

# SUB MODELS FOR OPTIMAL CONTINUOUS COVER MULTI SPECIES FORESTRY IN IRAN

*(One part of the joint presentation by Soleiman Mohammadi Limaiei, Peter Lohmander and Leif Olsson )*

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**The 8th International Conference of  
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- The available empirical data was used to **estimate a modified logistic growth model where stand density, altitude and species mix were considered as explanatory variables**. Logistics growth models have been found useful in continuous cover forest management optimization and examples of such studies are found in Lohmander [3] and Lohmander and Mohammadi [4].
- The **general dynamics of forests based on such models** was analyzed and dynamic equilibrium conditions (stand densities and species mixes) for different altitudes were determined.
- In some cases, dynamic multi species **model parameters are possible to determine via steady state observations** of unmanaged forests.
- **Optimization of management decisions in a changing and not perfectly predictable world, should always be based on adaptive optimization**. Lohmander [2] describes these principles and typical implications for optimal forestry decisions. Adaptable logistic growth functions work well in such cases.

## References

- [1] M. Kalinina, L. Olsson, A. Larsson, A Multi objective Chance Constrained Programming Model for Intermodal Logistics with Uncertain Time, *International Journal of Computer Science Issues* 10(2013), 35-44.
- [2] P. Lohmander, Optimal sequential forestry decisions under risk, *Annals of Operations Research* 95(2000), 217-228.
- [3] P. Lohmander, Adaptive Optimization of Forest Management in a Stochastic World, *Handbook of Operations Research in Natural Resources*, (Weintraub et al., eds.) Springer, Springer Science, International Series in Operations Research and Management Science, New York, USA, (2007), 525-544.
- [4] P. Lohmander and S. Mohammadi Limaei, Optimal Continuous Cover Forest Management in an Uneven-Aged Forest in the North of Iran, *Journal of Applied Sciences* 8(2008), 1995-2007.
- [5] S. Mohammadi Limaei, Mixed strategy game theory, application in forest industry, *Forest Policy and Economics* 12(2010), 527-531.

## Part 1.

- **The available empirical data was used to estimate a modified logistic growth model where stand density, altitude and species mix were considered as explanatory variables. Logistics growth models have been found useful in continuous cover forest management optimization and examples of such studies are found in Lohmander [3] and Lohmander and Mohammadi [4].**

The available empirical data was used to estimate a modified logistic growth model where stand density, altitude and species mix were considered as explanatory variables.

$$\#1: s \cdot x \cdot \left(1 - \frac{x}{c}\right) - k \cdot h$$

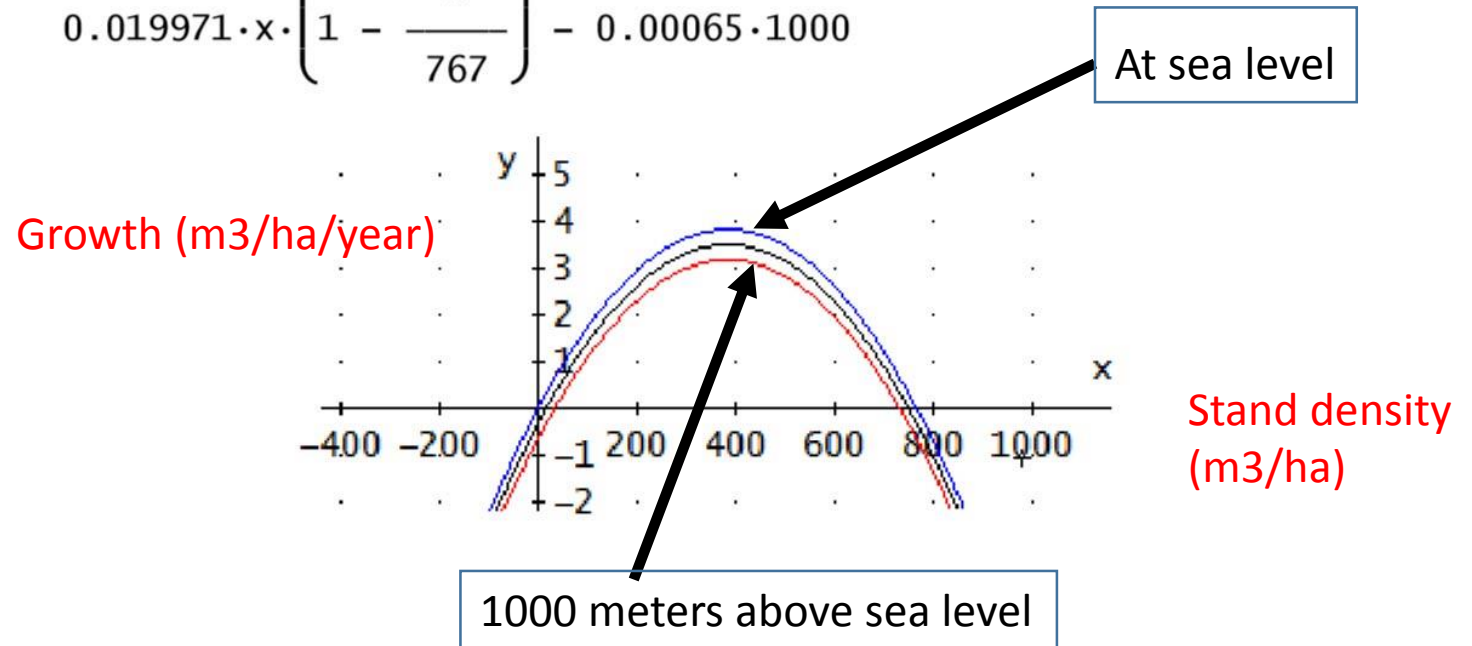
$$\#2: 0.019971 \cdot x \cdot \left(1 - \frac{x}{767}\right) - 0.00065 \cdot 0$$

$$\#3: 0.019971 \cdot x \cdot \left(1 - \frac{x}{767}\right) - 0.00065 \cdot 500$$

$$\#4: 0.019971 \cdot x \cdot \left(1 - \frac{x}{767}\right) - 0.00065 \cdot 1000$$

### Definitions

x = Stand density (m<sup>3</sup>/ha)  
s = Intrinsic growth rate  
c = Carrying capacity (m<sup>3</sup>/ha)  
h = Altitude (meters)  
k = Altitude parameter



<b>Annual growth as a function of stand density and elevation</b>							
Peter Lohmander 140730							
Below, we find that the annual growth can be expressed as a quadratic function of stand density.							
Furthermore, growth is (most likely) reduced with elevation.							
(The elevation effect is however not statistically significant at the 95% level.)							
$dX/dt = sX(1-X/K) + kE$							
X =	Stock level	(m <sup>3</sup> /ha)					
E =	Elevation	(m)					
s =	0,019971						
K =	767	(m <sup>3</sup> /ha)				(Carrying capacity at sea level)	
k =	-0,00065	(m <sup>3</sup> /ha/year/m)					

<i>Regressionsstatistik</i>								
Multipel-R	0,998098891							
R-kvadrat	0,996201397							
Justerad R-kvadrat	0,870251746							
Standardfel	0,203872045							
Observationer	11							
ANOVA								
	<i>fg</i>	<i>KvS</i>	<i>MKv</i>	<i>F</i>	<i>p-värde för F</i>			
Regression	3	87,20241936	29,06747312	699,3457225	4,93053E-09			
Residual	8	0,332510484	0,041563811					
Totalt	11	87,53492984						
	<i>Koefficienter</i>	<i>Standardfel</i>	<i>t-kvot</i>	<i>p-värde</i>	<i>Nedre 95%</i>	<i>Övre 95%</i>	<i>Nedre 95,0%</i>	<i>Övre 95,0%</i>
Konstant	0	#SAKNAS!	#SAKNAS!	#SAKNAS!	#SAKNAS!	#SAKNAS!	#SAKNAS!	#SAKNAS!
VOL	0,019970746	0,004215938	4,736963898	0,001469688	0,010248776	0,029692717	0,010248776	0,029692717
VOL2	-2,60227E-05	1,11457E-05	-2,334774567	0,047804052	-5,17247E-05	-3,20666E-07	-5,17247E-05	-3,20666E-07
Elev	-0,000646858	0,000452609	-1,429175526	0,190816399	-0,001690577	0,000396861	-0,001690577	0,000396861

# Empirical data

VOL	VOL2	Elev	Growth
306	93636	580	3,243756
220,7	48708,49	580	2,903158
320	102400	640	3,34
255,3	65178,09	800	2,910435
202	40804	1010	2,121818
259	67081	1080	2,972727
206	42436	875	2,76
218,5	47742,25	800	2,4
271	73441	850	3
299	89401	680	3,04
188	35344	985	2



# Competition within the forest stands

*(This particular slide is based on the results reported by Schutz (2006))*

Growth function analysis based on Schutz, J.P., Modelling the demographic sustainability of pure beech plenter forests in Eastern Germany, Ann.For.Sci. 63 (2006), 93–100, 93

$$\#2: \quad y = 1.506969 + 0.94225 \cdot \ln(20) - 0.000183455 \cdot x^3$$

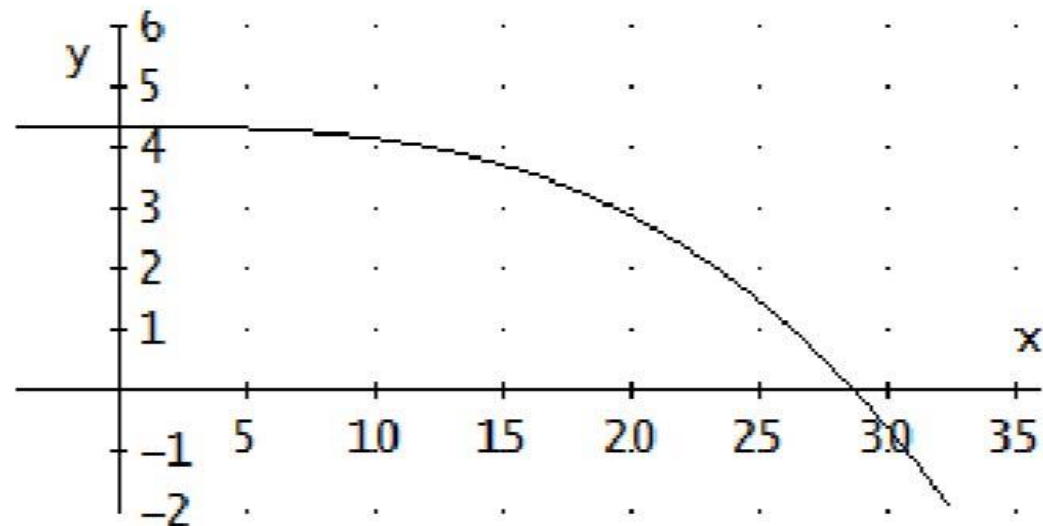


Figure 1: Diameter growth (according to the Schutz function) as a function of X, the basal area of bigger trees. (d = 20 cm).

# Area growth of individual trees

$$y = 0.1586038 \text{ xxLN}(x) + 0.07038919999 x$$

$x$  = area of individual tree (before growth) (m<sup>2</sup>)

$y$  = area growth of individual tree (as area growth during the coming ten years divided by 10.) (m<sup>2</sup>/year)

Regression statistics:

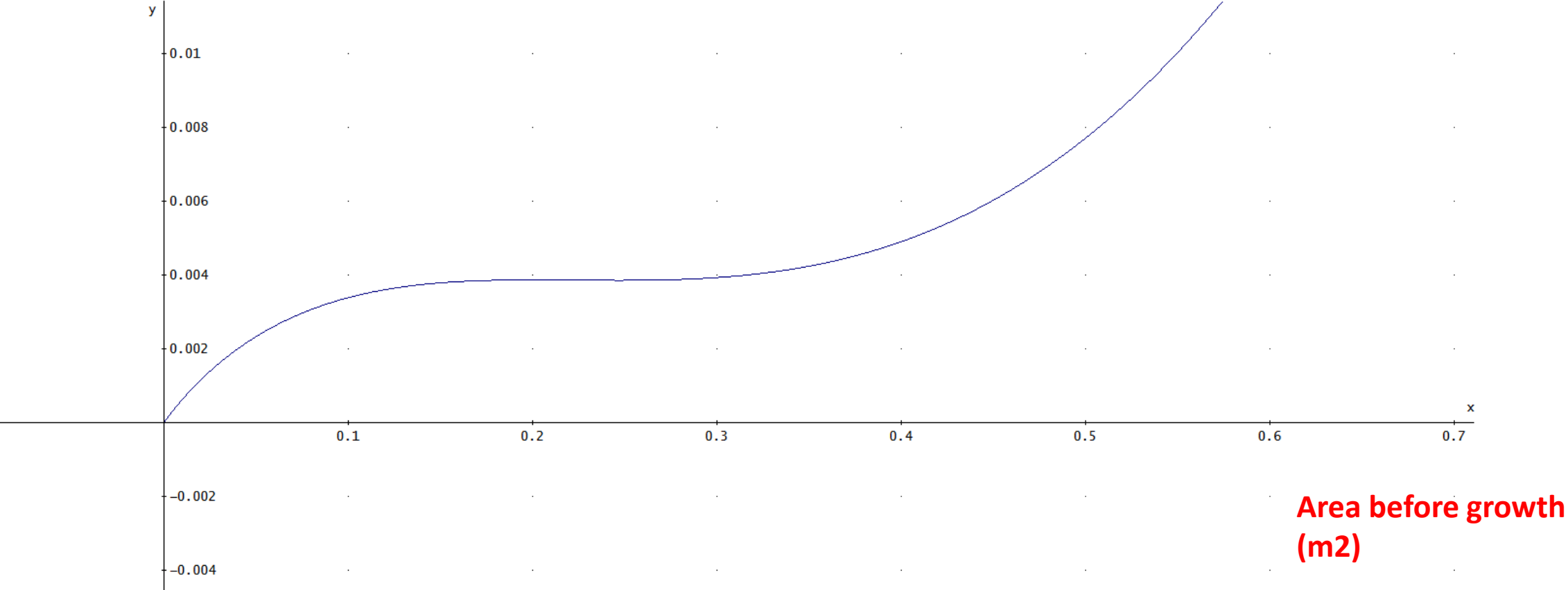
F = 211.8, R<sup>2</sup> = 0.872

t(x) = 13.09, t(xxLN(x)) = 9.56

<i>Regressionsstatistik</i>								
Multipel-R	0,93399518							
R-kvadrat	0,872346995							
Justerad R-kvadrat	0,854159044							
Standardfel	0,009200787							
Observationer	64							
<b>ANOVA</b>								
	<i>fg</i>	<i>KvS</i>	<i>Mkv</i>	<i>F</i>	<i>p-värde för F</i>			
Regression	2	0,035867397	0,017933698	211,8458311	3,51448E-28			
Residual	62	0,005248578	8,46545E-05					
Totalt	64	0,041115975						
	<i>Koefficienter</i>	<i>Standardfel</i>	<i>t-kvot</i>	<i>p-värde</i>	<i>Nedre 95%</i>	<i>Övre 95%</i>	<i>Nedre 95,0%</i>	<i>Övre 95,0%</i>
Konstant	0	#SAKNAS!	#SAKNAS!	#SAKNAS!	#SAKNAS!	#SAKNAS!	#SAKNAS!	#SAKNAS!
alallna	1,586037998	0,165941564	9,557810345	8,31339E-14	1,254325538	1,917750458	1,254325538	1,917750458
a1	0,703892375	0,053761805	13,09279651	1,6611E-19	0,596424057	0,811360692	0,596424057	0,811360692

# Area growth per tree per year as a function of the area before growth (per tree)

**Growth of area of individual tree  
(m<sup>2</sup>/year)**



## Observations and suggestions for future estimations:

- 1. It would have been valuable to have more variation in the raw data. Now, the degrees of competition and the stand densities have low degrees of variation.**
- 2. In the presently analyzed raw data, there are correlations different from zero between the possibly explaining variables "direction" and "altitude". In the future, such correlations should be removed.**
- 3. In the present data, there are also correlations different from zero between species and elevation. For instance, Beech is almost only found at the highest elevations.**

## Part 2.

- The **general dynamics of forests based on such models** was analyzed and dynamic equilibrium conditions (stand densities and species mixes) for different altitudes were determined.

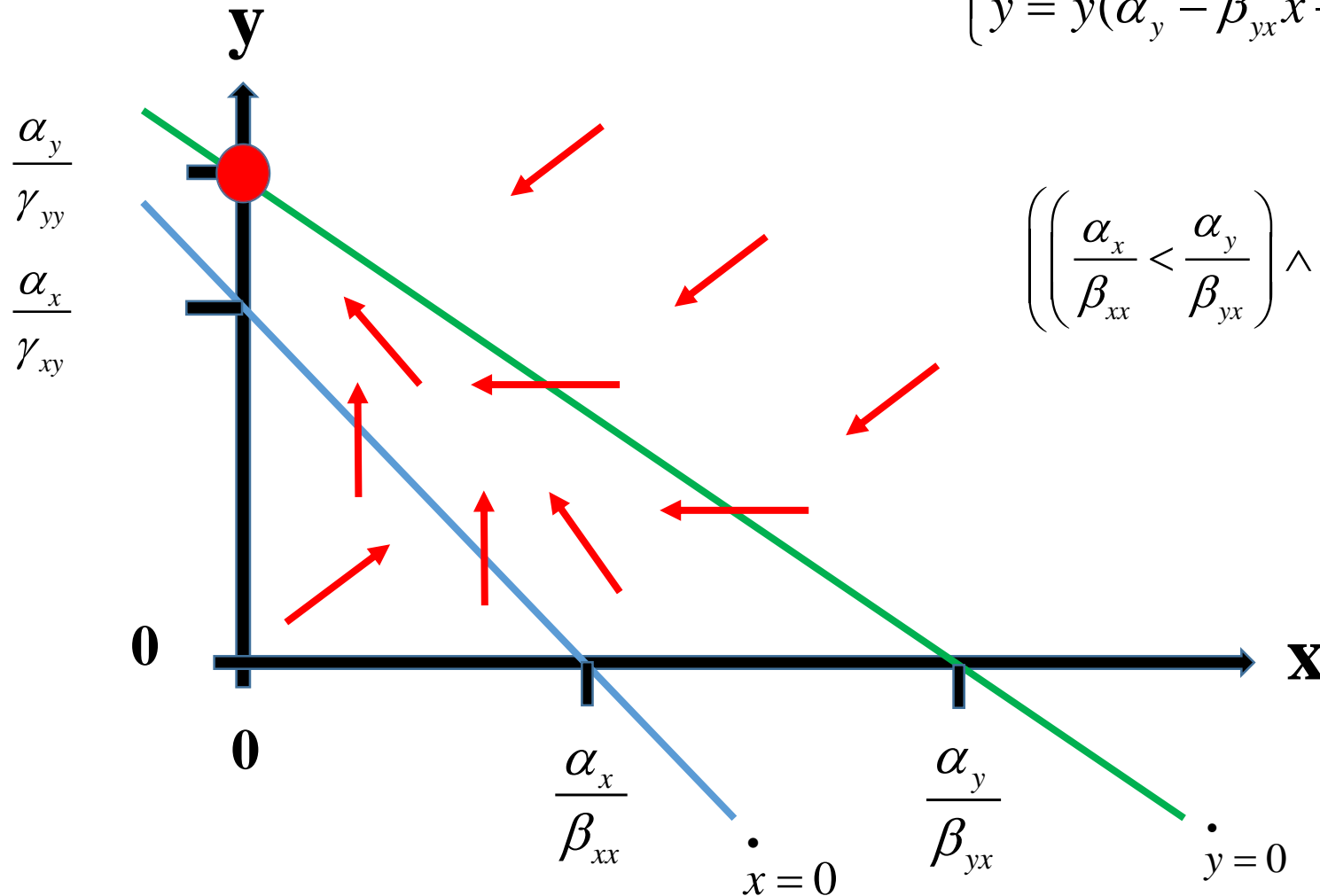
# *A dynamic two species model with competition*

*(A system of two "extended logistic models")*

$$\begin{cases} \dot{x} = \frac{dx}{dt} = x(\alpha_x - \beta_{xx}x - \gamma_{xy}y) \\ \dot{y} = \frac{dy}{dt} = y(\alpha_y - \beta_{yx}x - \gamma_{yy}y) \end{cases}$$

# CASE 1.

$$\begin{cases} \dot{x} = x(\alpha_x - \beta_{xx}x - \gamma_{xy}y) & \alpha_x > 0; \beta_{xx} > 0; \gamma_{xy} > 0 \\ \dot{y} = y(\alpha_y - \beta_{yx}x - \gamma_{yy}y) & \alpha_y > 0; \beta_{yx} > 0; \gamma_{yy} > 0 \end{cases}$$



$$\left( \left( \frac{\alpha_x}{\beta_{xx}} < \frac{\alpha_y}{\beta_{yx}} \right) \wedge \left( \frac{\alpha_x}{\gamma_{xy}} < \frac{\alpha_y}{\gamma_{yy}} \right) \right) \Rightarrow (x_e, y_e) = \left( 0, \frac{\alpha_y}{\gamma_{yy}} \right)$$

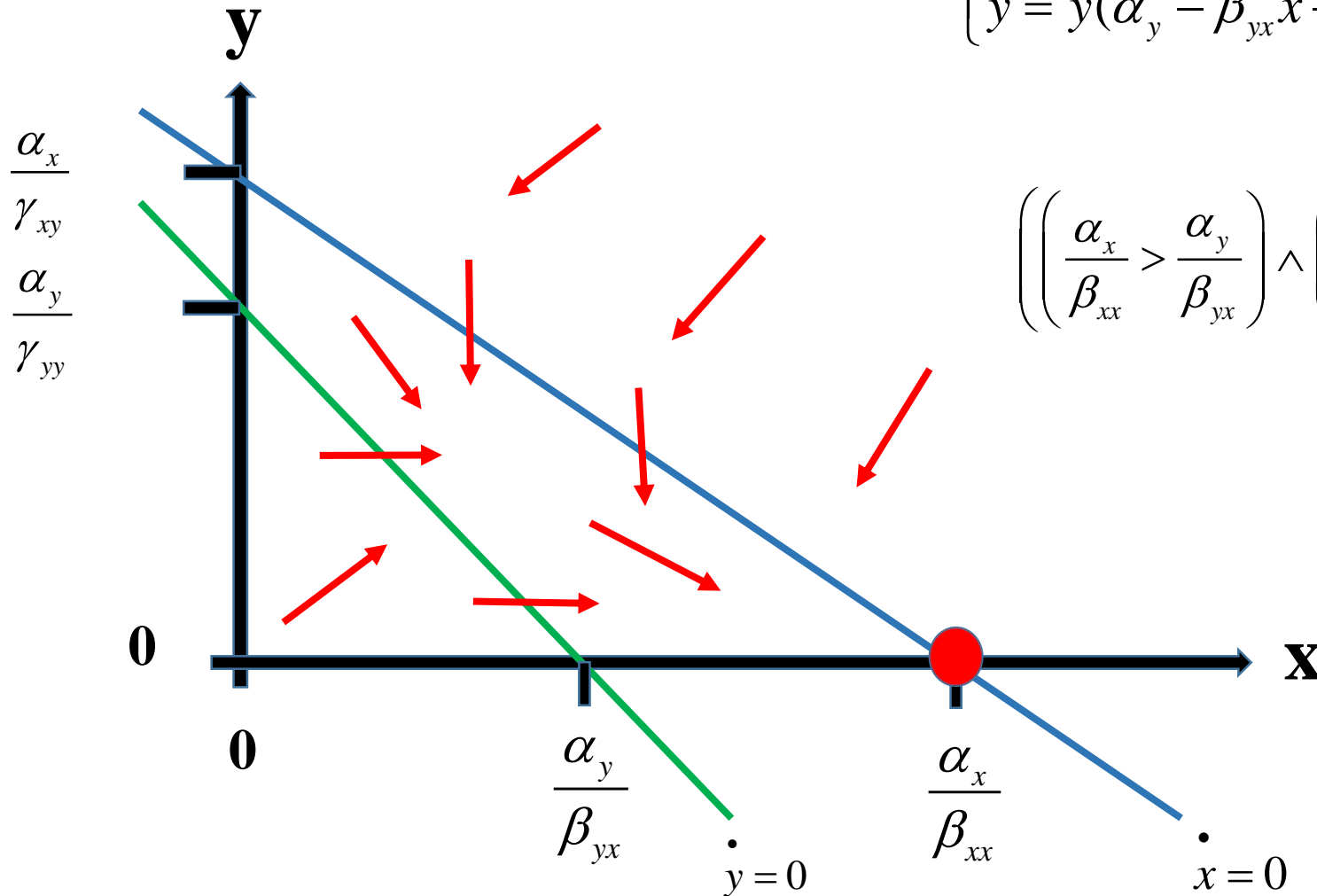
**Unique  
and  
Stable**



## CASE 2.

$$\begin{cases} \dot{x} = x(\alpha_x - \beta_{xx}x - \gamma_{xy}y) & \alpha_x > 0; \beta_{xx} > 0; \gamma_{xy} > 0 \\ \dot{y} = y(\alpha_y - \beta_{yx}x - \gamma_{yy}y) & \alpha_y > 0; \beta_{yx} > 0; \gamma_{yy} > 0 \end{cases}$$

$$\left( \left( \frac{\alpha_x}{\beta_{xx}} > \frac{\alpha_y}{\beta_{yx}} \right) \wedge \left( \frac{\alpha_x}{\gamma_{xy}} > \frac{\alpha_y}{\gamma_{yy}} \right) \right) \Rightarrow (x_e, y_e) = \left( \frac{\alpha_x}{\beta_{xx}}, 0 \right)$$

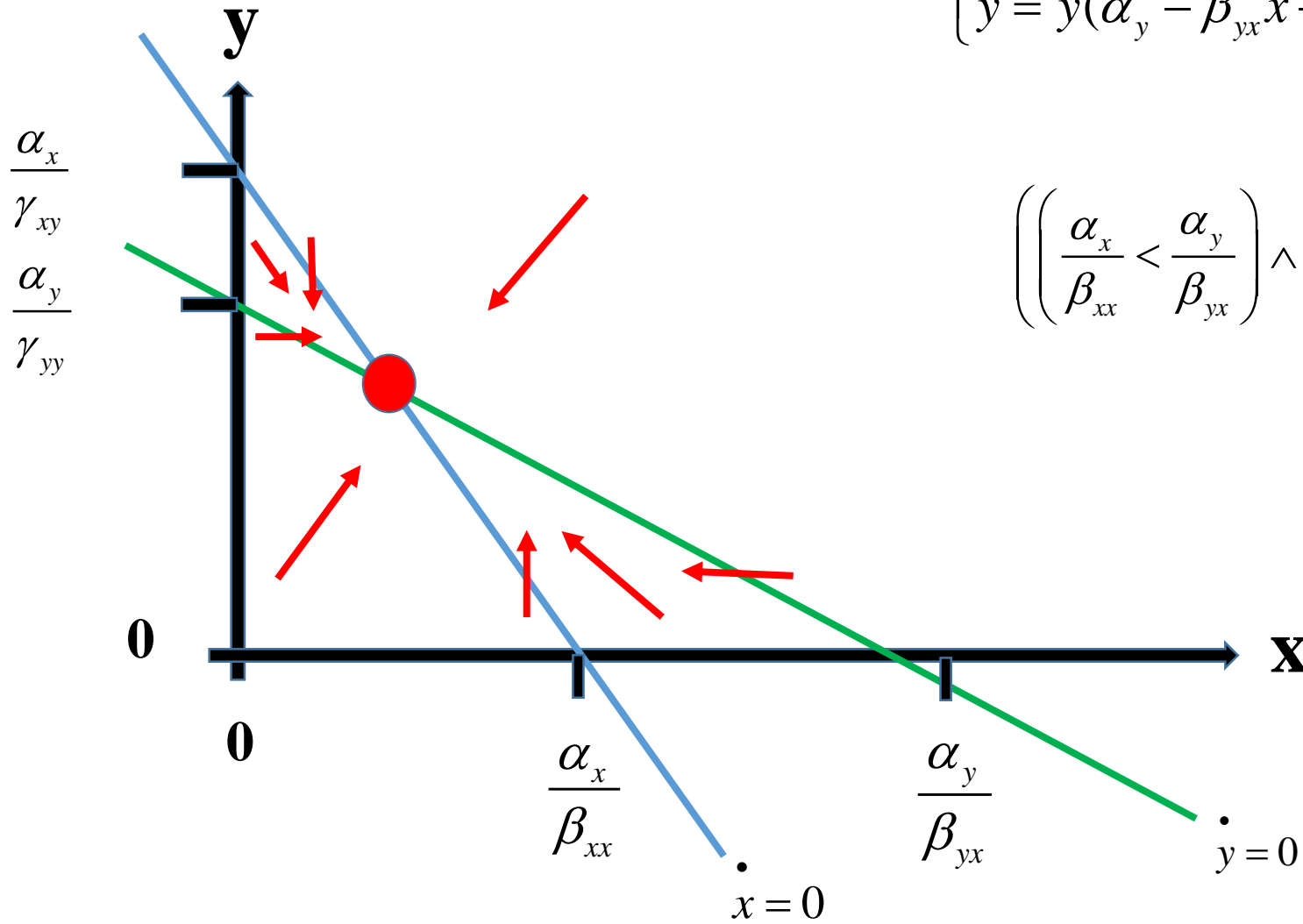


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Stable**

### CASE 3.

$$\begin{cases} \dot{x} = x(\alpha_x - \beta_{xx}x - \gamma_{xy}y) & \alpha_x > 0; \beta_{xx} > 0; \gamma_{xy} > 0 \\ \dot{y} = y(\alpha_y - \beta_{yx}x - \gamma_{yy}y) & \alpha_y > 0; \beta_{yx} > 0; \gamma_{yy} > 0 \end{cases}$$

$$\left( \left( \frac{\alpha_x}{\beta_{xx}} < \frac{\alpha_y}{\beta_{yx}} \right) \wedge \left( \frac{\alpha_x}{\gamma_{xy}} > \frac{\alpha_y}{\gamma_{yy}} \right) \right) \Rightarrow (x_e, y_e) = (x_e^0, y_e^0)$$



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## *CASE 3 and 4:* Interior equilibrium equations

$$\begin{cases} \dot{x} = x(\alpha_x - \beta_{xx}x - \gamma_{xy}y) & \alpha_x > 0; \beta_{xx} > 0; \gamma_{xy} > 0 \\ \dot{y} = y(\alpha_y - \beta_{yx}x - \gamma_{yy}y) & \alpha_y > 0; \beta_{yx} > 0; \gamma_{yy} > 0 \end{cases}$$

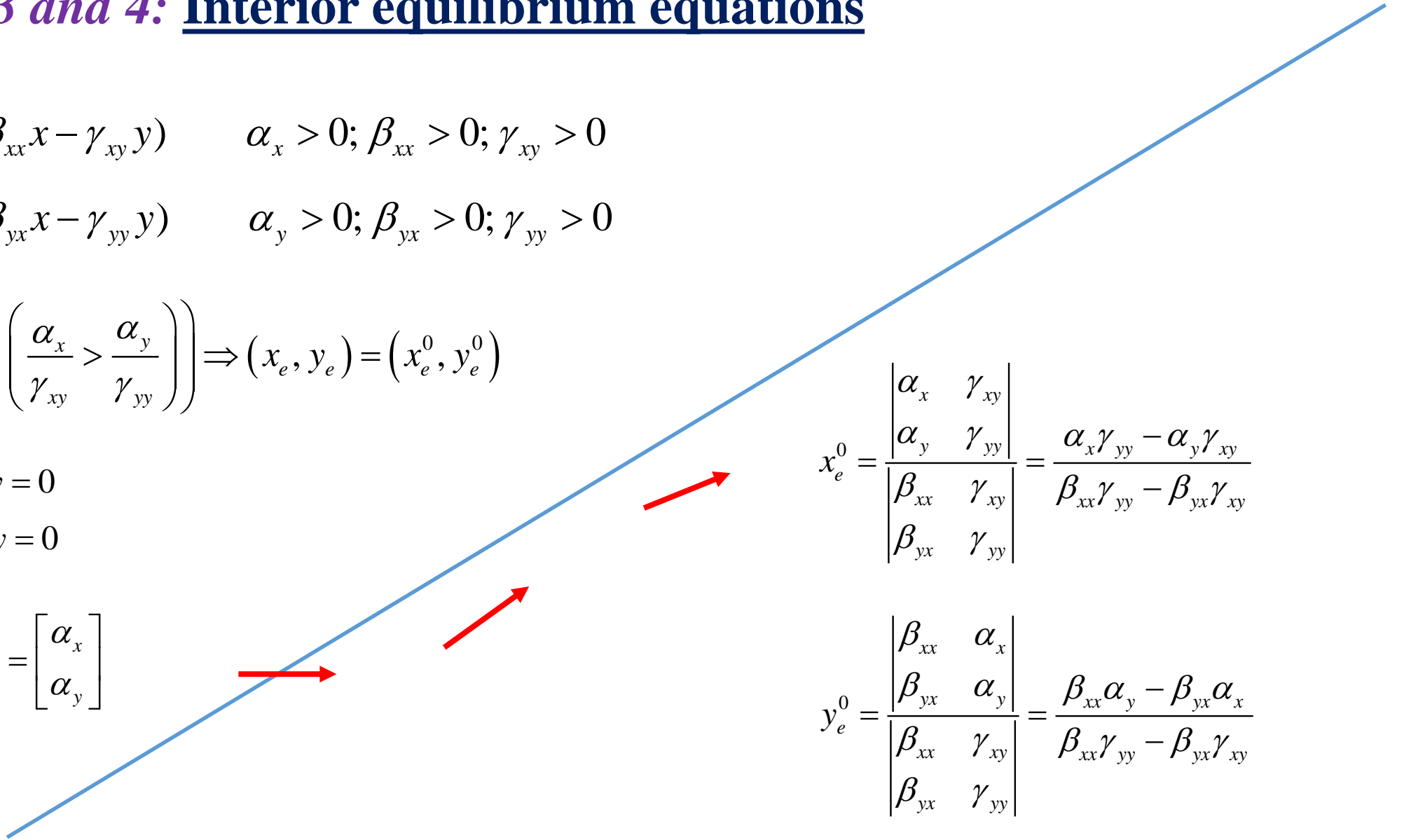
$$\left( \left( \frac{\alpha_x}{\beta_{xx}} < \frac{\alpha_y}{\beta_{yx}} \right) \wedge \left( \frac{\alpha_x}{\gamma_{xy}} > \frac{\alpha_y}{\gamma_{yy}} \right) \right) \Rightarrow (x_e, y_e) = (x_e^0, y_e^0)$$

$$\begin{cases} \alpha_x - \beta_{xx}x - \gamma_{xy}y = 0 \\ \alpha_y - \beta_{yx}x - \gamma_{yy}y = 0 \end{cases}$$

$$\begin{bmatrix} \beta_{xx} & \gamma_{xy} \\ \beta_{yx} & \gamma_{yy} \end{bmatrix} \begin{bmatrix} x_e^0 \\ y_e^0 \end{bmatrix} = \begin{bmatrix} \alpha_x \\ \alpha_y \end{bmatrix}$$

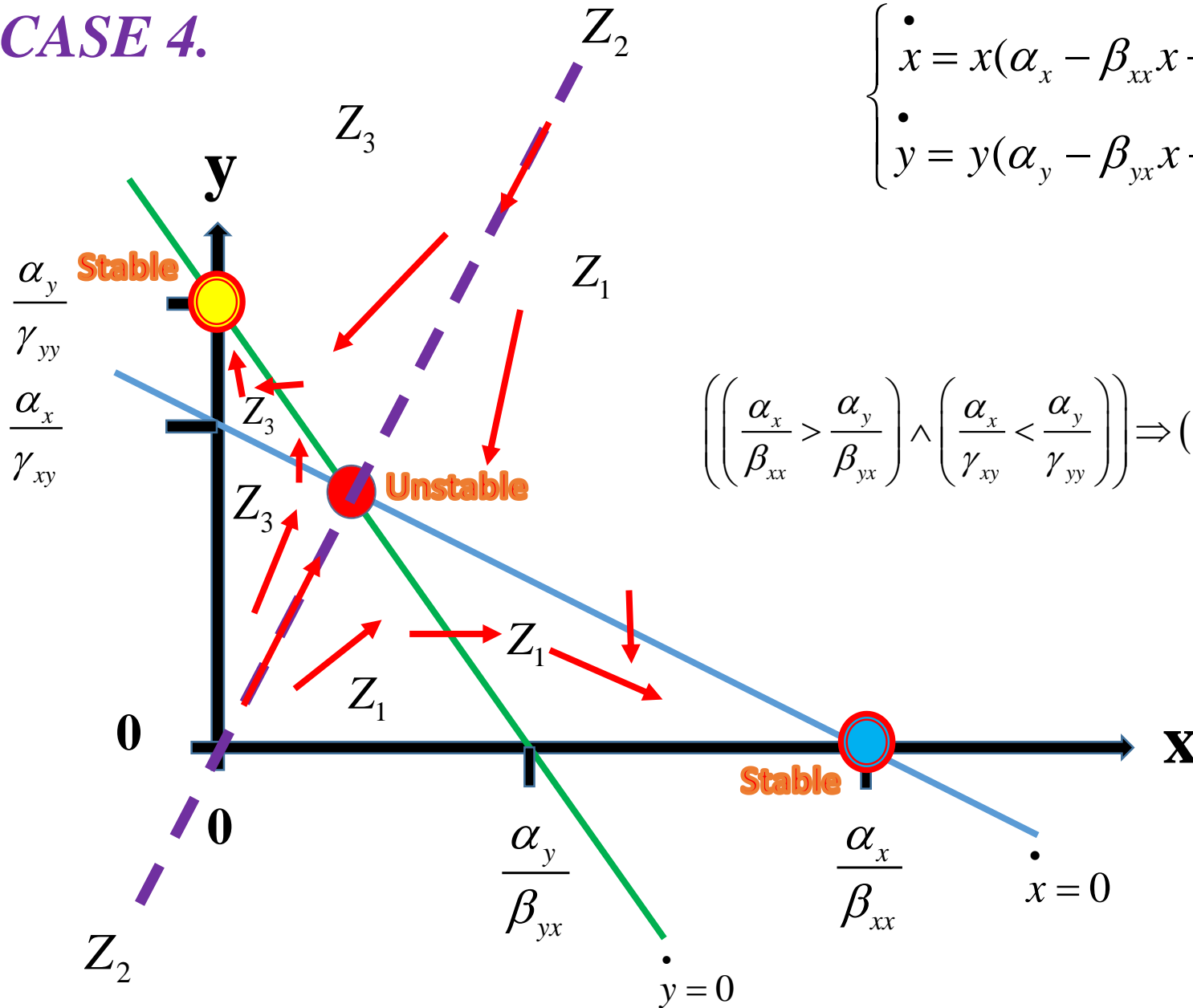
$$x_e^0 = \frac{\begin{vmatrix} \alpha_x & \gamma_{xy} \\ \alpha_y & \gamma_{yy} \end{vmatrix}}{\begin{vmatrix} \beta_{xx} & \gamma_{xy} \\ \beta_{yx} & \gamma_{yy} \end{vmatrix}} = \frac{\alpha_x \gamma_{yy} - \alpha_y \gamma_{xy}}{\beta_{xx} \gamma_{yy} - \beta_{yx} \gamma_{xy}}$$

$$y_e^0 = \frac{\begin{vmatrix} \beta_{xx} & \alpha_x \\ \beta_{yx} & \alpha_y \end{vmatrix}}{\begin{vmatrix} \beta_{xx} & \gamma_{xy} \\ \beta_{yx} & \gamma_{yy} \end{vmatrix}} = \frac{\beta_{xx} \alpha_y - \beta_{yx} \alpha_x}{\beta_{xx} \gamma_{yy} - \beta_{yx} \gamma_{xy}}$$



# CASE 4.

$$\begin{cases} \dot{x} = x(\alpha_x - \beta_{xx}x - \gamma_{xy}y) & \alpha_x > 0; \beta_{xx} > 0; \gamma_{xy} > 0 \\ \dot{y} = y(\alpha_y - \beta_{yx}x - \gamma_{yy}y) & \alpha_y > 0; \beta_{yx} > 0; \gamma_{yy} > 0 \end{cases}$$



$$\left( \left( \frac{\alpha_x}{\beta_{xx}} > \frac{\alpha_y}{\beta_{yx}} \right) \wedge \left( \frac{\alpha_x}{\gamma_{xy}} < \frac{\alpha_y}{\gamma_{yy}} \right) \right) \Rightarrow (x_e, y_e) = \begin{cases} \left( \frac{\alpha_x}{\beta_{xx}}, 0 \right) & \text{for } (x(0), y(0)) \in Z_1 \\ (x_e^0, y_e^0) & \text{for } (x(0), y(0)) \in Z_2 \\ \left( 0, \frac{\alpha_y}{\gamma_{yy}} \right) & \text{for } (x(0), y(0)) \in Z_3 \end{cases}$$

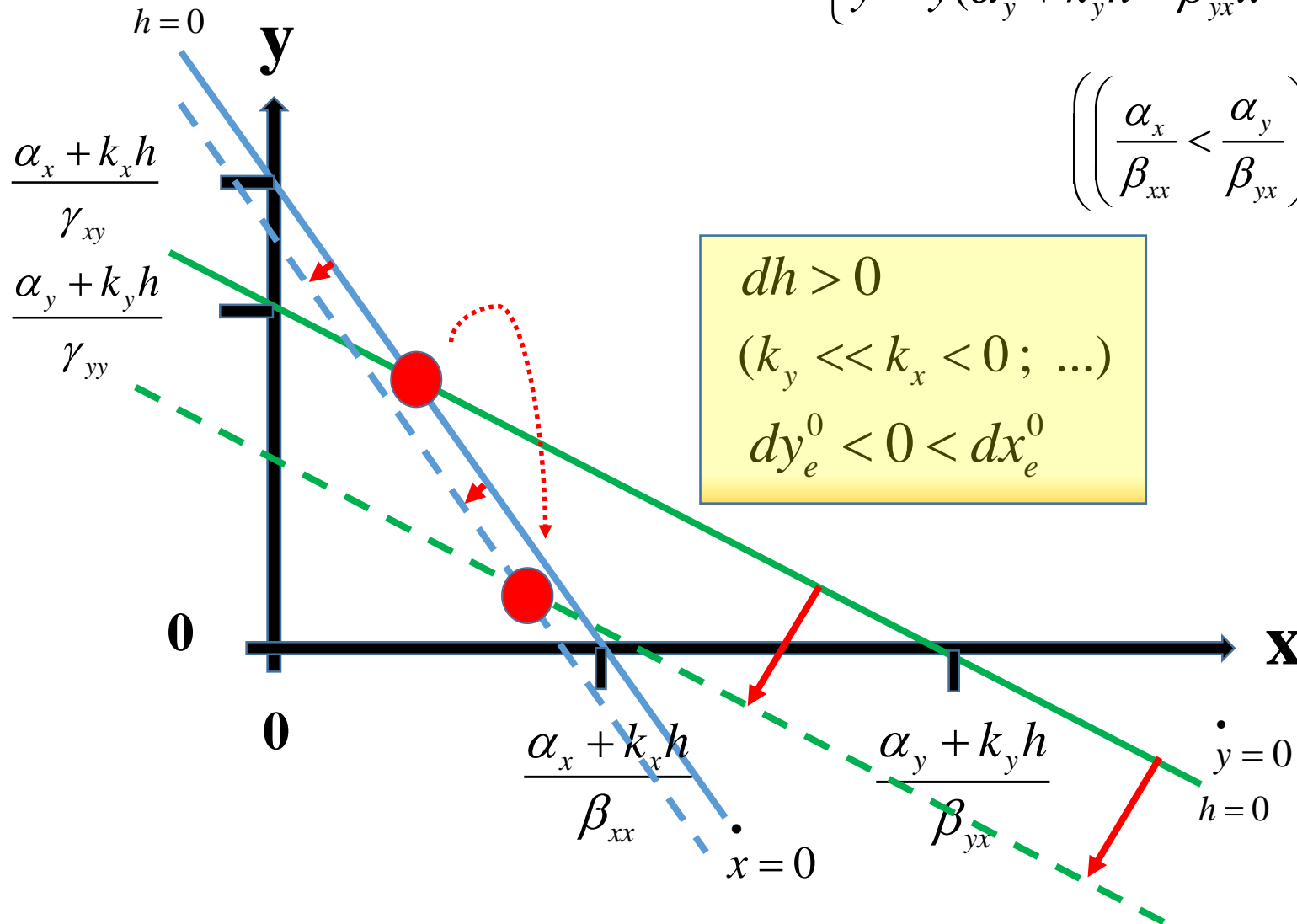
**Observation:**  
 The equilibrium is a function of the initial conditions.

### CASE 3b.

$$\begin{cases} \dot{x} = x(\alpha_x + k_x h - \beta_{xx} x - \gamma_{xy} y) & \alpha_x > 0; \beta_{xx} > 0; \gamma_{xy} > 0 \\ \dot{y} = y(\alpha_y + k_y h - \beta_{yx} x - \gamma_{yy} y) & \alpha_y > 0; \beta_{yx} > 0; \gamma_{yy} > 0 \end{cases}$$

$$\left( \left( \frac{\alpha_x}{\beta_{xx}} < \frac{\alpha_y}{\beta_{yx}} \right) \wedge \left( \frac{\alpha_x}{\gamma_{xy}} > \frac{\alpha_y}{\gamma_{yy}} \right) \right) \Rightarrow (x_e, y_e) = (x_e^0, y_e^0)$$

**Unique and Stable**



### Observation:

Some species are more sensitive to changes in  $h$  than others. The growth function of  $x$  is negatively affected by  $h$  but the equilibrium value of  $x$  still increases if  $h$  increases.

## Part 3.

- **In some cases, dynamic multi species model parameters are possible to determine via steady state observations of unmanaged forests.**
- **If we can observe  $x$  and  $y$  in several equilibria, we can in some cases estimate relations between the parameters.**
- **We can introduce altitude and direction in the "parameters" and evaluate the equilibria at different altitudes and directions.**
- **If we can observe  $x$  and  $y$  in several equilibria, at different altitudes and directions, we can in some cases estimate relations between the parameters and simultaneously determine the species specific sensitivities to altitude and direction.**

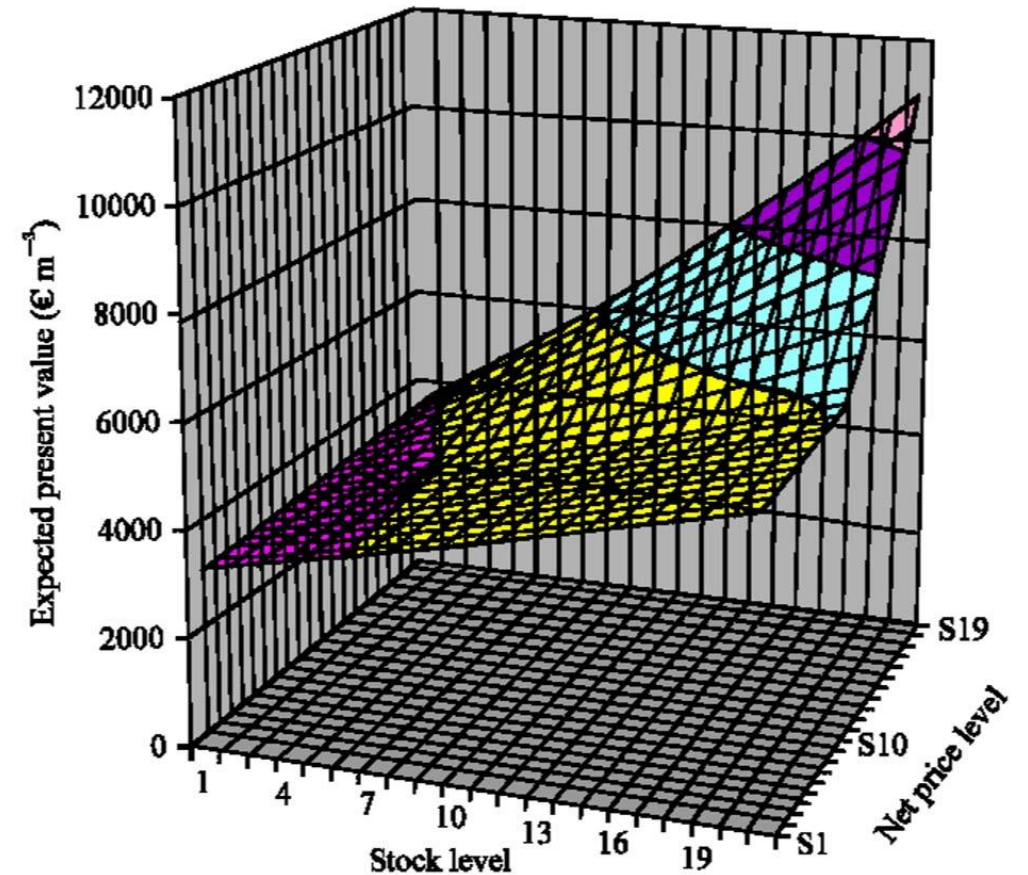
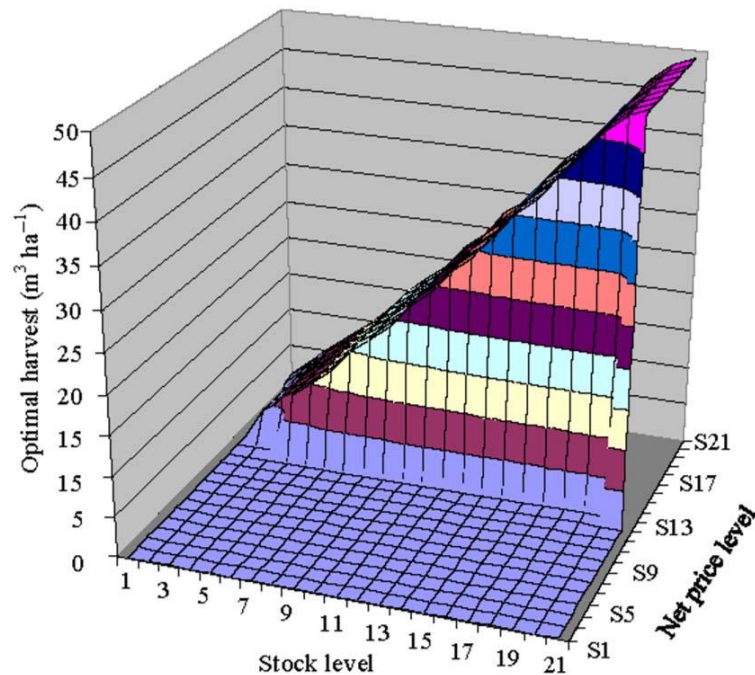
## Part 4.

- **Optimization of management decisions in a changing and not perfectly predictable world, should always be based on adaptive optimization.** Lohmander [2] describes these principles and typical implications for optimal forestry decisions. Adaptable logistic growth functions work well in such cases.

# Lohmander and Mohammadi (2008)

stochastic dynamic programming:

$$f_t(m) = \max_{u \in U(m)} \left\{ R_t(m, u) + d \sum_n p(n|m, u) f_{t+1}(n) \right\} \quad \forall m \in M$$





## ABSTRACT

Forestry in Iran is based on continuous cover forestry (CCF) management principles. CCF often leads to higher expected present values than rotation forestry (RF) with clear cuts. Furthermore, CCF has environmental advantages of several kinds. Many different species of trees grow together in large parts of these forests in Iran. Mixed species forests give advantages compared to monocultures, such as options to adapt harvesting of different species to changes in market prices, climate, species specific damages etc. In order to optimize multi species CCF in Iran, it is necessary to develop mathematical models for operations research studies that represent the relevant parts of the Iranian forestry planning problem. This presentation includes central components in this modelling process: Forest statistics, growth function estimations and the links to forest harvesting, logistics and the forest industry mills.

**Keywords:** Forest statistics; Caspian forest; Dynamic growth models; Optimization; Logistics.

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### ABSTRACT

Forestry in Iran is based on continuous cover forestry (CCF) management principles. CCF often leads to higher expected present values than rotation forestry (RF) with clear cuts. Furthermore, CCF has environmental advantages of several kinds. Many different species of trees grow together in large parts of these forests in Iran. Mixed species forests give advantages compared to monocultures, such as options to adapt harvesting of different species to changes in market prices, climate, species specific damages etc. In order to optimize multi species CCF in Iran, it is necessary to develop mathematical models for operations research studies that represent the relevant parts of the Iranian forestry planning problem. This presentation includes central components in this modelling process: Forest statistics, growth function estimations and the links to forest harvesting, logistics and the forest industry mills.

**Keywords:** Forest statistics; Caspian forest; Dynamic growth models; Optimization; Logistics.

### 1. INTRODUCTION

The area of natural forest in Iran is approximately 12.4 million ha of which about 1.9 million ha is managed as commercial forest called Iranian Caspian forest in northern Iran. The forests of Iran represent 7.5 percent of the total area of the country. Iranian Caspian forests are located on the south coast of the Caspian Sea and the northern slopes of the Alborz Mountain range from sea level to 2800 m. These forests grow in a strip 800 km in length and 2070 km wide. These are the most valuable forests in Iran. Industrial harvesting occurs only in the Caspian forest. Because of the severe climatic conditions and forest degradation, forests in other regions are not exploited for industrial wood production. Forest industries in Iran produce sawnwood and wood-based panels as well as pulp and paper from hardwood species. Moderate volumes of forest products, mainly paper, are imported. Modest quantities of wood are burned as fuel [5]. The number of options to model the dynamics of forests is almost unlimited. You may use stand models, models of individual trees, diameter class models, models that describe altitude, slope and directions, models in continuous time, in discrete time etc. The international demand for forest sector products has started to change. Some paper qualities have sharp demand decreases, mainly depending of lowered demand in the new internet based society. This transformation affects the whole forestry supply chain, which has to be considered as an integrated dynamical system with a lot of disturbances. Hence, when to manage the forest using multi species CCF, in our case in Iran, the main question for the forest industry is how much better this management regime is if the whole supply chain is considered and not only the management of the forest.

### 2. MAIN RESULTS

In order to collect volume and growth data, district number 2 of Losara forests that is located in Ploroad watershed, east of Guilan province was chosen. Its latitude ranges from 36 57' 38" N to 36

59' 40" N and its longitude ranges from 50 12' 10" E to 50 16' 40" E. These forests are located in Caspian mountainous area and its altitude ranges from 400 to 1200 meters. These forests are uneven-aged and the main species are: hornbeam (*Carpinus* sp), beech (*Fagus orientalis*), oak (*Quercus* sp), alder (*Alnus* sp) etc. The inventory area was 576 ha. A systematic random sampling method with network of 150\*200 m was used for inventory. The area of each sample plot was 1000 square meters. Therefore, 201 sample plots were determined and some items such as number of tree, tree diameter at breast height (DBH), trees height were measured at each sample plot. Furthermore, 3 sample plots at different 3 elevations were chosen to measure the tree increment. The available empirical data was used to estimate a modified logistic growth model where stand density, altitude and species mix were considered as explanatory variables. Logistic growth models have been found useful in continuous cover forest management optimization and examples of such studies are found in Lohmander [3] and Lohmander and Mohammadi [4]. The general dynamics of forests based on such models was analyzed and dynamic equilibrium conditions (stand densities and species mixes) for different altitudes were determined. In some cases, dynamic multi species model parameters are possible to determine via steady state observations of unmanaged forests. Optimization of management decisions in a changing and not perfectly predictable world, should always be based on adaptive optimization. Lohmander [2] describes these principles and typical implications for optimal forestry decisions. Adaptable logistic growth functions work well in such cases. The total forest sector model of this problem has to handle some criteria, maybe in conflict, such as the present value and the environmental value, as well as, the most influential disturbances. We shortly present how this can be modelled using a ranked multi criteria stochastic mixed integer programming model and why that approach is selected. We have not seen earlier applications of such models in this context. However, in distribution logistics, one method has recently been described in Kalinina et al.,[1].

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