OPTIMAL PRESENT RESOURCE EXTRACTION UNDER THE INFLUENCE OF FUTURE RISK

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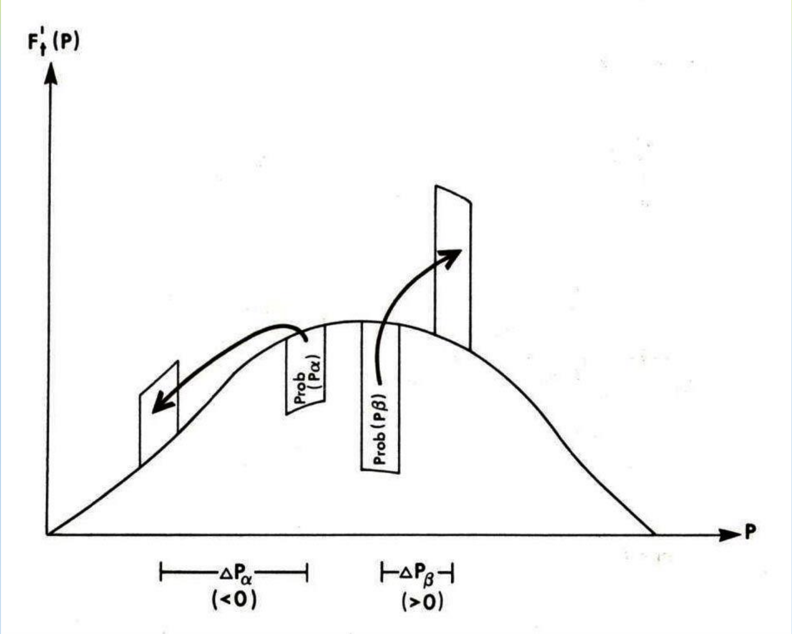
ABSTRACT

The analysis concerns determination of the optimal present extraction of a natural resouce and how this is affected by different kinds of risk in the future. The most general definition of increasing risk, according to Rothschild and Stiglitz, is used. It can be applied to all types of statistical distributions. The approach is much more general than, for instance, increasing variance. The analysis is performed via general function multi dimensional analyical optimization and comparative dynamics analysis in discrete time. It is found that most of the analytical results can be derived via comparative dynamics in a system with three equations in combination with supporting general function analysis. The general analytical results are illustrated via computer solutions to numerically specified special cases.

Keywords: Optimal stochastic control; Risk; Natural resource management; Forestry; Third order derivatives.



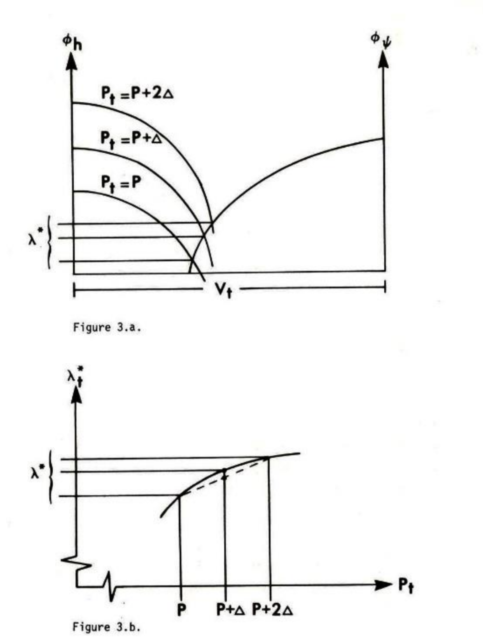


Figure 3.a.

Figure 3.b.

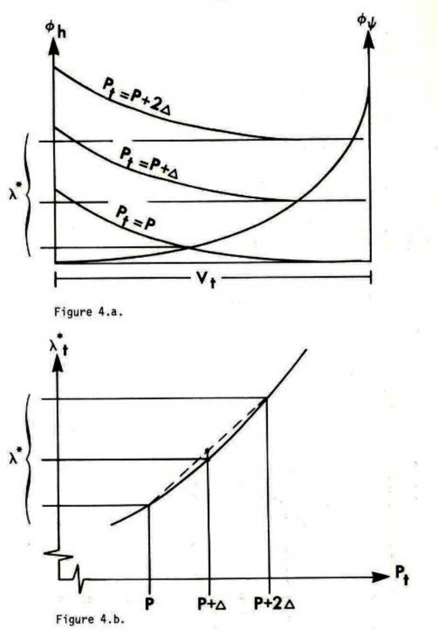


Figure 4.a.

Figure 4.b.

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1. Introduction via one dimensional optimization in dynamic problems, comparative statics analysis, probabilities, increasing risk and the importance of third order derivatives.
2. Explicit multi period analysis, stationarity and corner solutions.
3. Multi period problems and model structure with sequential adaptive decisions and risk.
4. Optimal decisions under future price risk.
5. Optimal decisions under future risk in the volume process (growth risk).
6. Optimal decisions under future price risk with mixed species.

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- In the following analyses, we study the solutions to maximization problems. The objective functions are the total expected present values. In particular, we study how the optimal decisions at different points in time are affected by stochastic variables, increasing risk and optimal adaptive future decisions.

- In all derivations in this document, continuously differentiable functions are assumed. In the optimizations and comparative statics calculations, local optima and small moves of these optima under the influence of parameter changes, are studied. For these reasons, derivatives of order four and higher are not considered. Derivatives of order three and lower can however not be neglected. We should be aware that functions that are not everywhere continuously differentiable may be relevant in several cases.

Introduction with a simplified problem

Objective function:

$$y(x) = P\big(f(x) + g(x,h)\big) + (1-P)\big(f(x) + g(x,0)\big)$$

Definitions:

x	Present extraction level
$y(x)$	Expected present value (expected discounted value) of present and future extraction
$f(x)$	Economic value of present extraction
h	Risk parameter
P	Probability that the expected present value of future extraction is affected by the risk parameter h
$g(x,h)$	Expected present value of future extraction (in case the expected present value of future extraction is affected by risk parameter h)
$g(x,0)$	Expected present value of future extraction (in case the expected present value of future extraction is not affected by risk parameter h)

The objective function can be rewritten as:

$$y(x) = f(x) + Pg(x,h) + (1-P)g(x,0)$$

Let us maximize $y(x)$ with respect to x. The first order optimum condition is:

$$\frac{dy}{dx} = \frac{df(x)}{dx} + P\frac{dg(x,h)}{dx} + (1-P)\frac{dg(x,0)}{dx} = 0$$

Optimal values are marked by stars. x^* is assumed to exist and be unique.

$$\frac{d^2y}{dx^2} < 0$$

How is the optimal value of x , x^*, affected by the value of the risk parameter h, ceteres paribus?

Differentiation of the first order optimum condition with respect to x^* and h gives:

$$d\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}dx^* + \frac{d^2y}{dxdh}dh = 0$$

$$\frac{d^2y}{dx^2}dx^* = -\frac{d^2y}{dxdh}dh$$

$$\boxed{\frac{dx^*}{dh} = -\frac{\left(\frac{d^2y}{dxdh}\right)}{\left(\frac{d^2y}{dx^2}\right)}}$$

$$\left(\frac{d^2y}{dx^2}<0\right)\Rightarrow\left(\operatorname{sgn}\left(\frac{dx^*}{dh}\right)=\operatorname{sgn}\left(\frac{d^2y}{dxdh}\right)\right)$$

$$\frac{d^2y}{dxdh}=P\frac{d^2g(x,h)}{dxdh}$$

$$P>0$$

$$\operatorname{sgn}\left(\frac{dx^*}{dh}\right)=\operatorname{sgn}\left(\frac{d^2g(x,h)}{dxdh}\right)$$

Result:

$$\frac{dx^*}{dh}\begin{cases}<0 & if \quad \dfrac{d^2g(x,h)}{dxdh}<0\\ =0 & if \quad \dfrac{d^2g(x,h)}{dxdh}=0\\ >0 & if \quad \dfrac{d^2g(x,h)}{dxdh}>0\end{cases}$$

How can this be interpreted?

Our objective function was initially defined as:

$$y(x)=f(x)+Pg(x,h)+(1-P)g(x,0)$$

Let us define marginally redefine the optimization problem:

$$y(x)=f(x)+PK(L(x),h)+(1-P)K(L(x),0)$$

$$K(L(x),h) = g(x,h)$$

Here, K replaces g and we have the function $L(x)$ that represents the resource available for future extraction as a function of the present extraction level. With growth and/or without growth, we usually find that:

$$\frac{dL}{dx} < 0$$

$$\frac{d^2 f}{dx^2} < 0$$

$$\frac{d^2 K}{dL^2} < 0$$

The first order optimum condition then becomes:

$$\frac{dy}{dx} = \frac{df}{dx} + \left(P \frac{dK(L(x),h)}{dL} + (1-P) \frac{dK(L(x),0)}{dL} \right) \frac{dL}{dx} = 0$$

A special case is when there is no growth of the resource. Then, $\frac{dL}{dx} = -1$.

Then, we get:

$$\frac{df}{dx} = \left(P \frac{dK(L(x),h)}{dL} + (1-P) \frac{dK(L(x),0)}{dL} \right)$$

This means that the expected marginal present value of the resource used for extraction should be the same in the present period and in the future. Then, if $\frac{d^2 K(L,h)}{dLdh} > 0$, and the value of the future risk parameter h increases, this makes the expected marginal present value of future extraction, $\frac{dK(L(x),h)}{dL}$ increase.

Then, $\frac{df}{dx}$ also has to increase. Since $\frac{d^2y}{dx^2} < 0$, the only way to make the first order optimum condition hold, is to reduce the present extraction level, x^*.

Since $\frac{d^2 f}{dx^2} < 0$, $\frac{df}{dx}$ increases if x is reduced. Then, the expected marginal present values of present and future extrations can again be set equal.

$$\frac{dx^*}{dh}\begin{cases} < 0 & if \quad \frac{d^2K(L,h)}{dLdh} > 0 \\ = 0 & if \quad \frac{d^2K(L,h)}{dLdh} = 0 \\ > 0 & if \quad \frac{d^2K(L,h)}{dLdh} < 0 \end{cases}$$

This illustrates the earlier found result:

$$\frac{dx^*}{dh}\begin{cases} < 0 & if \quad \frac{d^2g(x,h)}{dxdh} < 0 \\ = 0 & if \quad \frac{d^2g(x,h)}{dxdh} = 0 \\ > 0 & if \quad \frac{d^2g(x,h)}{dxdh} > 0 \end{cases}$$

Probabilities and outcomes:

Now, let us more explicitly define increasing risk and derive the conditional effects on the optimal value of x. In the next period, the outcome of a stochastic variable, s, will be known. This stochastic variable can represent different things, such as growth, price, environmental state etc.. More explicit cases will be defined in the later part of this analysis.

The original objective function was:

$$y(x) = P\left(f(x) + g(x,h)\right) + (1-P)\left(f(x) + g(x,0)\right)$$

Now, we get this objective function:

$$\boxed{y(x) = f(x) + \sum_{i=1}^{I} \phi(s_i)\varphi(x, s_i, \omega(h, s_i))}$$

The objective function $y(x)$ is the sum of the expected present values of present and future extraction, before future stochastic outcomes have been observed. $y(x)$ is a function of the extraction level x in the first period, period 1.

Increasing risk:

Definitions:

$\phi(s_i)$	Probability that the stochastic variable takes the value s_i in period 2.
	(The decision concerning x is taken in period 1, before s_i is known.)
$s_u \wedge s_v$	Two particular values the stochastic variable s. $s_u < s_v$
h	During a "mean preserving spread", s_u decreases by h and s_v increases by h. $h \geq 0$.
$\bar{\phi}$	$\bar{\phi} = \phi(s_u) = \phi(s_v) > 0$
$E(ds)$	Expected change of s as a result of a mean preserving spread. $E(ds) = -\phi(s_u)h + \phi(s_v)h = 0$
$\omega(h, s_i)$	The change of s_i as a result of a mean preserving spread.
$\varphi(x, s_i, \omega(h, s_i))$	Expected present value of future extraction when the value s_i is known (Of course, x and h are also known.)

Probability density

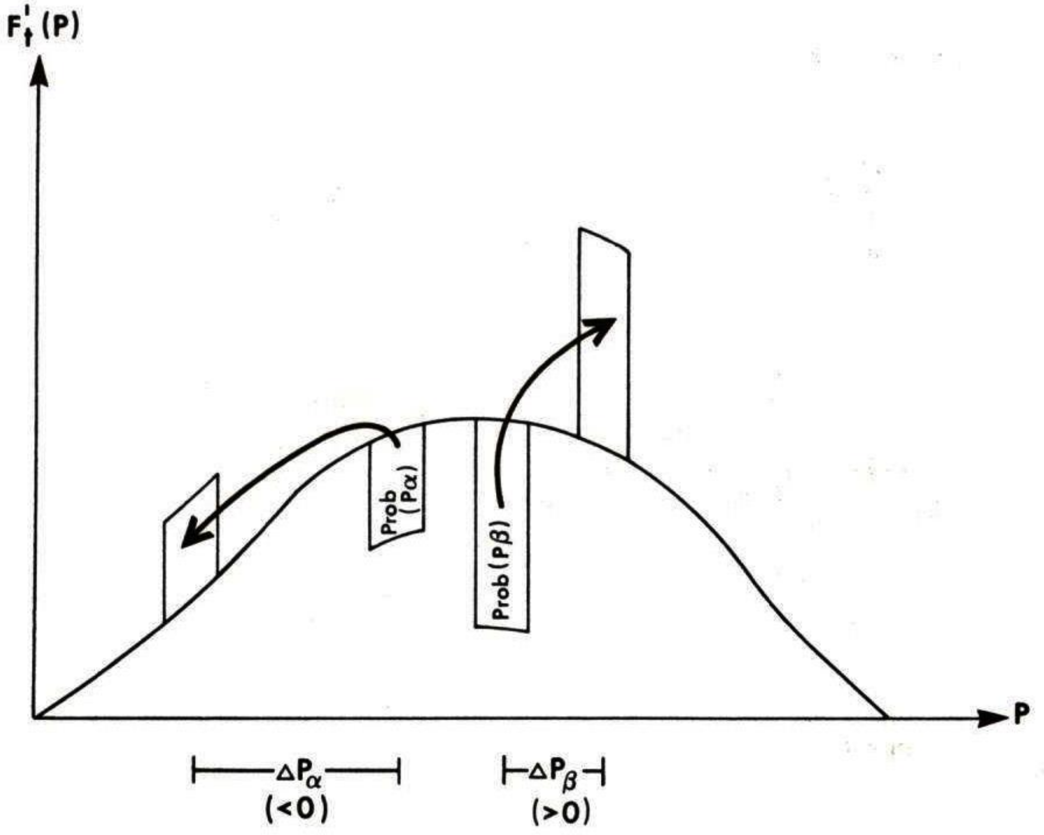

Figure M.2.10.1

A mean preserving spread (MPS) according to Rotschild and Stiglitz.
"Probability density" is moved from the center of the distribution to the tails in such a way that the expected value of the stochastic variable is not changed.

i	$\omega(h, s_i)$
1	0
.	.
u	-h
.	.
v	+h
.	.
l	0

Remark:

An almost identical analysis could be made with even more general mean preserving spreads, such that: $\boxed{E(ds) = -\phi(s_u)h_u + \phi(s_v)h_v = 0}$. Then, s_u would be reduced by h_u and s_v would be increased by h_v.

$\phi(s_u)h_u = \phi(s_v)h_v$ and $\dfrac{h_u}{h_v} = \dfrac{\phi(s_v)}{\phi(s_u)}$

In such a case, we would not need the constraint $\phi(s_u) = \phi(s_v)$. The notation would however become more confusing and the results of interest to this analysis would be the same as with the present analysis.

Let us define the function $\eta(x, s)$, as the expected present value of future (from period 2) extraction as a function of x and of the stochastic variable s, adjusted by the increasing risk in the probability distribution via the mean preserving spread.

$$\boxed{\begin{aligned} s_{u_2} &= s_u - h \\ s_{v_2} &= s_v + h \end{aligned}}$$

$$\eta(x,s_i)=\varphi(x,s_i,\omega(h,s_i))\;\forall\, s_i\big|_{i\neq u\wedge i\neq v}$$

$$\eta(x,s_u-h)=\varphi(x,s_u,\omega(h,s_u))$$

$$\eta(x,s_v+h)=\varphi(x,s_v,\omega(h,s_v))$$

$$\boxed{\begin{aligned}&\eta(x,s_{u_2})=\varphi(x,s_u,\omega(h,s_u))\\&\eta(x,s_{v_2})=\varphi(x,s_v,\omega(h,s_v))\end{aligned}}$$

First order optimum condition:

$$\frac{dy}{dx}=\frac{df}{dx}+\sum_i \phi(s_i)\frac{d\varphi(x,s_i,\omega(h,s_i))}{dx}=0$$

A unique interior maximum is assumed:

$$\frac{d^2y}{dx^2}=\frac{d^2f}{dx^2}+\sum_i \phi(s_i)\frac{d^2\varphi(x,s_i,\omega(h,s_i))}{dx^2}<0$$

$$\frac{d^2y}{dxdh}=\sum_i \phi(s_i)\frac{d^2\varphi(x,s_i,\omega(h,s_i))}{dxdh}$$

$$\frac{d^2y}{dxdh}=\phi(s_u)\frac{d^2\varphi(x,s_u,\omega(h,s_u))}{dxdh}+\phi(s_v)\frac{d^2\varphi(x,s_v,\omega(h,s_v))}{dxdh}$$

$$\varphi(x, s_u, \omega(h, s_u)) = \eta(x, s_{u_2}(s_u, h)) = \eta(x, s_u - h)$$

$$\varphi(x, s_v, \omega(h, s_v)) = \eta(x, s_{v_2}(s_v, h)) = \eta(x, s_v + h)$$

$$\boxed{\frac{d^2 y}{dxdh} = \bar{\phi}\left(\frac{d^2\eta(x, s_{u_2}(s_u, h))}{dxdh} + \frac{d^2\eta(x, s_{v_2}(s_v, h))}{dxdh}\right)}$$

Can the sign of $\frac{d^2 y}{dxdh}$ be determined ? (We remember that $h \geq 0$.)

$$\frac{d^2 y}{dxdh} = \bar{\phi}\left(\frac{d^2\eta(x, s_{u_2}(s_u, h))}{dxdh} + \frac{d^2\eta(x, s_{v_2}(s_v, h))}{dxdh}\right)$$

$$\frac{d^2 y}{dxdh} = \bar{\phi}\left(\frac{d^2\eta(x, s_{u_2})}{dxds}\frac{ds_{u_2}}{dh} + \frac{d^2\eta(x, s_{v_2})}{dxds}\frac{ds_{v_2}}{dh}\right)$$

$$\frac{d^2 y}{dxdh} = \bar{\phi}\left(\frac{d^2\eta(x, s_{u_2})}{dxds}(-1) + \frac{d^2\eta(x, s_{v_2})}{dxds}(+1)\right)$$

$$\boxed{\frac{d^2 y}{dxdh} = \bar{\phi}\left(-\frac{d^2\eta(x, s_{u_2})}{dxds} + \frac{d^2\eta(x, s_{v_2})}{dxds}\right)}$$

$$\left((s_u < s_v) \wedge (h \geq 0)\right) \Rightarrow \left(s_{u_2} < s_{v_2}\right)$$

$$\operatorname{sgn}\left(\frac{d^2y}{dxdh}\right)=\operatorname{sgn}\left(\frac{d^3\eta(x,s)}{dxds^2}\right)$$

$$\frac{dx^*}{dh}=-\frac{\left(\frac{d^2y}{dxdh}\right)}{\left(\frac{d^2y}{dx^2}\right)}$$

$$\boxed{\operatorname{sgn}\left(\frac{dx^*}{dh}\right)=\operatorname{sgn}\left(\frac{d^3\eta}{dxds^2}\right)}$$

The sign of this third order derivative determines the optimal direction of change of our present extraction level under the influence of increasing risk in the future.

How can these results be interpreted?

$$\frac{d^3\eta}{dxds^2} = \frac{d^2\left(\frac{d\eta}{dx}\right)}{ds^2}$$

We note that $\frac{d\eta}{dx}$ is the derivative of the expected present value of future (from period 2) extraction as a function of x with respect to the present extraction level. If $\frac{d^3\eta}{dxds^2} = \frac{d^2\left(\frac{d\eta}{dx}\right)}{ds^2}$ >0, then $\frac{d\eta}{dx}$ is a strictly convex function of the stochastic variable. Then, Jensen's inequality tells us that the expected value of $\frac{d\eta}{dx}$ increases if the risk of the stochastic variable increases. Hence, if the risk increases and $\frac{d^3\eta}{dxds^2} > 0$, it is rational that x^* increases.

Furthermore, if the risk increases and $\frac{d^3\eta}{dxds^2} < 0$, x^* decreases. If the risk increases and $\frac{d^3\eta}{dxds^2} = 0$, x^* remains unchanged.

$$\frac{d^3\eta(x,s)}{dxds^2}\begin{Bmatrix} > \\ (=) \\ < \end{Bmatrix}0$$ means that the marginal value of the resource used for present extraction increases (is unchanged) (decreases) in relation to the expected marginal present value of the resource used for future extraction, in case the future risk increases.

Then, it is obvious that the present extraction should increase (be unchanged)(decrease) as a result of increasing risk in the future.

Obviously, $\frac{d^3\eta(x,s)}{dxds^2}$ is of central importance to optimal extraction under risk. In the next sections, we will investigate how $\frac{d^3\eta(x,s)}{dxds^2}$ is affected by multi period settings and dynamic properties such as stationarity in the stochastic processes of relevance to the problem. Different constraints such as extraction volume constraints may also affect the results, in particular in multi species problems.

In order to discover the true and relevant effects of future risk on the optimal present decisions, it is necessary to let the future decisions be optimized conditional on the outcomes of stochastic events that will be observed before the future decisions are taken. The lowest number of periods that a resource extraction optimization problem must contain in order to discover, capture and analyze these effects is three.

For this reason, the rest of this analysis is based on three period versions of the problems. With more periods than three, the essential problem properties and results are the same but the results are more difficult to discover because of the large numbers of variables and equations. Earlier studies of related multi period problems have been made with stochastic dynamic programming and arbitrary numbers of periods. Please consult Lohmander (1987) and Lohmander (1988) for more details.

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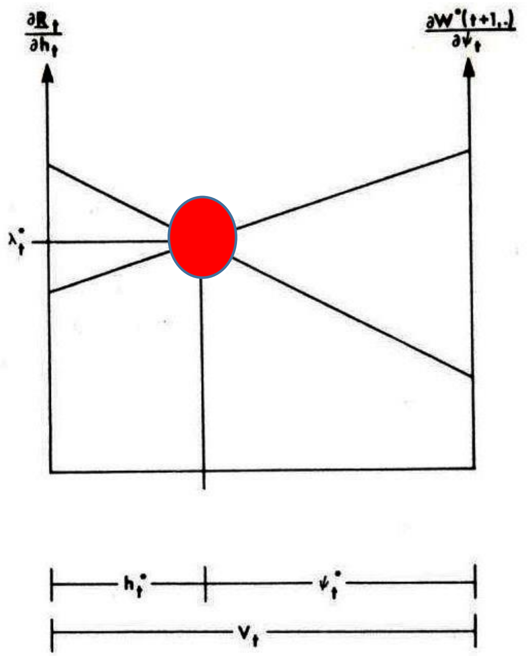
Towards multi period analysis

Marginal resource value In period t



Expected marginal resource value In period t+1

Figure 1.

At time t, an arbitrary period, ϕ_t is maximized. ϕ_t is the sum of $R_t(.)$, the present value of the profit in period t, and $W^*(t+1,.)$, the expected present value of the profits in the periods t+1, t+2,...,T. Optimal decisions are assumed in the periods t+1,...,T. Hence, ϕ_t is the expected present value at time t of the profits in the periods t, t+1,...,T when optimal decisions are assumed in the periods t+1,...,T. The maximization problem is :

$$\max_{h_t, \Psi_t} \phi_t = e^{-rt} R_t(h_t, P_t) + W^*(t+1, P_t, \Psi_t)$$

$$\text{s.t} \qquad h_t + \Psi_t = V_t$$

On stationary and nonstationary processes

Marginal resource value In period t

Expected marginal resource value In period t+1

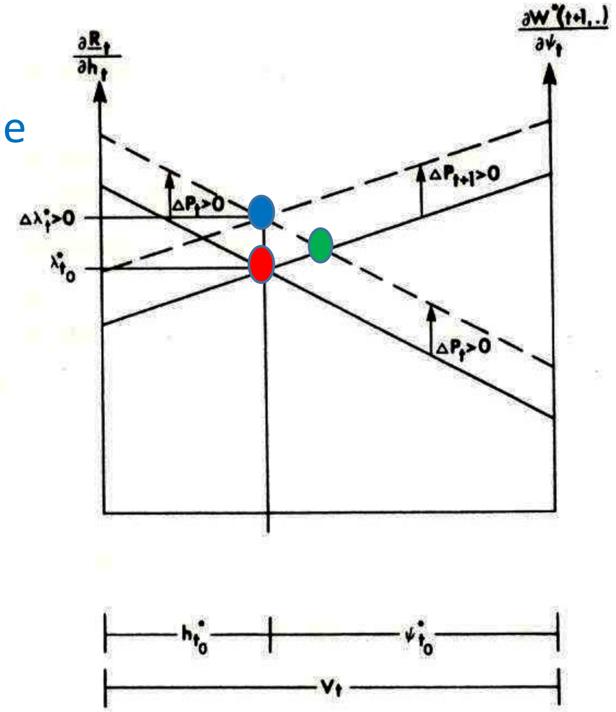
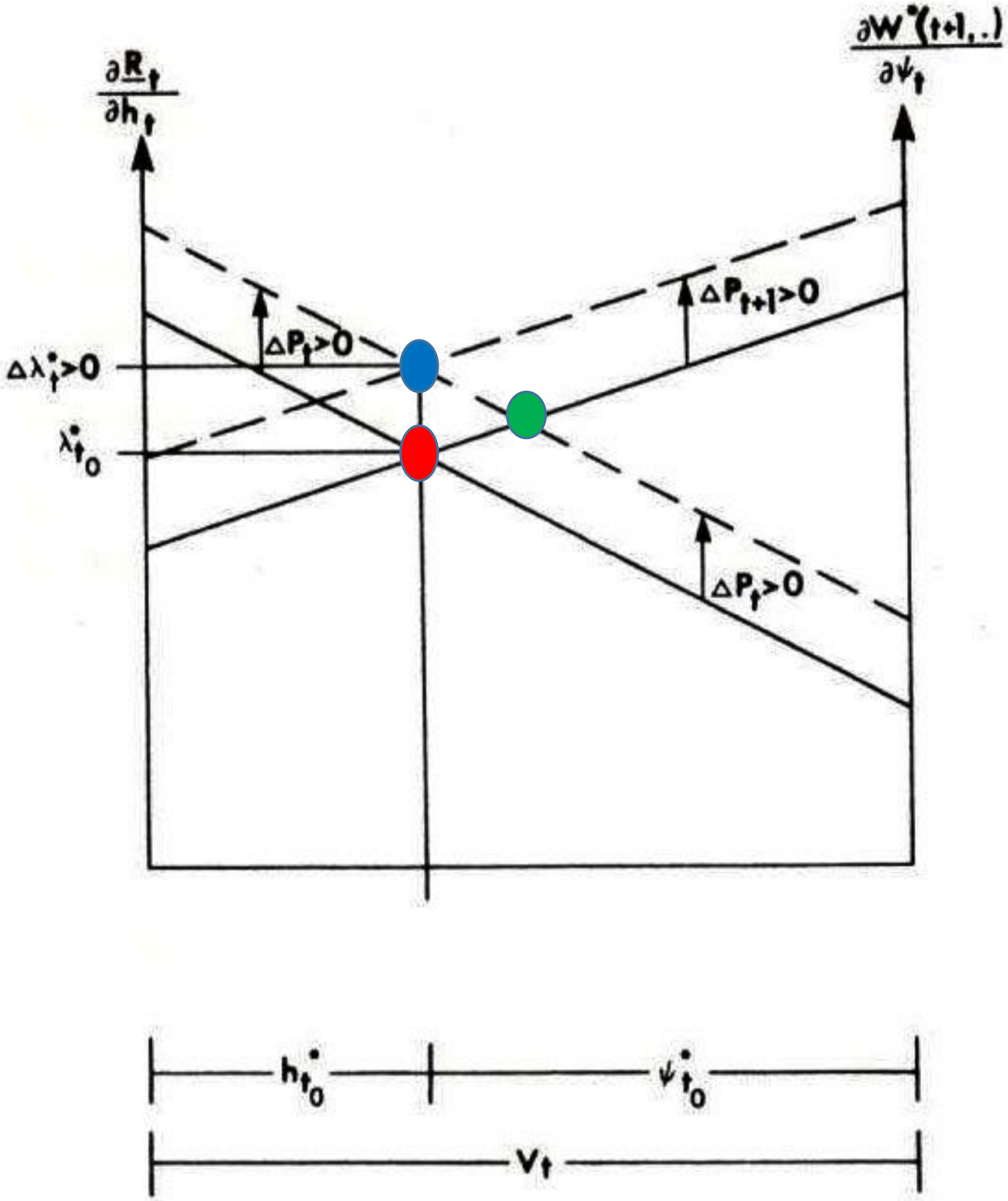


Figure 2.

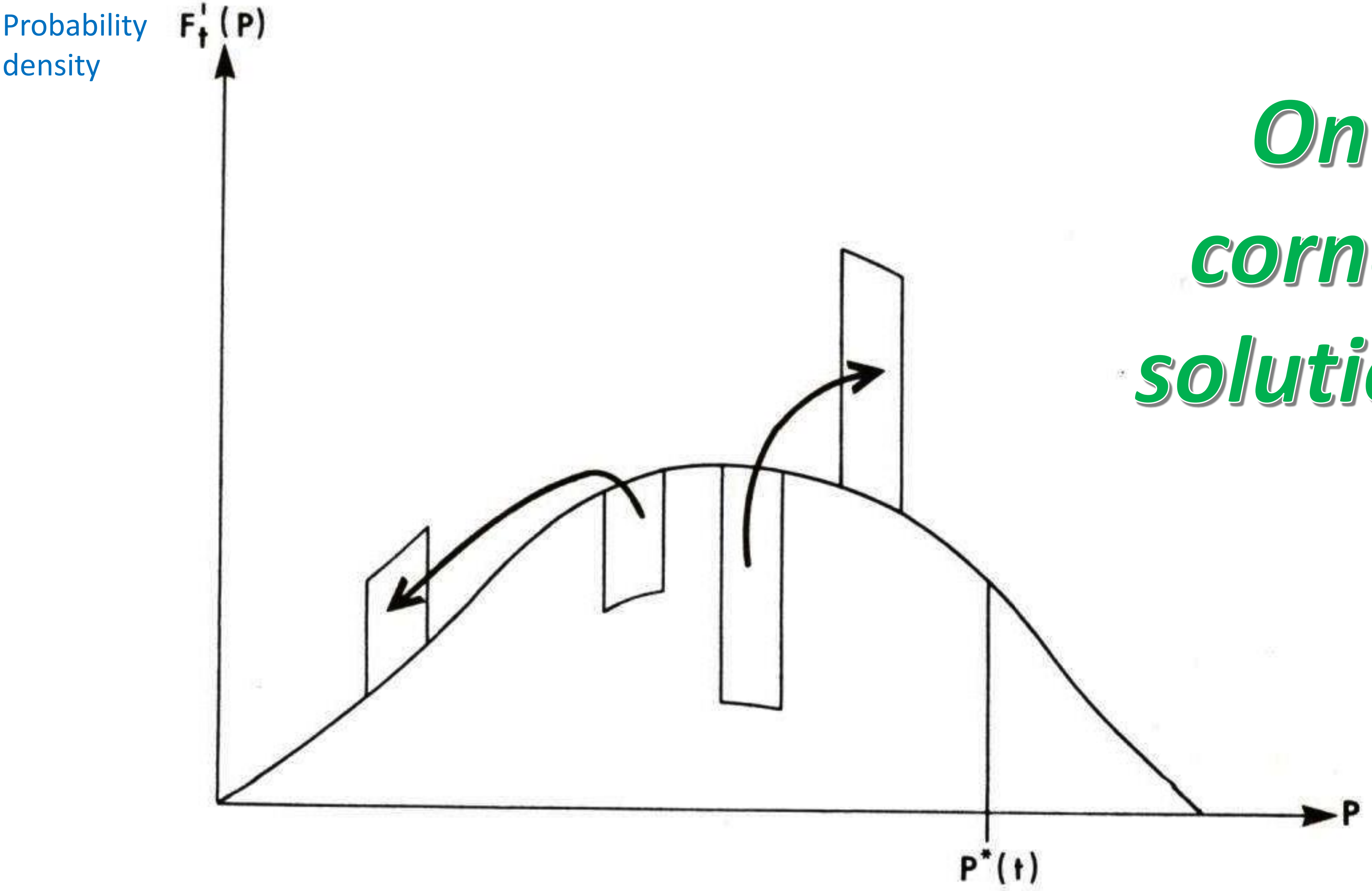
The graph shows a situation where the expected price in the future periods is the same as the revealed price in period t. (Price is a stochastic process called a Martingale.) Then, if the price in period t increases, the price is expected to increase also in the future periods. It is possible that the optimal harvest level in period t is not an increasing function of the price in period t in case price is a Martingale. However, if price is stationary around a predictable trend, the optimal harvest level in period t is generally an increasing function of the price in the same time period.

Marginal resource value In period t

Expected marginal resource value In period t+1



On corner solutions



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Optimization in multi period problems

The multi period problem

We maximize Z, the total expected present value. $R_t(.)$ and $C_t(.)$ denote discounted revenue and cost functions in period t.

Now, we introduce a three period problem. In period 1, x_1, the extraction level, is determined before the stochastic event in period 2 takes place. In period 2, the outcome of the stochastic event is observed before the extraction level is period 2, x_2, is determined. With probability ω, the discounted price in period 2 increases with h in relation to what was earlier assumed according to the revenue function. With probability $(1-\omega)$, the discounted price in period 2 decreases by h. In the first case, we select $x_2 = x_{21}$ and in the second case, we select $x_2 = x_{22}$. The resource available for extration in period 3, x_3, is of course affected by the decisions in period 2. If $x_2 = x_{21}$, then $x_3 = x_{31}$. If $x_2 = x_{22}$, then $x_3 = x_{32}$.

$$
\begin{aligned}
Z = \quad & R_1(x_1) - C_1(x_1) + \\
+ \quad & \omega\left(R_2(x_{21}) + hx_{21} - C_2(x_{21})\right) + \quad (1-\omega)\left(R_2(x_{22}) - hx_{22} - C_2(x_{22})\right) \\
+ \quad & \omega\left(R_3(x_{31}) - C_3(x_{31})\right) \qquad + \quad (1-\omega)\left(R_3(x_{32}) - C_3(x_{32})\right)
\end{aligned}
$$

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We may also study the effects of risk in the resource volume process, growth risk, with the same basic structure. Then, g serves as the risk parameter. With some probability, the volume increases by g and with some probability, the volume decreases by g, in relation to what was earlier expected.

Let us study a special case:

$$\max Z$$

$$\textit{subject to}$$

$$\alpha x_1 + \beta x_{21} + x_{31} = A + g$$

$$\alpha x_1 + \beta x_{22} + x_{32} = A - g$$

We note that we have five decision variables. In period 1, we only have one decision, the optimal extratction level, x_1. In period 2, we have two alternative optimal extraction levels, x_{21} or x_{22}, depending on the outcome of the stochastic event. In period 3, the optimal extration level x_{31} or x_{32}, is conditional on all earlier extraction levels and outcomes.

We may instantly solve for x_{31} and x_{32}.

$$x_{31} = A - \alpha x_1 - \beta x_{21} + g$$

$$x_{32} = A - \alpha x_1 - \beta x_{22} - g$$

$$\pi_t(.) = R_t(.) - C_t(.)$$

$$\begin{aligned}
Z \quad &= \quad \pi_1(x_1) + \\
&+ \quad \omega\left(\pi_2(x_{21}) + hx_{21}\right) + \quad (1-\omega)\left(\pi_2(x_{22}) - hx_{22}\right) \\
&+ \quad \omega\pi_3(x_{31}) \qquad\qquad + \quad (1-\omega)\pi_3(x_{32})
\end{aligned}$$

$$\begin{aligned}
Z \quad &= \quad \pi_1(x_1) + \\
&+ \quad \omega\left(\pi_2(x_{21}) + hx_{21}\right) \quad + \quad (1-\omega)\left(\pi_2(x_{22}) - hx_{22}\right) \\
&+ \quad \omega\pi_3(A - \alpha x_1 - \beta x_{21} + g) + \quad (1-\omega)\pi_3(A - \alpha x_1 - \beta x_{22} - g)
\end{aligned}$$

Three free decision variables and three first order optimum conditions

Optimization:

We have three first order optimum conditions since two of the five decision variables can be determined via the constraints and the other decision variables.

The first order optimum conditions are: $\frac{dZ}{dx_1} = 0$, $\frac{dZ}{dx_{21}} = 0$ and $\frac{dZ}{dx_{22}} = 0$. These may be expressed as:

$$\frac{dZ}{dx_1} = \quad \frac{d\pi_1(x_1)}{dx_1} - \omega\alpha \frac{d\pi_3(A-\alpha x_1 - \beta x_{21} + g)}{dx_3} - (1-\omega)\alpha \frac{d\pi_3(A-\alpha x_1 - \beta x_{22} - g)}{dx_3} = 0$$

$$\frac{dZ}{dx_{21}} = \quad \omega\left(\frac{d\pi_2(x_{21})}{dx_2} + h\right) \quad - \quad \omega\beta \frac{d\pi_3(A-\alpha x_1 - \beta x_{21} + g)}{dx_3} = 0$$

$$\frac{dZ}{dx_{22}} = \quad (1-\omega)\left(\frac{d\pi_2(x_{22})}{dx_2} - h\right) \quad - \quad (1-\omega)\beta \frac{d\pi_3(A-\alpha x_1 - \beta x_{22} - g)}{dx_3} = 0$$

Let us differentiate the first order optimum conditions with respect to the decision variables and the risk parameters:

$$\frac{d^2Z}{dx_1^{\ 2}}dx_1^{\ *}+\frac{d^2Z}{dx_1dx_{21}}dx_{21}^{\ *}+\frac{d^2Z}{dx_1dx_{22}}dx_{22}^{\ *}+\frac{d^2Z}{dx_1dg}dg=0$$

$$\frac{d^2Z}{dx_{21}dx_1}dx_1^{\ *}+\frac{d^2Z}{dx_{21}^{\ 2}}dx_{21}^{\ *}+\frac{d^2Z}{dx_{21}dx_{22}}dx_{22}^{\ *}+\frac{d^2Z}{dx_{21}dh}dh+\frac{d^2Z}{dx_{21}dg}dg=0$$

$$\frac{d^2Z}{dx_{22}dx_1}dx_1^{\ *}+\frac{d^2Z}{dx_{22}dx_{21}}dx_{21}^{\ *}+\frac{d^2Z}{dx_{22}^{\ 2}}dx_{22}^{\ *}+\frac{d^2Z}{dx_{22}dh}dh+\frac{d^2Z}{dx_{22}dg}dg=0$$

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4. **Optimal decisions under future price risk.**
5. Optimal decisions under future risk in the volume process (growth risk).
6. Optimal decisions under future price risk with mixed species.

The effects of increasing future price risk:

Now, we will investigate how the optimal values of the decision variables change if h increases.

$$\frac{d^2 Z}{dx_{21} dh} = \omega > 0$$

$$\frac{d^2 Z}{dx_{22} dh} = -(1-\omega) < 0$$

$$\omega = \frac{1}{2}$$

$$[D]\begin{bmatrix} dx_1^* \\ dx_{21}^* \\ dx_{22}^* \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{1}{2} dh \\ +\frac{1}{2} dh \end{bmatrix}$$

$$\left[D\right]=\begin{bmatrix} \left(\dfrac{d^2\pi_1}{dx_1{}^2}+\alpha^2 E\left(\dfrac{d^2\pi_3}{dx_3{}^2}\right)\right) & \left(\dfrac{\alpha\beta}{2}\dfrac{d^2\pi_3\left(x_{31}\right)}{dx_3{}^2}\right) & \left(\dfrac{\alpha\beta}{2}\dfrac{d^2\pi_3\left(x_{32}\right)}{dx_3{}^2}\right) \\ \left(\dfrac{\alpha\beta}{2}\dfrac{d^2\pi_3\left(x_{31}\right)}{dx_3{}^2}\right) & \left(\dfrac{1}{2}\dfrac{d^2\pi_2\left(x_{21}\right)}{dx_2{}^2}+\dfrac{\beta^2}{2}\dfrac{d^2\pi_3\left(x_{31}\right)}{dx_3{}^2}\right) & 0 \\ \left(\dfrac{\alpha\beta}{2}\dfrac{d^2\pi_3\left(x_{32}\right)}{dx_3{}^2}\right) & 0 & \left(\dfrac{1}{2}\dfrac{d^2\pi_2\left(x_{22}\right)}{dx_2{}^2}+\dfrac{\beta^2}{2}\dfrac{d^2\pi_3\left(x_{32}\right)}{dx_3{}^2}\right) \end{bmatrix}$$

$$\frac{dx_1^*}{dh}=\frac{\begin{vmatrix} 0 & \left(\frac{\alpha\beta}{2}\frac{d^2\pi_3(x_{31})}{dx_3^{\,2}}\right) & \left(\frac{\alpha\beta}{2}\frac{d^2\pi_3(x_{32})}{dx_3^{\,2}}\right) \\ -\frac{1}{2} & \left(\frac{1}{2}\frac{d^2\pi_2(x_{21})}{dx_2^{\,2}}+\frac{\beta^2}{2}\frac{d^2\pi_3(x_{31})}{dx_3^{\,2}}\right) & 0 \\ +\frac{1}{2} & 0 & \left(\frac{1}{2}\frac{d^2\pi_2(x_{22})}{dx_2^{\,2}}+\frac{\beta^2}{2}\frac{d^2\pi_3(x_{32})}{dx_3^{\,2}}\right) \end{vmatrix}}{|D|}$$

$$\frac{dx_1^*}{dh}=\frac{1}{|D|}\left\{\begin{array}{l} -\left(\frac{\alpha\beta}{2}\frac{d^2\pi_3(x_{31})}{dx_3^{\,2}}\right)\left(-\frac{1}{2}\right)\left(\frac{1}{2}\frac{d^2\pi_2(x_{22})}{dx_2^{\,2}}+\frac{\beta^2}{2}\frac{d^2\pi_3(x_{32})}{dx_3^{\,2}}\right) \\ -\left(\frac{\alpha\beta}{2}\frac{d^2\pi_3(x_{32})}{dx_3^{\,2}}\right)\left(\frac{1}{2}\frac{d^2\pi_2(x_{21})}{dx_2^{\,2}}+\frac{\beta^2}{2}\frac{d^2\pi_3(x_{31})}{dx_3^{\,2}}\right)\left(\frac{1}{2}\right) \end{array}\right\}$$

$$\frac{dx_1^*}{dh} = \frac{\alpha\beta}{8|D|}\left\{\left(\frac{d^2\pi_3(x_{31})}{dx_3{}^2}\right)\left(\frac{d^2\pi_2(x_{22})}{dx_2{}^2} + \beta^2\frac{d^2\pi_3(x_{32})}{dx_3{}^2}\right) - \left(\frac{d^2\pi_3(x_{32})}{dx_3{}^2}\right)\left(\frac{d^2\pi_2(x_{21})}{dx_2{}^2} + \beta^2\frac{d^2\pi_3(x_{31})}{dx_3{}^2}\right)\right\}$$

Simplification gives:

$$\frac{dx_1^*}{dh} = \frac{\alpha\beta}{8|D|}\left\{\left(\frac{d^2\pi_3(x_{31})}{dx_3{}^2}\right)\left(\frac{d^2\pi_2(x_{22})}{dx_2{}^2}\right) - \left(\frac{d^2\pi_3(x_{32})}{dx_3{}^2}\right)\left(\frac{d^2\pi_2(x_{21})}{dx_2{}^2}\right)\right\}$$

A unique maximum is assumed.

$$|D| < 0$$

Observation:

$$\operatorname{sgn}\left(\frac{dx_1^*}{dh}\right)=\operatorname{sgn}\left(\left(\frac{d^2\pi_2\left(x_{21}\right)}{dx_2^{\ 2}}\right)\left(\frac{d^2\pi_3\left(x_{32}\right)}{dx_3^{\ 2}}\right)-\left(\frac{d^2\pi_2\left(x_{22}\right)}{dx_2^{\ 2}}\right)\left(\frac{d^2\pi_3\left(x_{31}\right)}{dx_3^{\ 2}}\right)\right)$$

We assume decreasing marginal profits in all periods.

$$\frac{d^2\pi_2(.)}{dx_2^{\ 2}}<0$$

$$\frac{d^2\pi_3(.)}{dx_3^{\ 2}}<0$$

The following results follow from optimization:

$$(h>0)\Rightarrow\left(\left(x_{21}>x_{22}\right)\wedge\left(x_{31}<x_{32}\right)\right)$$

$$\left.\begin{array}{l}
\left(\dfrac{d^3\pi_2}{dx_2{}^3}<0 \quad \wedge \quad \dfrac{d^3\pi_3}{dx_3{}^3}<0\right)\\
\left(\dfrac{d^3\pi_2}{dx_2{}^3}=0 \quad \wedge \quad \dfrac{d^3\pi_3}{dx_3{}^3}=0\right)\\
\left(\dfrac{d^3\pi_2}{dx_2{}^3}>0 \quad \wedge \quad \dfrac{d^3\pi_3}{dx_3{}^3}>0\right)
\end{array}\right\}\Rightarrow\left\{\begin{array}{l}
\dfrac{dx_1^{\,*}}{dh}>0\\
\dfrac{dx_1^{\,*}}{dh}=0\\
\dfrac{dx_1^{\,*}}{dh}<0
\end{array}\right.$$

$$\left.\begin{array}{l}\left(\dfrac{d^3\pi_2}{d{x_2}^3}<0 \quad \wedge \quad \dfrac{d^3\pi_3}{d{x_3}^3}=0\right)\\ \left(\dfrac{d^3\pi_2}{d{x_2}^3}=0 \quad \wedge \quad \dfrac{d^3\pi_3}{d{x_3}^3}=0\right)\\ \left(\dfrac{d^3\pi_2}{d{x_2}^3}>0 \quad \wedge \quad \dfrac{d^3\pi_3}{d{x_3}^3}=0\right)\end{array}\right\} \Rightarrow \begin{cases}\dfrac{d{x_1}^*}{dh}>0\\ \dfrac{d{x_1}^*}{dh}=0\\ \dfrac{d{x_1}^*}{dh}<0\end{cases}$$

$$\left.\begin{array}{l}\left(\dfrac{d^3\pi_2}{d{x_2}^3}=0 \quad \wedge \quad \dfrac{d^3\pi_3}{d{x_3}^3}<0\right)\\ \left(\dfrac{d^3\pi_2}{d{x_2}^3}=0 \quad \wedge \quad \dfrac{d^3\pi_3}{d{x_3}^3}=0\right)\\ \left(\dfrac{d^3\pi_2}{d{x_2}^3}=0 \quad \wedge \quad \dfrac{d^3\pi_3}{d{x_3}^3}>0\right)\end{array}\right\} \Rightarrow \begin{cases}\dfrac{d{x_1}^*}{dh}>0\\ \dfrac{d{x_1}^*}{dh}=0\\ \dfrac{d{x_1}^*}{dh}<0\end{cases}$$

The results may also be summarized this way:

$$\left.\begin{array}{c}
\left(\dfrac{d^3\pi_2}{dx_2^{\ 3}} \le 0 \quad \wedge \quad \dfrac{d^3\pi_3}{dx_3^{\ 3}} \le 0 \quad \wedge \quad \left(\dfrac{d^3\pi_2}{dx_2^{\ 3}}\right) + \left(\dfrac{d^3\pi_3}{dx_3^{\ 3}}\right) \ne 0\right) \\
\left(\dfrac{d^3\pi_2}{dx_2^{\ 3}} = 0 \quad \wedge \quad \dfrac{d^3\pi_3}{dx_3^{\ 3}} = 0\right) \\
\left(\dfrac{d^3\pi_2}{dx_2^{\ 3}} \ge 0 \quad \wedge \quad \dfrac{d^3\pi_3}{dx_3^{\ 3}} \ge 0 \quad \wedge \quad \left(\dfrac{d^3\pi_2}{dx_2^{\ 3}}\right) + \left(\dfrac{d^3\pi_3}{dx_3^{\ 3}}\right) \ne 0\right)
\end{array}\right\} \Rightarrow \left\{\begin{array}{l}
\dfrac{dx_1^{\ *}}{dh} > 0 \\
\dfrac{dx_1^{\ *}}{dh} = 0 \\
\dfrac{dx_1^{\ *}}{dh} < 0
\end{array}\right.$$

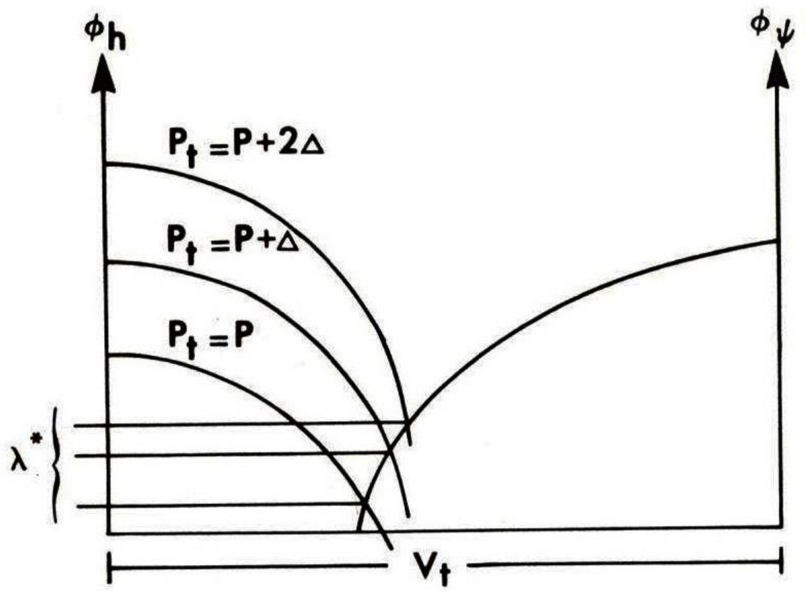


Figure 3.a.

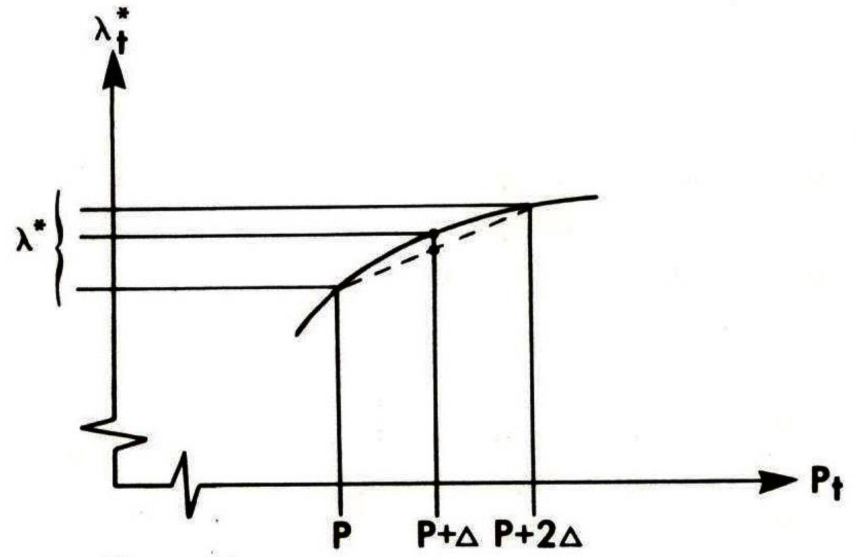


Figure 3.b.

$$\frac{d^3K}{dLdP^2} = \frac{d^2\left(\frac{dK}{dL}\right)}{dP^2} = \frac{d^2\lambda}{dP^2} < 0$$

$$E(\lambda) = E\left(\frac{dK}{dL}\right) decreases$$

$$if\ the\ risk\ in\ P\ increases.$$

The expected future marginal resource value decreases from increasing price risk and we should increase present extraction.

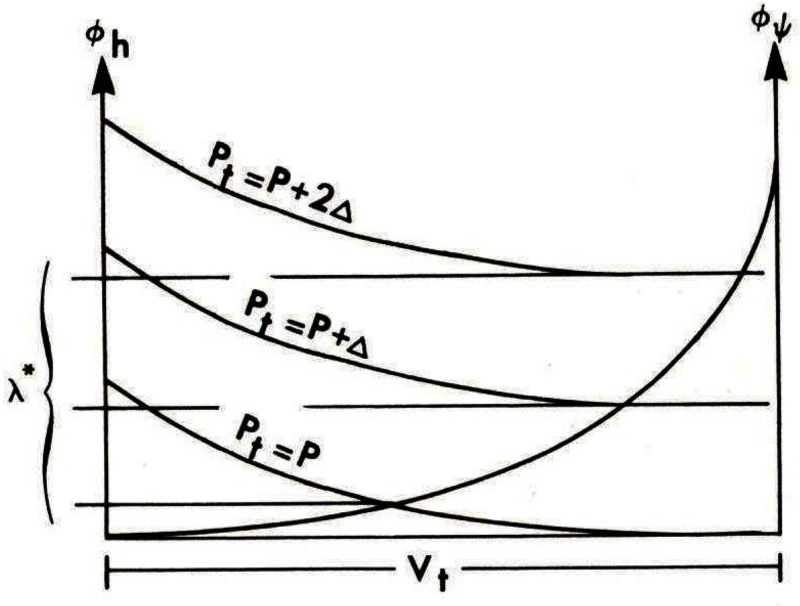


Figure 4.a.

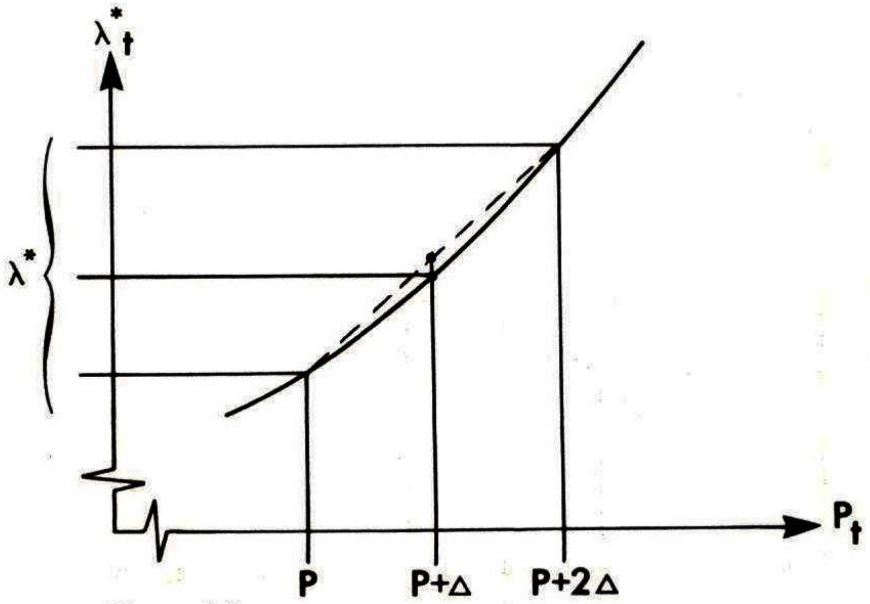


Figure 4.b.

$$\frac{d^3K}{dLdP^2} = \frac{d^2\left(\frac{dK}{dL}\right)}{dP^2} = \frac{d^2\lambda}{dP^2} > 0$$

$$E(\lambda) = E\left(\frac{dK}{dL}\right) increases$$

$$if\ the\, risk\, in\, P\, increases.$$

The expected future marginal resource value increases from increasing price risk and we should decrease present extraction.

Some results of increasing risk in the price process:

- If the future risk in the price process increases, we should increase the present extraction level in case the third order derivatives of profit with respect to volume are strictly negative.
- If the future risk in the price process increases, we should not change the present extraction level in case the third order derivatives of profit with respect to volume are zero.
- If the future risk in the price process increases, we should decrease the present extraction level in case the third order derivatives of profit with respect to volume are strictly positive.

Contents:

The effects of increasing future risk in the volume process:

Now, we will investigate how the optimal values of the decision variables change if g increases.
We recall these first order derivatives:

$$\frac{dZ}{dx_1} = \frac{d\pi_1(x_1)}{dx_1} - \omega\alpha \frac{d\pi_3(A-\alpha x_1 - \beta x_{21} + g)}{dx_3} - (1-\omega)\alpha \frac{d\pi_3(A-\alpha x_1 - \beta x_{22} - g)}{dx_3} = 0$$

$$\frac{dZ}{dx_{21}} = \omega\left(\frac{d\pi_2(x_{21})}{dx_2} + h\right) - \omega\beta \frac{d\pi_3(A-\alpha x_1 - \beta x_{21} + g)}{dx_3} = 0$$

$$\frac{dZ}{dx_{22}} = (1-\omega)\left(\frac{d\pi_2(x_{22})}{dx_2} - h\right) - (1-\omega)\beta \frac{d\pi_3(A-\alpha x_1 - \beta x_{22} - g)}{dx_3} = 0$$

$$\frac{d^2 Z}{dx_1 dg} = \quad -\omega\alpha \frac{d^2 \pi_3 (A - \alpha x_1 - \beta x_{21} + g)}{dx_3{}^2} + (1-\omega)\alpha \frac{d^2 \pi_3 (A - \alpha x_1 - \beta x_{22} - g)}{dx_3{}^2}$$

$$\frac{d^2 Z}{dx_{21} dg} = \quad -\omega\beta \frac{d^2 \pi_3 (A - \alpha x_1 - \beta x_{21} + g)}{dx_3{}^2}$$

$$\frac{d^2 Z}{dx_{22} dg} = \quad +(1-\omega)\beta \frac{d^2 \pi_3 (A - \alpha x_1 - \beta x_{22} - g)}{dx_3{}^2}$$

With more simple notation, we get:

$$\frac{d^2Z}{dx_1dg} = \quad -\omega\alpha\frac{d^2\pi_3(x_{31})}{dx_3{}^2} + (1-\omega)\alpha\frac{d^2\pi_3(x_{32})}{dx_3{}^2}$$

$$\frac{d^2Z}{dx_{21}dg} = \quad -\omega\beta\frac{d^2\pi_3(x_{31})}{dx_3{}^2}$$

$$\frac{d^2Z}{dx_{22}dg} = \quad +(1-\omega)\beta\frac{d^2\pi_3(x_{32})}{dx_3{}^2}$$

We have already differentiated the first order optimum conditions with respect to the decision variables and the risk parameters:

$$\frac{d^2Z}{dx_1{}^2}dx_1{}^* + \frac{d^2Z}{dx_1dx_{21}}dx_{21}{}^* + \frac{d^2Z}{dx_1dx_{22}}dx_{22}{}^* \qquad + \frac{d^2Z}{dx_1dg}dg = 0$$

$$\frac{d^2Z}{dx_{21}dx_1}dx_1{}^* + \frac{d^2Z}{dx_{21}{}^2}dx_{21}{}^* + \frac{d^2Z}{dx_{21}dx_{22}}dx_{22}{}^* + \frac{d^2Z}{dx_{21}dh}dh + \frac{d^2Z}{dx_{21}dg}dg = 0$$

$$\frac{d^2Z}{dx_{22}dx_1}dx_1{}^* + \frac{d^2Z}{dx_{22}dx_{21}}dx_{21}{}^* + \frac{d^2Z}{dx_{22}{}^2}dx_{22}{}^* + \frac{d^2Z}{dx_{22}dh}dh + \frac{d^2Z}{dx_{22}dg}dg = 0$$

Now, we will investigate how the optimal values of the decision variables change if g increases. ($dh = 0$.)

$$\frac{d^2Z}{dx_1^{\,2}} dx_1^{\,*} + \frac{d^2Z}{dx_1 dx_{21}} dx_{21}^{\,*} + \frac{d^2Z}{dx_1 dx_{22}} dx_{22}^{\,*} = -\frac{d^2Z}{dx_1 dg} dg$$

$$\frac{d^2Z}{dx_{21} dx_1} dx_1^{\,*} + \frac{d^2Z}{dx_{21}^{\,2}} dx_{21}^{\,*} + \frac{d^2Z}{dx_{21} dx_{22}} dx_{22}^{\,*} = -\frac{d^2Z}{dx_{21} dg} dg$$

$$\frac{d^2Z}{dx_{22} dx_1} dx_1^{\,*} + \frac{d^2Z}{dx_{22} dx_{21}} dx_{21}^{\,*} + \frac{d^2Z}{dx_{22}^{\,2}} dx_{22}^{\,*} = -\frac{d^2Z}{dx_{22} dg} dg$$

$$[D]=\begin{bmatrix} \left(\dfrac{d^2\pi_1}{dx_1{}^2}+\alpha^2 E\left(\dfrac{d^2\pi_3}{dx_3{}^2}\right)\right) & \left(\dfrac{\alpha\beta}{2}\dfrac{d^2\pi_3\left(x_{31}\right)}{dx_3{}^2}\right) & \left(\dfrac{\alpha\beta}{2}\dfrac{d^2\pi_3\left(x_{32}\right)}{dx_3{}^2}\right) \\ \left(\dfrac{\alpha\beta}{2}\dfrac{d^2\pi_3\left(x_{31}\right)}{dx_3{}^2}\right) & \left(\dfrac{1}{2}\dfrac{d^2\pi_2\left(x_{21}\right)}{dx_2{}^2}+\dfrac{\beta^2}{2}\dfrac{d^2\pi_3\left(x_{31}\right)}{dx_3{}^2}\right) & 0 \\ \left(\dfrac{\alpha\beta}{2}\dfrac{d^2\pi_3\left(x_{32}\right)}{dx_3{}^2}\right) & 0 & \left(\dfrac{1}{2}\dfrac{d^2\pi_2\left(x_{22}\right)}{dx_2{}^2}+\dfrac{\beta^2}{2}\dfrac{d^2\pi_3\left(x_{32}\right)}{dx_3{}^2}\right) \end{bmatrix}$$

$$[D]\begin{bmatrix} dx_1{}^* \\ dx_{21}{}^* \\ dx_{22}{}^* \end{bmatrix}=\begin{bmatrix} \left(+\omega\alpha\dfrac{d^2\pi_3(x_{31})}{dx_3{}^2}-(1-\omega)\alpha\dfrac{d^2\pi_3(x_{32})}{dx_3{}^2}\right)dg \\ \left(\omega\,\beta\dfrac{d^2\pi_3(x_{31})}{dx_3{}^2}\right)dg \\ \left(-(1-\omega)\,\beta\dfrac{d^2\pi_3(x_{32})}{dx_3{}^2}\right)dg \end{bmatrix}$$

$$\omega=\frac{1}{2}$$

$$\frac{dx_1^*}{dg} = \frac{\begin{vmatrix} \left(+\frac{\alpha}{2}\frac{d^2\pi_3(x_{31})}{dx_3{}^2} - \frac{\alpha}{2}\frac{d^2\pi_3(x_{32})}{dx_3{}^2}\right) & \left(\frac{\alpha\beta}{2}\frac{d^2\pi_3\left(x_{31}\right)}{dx_3{}^2}\right) & \left(\frac{\alpha\beta}{2}\frac{d^2\pi_3\left(x_{32}\right)}{dx_3{}^2}\right) \\ \left(\frac{\beta}{2}\frac{d^2\pi_3(x_{31})}{dx_3{}^2}\right) & \left(\frac{1}{2}\frac{d^2\pi_2\left(x_{21}\right)}{dx_2{}^2} + \frac{\beta^2}{2}\frac{d^2\pi_3\left(x_{31}\right)}{dx_3{}^2}\right) & 0 \\ \left(-\frac{\beta}{2}\frac{d^2\pi_3(x_{32})}{dx_3{}^2}\right) & 0 & \left(\frac{1}{2}\frac{d^2\pi_2\left(x_{22}\right)}{dx_2{}^2} + \frac{\beta^2}{2}\frac{d^2\pi_3\left(x_{32}\right)}{dx_3{}^2}\right) \end{vmatrix}}{\left|D\right|}$$

Let us simplify notation:

$$U(.) = \frac{d^2\pi_2(.)}{dx_2{}^2}$$

$$W(.) = \frac{d^2\pi_3(.)}{dx_3{}^2}$$

$$\frac{dx_1{}^*}{dg} = \left(\frac{1}{8}\right)\frac{\begin{vmatrix} \alpha\left(W(x_{31})-W(x_{32})\right) & \left(\alpha\beta W(x_{31})\right) & \left(\alpha\beta W(x_{32})\right) \\ \left(\beta W(x_{31})\right) & \left(U(x_{21})+\beta^2 W(x_{31})\right) & 0 \\ \left(-\beta W(x_{32})\right) & 0 & \left(U(x_{22})+\beta^2 W(x_{32})\right) \end{vmatrix}}{|D|}$$

$$\frac{dx_1^*}{dg} = \frac{1}{8|D|}\begin{pmatrix} \alpha\left(W(x_{31}) - W(x_{32})\right)\left(U(x_{21}) + \beta^2 W(x_{31})\right)\left(U(x_{22}) + \beta^2 W(x_{32})\right) \\ -\left(\alpha\beta W(x_{31})\right)\left(\beta W(x_{31})\right)\left(U(x_{22}) + \beta^2 W(x_{32})\right) \\ -\left(\alpha\beta W(x_{32})\right)\left(U(x_{21}) + \beta^2 W(x_{31})\right)\left(-\beta W(x_{32})\right) \end{pmatrix}$$

Now, we simplify notation even further:

$$u_j = U(x_{ij})$$

$$w_j = W(x_{ij})$$

$$\frac{dx_1^*}{dg} = \frac{1}{8|D|}\begin{pmatrix} \alpha\left(w_1 - w_2\right)\left(u_1 + \beta^2 w_1\right)\left(u_2 + \beta^2 w_2\right) \\ -\left(\alpha\beta w_1\right)\left(\beta w_1\right)\left(u_2 + \beta^2 w_2\right) \\ +\left(\alpha\beta w_2\right)\left(u_1 + \beta^2 w_1\right)\left(\beta w_2\right) \end{pmatrix}$$

$$\phi = \alpha\left(w_1 - w_2\right)\left(u_1 + \beta^2 w_1\right)\left(u_2 + \beta^2 w_2\right) - \left(\alpha\beta w_1\right)\left(\beta w_1\right)\left(u_2 + \beta^2 w_2\right) + \left(\alpha\beta w_2\right)\left(u_1 + \beta^2 w_1\right)\left(\beta w_2\right)$$

We once again simplify notation to the following expression (where all variables appear in the same order as before and all indices are removed):

$$\phi = \mathrm{a(w - x)(u + bbw)(s + bbx) - (abw)(bw)(s + bbx) + (abx)(u + bbw)(bx)}$$

This expression can instantly be simplified to:

$$\phi = \mathrm{a(suw - x(b^2w(s - u) + su))}$$

This can be rearranged to:

$$\phi = \mathrm{a(su(w\text{-}x) - x(b^2w(s - u)))}$$

$$\phi = \mathrm{a}\left(\mathrm{su(w\text{-}x) + b^2wx(u\text{-}s)}\right)$$

Now, we slowly move back to our original notation:

$$\phi = \alpha\left(\mathrm{u}_1\mathrm{u}_2(\mathrm{w}_1\text{-}\mathrm{w}_2) + \beta^2\mathrm{w}_1\mathrm{w}_2(\mathrm{u}_1\text{-}\mathrm{u}_2)\right)$$

$$\phi = \alpha\Big(\quad \mathrm{U}(\mathrm{x}_{21})\mathrm{U}(\mathrm{x}_{22})\big(\mathrm{W}(\mathrm{x}_{31})\text{-}\mathrm{W}(\mathrm{x}_{32})\big) \quad + \quad \beta^2\mathrm{W}(\mathrm{x}_{31})\mathrm{W}(\mathrm{x}_{32})\big(\mathrm{U}(\mathrm{x}_{21})\text{-}\mathrm{U}(\mathrm{x}_{22})\big) \quad\Big)$$

$$\boxed{\phi = \alpha\left(\begin{array}{l} \dfrac{d^2\pi_2(\mathrm{x}_{21})}{dx_2{}^2}\dfrac{d^2\pi_2(\mathrm{x}_{22})}{dx_2{}^2}\left(\dfrac{d^2\pi_3(\mathrm{x}_{31})}{dx_3{}^2}-\dfrac{d^2\pi_3(\mathrm{x}_{32})}{dx_3{}^2}\right) \\ +\beta^2\dfrac{d^2\pi_3(\mathrm{x}_{31})}{dx_3{}^2}\dfrac{d^2\pi_3(\mathrm{x}_{32})}{dx_3{}^2}\left(\dfrac{d^2\pi_2(\mathrm{x}_{21})}{dx_2{}^2}-\dfrac{d^2\pi_2(\mathrm{x}_{22})}{dx_2{}^2}\right)\end{array}\right)}$$

Observations:

$$\frac{dx_1^*}{dg} = \frac{\phi}{8|D|}$$

We already know that $|D| < 0$.

$$\left((g>0)\wedge\left(\frac{d^2\pi_2}{dx_2{}^2}<0\right)\wedge\left(\frac{d^2\pi_3}{dx_3{}^2}<0\right)\right)\Rightarrow\left((x_{21}>x_{22})\wedge(x_{31}>x_{32})\right)$$

Results:

$$\left(\left(\frac{d^3\pi_2}{dx_2{}^3}\le 0\right)\wedge\left(\frac{d^3\pi_3}{dx_3{}^3}<0\right)\right)\Rightarrow\left(\frac{dx_1{}^*}{dg}>0\right)$$

$$\left(\left(\frac{d^3\pi_2}{dx_2{}^3}<0\right)\wedge\left(\frac{d^3\pi_3}{dx_3{}^3}\le 0\right)\right)\Rightarrow\left(\frac{dx_1{}^*}{dg}>0\right)$$

$$\left(\left(\frac{d^3\pi_2}{dx_2{}^3}\le 0\right)\wedge\left(\frac{d^3\pi_3}{dx_3{}^3}\le 0\right)\right)\Rightarrow\left(\frac{dx_1{}^*}{dg}\ge 0\right)$$

$$\left(\left(\frac{d^3\pi_2}{dx_2{}^3}=0\right)\wedge\left(\frac{d^3\pi_3}{dx_3{}^3}=0\right)\right)\Rightarrow\left(\frac{dx_1{}^*}{dg}=0\right)$$

$$\left(\left(\frac{d^3\pi_2}{dx_2{}^3}\ge 0\right)\wedge\left(\frac{d^3\pi_3}{dx_3{}^3}\ge 0\right)\right)\Rightarrow\left(\frac{dx_1{}^*}{dg}\le 0\right)$$

$$\left(\left(\frac{d^3\pi_2}{dx_2{}^3}\ge 0\right)\wedge\left(\frac{d^3\pi_3}{dx_3{}^3}>0\right)\right)\Rightarrow\left(\frac{dx_1{}^*}{dg}<0\right)$$

$$\left(\left(\frac{d^3\pi_2}{dx_2{}^3}>0\right)\wedge\left(\frac{d^3\pi_3}{dx_3{}^3}\ge 0\right)\right)\Rightarrow\left(\frac{dx_1{}^*}{dg}<0\right)$$

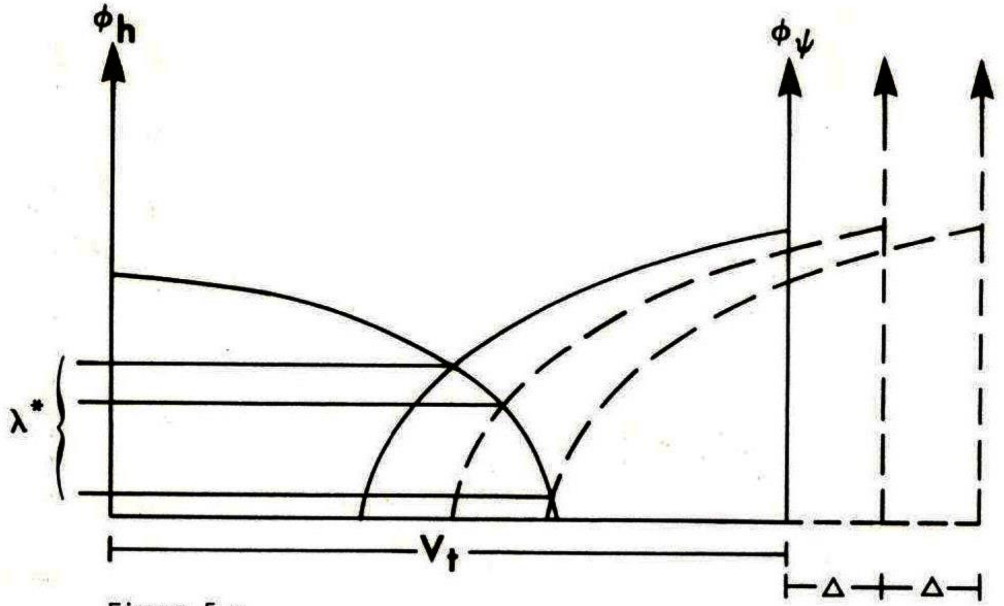


Figure 5.a.

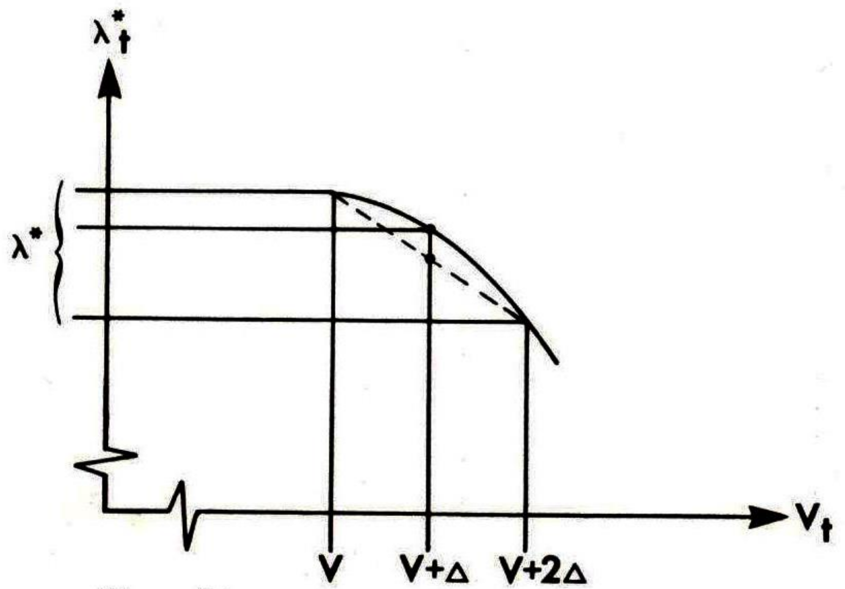


Figure 5.b.

$$\frac{d^3K}{dLdV^2} = \frac{d^2\left(\frac{dK}{dL}\right)}{dV^2} = \frac{d^2\lambda}{dV^2} < 0$$

$$E(\lambda) = E\left(\frac{dK}{dL}\right) decreases$$

if the risk in V increases.

The expected future marginal resource value decreases from increasing risk in the volume process (growth) and we should increase present extraction.

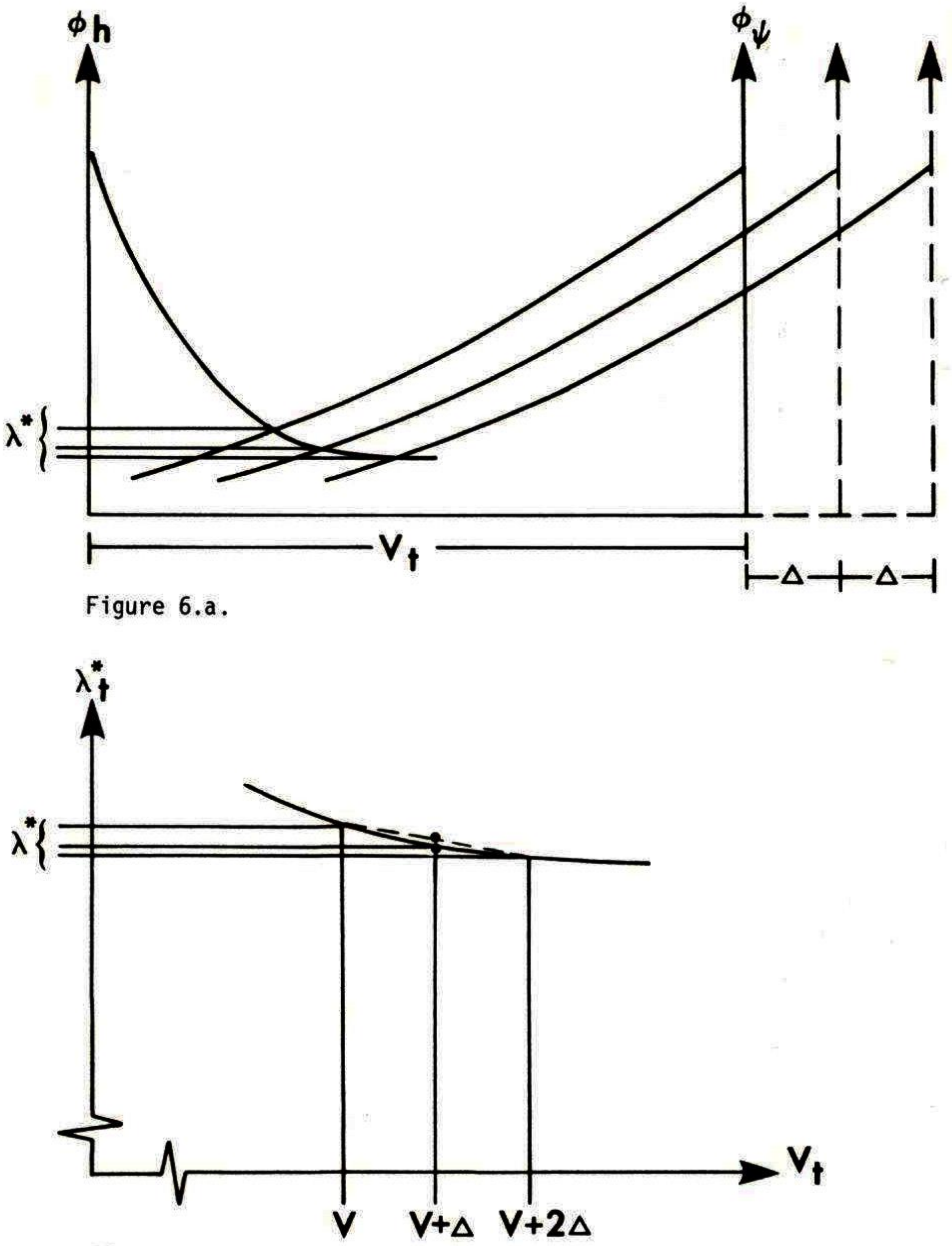


Figure 6.a.

Figure 6.b.

$$\frac{d^3K}{dLdV^2} = \frac{d^2\left(\frac{dK}{dL}\right)}{dV^2} = \frac{d^2\lambda}{dV^2} > 0$$

$$E(\lambda) = E\left(\frac{dK}{dL}\right) \textit{increases}$$

if the risk in V increases.

The expected future marginal resource value increases from increasing risk in the volume process (growth) and we should decrease present extraction.

Some results of increasing risk in the volume process (growth process):

- If the future risk in the volume process increases, we should increase the present extraction level in case the third order derivatives of profit with respect to volume are strictly negative.
- If the future risk in the volume process increases, we should not change the present extraction level in case the third order derivatives of profit with respect to volume are zero.
- If the future risk in the volume process increases, we should decrease the present extraction level in case the third order derivatives of profit with respect to volume are strictly positive.

Contents:

1. Introduction via one dimensional optimization in dynamic problems, comparative statics analysis, probabilities, increasing risk and the importance of third order derivatives.
2. Explicit multi period analysis, stationarity and corner solutions.
3. Multi period problems and model structure with sequential adaptive decisions and risk.
4. Optimal decisions under future price risk.
5. Optimal decisions under future risk in the volume process (growth risk).
6. **Optimal decisions under future price risk with mixed species.**

The mixed species case:

- A complete dynamic analysis of optimal natural resource management with several species should include decisions concerning total stock levels and interspecies competition.
- In the following analysis, we study a case with two species, where the growth of a species is assumed to be a function of the total stock level and the stock level of the individual species.
- The total stock level has however already indirectly been determined via binding constraints on total harvesting in periods 1 and 2.
- We start with a deterministic version of the problem and later move to the stochastic counterpart.

Π is the total present value. x_{it} denotes harves volume of species i in period t .

$\pi_{it}(x_{it})$ is the present value of harvesting species i in period t .

Each species has an intertemporal harvest volume constraint.

H_t denotes the total harvest volume in period t.

These total harvest volumes are constrained in periods 1 and 2,

because of harvest capacity constraints, constraints in logistics or other constraints,

maybe reflecting the desire to control the total stock level.

Period 1 Period 2 Period 3

$$\max \Pi = \boxed{\pi_{11}(x_{11}) + \pi_{21}(x_{21})} + \boxed{\pi_{12}(x_{12}) + \pi_{22}(x_{22})} + \boxed{\pi_{13}(x_{13}) + \pi_{23}(x_{23})}$$

$s.t.$

$$\alpha x_{11} + \beta x_{12} + x_{13} = C_1$$

$$\alpha x_{21} + \beta x_{22} + x_{23} = C_2$$

$$x_{11} + x_{21} = H_1$$

$$x_{12} + x_{22} = H_2$$

Consequences:

$$x_{21} = H_1 - x_{11}$$

$$x_{22} = H_2 - x_{12}$$

$$x_{13} = C_1 - \alpha x_{11} - \beta x_{12}$$

$$x_{23} = C_2 - \alpha x_{21} - \beta x_{22}$$

$$x_{23} = C_2 - \alpha(H_1 - x_{11}) - \beta(H_2 - x_{12})$$

$$\Pi = \pi_{11}(x_{11}) + \pi_{21}(x_{21}) + \pi_{12}(x_{12}) + \pi_{22}(x_{22}) + \pi_{13}(x_{13}) + \pi_{23}(x_{23})$$

$$\begin{aligned}\Pi = {} & \pi_{11}(x_{11}) + \pi_{21}(H_1 - x_{11}) + \pi_{12}(x_{12}) + \pi_{22}(H_2 - x_{12}) \\ & + \pi_{13}(C_1 - \alpha x_{11} - \beta x_{12}) + \pi_{23}(C_2 - \alpha(H_1 - x_{11}) - \beta(H_2 - x_{12}))\end{aligned}$$

Now, we move to a stochastic version of the same problem. Θ is the expected total present value under the influence of stochastic future events and optimal adaptive decisions. With probability ϕ, the discounted price of species 1 increases by h in period 2 and with probability $(1-\phi)$, the price decreases by the same amount. We define this a ”mean preserving spread” via the constraint $\phi = (1-\phi) = 1/2$.

x_{itp} =Harvest volume in species i, at time t, for price state p

<u>A: Consequences for harvest decisions in periods 2 and 3 of a price increase of species 1 in period 2:</u>

Consequences for harvest decisions for species 1:

If the price in period 2 of species 1 increases by h, then we harvest $x_{12} = x_{121}$ in period 2. In period 3, we get the conditional harvest $x_{13} = x_{131}$.

Consequences for harvest decisions for species 2:

If the price in period 2 of species 1 increases by h, then we harvest $x_{22} = x_{221}$ in period 2. In period 3, we get the conditional harvest $x_{23} = x_{231}$.

B: Consequences for harvest decisions in periods 2 and 3 of a price decrease of species 1 in period 2:

Consequences for harvest decisions for species 1:

If the price in period 2 of species 1 decreases by h, then we harvest $x_{12} = x_{122}$ in period 2. In period 3, we get the conditional harvest $x_{13} = x_{132}$.

Consequences for harvest decisions for species 2:

If the price in period 2 of species 1 decreases by h, then we harvest $x_{22} = x_{222}$ in period 2. In period 3, we get the conditional harvest $x_{23} = x_{232}$.

$$
\begin{aligned}
\Theta = \quad & \pi_{11}(x_{11}) + \pi_{21}(H_1 - x_{11}) \\
& + \phi\Big[\big(\pi_{12}(x_{121}) + hx_{121} + \pi_{22}(H_2 - x_{121})\big) + \big(\pi_{13}(x_{131}) + \pi_{23}(x_{231})\big)\Big] \\
& + (1-\phi)\Big[\big(\pi_{12}(x_{122}) - hx_{122} + \pi_{22}(H_2 - x_{122})\big) + \big(\pi_{13}(x_{132}) + \pi_{23}(x_{232})\big)\Big]
\end{aligned}
$$

$$
\begin{aligned}
x_{131} &= C_1 - \alpha x_{11} - \beta x_{121} \\
x_{231} &= C_2 - \alpha(H_1 - x_{11}) - \beta(H_2 - x_{121})
\end{aligned}
$$

$$
\begin{aligned}
x_{132} &= C_1 - \alpha x_{11} - \beta x_{122} \\
x_{232} &= C_2 - \alpha(H_1 - x_{11}) - \beta(H_2 - x_{122})
\end{aligned}
$$

$$\phi = \frac{1}{2}$$

$$\begin{aligned} Z = 2\Theta = \quad & 2\pi_{11}(x_{11}) + 2\pi_{21}(H_1 - x_{11}) \\ & + \left(\pi_{12}(x_{121}) + hx_{121} + \pi_{22}(H_2 - x_{121})\right) + \left(\pi_{13}(x_{131}) + \pi_{23}(x_{231})\right) \\ & + \left(\pi_{12}(x_{122}) - hx_{122} + \pi_{22}(H_2 - x_{122})\right) + \left(\pi_{13}(x_{132}) + \pi_{23}(x_{232})\right) \end{aligned}$$

$$x_{131} = C_1 - \alpha x_{11} - \beta x_{121}$$
$$x_{231} = C_2 - \alpha(H_1 - x_{11}) - \beta(H_2 - x_{121})$$

$$x_{132} = C_1 - \alpha x_{11} - \beta x_{122}$$
$$x_{232} = C_2 - \alpha(H_1 - x_{11}) - \beta(H_2 - x_{122})$$

$$
\begin{aligned}
Z = 2\Theta = \quad & 2\pi_{11}(x_{11}) + 2\pi_{21}(H_1 - x_{11}) \\
& + \pi_{12}(x_{121}) + hx_{121} + \pi_{22}(H_2 - x_{121}) \\
& + \pi_{12}(x_{122}) - hx_{122} + \pi_{22}(H_2 - x_{122}) \\
& + \pi_{13}(x_{131}) + \pi_{23}(x_{231}) \\
& + \pi_{13}(x_{132}) + \pi_{23}(x_{232})
\end{aligned}
$$

$$
\begin{aligned}
x_{131} &= C_1 - \alpha x_{11} - \beta x_{121} \\
x_{231} &= C_2 - \alpha(H_1 - x_{11}) - \beta(H_2 - x_{121})
\end{aligned}
$$

$$
\begin{aligned}
x_{132} &= C_1 - \alpha x_{11} - \beta x_{122} \\
x_{232} &= C_2 - \alpha(H_1 - x_{11}) - \beta(H_2 - x_{122})
\end{aligned}
$$

$$
\begin{aligned}
Z = 2\Theta = \quad & 2\pi_{11}(x_{11}) + 2\pi_{21}(H_1 - x_{11}) \\
& + \pi_{12}(x_{121}) + hx_{121} + \pi_{22}(H_2 - x_{121}) \\
& + \pi_{12}(x_{122}) - hx_{122} + \pi_{22}(H_2 - x_{122}) \\
& + \pi_{13}(C_1 - \alpha x_{11} - \beta x_{121}) \\
& + \pi_{23}(C_2 - \alpha(H_1 - x_{11}) - \beta(H_2 - x_{121})) \\
& + \pi_{13}(C_1 - \alpha x_{11} - \beta x_{122}) \\
& + \pi_{23}(C_2 - \alpha(H_1 - x_{11}) - \beta(H_2 - x_{122}))
\end{aligned}
$$

Now, there are three free decision variables and three first order optimum conditions:

$$\frac{dZ}{dx_{11}} = 2\pi'_{11}(x_{11}) - 2\pi'_{21}(H_1 - x_{11})$$
$$-\alpha\pi'_{13}(C_1 - \alpha x_{11} - \beta x_{121})$$
$$+\alpha\pi'_{23}(C_2 - \alpha(H_1 - x_{11}) - \beta(H_2 - x_{121}))$$
$$-\alpha\pi'_{13}(C_1 - \alpha x_{11} - \beta x_{122})$$
$$+\alpha\pi'_{23}(C_2 - \alpha(H_1 - x_{11}) - \beta(H_2 - x_{122})) \qquad = 0$$

$$\frac{dZ}{dx_{121}} = \pi'_{12}(x_{121}) + h - \pi'_{22}(H_2 - x_{121})$$
$$-\beta\pi'_{13}(C_1 - \alpha x_{11} - \beta x_{121})$$
$$+\beta\pi'_{23}(C_2 - \alpha(H_1 - x_{11}) - \beta(H_2 - x_{121})) \qquad = 0$$

$$\frac{dZ}{dx_{122}} = \pi'_{12}(x_{122}) - h - \pi'_{22}(H_2 - x_{122})$$
$$-\beta\pi'_{13}(C_1 - \alpha x_{11} - \beta x_{122})$$
$$+\beta\pi'_{23}(C_2 - \alpha(H_1 - x_{11}) - \beta(H_2 - x_{122})) \qquad = 0$$

$$
|D| = \begin{vmatrix}
\begin{pmatrix} 2\pi_{11}^{"}(x_{11}) + 2\pi_{21}^{"}(H_1 - x_{11}) \\ +\alpha^2\pi_{13}^{"}(C_1 - \alpha x_{11} - \beta x_{121}) \\ +\alpha^2\pi_{23}^{"}(C_2 - \alpha(H_1 - x_{11}) - \beta(H_2 - x_{121})) \\ +\alpha^2\pi_{13}^{"}(C_1 - \alpha x_{11} - \beta x_{122}) \\ +\alpha^2\pi_{23}^{"}(C_2 - \alpha(H_1 - x_{11}) - \beta(H_2 - x_{122})) \end{pmatrix} & \begin{pmatrix} \alpha\beta\pi_{13}^{"}(C_1 - \alpha x_{11} - \beta x_{121}) \\ +\alpha\beta\pi_{23}^{"}(C_2 - \alpha(H_1 - x_{11}) - \beta(H_2 - x_{121})) \end{pmatrix} & \begin{pmatrix} \alpha\beta\pi_{13}^{"}(C_1 - \alpha x_{11} - \beta x_{122}) \\ +\alpha\beta\pi_{23}^{"}(C_2 - \alpha(H_1 - x_{11}) - \beta(H_2 - x_{122})) \end{pmatrix} \\
\begin{pmatrix} \alpha\beta\pi_{13}^{"}(C_1 - \alpha x_{11} - \beta x_{121}) \\ +\alpha\beta\pi_{23}^{"}(C_2 - \alpha(H_1 - x_{11}) - \beta(H_2 - x_{121})) \end{pmatrix} & \begin{pmatrix} \pi_{12}^{"}(x_{121}) + \pi_{22}^{"}(H_2 - x_{121}) \\ +\beta^2\pi_{13}^{"}(C_1 - \alpha x_{11} - \beta x_{121}) \\ +\beta^2\pi_{23}^{"}(C_2 - \alpha(H_1 - x_{11}) - \beta(H_2 - x_{121})) \end{pmatrix} & 0 \\
\begin{pmatrix} \alpha\beta\pi_{13}^{"}(C_1 - \alpha x_{11} - \beta x_{122}) \\ +\alpha\beta\pi_{23}^{"}(C_2 - \alpha(H_1 - x_{11}) - \beta(H_2 - x_{122})) \end{pmatrix} & 0 & \begin{pmatrix} \pi_{12}^{"}(x_{122}) + \pi_{22}^{"}(H_2 - x_{122}) \\ +\beta^2\pi_{13}^{"}(C_1 - \alpha x_{11} - \beta x_{122}) \\ +\beta^2\pi_{23}^{"}(C_2 - \alpha(H_1 - x_{11}) - \beta(H_2 - x_{122})) \end{pmatrix}
\end{vmatrix}
$$

$$|D| = \begin{vmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & 0 \\ D_{31} & 0 & D_{33} \end{vmatrix}$$

$$|D| = D_{11}D_{22}D_{33} - D_{12}D_{21}D_{33} - D_{13}D_{22}D_{31} < 0$$

$$\begin{bmatrix} \dfrac{d^2 Z}{dx_{11} dh} \\ \dfrac{d^2 Z}{dx_{121} dh} \\ \dfrac{d^2 Z}{dx_{122} dh} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

$$[D]\begin{bmatrix} dx_{11}^* \\ dx_{121}^* \\ dx_{122}^* \end{bmatrix} = \begin{bmatrix} 0 \\ -1dh \\ +1dh \end{bmatrix}$$

$$\frac{dx_{11}^*}{dh} = \frac{\begin{vmatrix} 0 & D_{12} & D_{13} \\ -1 & D_{22} & 0 \\ 1 & 0 & D_{33} \end{vmatrix}}{\begin{vmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & 0 \\ D_{31} & 0 & D_{33} \end{vmatrix}} = \frac{D_{12}D_{33} - D_{13}D_{22}}{|D|}$$

$$U = D_{12}D_{33} - D_{13}D_{22}$$

$$U = \begin{pmatrix} \alpha\beta\pi''_{13}(C_1 - \alpha x_{11} - \beta x_{121}) \\ +\alpha\beta\pi''_{23}(C_2 - \alpha(H_1 - x_{11}) - \beta(H_2 - x_{121})) \end{pmatrix} \begin{pmatrix} \pi''_{12}(x_{122}) + \pi''_{22}(H_2 - x_{122}) \\ +\beta^2\pi''_{13}(C_1 - \alpha x_{11} - \beta x_{122}) \\ +\beta^2\pi''_{23}(C_2 - \alpha(H_1 - x_{11}) - \beta(H_2 - x_{122})) \end{pmatrix}$$

$$-\begin{pmatrix} \alpha\beta\pi''_{13}(C_1 - \alpha x_{11} - \beta x_{122}) \\ +\alpha\beta\pi''_{23}(C_2 - \alpha(H_1 - x_{11}) - \beta(H_2 - x_{122})) \end{pmatrix} \begin{pmatrix} \pi''_{12}(x_{121}) + \pi''_{22}(H_2 - x_{121}) \\ +\beta^2\pi''_{13}(C_1 - \alpha x_{11} - \beta x_{121}) \\ +\beta^2\pi''_{23}(C_2 - \alpha(H_1 - x_{11}) - \beta(H_2 - x_{121})) \end{pmatrix}$$

Assumptions:

$$\pi_{13}'' < 0 \wedge \pi_{23}'' < 0$$

In order to produce strong and relevant results, we assume that:

$$\pi_{13}''' \approx 0 \wedge \pi_{23}''' \approx 0$$

and even

$$\pi_{13}''' = 0 \wedge \pi_{23}''' = 0$$

In general, one should expect that $\left|\frac{\pi_{13}'''}{\pi_{12}'''}\right| < 1 \vee \left|\frac{\pi_{13}'''}{\pi_{12}'''}\right| \ll 1$ and $\left|\frac{\pi_{23}'''}{\pi_{22}'''}\right| < 1 \vee \left|\frac{\pi_{23}'''}{\pi_{22}'''}\right| \ll 1$, since the effects of volume increases on

the marginal profit level are usually less dramatic in the long run than in the short run. In the long run, there is more time available to adjust infrastructure capacity, logistics, labour force and industrial capacities to large volume changes.

$$\kappa > 0 \wedge \gamma > 0$$

Consequence:

$$U=\left(-\kappa\right)\begin{pmatrix}\pi''_{12}(x_{122})+\pi''_{22}(H_2-x_{122})\\-\gamma\end{pmatrix}$$

$$-\left(-\kappa\right)\begin{pmatrix}\pi''_{12}(x_{121})+\pi''_{22}(H_2-x_{121})\\-\gamma\end{pmatrix}$$

$$\bar{U}=\frac{U}{\kappa}=\left(\pi''_{12}(x_{121})+\pi''_{22}(H_2-x_{121})\right)-\left(\pi''_{12}(x_{122})+\pi''_{22}(H_2-x_{122})\right)$$

$$\bar{U}=\frac{U}{\kappa}=\left(\pi''_{12}(x_{121})-\pi''_{12}(x_{122})\right)+\left(\pi''_{22}(H_2-x_{121})-\pi''_{22}(H_2-x_{122})\right)$$

Observations:

$$\left(x_{121} > x_{122}\right) \quad \wedge \quad \left(H_2 - x_{121} < H_2 - x_{122}\right)$$

$$\operatorname{sgn}\left(\frac{dx_{11}^{*}}{dh}\right) = \operatorname{sgn}\left(-\bar{U}\right)$$

$$\operatorname{sgn}\left(\frac{dx_{21}^{*}}{dh}\right) = \operatorname{sgn}\left(-\frac{dx_{11}^{*}}{dh}\right) = \operatorname{sgn}\left(\bar{U}\right)$$

Multi species results:

CASE 1:

$$\left(\pi_{12}''' > \pi_{22}'''\right) \Rightarrow \left(\frac{dx_{11}^{*}}{dh} < 0 \quad \wedge \quad \frac{dx_{21}^{*}}{dh} > 0\right)$$

CASE 2:

$$\left(\pi_{12}''' \geq \pi_{22}'''\right) \Rightarrow \left(\frac{dx_{11}^{*}}{dh} \leq 0 \quad \wedge \quad \frac{dx_{21}^{*}}{dh} \geq 0\right)$$

CASE 3:

$$\left(\pi_{12}''' = \pi_{22}'''\right) \Rightarrow \left(\frac{dx_{11}^{*}}{dh} = 0 \quad \wedge \quad \frac{dx_{21}^{*}}{dh} = 0\right)$$

CASE 4:

$$\left(\pi_{12}''' \leq \pi_{22}'''\right) \Rightarrow \left(\frac{dx_{11}^{*}}{dh} \geq 0 \quad \wedge \quad \frac{dx_{21}^{*}}{dh} \leq 0\right)$$

CASE 5:

$$\left(\pi_{12}''' < \pi_{22}'''\right) \Rightarrow \left(\frac{dx_{11}^{*}}{dh} > 0 \quad \wedge \quad \frac{dx_{21}^{*}}{dh} < 0\right)$$

Some multi species results:

With multiple species and total harvest volume constraints:

Case 1:

If the future price risk of one species, A, increases, we should now harvest less of this species (A) and more of the other species, in case the third order derivative of the profit function of species A with respect to harvest volume is greater than the corresponding derivative of the other species.

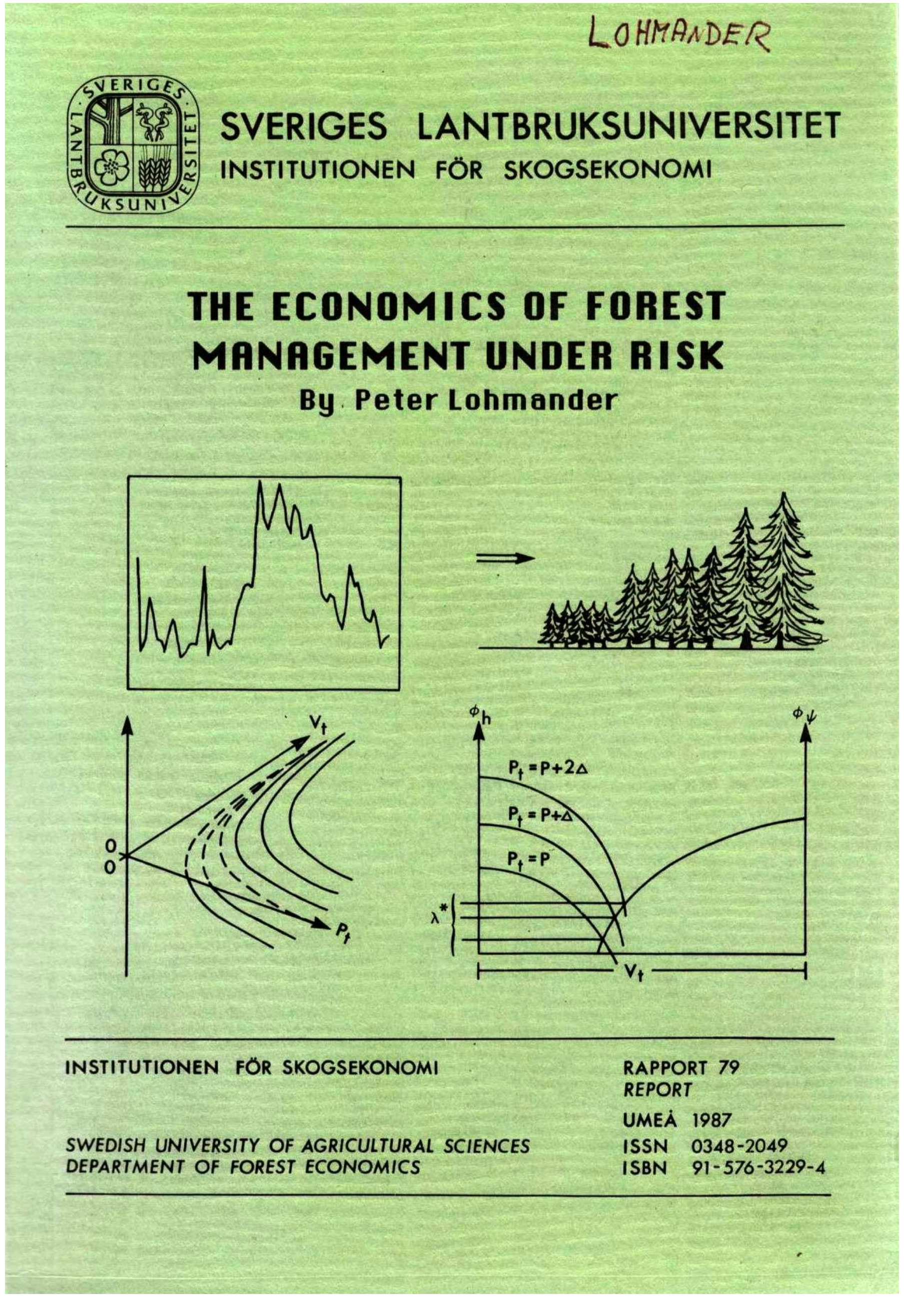
Case 3:

If the future price risk of one species, A, increases, we should not change the present harvest of this species (A) and not change the harvest of the other species, in case the third order derivative of the profit function of species A with respect to harvest volume is equal to the corresponding derivative of the other species.

Case 5:

If the future price risk of one species, A, increases, we should now harvest more of this species (A) and less of the other species, in case the third order derivative of the profit function of species A with respect to harvest volume is less than the corresponding derivative of the other species.

Related analyses, via stochastic dynamic programming, are found here: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THE ECONOMICS OF FOREST MANAGEMENT UNDER RISK

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Many more references, including this presentation, are found here:

http://www.lohmander.com/Information/Ref.htm

OPTIMAL PRESENT RESOURCE EXTRACTION UNDER THE INFLUENCE OF FUTURE RISK

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ABSTRACT

The analysis concerns determination of the optimal present extraction of a natural resouce and how this is affected by different kinds of risk in the future. The most general definition of increasing risk, according to Rothschild and Stiglitz, is used. It can be applied to all types of statistical distributions. The approach is much more general than, for instance, increasing variance. The analysis is performed via general function multi dimensional analyical optimization and comparative dynamics analysis in discrete time. It is found that most of the analytical results can be derived via comparative dynamics in a system with three equations in combination with supporting general function analysis. The general analytical results are illustrated via computer solutions to numerically specified special cases.

Keywords: Optimal stochastic control; Risk; Natural resource management; Forestry; Third order derivatives.

1. INTRODUCTION

In real production processes, continuous adjustments of all activitites are mostly not technically possible and would almost never be economically rational. The optimal extraction problems have often been studied with optimal stochastic control theory in continuous time. Then, however, the continuous time assumption and the common Wiener process assumption usually imply that derivatives of order three and higher are not needed in the derivations. The present analysis proves that, in discrete time, derivations of optimal decisions under risk have to take the third order derivatives into account. The most general definitions of increasing risk introduced and analyzed by Rothschild and Stiglitz [4] and [5] are used. Third order derivatives different from zero are often present in several parts of natural resource management problems in cost functions, growth functions and demand functions. Furthermore, third order derivatives different from zero can be very useful in approximation of capacity constraints and penalty functions. Earlier results of a similar nature have been derived via comparative dynamic analysis within stochastic dynamic programming problems. Such derivations are found in[1] and [2]. In those studies, you also find detalied analyses of the effects of stationarity in the stochastic processes. The effects of increasing future risk on the optimal present extraction level are usually also dependent on process stationarity. If the processes are stationary, the effects of increasing risk on optimal present extraction may be quite different from the case when the processes are nonstationary. These effects are also described in the present analysis.

2. MAIN RESULTS

In the first section, the price and/or cost risk in the next period increases. The direction of optimal adjustment of the present extraction level is then found to be a function of the third order derivatives of the profit functions in later time periods with respect to the extraction levels. If the signs of these derivatives are known and constant over time, it is possible to determine the sign of the optimal adjustment of the present extraction level. In the second section, the resource is growing. The optimal present extraction level is then studied under the influence of increasing risk in the growth process. The direction of optimal adjustment of the present extraction is found to be a function of the third order derivatives of the profit functions in later time periods with respect to the extraction levels. In some cases, it is possible to determine the sign of the optimal adjustment of the present extraction level. In the third section, the resource contains different species, growing together. Furthermore, the total harvest in each period is constrained. The question is how the optimal present harvest of these species is affected by increasing price risk in one of the species. Again, it turns out that the direction of adjustment of the present extraction is a function of the third order derivatives. In some cases, it is possible to determine the signs of the optimal adjustments of the present extraction levels in the different species. An alternative way to optimize similar but not identical stochastic dynamic multi species management management problems is reported in [3].

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