Statistics and Mathematics of General Control Function Optimization for Continuous Cover Forestry, with a Swedish Case Study based on Picea abies

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Forest data and functions

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http://www.Lohmander.com/PL_NF_ICSTC_2021.pptx (with included short movies)

http://www.Lohmander.com/PL_NF_ICSTC_2021.pdf (The included movies are not shown with the pdf version.)

Movie Version 211214_2040

The background to this presentation is found in the following five open access articles. More details can be obtained from the links found below.

[1] Lohmander, P., Optimal Stochastic Dynamic Control of Spatially Distributed Interdependent Production Units. In: Cao BY. (ed) Fuzzy Information and Engineering and Decision. IWDS 2016. ***Advances in Intelligent Systems and Computing***, vol 646. Springer, Cham, 2018 Print ISBN 978-3-319-66513-9, Online ISBN 978-3-319-66514-6, eBook Package: Engineering, https://doi.org/10.1007/978-3-319-66514-6_13

[2] Lohmander, P., A General Dynamic Function for the Basal Area of Individual Trees Derived from a Production Theoretically Motivated Autonomous Differential Equation, ***Iranian Journal of Management Studies*** (IJMS), Vol. 10, No. 4, Autumn 2017, pp. 917-928, https://ijms.ut.ac.ir/article_64225.html

[3] Lohmander, P., Control function optimization for stochastic continuous cover forest management, ***International Robotics and Automation Journal***, 2019;5(2), pages 85-89. https://medcraveonline.com/IRATJ/IRATJ-05-00178.pdf

[4] Lohmander, P., Market Adaptive Control Function Optimization in Continuous Cover Forest Management, ***Iranian Journal of Management Studies***, Article 1, Volume 12, Issue 3, Summer 2019, Page 335-361. DOI: 10.22059/IJMS.2019.267204.673348, https://ijms.ut.ac.ir/article_71443.html

[5] Lohmander, P., Optimal Adaptive Integer Pulse Control of Stochastic Nonlinear Systems: Application to the Wolf-Moose Predator Prey System, ***Asian Journal of Statistical Sciences***, 1(1), 2021, pages 23-38. https://arfjournals.com/abstract/41816_article_no_3.pdf

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Statistics and Mathematics of General Control Function Optimization for Continuous Cover Forestry, with a Swedish Case Study based on Picea abies

Abstract:

Continuous cover forests contain large numbers of spatially distributed trees of different sizes. The growth of a particular tree is a function of the properties of that tree and the neighbor trees, since they compete for light, water and nutrients. Such a dynamical system is highly nonlinear and multidimensional. Lohmander [1], [3] and [4] shows how dynamic management of such systems can be optimized, via optimal control functions, dynamically applied to each tree. Lohmander [5] applies a related method to optimize the management of a dynamic multi species system with animals. In this paper, a particular tree is instantly harvested if a control function based on two local state variables, S and Q, is satisfied, where S represents the size of the particular tree and Q represents the level of local competition. The control function has two parameters. An explicit nonlinear present value function, representing the total value of all forestry activities over time, is suggested. This is based on the parameters in the control function, now treated as variables, and six new parameters. The functional form is motivated by a set of explicit hypotheses. Then, explicit functions for the optimal values of the two parameters in the control function are determined via optimization of the present value function. Two equilibria are obtained, where one is a unique local maximum and the other is a saddle point. An equation is determined that defines the region where the solution is a unique local maximum. Then, a case study with a continuous cover Picea abies forest, in southern Sweden, is presented. A new growth function, which is an extended version of Lohmander [2], is estimated and used in the simulations. The following procedure is repeated for five alternative levels of the interest rate: The total present value of all forest management activities in the forest, during 300 years, is determined for 1000 complete system simulations. In each system simulation, different random combinations of control function parameters are used and the total present value of all harvest activities is determined. Then, the parameters of the present value function are estimated via multivariate regression analysis. All parameters are determined with high precision and high absolute t-values. The present value function fits the data very well and no hypothesis can be rejected. Then, the optimal control function parameters and the optimal present values are analytically determined for alternative interest rates. The optimal solutions found within the relevant regions are shown to be unique maxima and the solutions that are saddle points are located far outside the relevant regions.

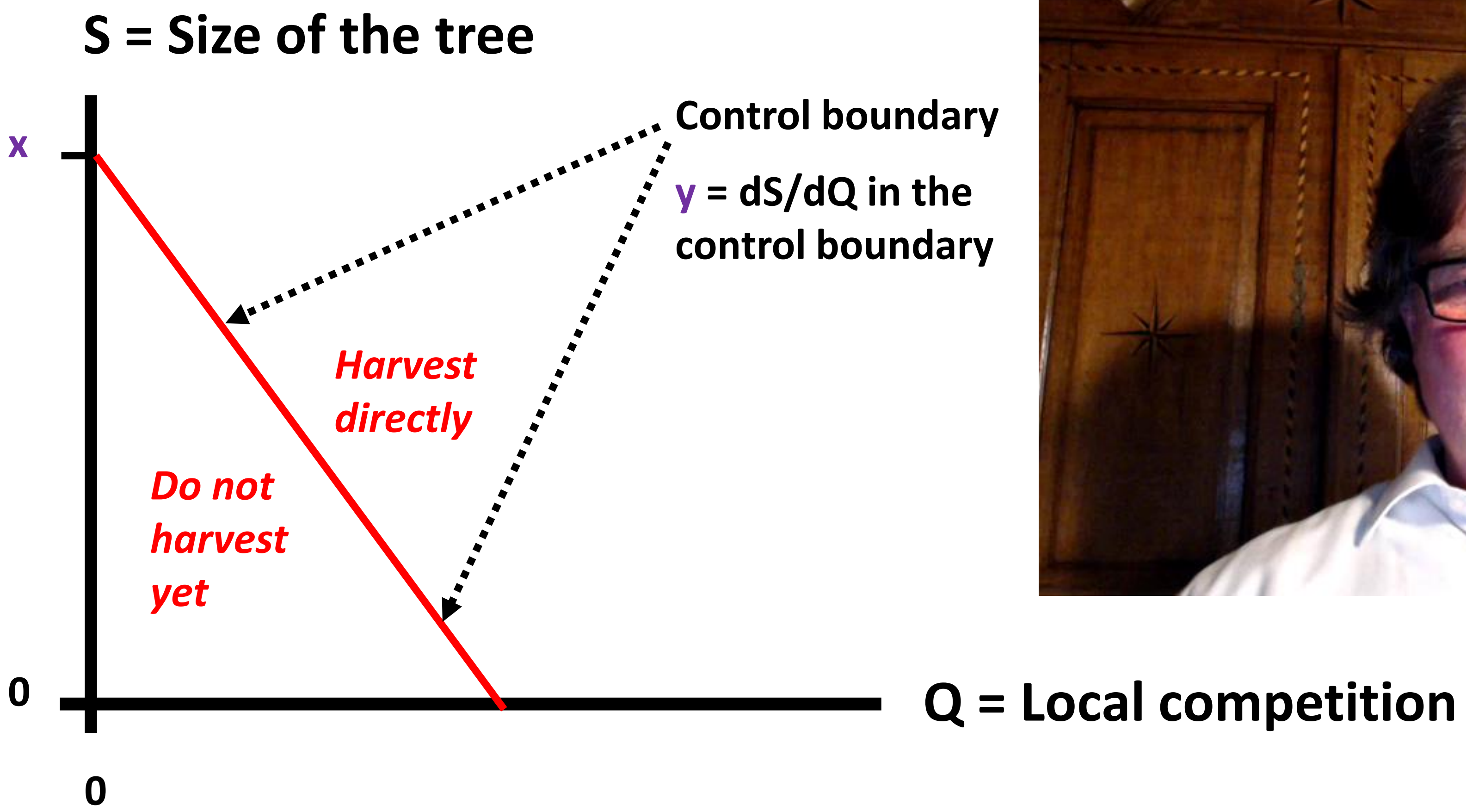
How should we optimally manage continuous cover forests?

Continuous cover forests contain large numbers of spatially distributed trees of different sizes. The growth of a particular tree is a function of the properties of that tree and the neighbor trees, since they compete for light, water and nutrients. Such a dynamical system is highly nonlinear and multidimensional.

Lohmander [1], [3] and [4] shows how dynamic management of such systems can be optimized, via optimal control functions, dynamically applied to each tree.

Lohmander [5] applies a related method to optimize the management of a dynamic multi species system with animals.

In this paper, a particular tree is instantly harvested if a control function based on two local state variables, S and Q, is satisfied, where S represents the size of the particular tree and Q represents the level of local competition. The control function has two parameters (x, and y).



Special case and optimal resource distribution interpretation:

- We may define S, the size of the tree, as the diameter at breast height, ”D”, 1.3 meters above ground.
- Then, we may interpret x as the ”diameter limit”, ”DL”, in the case of no local competition.
- If ”D” is >= ”DL”, and no local competition exists, then the tree should be instantly harvested.
- If ”D” < ”DL”, then the tree should not yet be harvested. It should continue to grow until it reaches ”DL”.
- ”DL” is a decreasing function of local competition. ($y < 0$.)
- This is reasonable since growth resources such as nutrients, light and water are locally constrained. The particular tree negatively affects the competitors and the competitors negatively affect the particular tree. In order to avoid these negative competition effects, the trees with local competition should be harvested when the diameters are smaller than the ”DL” determined without competition.

An explicit nonlinear present value function, representing the total value of all forestry activities over time, is suggested. $f(x,y)$

This is based on the parameters in the control function, now treated as variables, and six new parameters.

$$f(x,y) = k + ax - bx^{1.5} + cy - gy^2 - hxy$$

$$\left(a,b,c,g,h > 0\right)$$

The functional form is motivated by a set of explicit hypotheses.

If there are no exogenous disturbances, such as competition, f(.) is a strictly concave function of x.

$$f_1(x) = f(x,y)\big|_{y=0} = k + ax - bx^{1.5}$$

$$\frac{df_1(x)}{dx} = a - 1.5bx^{0.5}$$

$$\frac{d^2 f_1(x)}{dx^2} = -0.75bx^{-0.5} < 0$$

$$(x > 0) \Rightarrow \left(\frac{d^2 f_1(x)}{dx^2} < 0 \right)$$

Hence, if there are no exogenous disturbances, such as competition, f(.) is a strictly concave function of x, for strictly positive values of x.

The functional form is motivated by a set of explicit hypotheses.

The optimal value of x is unique and maximizes f. This optimal value is greater than 0 and less than infinity.

$$\frac{df_1(x)}{dx} = a - 1.5bx^{0.5} = 0$$

$$x^{0.5} = \frac{2a}{3b}$$

$$0 < x = \frac{4a^2}{9b^2} < \infty$$

The functional form is motivated by a set of explicit hypotheses.

The optimal value of y is unique and maximizes f.

$$\frac{df(x,y)}{dy} = c - 2gy - hx = 0$$ **The first order optimum condition.**

$$\frac{d^2 f(x,y)}{dy^2} = -2g < 0$$ **The second order maximum condition is always satisfied.**

$$\left(\frac{df(x,y)}{dy} = 0\right) \Rightarrow \left(y = \frac{c - hx}{2g}\right)$$ **The optimal value of y is unique and conditional on x.**

The functional form is motivated by a set of explicit hypotheses.

The optimal value of x is a strictly decreasing function of y.

$$f(x,y) = k + ax - bx^{1.5} + cy - gy^2 - hxy$$

$$\frac{df(x,y)}{dx} = a - 1.5bx^{0.5} - hy$$

$$\frac{d^2 f(x,y)}{dxdy} = -h < 0$$

We see that the functional form is consistent with this hypothesis, if h > 0.

The following functional form was consistent with the hypotheses and suggested. Of course, marginally different functional forms can also be found that are also consistent with the hypotheses.

$$f(x,y) = k + ax \boxed{-bx^{1.5}} + cy - gy^2 - hxy$$

Observations of empirical data and regression analyses showed that the following function, which also satisfies the hypotheses, does not fit the empirical data as well as the suggested function, found above.

$$f(x,y) = k + ax \boxed{-bx^{2}} + cy - gy^2 - hxy$$

Then, explicit functions for the optimal values of the two parameters in the control function are determined via optimization of the present value function.

If x is a free variable and y=0, then:

$$f(x,y) = k + ax - bx^{1.5} + cy - gy^2 - hxy$$

$$f_1(x) = f(x,y)\big|_{y=0} = k + ax - bx^{1.5}$$

First order optimum condition:

$$\frac{df_1}{dx} = a - \frac{3b}{2}\sqrt{x} = 0$$

$$\left(\frac{df_1}{dx} = a - \frac{3b}{2}\sqrt{x} = 0 \right) \Rightarrow \left(\sqrt{x} = \frac{2a}{3b} \right)$$

$$x = \frac{4a^2}{9b^2} > 0$$

$$\frac{d^2 f_1}{dx^2} = -\frac{3b}{4\sqrt{x}} < 0$$

Unique global maximum

If x and y are free variables, then:

$$f(x,y) = k + ax - bx^{1.5} + cy - gy^2 - hxy$$

First order optimum conditions

$$\begin{cases} \dfrac{df}{dx} = a - 1.5b\sqrt{x} - hy = 0 \\ \dfrac{df}{dy} = c - hx - 2gy = 0 \end{cases}$$

$$\left(\frac{df}{dy} = c - hx - 2gy = 0\right) \Rightarrow \left(y = \frac{c - hx}{2g}\right)$$

$$\left(\frac{df}{dx} = a - 1.5b\sqrt{x} - hy = 0\right)$$

$$a - 1.5b\sqrt{x} - h\left(\frac{c - hx}{2g}\right) = 0$$

$$a - 1.5b\sqrt{x} - \frac{ch}{2g} + \frac{h^2}{2g}x = 0$$

Thank you Nils Fagerberg for discovering a misprint in the earlier version of this page!

$$a-1.5b\sqrt{x}-\frac{ch}{2g}+\frac{h^2}{2g}x=0$$

$$\frac{h^2}{2g}x-\frac{3bg}{2g}\sqrt{x}+\frac{2ag-ch}{2g}=0$$

$$\left(g>0\right)\Rightarrow \quad h^2x-3bg\sqrt{x}+\left(2ag-ch\right)=0$$

$$\left(z=\sqrt{x}\right)\Rightarrow \quad h^2z^2-3bgz+\left(2ag-ch\right)=0$$

$$\left(h>0\right)\Rightarrow \quad z^2-\frac{3bg}{h^2}z+\frac{2ag-ch}{h^2}=0$$

$$z^2 \quad \underbrace{\boxed{-\frac{3bg}{h^2}}}_{p} \quad z \quad + \quad \underbrace{\boxed{\frac{2ag-ch}{h^2}}}_{q} \quad = 0$$

$$z^2 + pz + q = 0$$

$$z = \frac{-p}{2} \pm \sqrt{\left(\frac{-p}{2}\right)^2 - q}$$

$$z = \frac{3bg}{2h^2} \pm \sqrt{\left(\frac{3bg}{2h^2}\right)^2 + \frac{ch - 2ag}{h^2}}$$

$$z = \frac{3bg \pm \sqrt{9b^2g^2 + 4ch^3 - 8agh^2}}{2h^2}$$

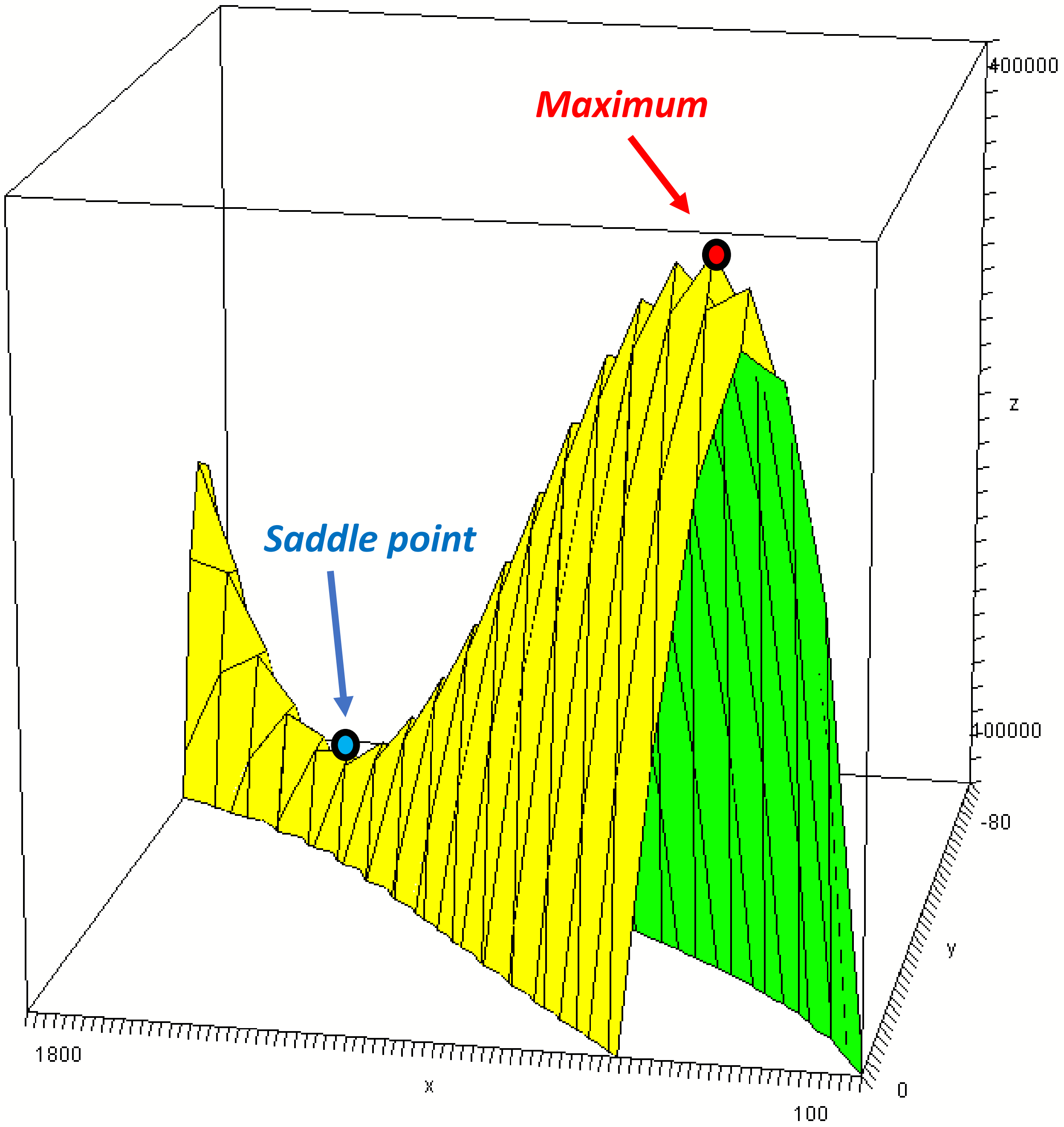
$$z_1 = \frac{3bg - \sqrt{9b^2g^2 + 4ch^3 - 8agh^2}}{2h^2}$$

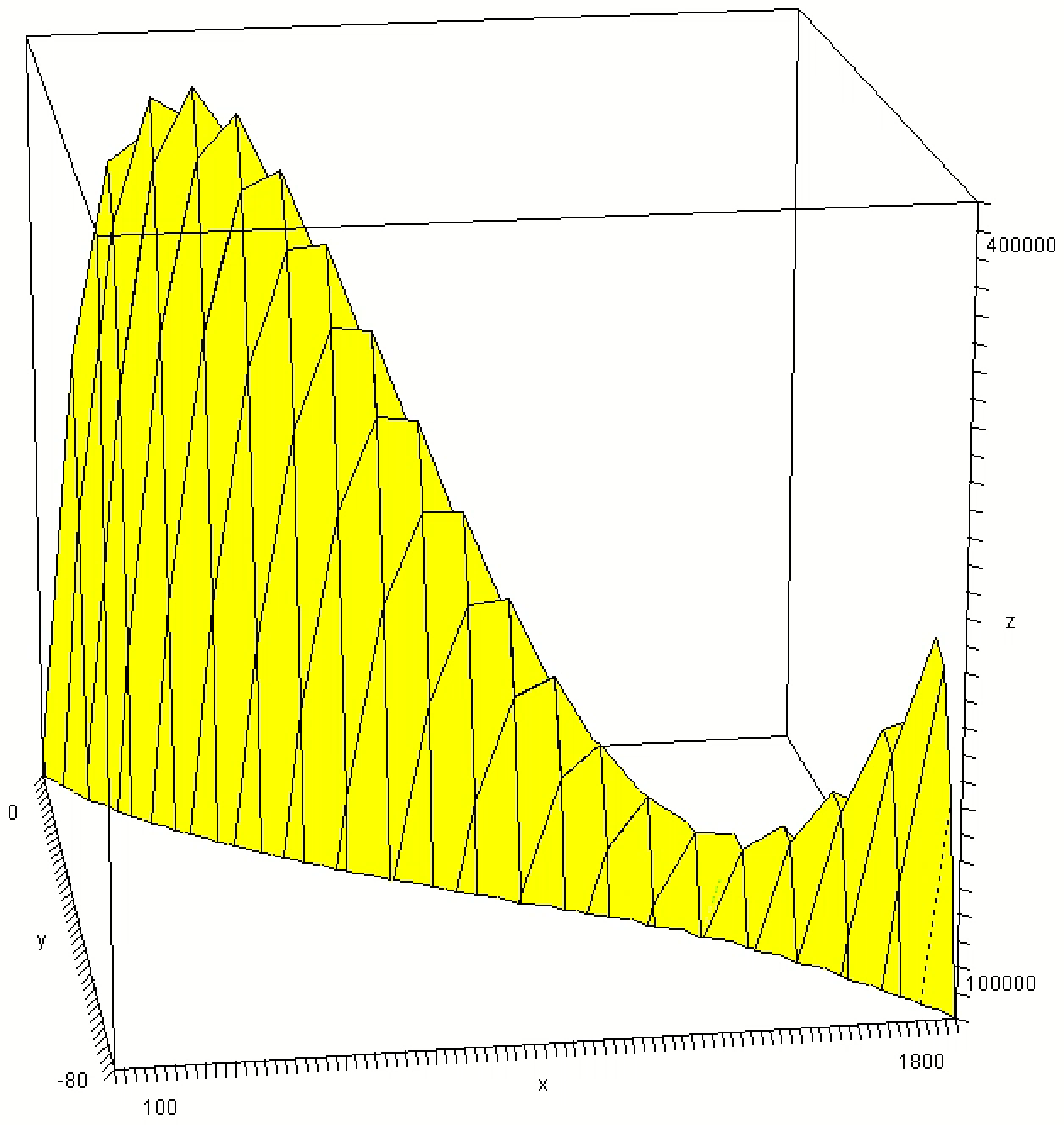
$$z_2 = \frac{3bg + \sqrt{9b^2g^2 + 4ch^3 - 8agh^2}}{2h^2}$$

Hence, there are two different solutions to the first order optimum conditions.
We have to investigate if one of these is a maximum and what the other solution is.

$$\begin{cases} x_1 = (z_1)^2 \\ y_1 = \dfrac{c - h(z_1)^2}{2g} \end{cases}$$

$$\begin{cases} x_2 = (z_2)^2 \\ y_2 = \dfrac{c - h(z_2)^2}{2g} \end{cases}$$


Maximum
Saddle point
400000
z
100000
-80
y
0
1800
x
100



r = 1%

Short movie #0. Click below.

Two equilibria are obtained, where one is a unique local maximum and the other is a saddle point.

$$\begin{cases} \dfrac{df}{dx} = a - \dfrac{3}{2} b\sqrt{x} - hy = 0 \\ \dfrac{df}{dy} = c - hx - 2gy = 0 \end{cases}$$

$$[D] = \begin{bmatrix} \dfrac{d^2 f}{dx^2} & \dfrac{d^2 f}{dxdy} \\ \dfrac{d^2 f}{dydx} & \dfrac{d^2 f}{dy^2} \end{bmatrix} = \begin{bmatrix} -\dfrac{3b}{4\sqrt{x}} & -h \\ -h & -2g \end{bmatrix}$$

Second order conditions of local maximum:

$$|D_1| = \left|\frac{d^2 f}{dx^2}\right| = \left|-\frac{3b}{4\sqrt{x}}\right| = \left(-\frac{3b}{4\sqrt{x}}\right) < 0$$

This inequality is satisfied everywhere, for x>0 and all y.

$$|D| = \begin{vmatrix} \frac{d^2 f}{dx^2} & \frac{d^2 f}{dxdy} \\ \frac{d^2 f}{dydx} & \frac{d^2 f}{dy^2} \end{vmatrix} = \begin{vmatrix} -\frac{3b}{4\sqrt{x}} & -h \\ -h & -2g \end{vmatrix} = \frac{3bg}{2\sqrt{x}} - h^2 > 0$$

This inequality is not satisfied for all values of x. The limiting value of x will be determined from the inequality.

The solution to the first order optimum conditions is a locally unique maximum if this condition is satisfied:

$$\frac{3bg}{2\sqrt{x}} - h^2 > 0$$

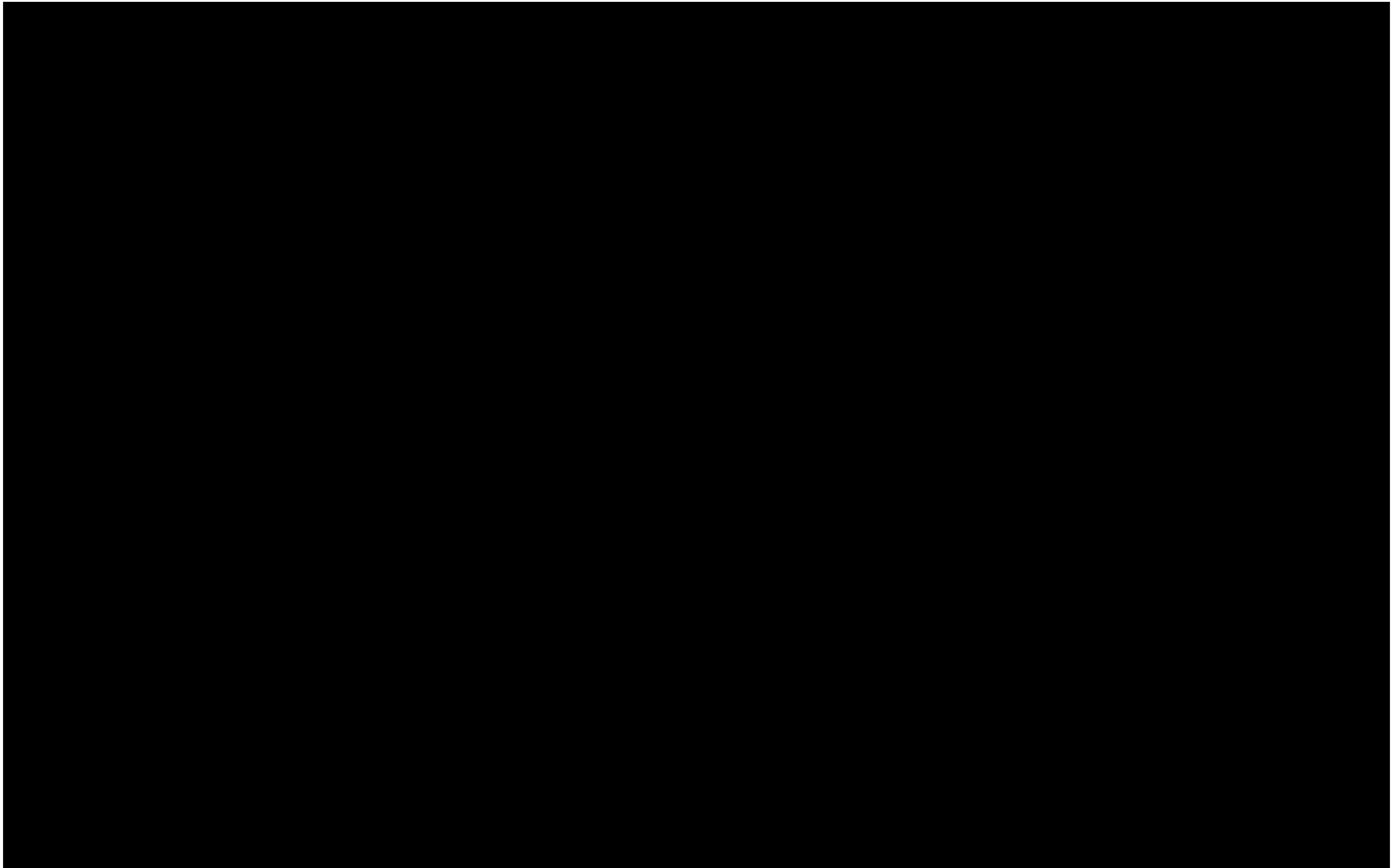
$$3bg > 2h^2\sqrt{x}$$

$$2h^2\sqrt{x} < 3bg$$

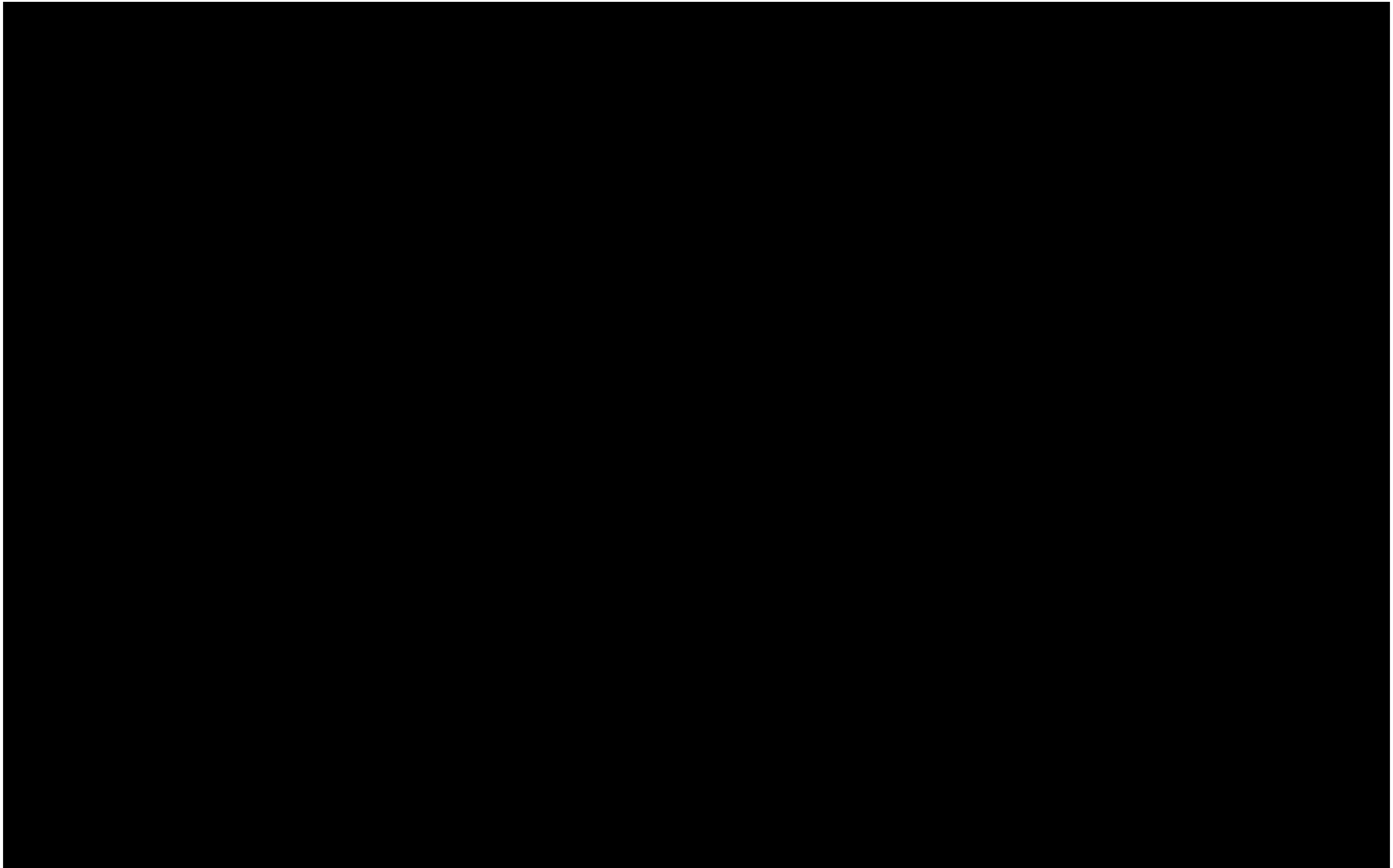
$$\sqrt{x} < \frac{3bg}{2h^2}$$

$$\boxed{x < \frac{9b^2g^2}{4h^4}}$$

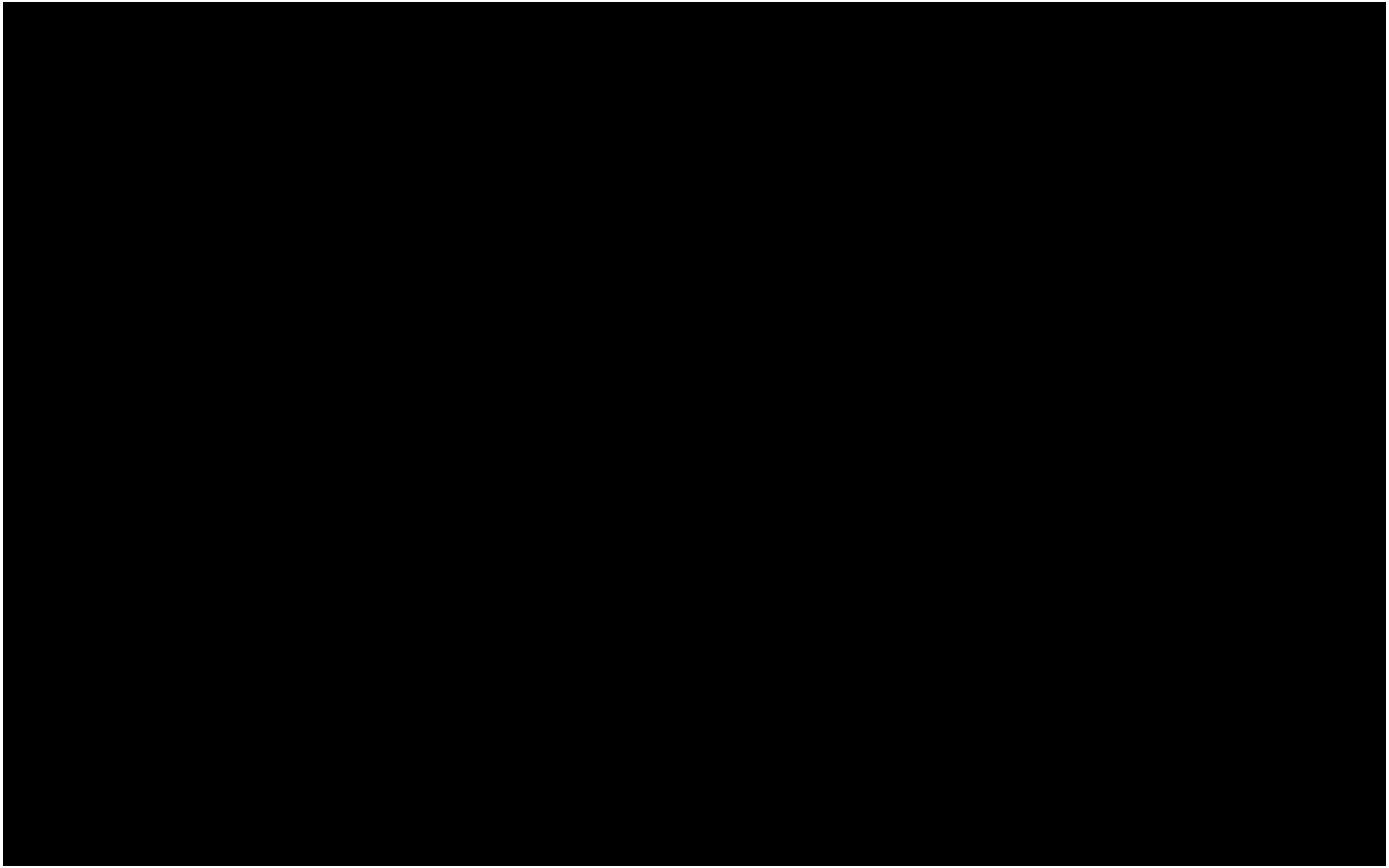
A case study with a continuous cover Picea abies forest, in southern Sweden, has been made. A new growth function, which is an extended version of Lohmander [2], is estimated and used in the simulations. On the following three pages, Nils Fagerberg, Linnaeus university, presents some background to the case study field work, data collection and function estimations.



Short Movie #1. Click below.



Short Movie #2. Click below.



Short Movie #3. Click below.

The following procedure is repeated for five alternative levels of the interest rate:

The total present value of all forest management activities in the forest, during 300 years, is determined for 1000 complete system simulations.

In each system simulation, different random combinations of control function parameters are used and the total present value of all harvest activities is determined.

Then, the parameters of the present value function are estimated via multivariate regression analysis.

First definition of competition:

The competition is defined as the total basal area of competing trees, square meters per hectare, within a circle with radius 10 meters. The subject tree is in the center of the circle.

With that definition of competition:

All parameters of relevance to the determination of the optimal control function are determined with high precision and high absolute t-values.

The present value function fits the data very well and no hypothesis can be rejected.

This is found in the following pictures, representing rates of interest 1% to 3%.

r (%)	1	1,5	2	2,5	3
	t-values in the regression analyses				
k	-8,97041	-6,11472	-1,14039	6,363739	16,74447
x	27,66237	34,24437	41,18366	45,60594	46,64495
x^1.5	-30,5576	-38,3753	-46,6355	-51,7893	-52,7345
y	32,01628	35,58006	38,24762	37,00359	32,11392
y^2	-25,4535	-27,9865	-28,9649	-26,5497	-21,8865
xy	-53,3308	-62,2809	-69,5523	-69,3717	-62,1029
	p-values in the regression analyses				
k	1,44E-18	1,39E-09	0,2544	3E-10	1,25E-55
x	2,3E-125	2,2E-170	4,1E-217	6,4E-246	1,5E-252
x^1.5	3,9E-145	2,2E-198	1,7E-252	1,5E-284	2,8E-290
y	3,9E-155	1,8E-179	1,6E-197	3,9E-189	8,4E-156
y^2	1,9E-110	1,4E-127	3,1E-134	8E-118	5,6E-87
xy	7,2E-294	0	0	0	0

In the following detailed regression tables,
The notation is this:

DL0 = x

DLC = y

Regression
r = 1.0 %

Regressionsstatistik	
Multipel-R	0,90606
R-kvadrat	0,820946
Justerad R-kvadrat	0,820045
Standardfel	11572,18
Observationer	1000

ANOVA

	fg	KvS	MKv	F
Regression	5	6,1E+11	1,22E+11	911,4768
Residual	994	1,33E+11	1,34E+08	
Totalt	999	7,43E+11		

		Koefficienter	Standardfel	t-kvot	p-värde
	Konstant	-235947	26302,77	-8,97041	1,44E-18
x	DL0	5501,829	198,8921	27,66237	2,3E-125
x^1.5	(DL0)^1.5	-202,919	6,64056	-30,5576	3,9E-145
y	DLC	31104,54	971,5226	32,01628	3,9E-155
y^2	(DLC)^2	-1255,7	49,33315	-25,4535	1,9E-110
xy	DL0*DLC	-114,217	2,141676	-53,3308	7,2E-294

Regression
r = 1.5 %

Regressionsstatistik	
Multipel-R	0,933819
R-kvadrat	0,872018
Justerad R-kvadrat	0,871374
Standardfel	6448,06
Observationer	1000

ANOVA

	fg	*KvS*	*MKv*	*F*
Regression	5	2,82E+11	5,63E+10	1354,545
Residual	994	4,13E+10	41577479	
Totalt	999	3,23E+11		

	Koefficienter	*Standardfel*	*t-kvot*	*p-värde*
Konstant	-78658,9	12863,85	-6,11472	1,39E-09
DL0	3555,597	103,8301	34,24437	2,2E-170
(DL0)^1.5	-137,408	3,580645	-38,3753	2,2E-198
DLC	18364,61	516,1491	35,58006	1,8E-179
(DLC)^2	-769,31	27,48863	-27,9865	1,4E-127
DL0*DLC	-74,3229	1,193349	-62,2809	0

Regression r = 2.0 %

Regressionsstatistik	
Multipel-R	0,951244
R-kvadrat	0,904865
Justerad R-kvadrat	0,904386
Standardfel	4230,064
Observationer	1000

ANOVA

	fg	*KvS*	*MKv*	*F*
Regression	5	1,69E+11	3,38E+10	1890,853
Residual	994	1,78E+10	17893438	
Totalt	999	1,87E+11		

	Koefficienter	*Standardfel*	*t-kvot*	*p-värde*
Konstant	-8370,16	7339,752	-1,14039	0,2544
DL0	2616,173	63,52455	41,18366	4,1E-217
(DL0)^1.5	-105,759	2,26778	-46,6355	1,7E-252
DLC	12331,92	322,4232	38,24762	1,6E-197
(DLC)^2	-522,328	18,03314	-28,9649	3,1E-134
DL0*DLC	-54,4498	0,782862	-69,5523	0

Regression
r = 2.5 %

Regressionsstatistik	
Multipel-R	0,955078
R-kvadrat	0,912175
Justerad R-kvadrat	0,911733
Standardfel	3236,303
Observationer	1000

ANOVA

	fg	*KvS*	*MKv*	*F*
Regression	5	1,08E+11	2,16E+10	2064,789
Residual	994	1,04E+10	10473660	
Totalt	999	1,19E+11		

	Koefficienter	*Standardfel*	*t-kvot*	*p-värde*
Konstant	30756,29	4833,054	6,363739	3E-10
DL0	2056,22	45,08667	45,60594	6,4E-246
(DL0)^1.5	-86,5134	1,670489	-51,7893	1,5E-284
DLC	8681,028	234,5996	37,00359	3,9E-189
(DLC)^2	-366,297	13,79667	-26,5497	8E-118
DL0*DLC	-41,5499	0,598945	-69,3717	0

Regression
r = 3.0 %

Regressionsstatistik	
Multipel-R	0,94809
R-kvadrat	0,898875
Justerad R-kvadrat	0,898366
Standardfel	2741,975
Observationer	1000

ANOVA

	fg	*KvS*	*MKv*	*F*
Regression	5	6,64E+10	1,33E+10	1767,077
Residual	994	7,47E+09	7518429	
Totalt	999	7,39E+10		

	Koefficienter	*Standardfel*	*t-kvot*	*p-värde*
Konstant	58298,9	3481,681	16,74447	1,25E-55
DL0	1642,834	35,21998	46,64495	1,5E-252
(DL0)^1.5	-71,6322	1,358356	-52,7345	2,8E-290
DLC	6064,113	188,8313	32,11392	8,4E-156
(DLC)^2	-255,838	11,68932	-21,8865	5,6E-87
DL0*DLC	-31,5147	0,507459	-62,1029	0

What happens if we redefine competition?

The first definition of competition, which we have now investigated, was:

The competition is defined as the total basal area of competing trees, square meters per hectare, within a circle with radius 10 meters. The subject tree is in the center of the circle.

It *could* be convenient if we did not have to investigate the basal area of the competitors. Let us now see how the results change if we *temporarily* redefine competition and only base that on information about the "closest and biggest neighbor tree".

Regression r = 2.0 % (if we only consider the competition from the neighbor tree with the largest value of the ratio ”diameter divided by distance”. ”Competition”, Q, is then defined as the maximum value of diameter divided by distance.)

Here, DLQ = y.

	Koefficienter	*Standardfel*	*t-kvot*	*p-värde*
Konstant	-236108,5059	2260,890939	-104,4316211	0
DL0	4950,345662	19,56770105	252,9855525	0
(DL0)^1.5	-195,6973229	0,698552758	-280,1468044	0
DLQ	-156,2041181	99,31721369	-1,572779907	0,116088016
(DLQ)^2	-5,168695365	5,554815217	-0,930489164	0,352343795
DL0*DLQ	0,321986381	0,241147784	1,335224299	0,18210864

Observation:

This alternative definition of competition does not give satisfactory t-values and p-values of the estimated parameters. This is understandable, since if we only consider the competition from the ”biggest and closest” competitor, we do not consider all of the other trees in the local area, that also are competitors.

Now, the optimal control function parameters and the optimal present values are analytically determined for alternative interest rates.

The optimal solutions found within the relevant regions are shown to be unique maxima and the solutions that are saddle points are located far outside the relevant regions.

The regression analysis gave these parameter values:

r (%)	1	1,5	2	2,5	3
k	-235946,7178	-78658,85812	-8370,156496	30756,29305	58298,89751
x	5501,829188	3555,596824	2616,173465	2056,220151	1642,834135
x^1.5	-202,9194332	-137,408258	-105,7591839	-86,51339767	-71,63220229
y	31104,53869	18364,61462	12331,92025	8681,028154	6064,113391
y^2	-1255,699477	-769,3098941	-522,3284433	-366,2971758	-255,8380047
xy	-114,2172637	-74,32288849	-54,44982093	-41,549852	-31,51469255

From the regression analysis parameters, the parameter values of the objective function f(x,y) could be determined. (Note the signs of the parameters.)

r (%)	1	1,5	2	2,5	3
a	5501,829188	3555,596824	2616,173465	2056,220151	1642,834135
b	202,9194332	137,408258	105,7591839	86,51339767	71,63220229
c	31104,53869	18364,61462	12331,92025	8681,028154	6064,113391
g	1255,699477	769,3098941	522,3284433	366,2971758	255,8380047
h	114,2172637	74,32288849	54,44982093	41,549852	31,51469255

$$f(x,y) = k + ax - bx^{1.5} + cy - gy^2 - hxy$$

We remember that:

$$z_1 = \frac{3bg - \sqrt{9b^2g^2 + 4ch^3 - 8agh^2}}{2h^2}$$

$$z_2 = \frac{3bg + \sqrt{9b^2g^2 + 4ch^3 - 8agh^2}}{2h^2}$$

These values will be used to derive the results on the next page.

r (%)	1	1,5	2	2,5	3
p	-58,59588331	-57,41035693	-55,89720722	-55,06799078	-55,35648026
q	786,8266608	743,280392	695,3401909	663,6273957	653,9528201
Discriminant	71,54272451	80,70687872	85,78425297	94,49350648	112,1321566
z1	20,83964842	19,72147774	18,68662472	17,81321834	17,08899291
z2	37,75623489	37,68887919	37,2105825	37,25477244	38,26748735

We remember that:

$$\begin{cases} x_1 = (z_1)^2 \\ y_1 = \dfrac{c - h(z_1)^2}{2g} \end{cases}$$

and

$$\begin{cases} x_2 = (z_2)^2 \\ y_2 = \dfrac{c - h(z_2)^2}{2g} \end{cases}$$

(x1, y1) *is the optimal solution which maximizes* **f(x,y)**.
(x2,y2) *is a saddle point, which does not maximize* **f(x,y)**.
f(.) *is a strictly concave function for x < "***x_lim for maximum***".*

r (%)	1	1,5	2	2,5	3
x_lim for maximum	858,3693853	823,9872707	781,1244439	758,1209022	766,0849767
x1	434,2909462	388,9366842	349,1899435	317,3107476	292,0336788
y1	-7,36600802	-6,851779285	-6,395793422	-6,146903042	-6,13520695
x2	1425,533273	1420,451615	1384,62745	1387,91807	1464,400588
y2	-52,44725093	-56,67901388	-60,36507994	-66,86751287	-78,34258428
f(x1,y1)	385063,3506	286382,7313	236439,9027	208055,4769	190206,8231
f(x2,y2)	139478,344	87127,99343	68381,80349	49121,6605	20095,36724

We remember that:

$$x = \frac{4a^2}{9b^2} > 0$$

$$\frac{d^2 f_1}{dx^2} = -\frac{3b}{4\sqrt{x}} < 0$$

Unique global maximum if only x is a free variable

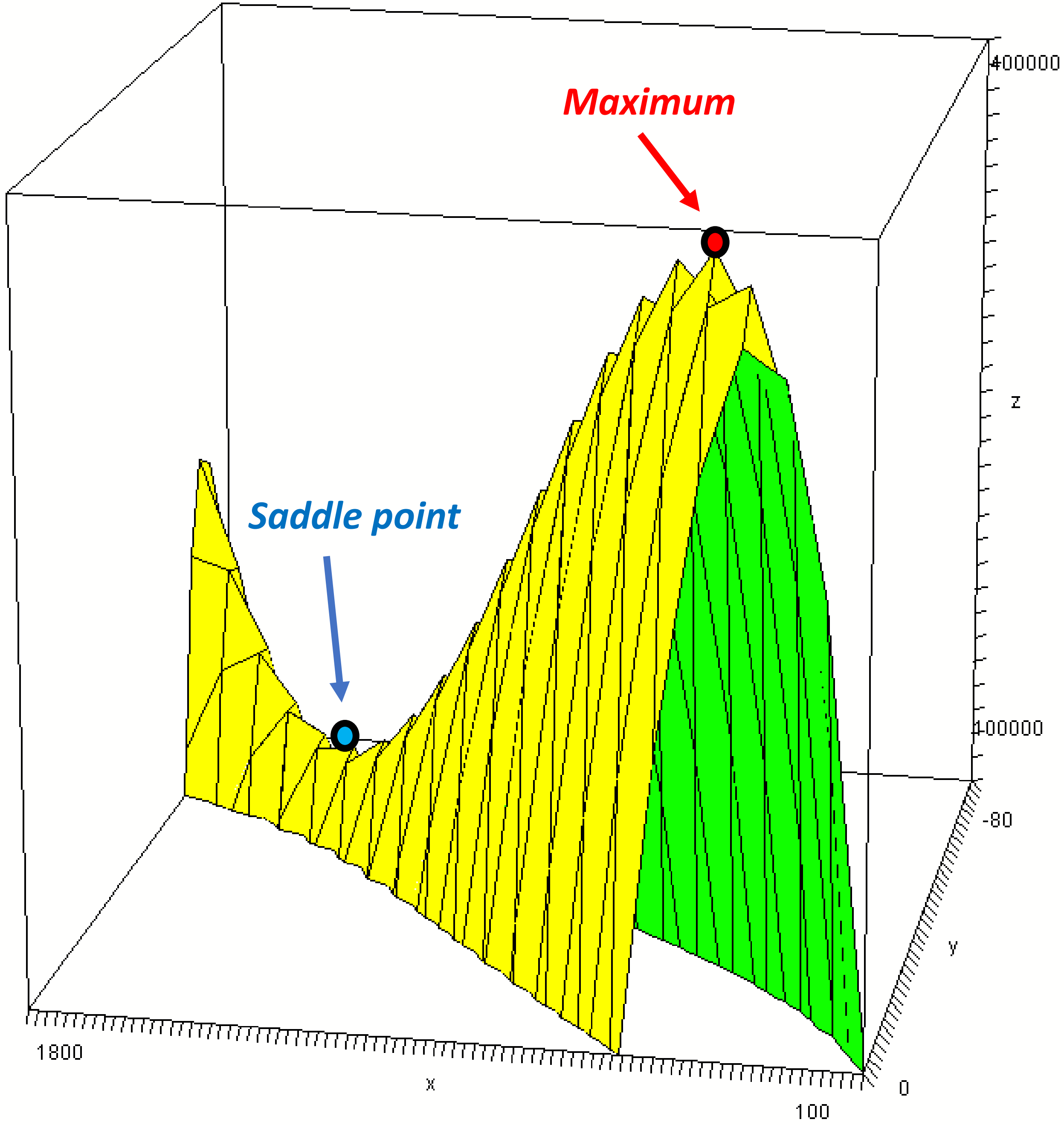
x(y=0) *is the optimal value of x, which maximizes f(x,y), in case y = 0.*

x1-x(y=0) *says how much higher the optimal x-value should be, in case we also introduce competition as a component in the control function.*

f1(x,y=0) *gives the optimal objective function value in case we do not consider competition in the decisions.*

f(x1,y1)-f1(.) *shows how much we expect to gain if we first only consider the size of the subject tree in the control function, and then start to also consider the competition.*

r (%)	1	1,5	2	2,5	3
x(y=0)	326,7265347	297,5889722	271,965849	251,0671928	233,7697494
x1-x(y=0)	107,5644115	91,34771205	77,22409448	66,24355478	58,26392943
f1(x,y=0)	363251,1438	274043,2767	228799,7893	202839,4334	186313,8722
f(x1,y1)-f1(.)	21812,20684	12339,45466	7640,11335	5216,043419	3892,950946

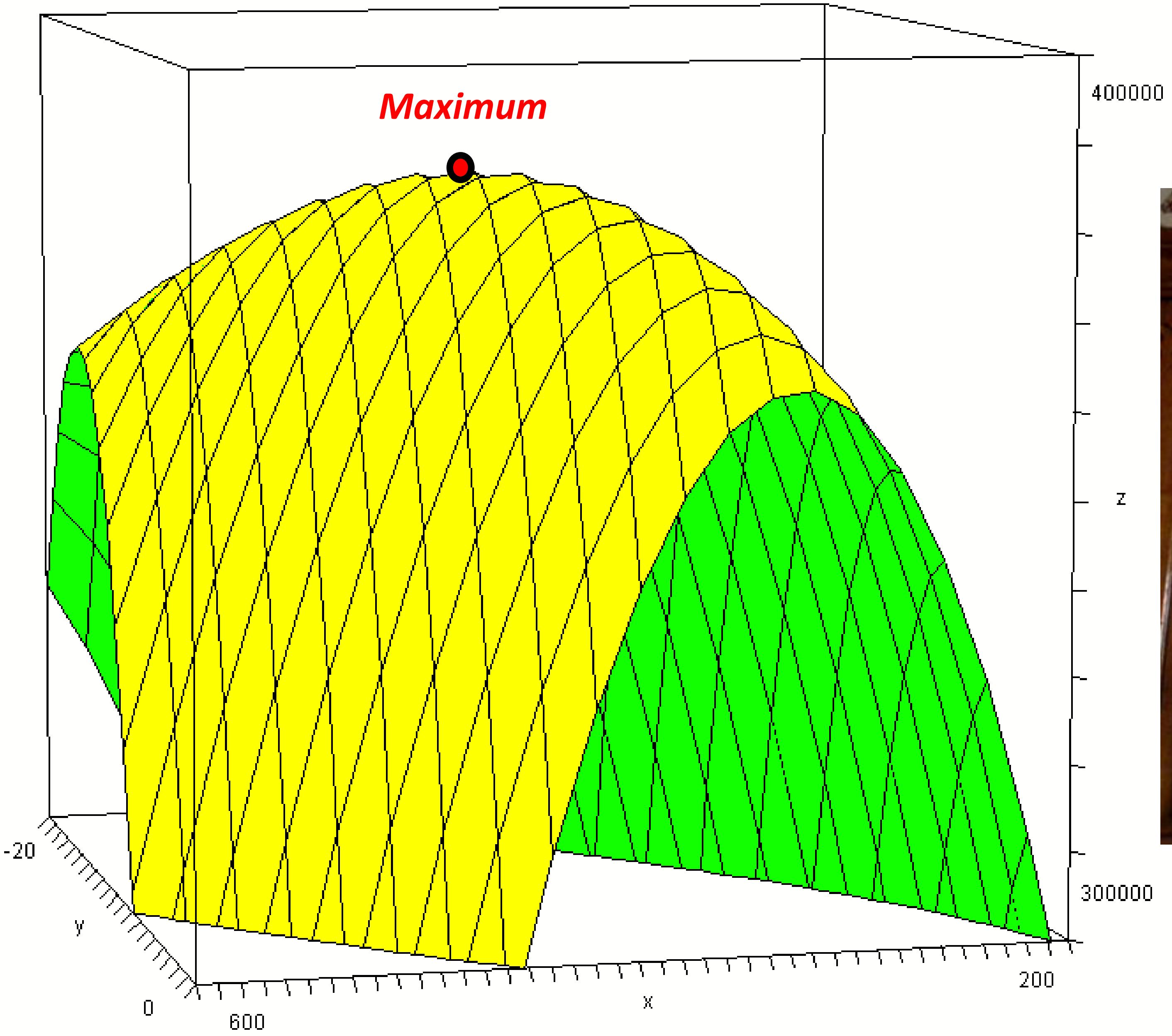


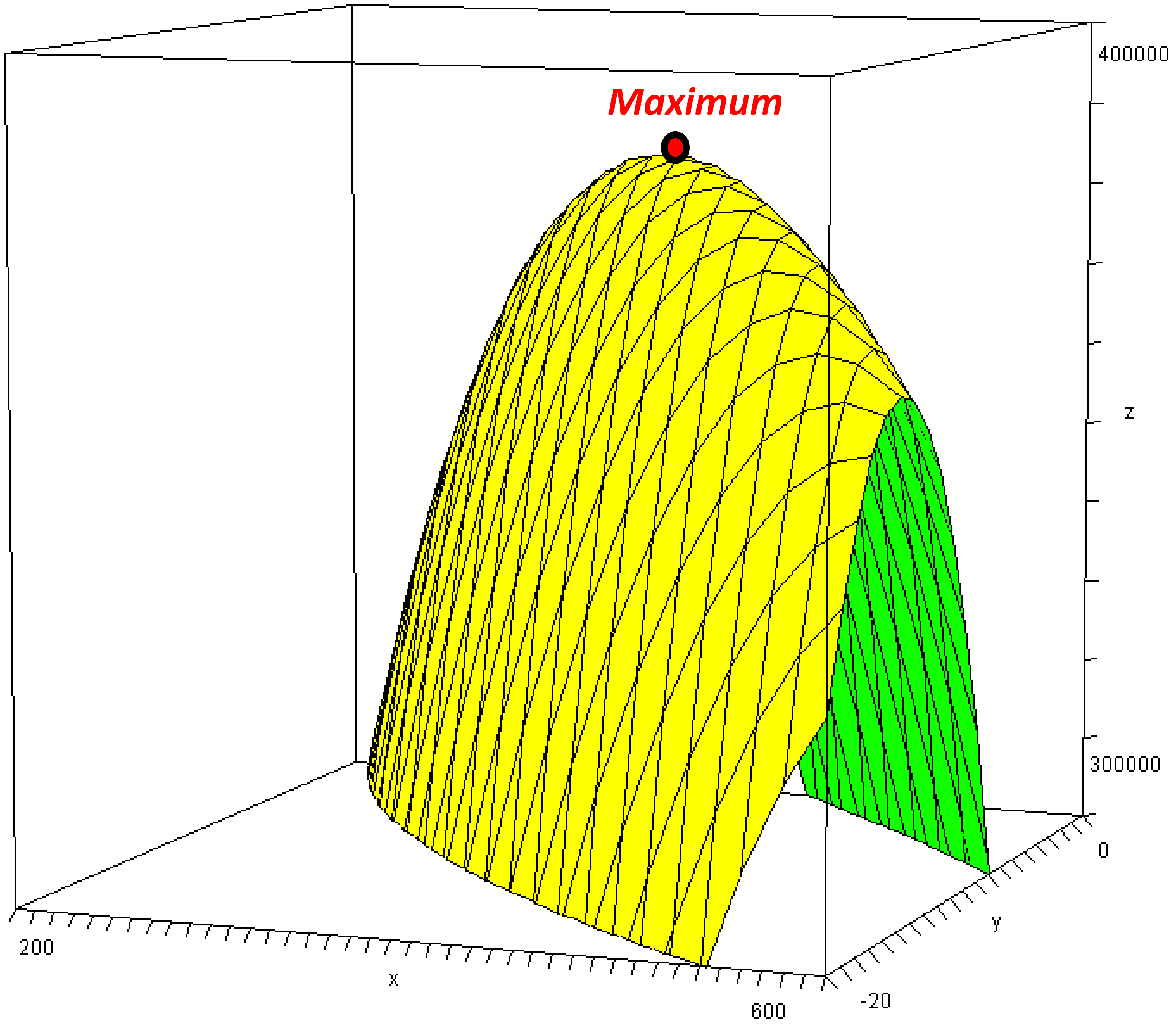
r = 1%

The solution to the first
order optimum conditions
in the empirically relevant region is
a maximum.

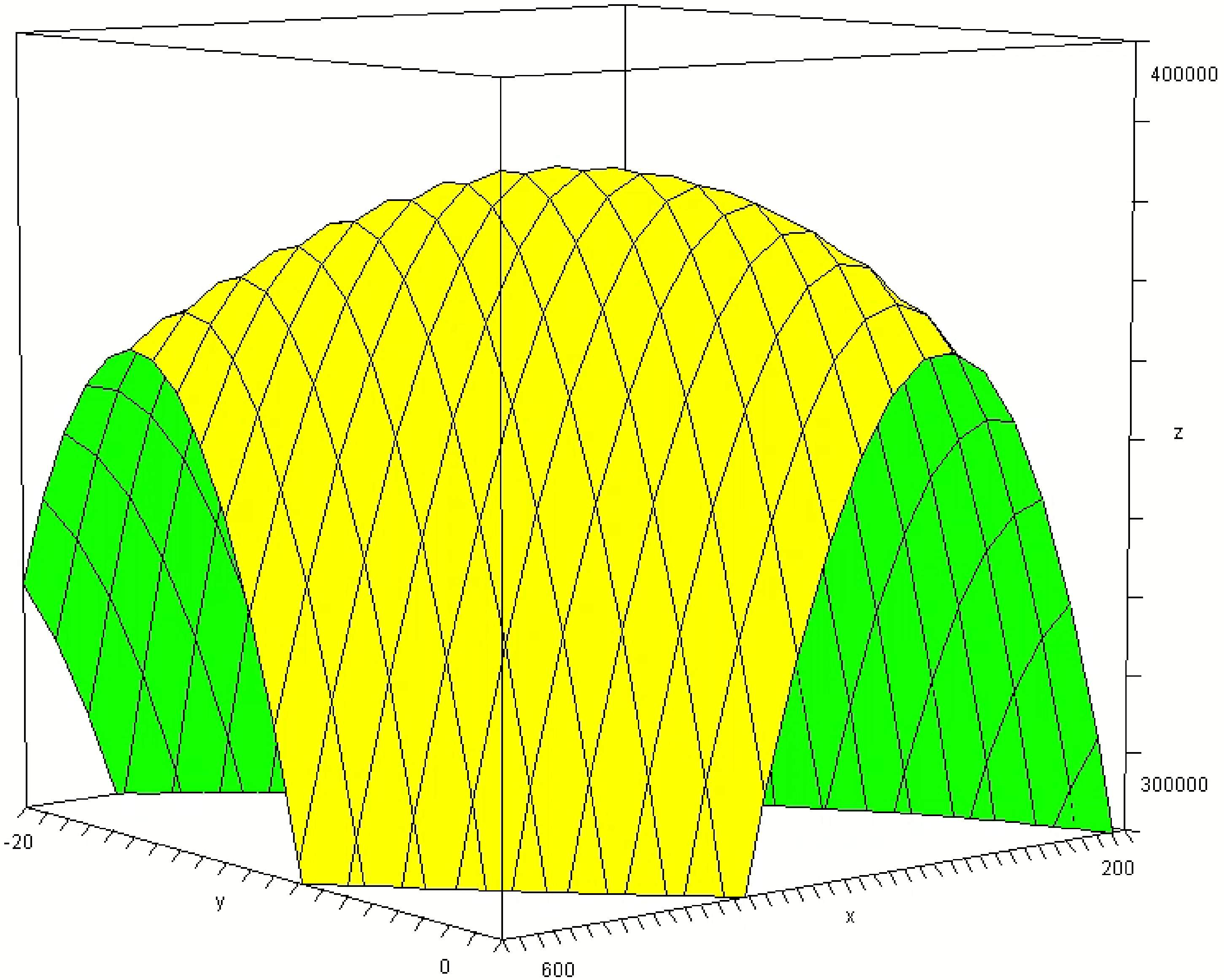
The other solution is
a saddle point.

r = 1%

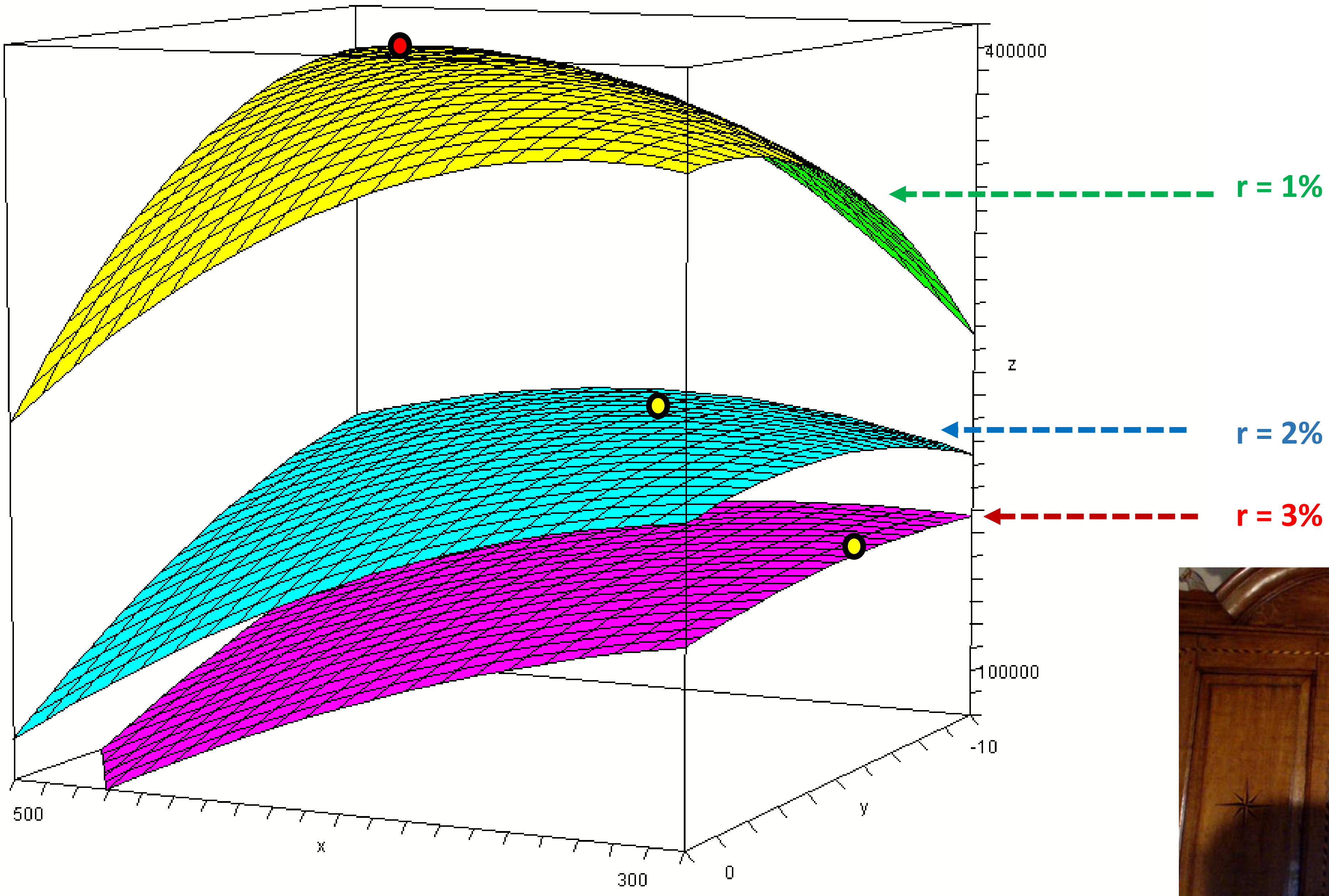


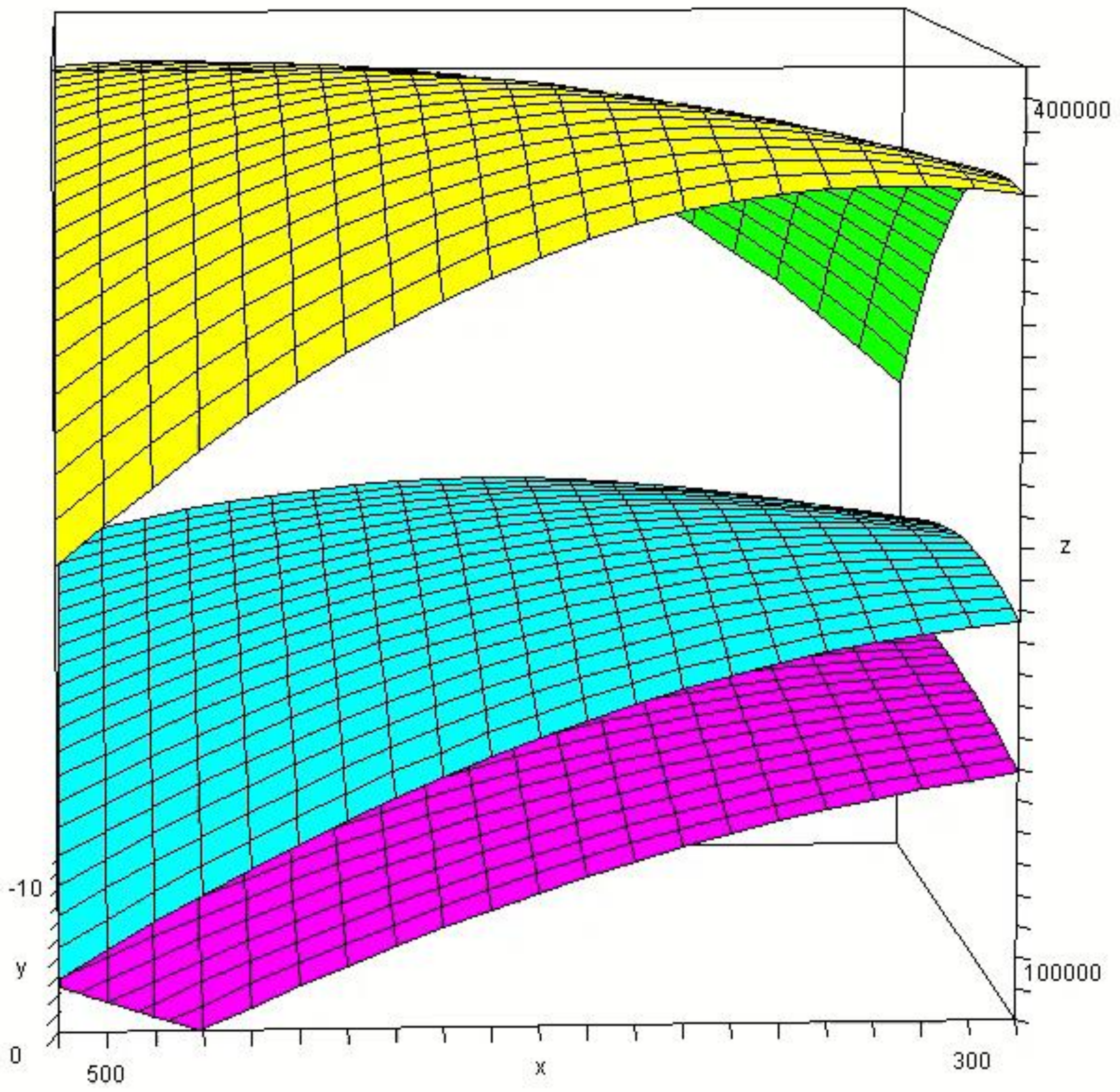


r = 1%



Short movie #4. Click below.

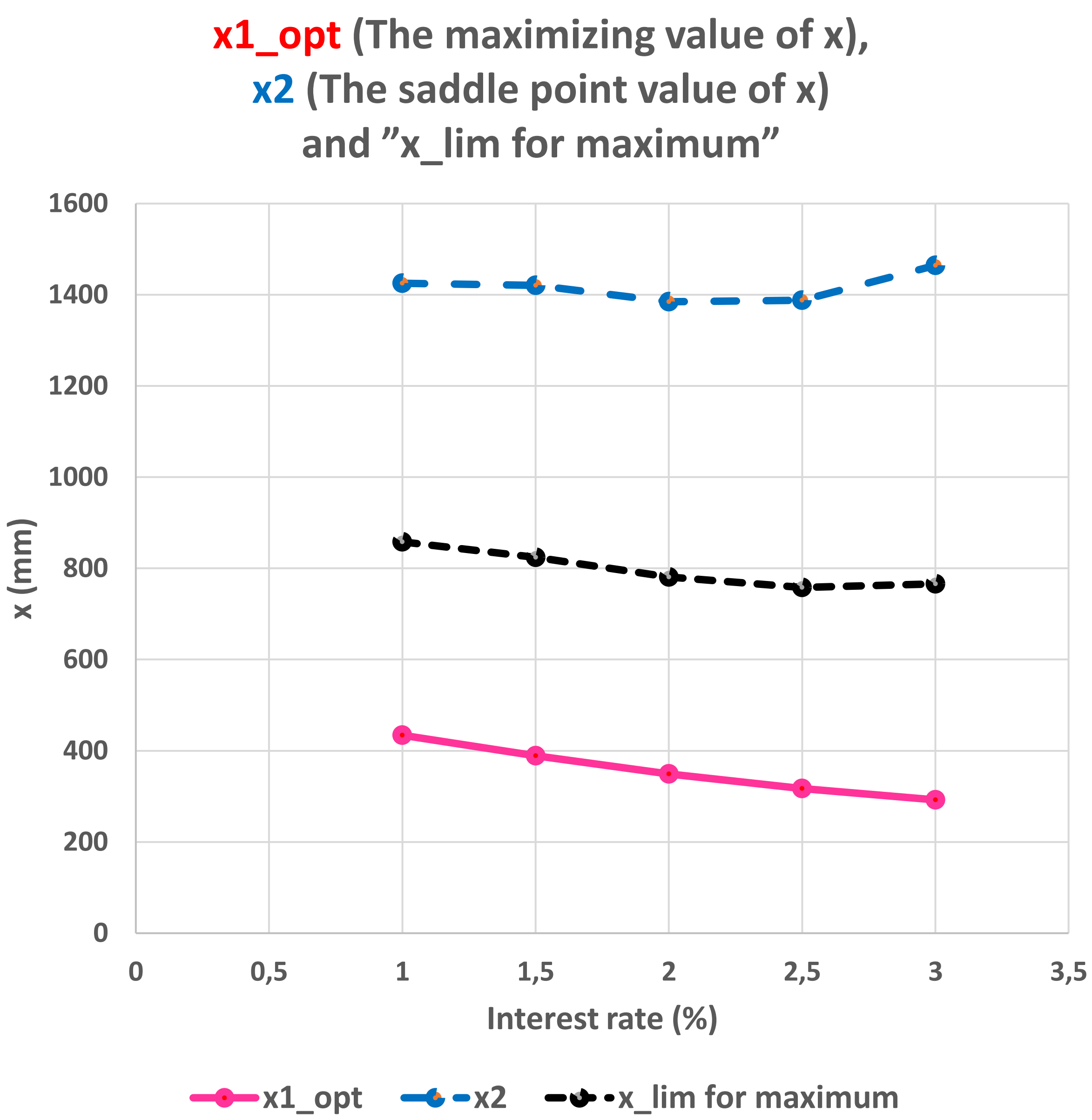

400000
z
100000
-10
y
0
500
x
300
r = 1%
r = 2%
r = 3%



Short movie #5. Click below.

Not empirically relevant

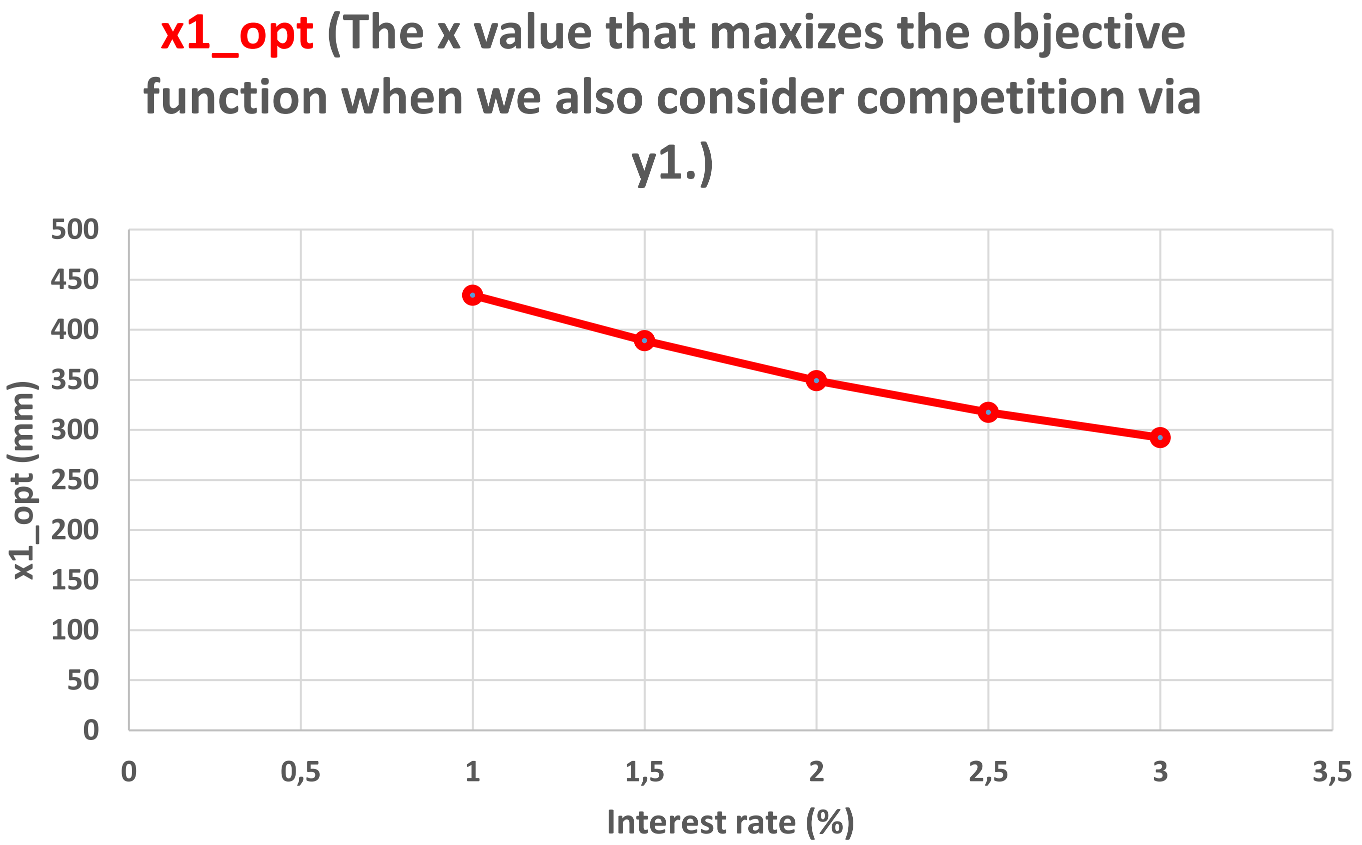
Empirically relevant



These x-values belong to the saddle point

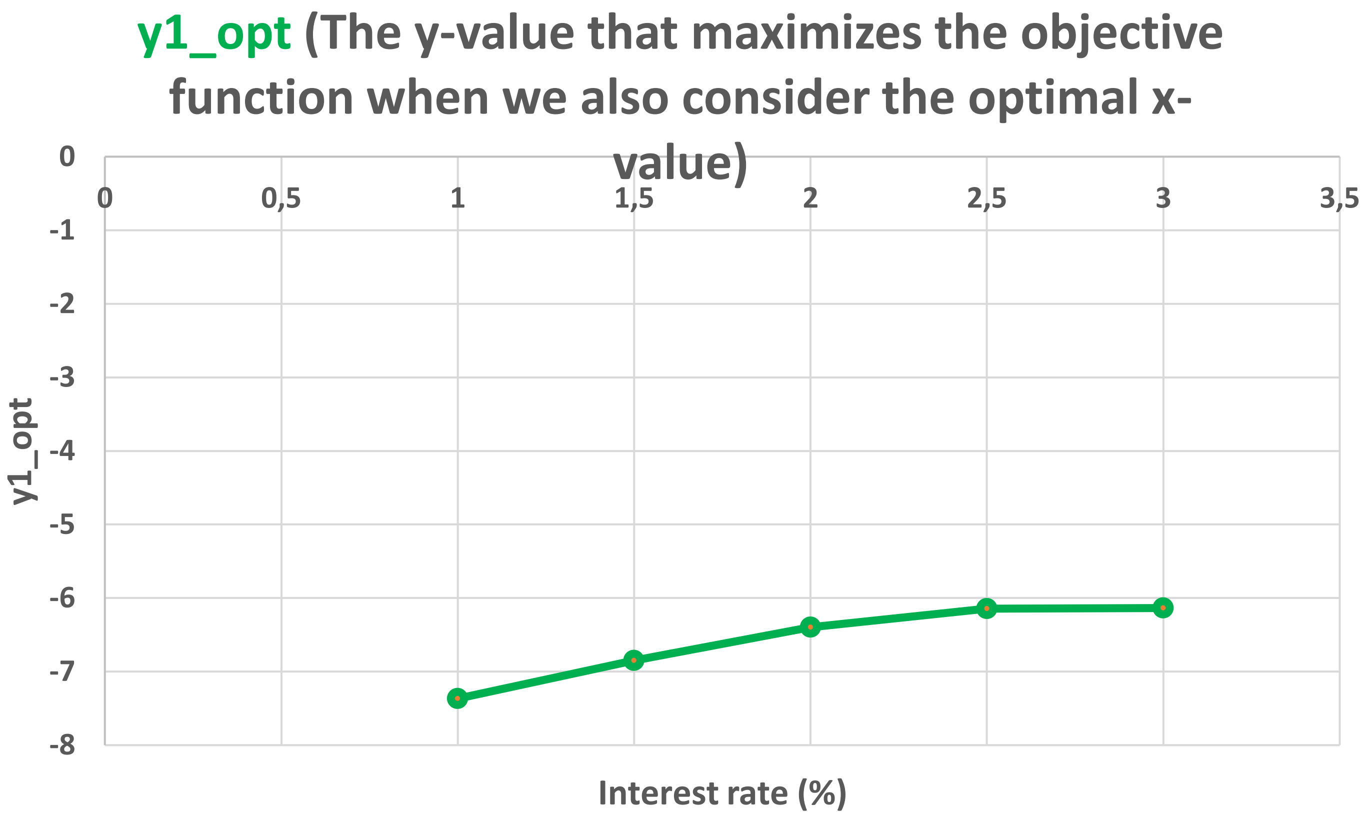
Maxima are only found below this line.

These x-values maximize The objective function



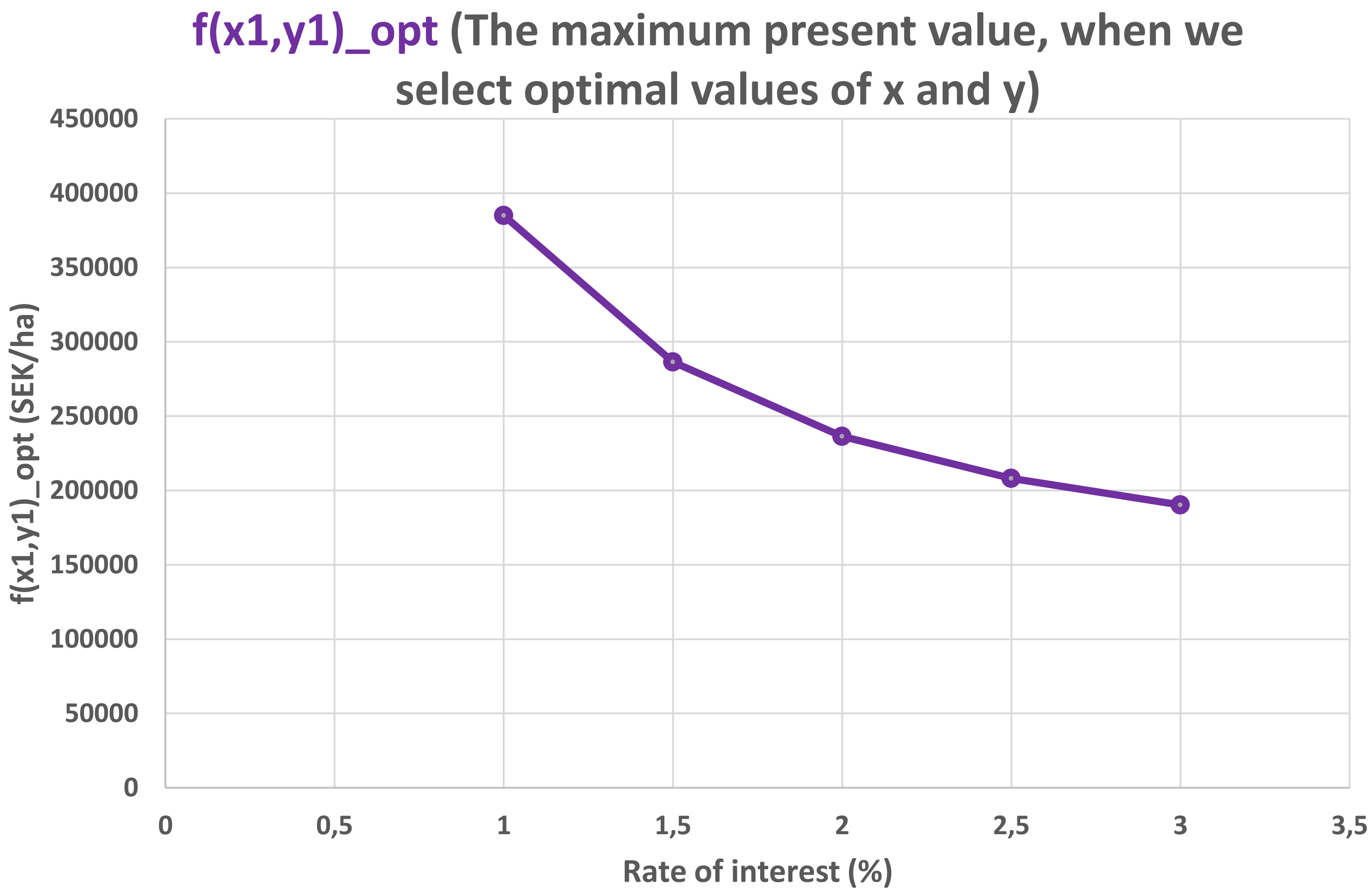
$$x_1 \approx 499 - 71.2\,r$$

	Koefficiente	Standardfe	t-kvot	p-värde
Konstant	498,8086	9,949824	50,1324	1,75E-05
r (%)	-71,2281	4,690392	-15,186	0,00062



$$y_1 \approx -7.85 + 0.633 r$$

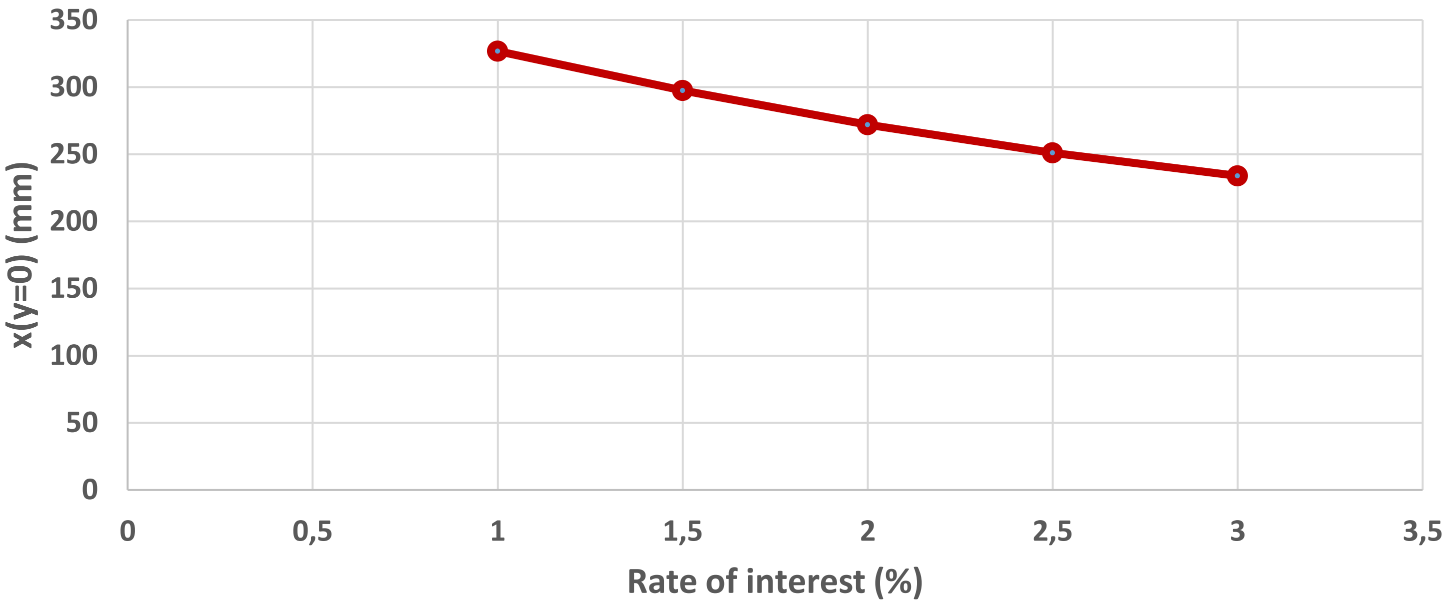
	Koefficiente	Standardfe	t-kvot	p-värde
Konstant	-7,84573	0,254978	-30,7702	7,54E-05
r (%)	0,633296	0,120198	5,268782	0,013326



$$f(x_1, y_1) \approx 90917 + 293641 r^{-1}$$

	Koefficiente	*tandardfe*	*t-kvot*	*p-värde*
Konstant	90917,78	1378,804	65,93959	7,69E-06
1/r	293641,2	2199,1	133,5279	9,26E-07

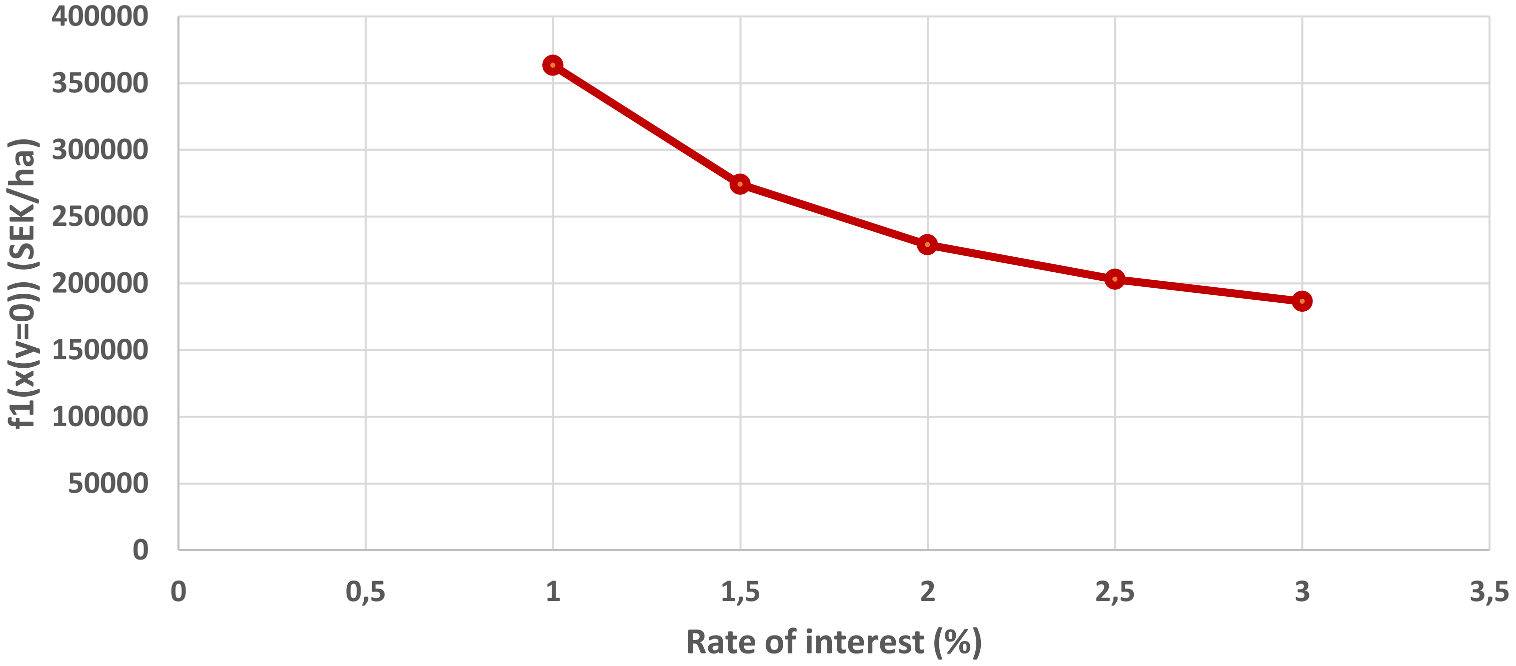
x(y=0) The value of x that maximizes the objective function in case we do not consider competition at all



$$x(y=0) \approx 369 - 46.5\,r$$

	Koefficiente	Standardfe	t-kvot	p-värde
Konstant	369,1978	5,884338	62,74246	8,92E-06
r (%)	-46,4871	2,773903	-16,7587	0,000463

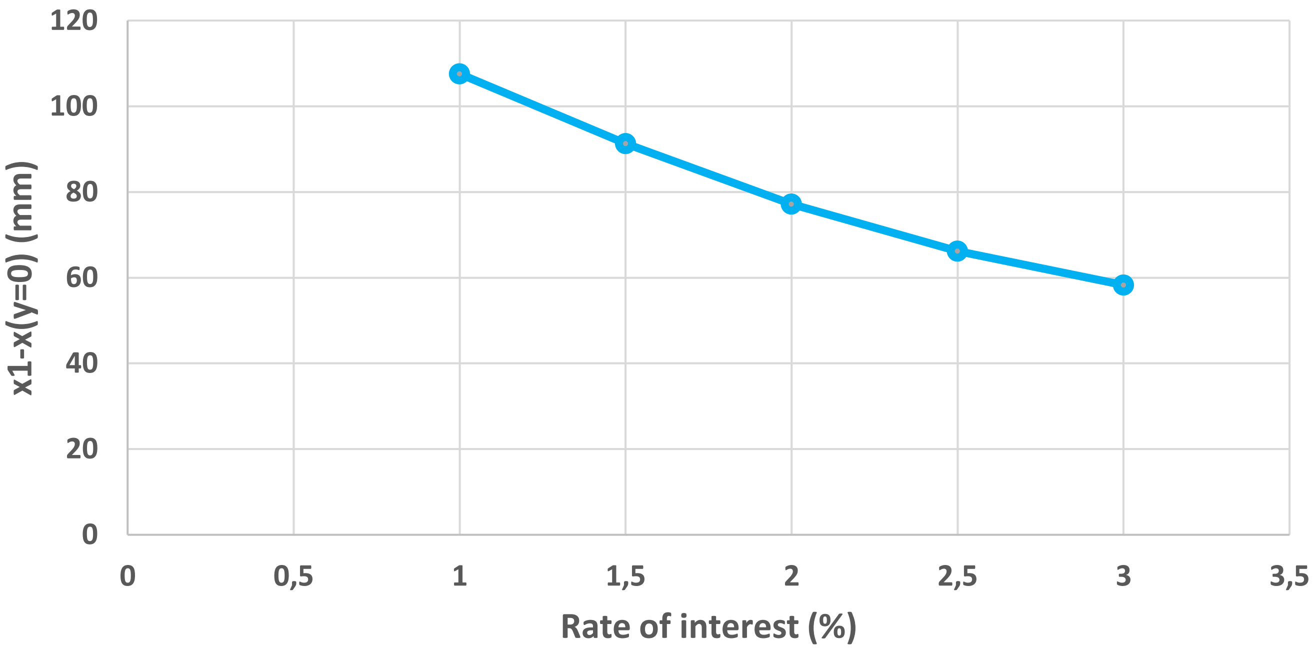
f1(x(y=0)) (The maximum present value, when we do not consider competition at all)



$$f_1(x(y=0)) \approx 96529 + 266415\, r^{-1}$$

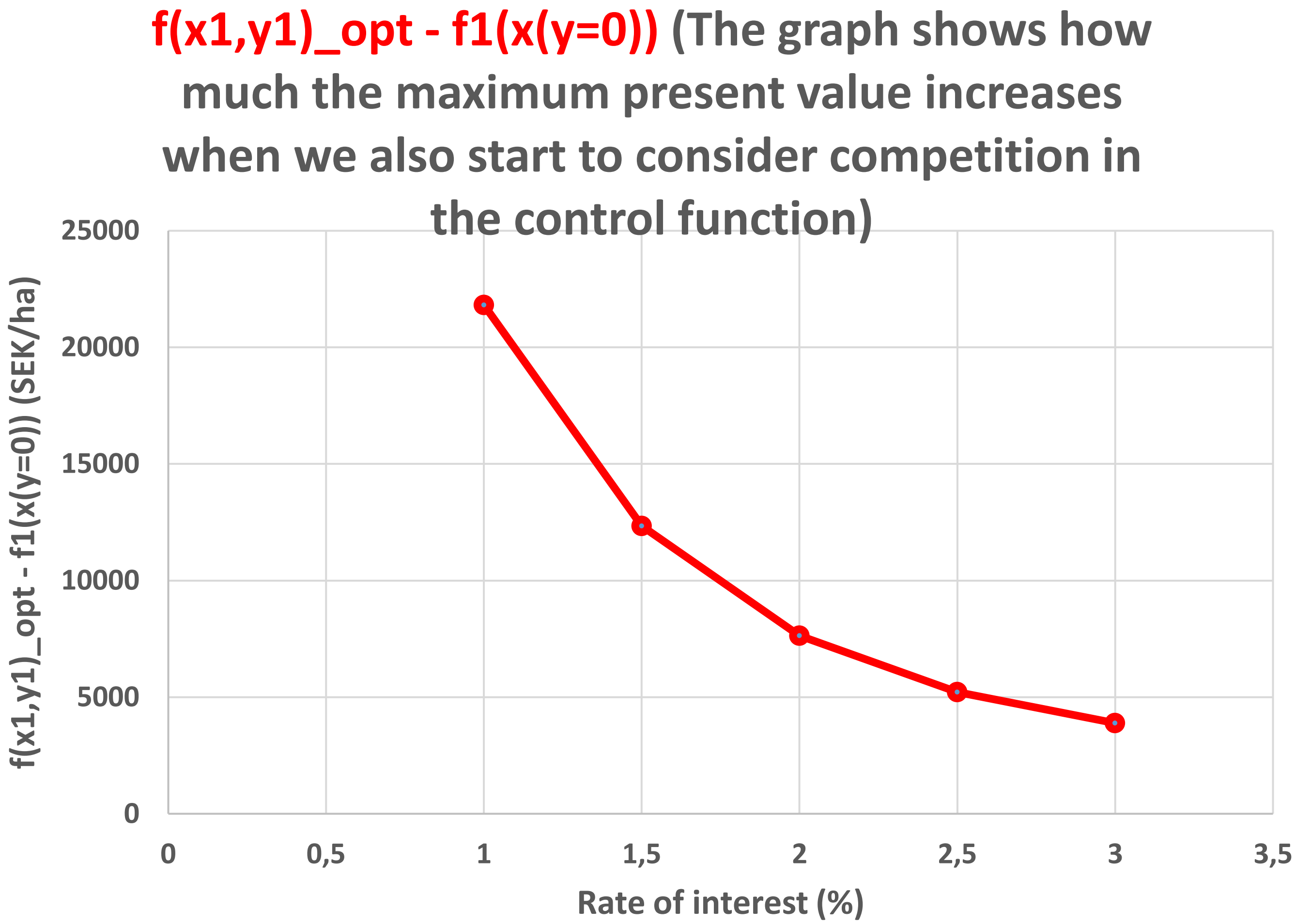
	Koefficiente	Standardfe	t-kvot	p-värde
Konstant	96528,92	962,91	100,2471	2,19E-06
1/r	266414,8	1535,777	173,4724	4,22E-07

x1-x(y=0) (The graph shows how much the optimal value of x increases when we also start to consider competition in the control function)



$$x_1 - x(y=0) \approx 130 - 24.7\,r$$

	Koefficiente	*Standardfe*	*t-kvot*	*p-värde*
Konstant	129,6108	4,068733	31,85532	6,8E-05
r (%)	-24,741	1,918019	-12,8993	0,001006



	Koefficiente	*Standardfe*	*t-kvot*	*p-värde*
Konstant	-5611,14	428,683	-13,0893	0,000963
1/r	27226,37	683,7206	39,8209	3,48E-05

$$f(x_1, y_1) - f_1(x(y=0)) \approx -5611 + 27226 r^{-1}$$

Conclusions

- A control function based on the size of the tree and the local competition can be used to optimize continuous cover forestry.
- The mathematical principles and statistical methods of determination of the optimal parameters of such control functions have been presented.
- Numerical values of optimal control function parameters of relevance to a particular case study forest with the species picea Abies have been determined.
- Generalized versions of the method can be used to optimize management of multi species forests, also with stochastic prices and adaptive decisions, as shown by Lohmander [1], [3] and [4].
- The readers are encouraged to use the methodology to optimize management of continuous cover forestry in all regions of the world.

THANK YOU VERY MUCH FOR YOUR TIME AND A MOST INTERESTING CONFERENCE!