleventh International Conference on Statistics for the Twenty-First Century ICSTC 2025

OPTIMAL TARIFFS AND INTERNATIONAL TRADE,
Invited presentation, Eleventh International Conference on Statistics for the Twenty-First Century, ICSTC 2025, December 2025.

Peter Lohmander

Abstract

Optimal tariff strategies, with and without optimal responses, are derived, based on a linear partial equilibrium trade model, A, and a nonlinear general equilibrium trade model with two nations, B.

Under free trade, models A and B are applied to prove that it is always profitable for one nation, N1, to introduce a strictly positive tariff on imports, T1, as long as other nations do not introduce tariffs.

With model B, it is proved that it is rational also for nation N2, to introduce a tariff, T2, if N1, introduces a tariff, T1.

The Nash equilibrium tariff combination is unique and stable. The economic results, in both nations, are however higher with free trade, than in the Nash equilibrium tariff solution.

If both nations know that the other nations will respond to increasing tariffs, and select the optimal tariff, then it is not rational for any nation to leave free trade and introduce tariffs. Free trade agreements are rational for the participants.

It is also shown that it may be rational for a dictator, to introduce a tariff, even if this is not rational for the consumers in the same nation. The dictator may keep the tariff revenues and use these for other purposes. In such a case, the tariff reduces the consumer surpluses in both nations.

Thank you for your time!

Questions?

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References:
https://www.lohmander.com/Information/Ref.htm

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