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THE MULTI SPECIES FOREST STAND, STOCHASTIC PRICES AND ADAPTIVE SELECTIVE THINNING

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It is shown that and how it is possible to benefit from flexible initial investments and stochastic changes in Martingale prices in sequential information and decision problems that are typical in forestry. At $T0$, several species are planted in a mixed forest stand. At $T1$ ($T1 > T0$), the prices of the different species are observed and one species is selected for continued production. A better selection can be made than earlier. The final harvest takes place at $T2$ ($T2 > T1$). The expected price at $T2$ is the price at $T1$. However, also here the price variability and the time difference $T2 - T1$ increase the expected profit thanks to the Jensen Inequality. If prices increase, this increases the profit but the profit does not decrease by net price drops below zero. We may stop harvesting in that case and the minimum profit is zero.

KEY WORDS Forest modelling, decisions support in forestry.

1. INTRODUCTION, GENERAL QUESTIONS

—Is it possible to increase the expected profit by replacing decisions based on deterministic models by adaptive decisions in the presence of stochastic prices that are Martingales?
—How is the expected profit affected by the level of flexibility determined by the properties of the initial investment?
—Do the planning horizon and the initial states of the stochastic processes matter?

These questions will be addressed and answered via the analysis of an adaptive multi period information and decision model with Martingale price processes. The principles and qualitative results will be discussed via a general model. Finally, numerical answers relevant to multi species forestry are derived and presented. The stochastic properties of real pine pulpwood export prices and real birch pulpwood export prices are determined and discussed.

The problems discussed in this paper are in several ways very different from other stochastic resource management problems that have been studied through adaptive optimization. Most articles on adaptive resource management that are found in the literature are devoted to the following issues:

i. Continuous extraction of a homogeneous resource stock with or without growth. The growth and/or the price is a stochastic process. Gleit [4], Kaya and Buongiorno [7], Lohmander [9, 11, 14, 21], Pindyck [28].

ii. Pulse extraction of a homogeneous resource stock with or without growth. The growth and/or the price is a stochastic process. Brazee and Mendelsohn [1], Johansson and Löfgren [8], Lohmander [9, 12], Norstrom [27], Risvand [30].

The first class, i., finds the theoretical foundations in the stochastic optimal control literature. The second class, ii., consists of applications of optimal stopping theory. In most cases, it is also important to understand the behaviour of the stochastic processes and how to empirically estimate such. In recent years, new tools have been found in the pure and applied chaos theory, a good example of which is May, Beddington, Harwood and Shepherd [24]. Among the introduced classes, we should mention:

iii. Probability, random processes and estimation. Ito and McKean [6], Grimmet and Stirzaker [5], Malliaris and Brock [23], Pindyck and Rubinfeld [29]
iv. Optimal statistical decisions. DeGroot [2], Fleming and Rishel [3]

In the papers in classes i. and ii., the harvest (extraction) levels have been optimized and the resource has been completely described through the stock size or the age distribution. In this paper, the structure of the problem is quite different. The species composition is a temporal decision variable and the prices of the different species are described as stochastic processes. Hence, the methodology used and the normative results have not been possible to find in the literature. Still, the author is convinced that the reader will find the results reasonable in applied forestry planning situations.

1.1. Results from the Theory of Finance

It is a well known and seldom questioned result that one can not gain an expected profit from adaptive behaviour in the financial markets if the prices are in fact Martingales. This idea can be found in Samuelson [32] and the title of the article "Proof that properly anticipated prices fluctuate randomly" reveals the content. If a stochastic process P is a Martingale in discrete time this means that the expected value of P at time $t+1$ is equal to the value of P at time t.

The idea presented by Samuelson [32] is the following: Assume that we own one unit of something called X in period t, the physical properties of which do not change over time. If it could be calculated, via past information including the price P (available at t), that the expected price of X is higher in period $t+1$, then we should not sell X in period t. In order to sell X already in period t, we would like the price in period t to be at least as high as the calculated expected price in $t+1$. (Note that all problems associated with risk aversion etc. have been ignored.) In other words: If we expect the price to increase, it has already increased.

In the same way: If the expected price in period $t+1$ is lower than in period t, then we should sell X already in period t even if the price in period t would have been as low as the expected price in period $t+1$: If we expect the price to decrease, it has already decreased.

Ever since Samuelson wrote his famous article, the Martingale price hypothesis (with minor modifications because of the rate of interest, inflation etc.) has dominated the stock market theory. If prices have not been Martingales, the market has been said not to be "fair".

Clearly, if prices really are Martingales, we do not know if we should buy or sell today or tomorrow in the pure stock market situation discussed by Samuelson. However, the usual "intuitive" interpretation that we can not benefit from adaptive decision making in the presence of Martingale prices is not correct in the more interesting but very common multi stage information and decision problem to be discussed below.

1.2. Two Reasons why we may Benefit from Stochastic Martingales

In fact, there are two distinct reasons why we may benefit from adaptive decision making in the presence of stochastic Martingale prices in temporal production problems.

—We may, via a flexible initial investment, delay the final production decision. This way, we have time to observe the development of the product prices. In a late stage of production, we can select the product that turned out to have the most positive price development. The higher the price variability, the more important is the flexibility obtained via the initial investment.

—The profit is a kinked convex function of price: If it turns out that the final net price (price minus variable production costs) falls below zero, further price decreases will not decrease the profit, since we may stop production (harvesting). Hence, via the Jensen inequality, we easily find that the expected profit increases with the price risk (proportional to standard deviation in a Normal distribution). When we have Martingale prices, the risk increases with the standard deviation of the stochastic change (positive or negative "increment") each period and the number of periods until the final point in time when the product should be sold (and/or, in forestry problems, harvested).

The concrete forestry example is the following: We plant several species in the initial stand (which represents a flexible investment). Several decades later we select one species for continued production using information concerning the latest prices of the different species. Hence, we gain from stochastic variability in the Martingale prices until the selection time. We also gain from variability in the final stage of production thanks to the "Jensen inequality effect" discussed above.

2. QUALITATIVE ANALYSIS

In this section, a qualitative analysis will be made of a model which is simple but still sufficiently complex to give results that are relevant answers to the questions raised in this paper. The main ambition is to present the general principles to the reader in the most simple and transparent form. The presented qualitative graphs will make it possible for the reader to investigate the relevance and applicability of the principles.

2.1. General Problem Structure

Figure 1 illustrates the structure of the multi period information and decision problem: PAt and PBt denote the net prices (price-variable harvest and

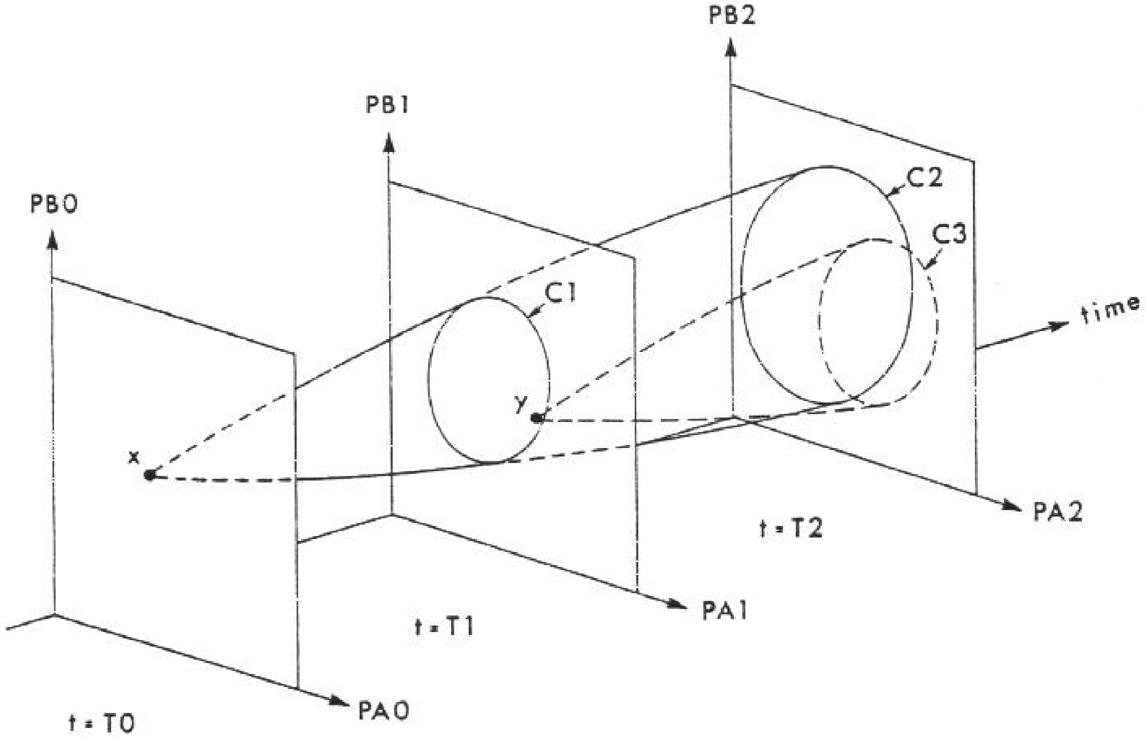

Figure 1 In forestry, we should not select only one species already at the time of plantation, at *T*0. We should have many different species in the first stage of production. Then, later on, at *T*1, we can make a better prediction of future prices and may perform a selective adaptive thinning based on the latest price information. The figure illustrates a typical situation: *PAt* and *PBt* denote the deflated net prices (price-variable costs in harvesting and transportation) of the species *A* and *B*. If the state at *T*0 is *x*, then the one standard deviation boundary of the prediction of future states are *C*1 and *C*2 in the periods *T*1 and *T*2 respectively. If we make a new prediction at *T*2 based on the revealed and observed state *y*, then the prediction of the state at *T*2 is quite different. The one standard deviation boundary of the prediction in period *T*2 is then *C*3.

transportation costs) of species *A* and *B* in period *t*. In order to simplify notation, we denote PAt and PBt "prices". The two dimensional state space, with the dimensions PAt and PBt, is shown for $t = T0$, *T*1 and *T*2. PAt and PBt are stochastic processes. Later on, we will discuss the stochastic properties of these in more detail. An *initial* assumption is that they are two (possibly correlated) Martingale processes. At *T*0, we do not know the future behavior of the prices. However, using information concerning the stochastic properties of the processes, we may at *T*0 estimate the probability density functions of the state in the future periods *T*1 and *T*2. In case the processes are in fact Martingales, the figure shows the typical development of the boundary (the circles *C*1 and *C*2) representing a fix number of standard deviations from the expected value. Most importantly, when we are dealing with Martingale processes, the best prediction of the future value of the process is the latest observed value of the process (at the time of prediction). The expected error of the prediction increases with the prediction horizon. Note that the radius of *C*2 is larger than that of *C*1.

2.2. Selection of species at T0, the "stiff approach"

We have to select what species (only one) to place in the forest stand at *T*0. Then, if trees of the two possible species *A* and *B* have the same growth

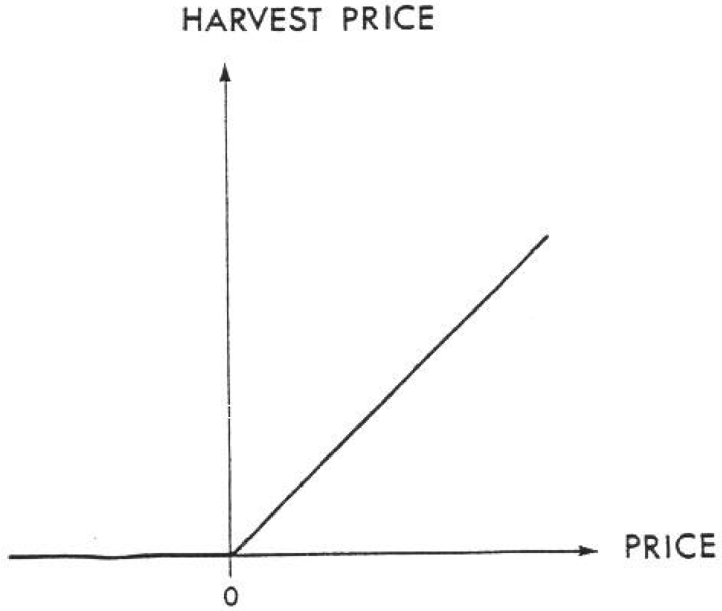

Figure 2 The "harvest price" is defined as the maximum of the (net) price at $T2$ and zero. If the price is below zero, it is possible to stop harvesting and the profit is the same as if the price would have been zero.

development and have the same properties in every sense except that the prices are PAt and PBt, then the optimal selection should be based on the estimated prices at the time of final harvest, $T2$. $T2$ is assumed to be fixed in this discussion but we *may* benefit from adaptive behaviour. If the price happens to be negative at $T2$, we do not have to harvest at all. Hence, the "price" is bounded from below by zero. The "harvest price" is defined as the maximum of zero and the price at $T2$. This is shown in Figure 2. The harvest price is a kinked convex function of the price at $T2$.

Figure 3 shows how the expected harvest price is affected by increasing risk in the price distribution at $T2$. If the expected price is close to zero, $a0$, then increasing risk implies that the expected harvest price increases. $H(a0, a0) < H(a1, a2) < H(a3, a4)$. This is a typical result in forestry in remote areas. If the expected price is strongly positive, $b0$, then the expected harvest price is the same before and after increasing price risk in the illustration. $H(b0, b0) = H(b1, b2) = H(b3, b4)$. If the stand is located far away from the mills, this means that the transportation cost is high and that the expected net price, "price", may be close to zero. However, since we have the option to stop harvesting in case price is below zero, we benefit from price variability. Only the positive deviations from zero affect the expected harvest price. This observation is closely related to observations by McDonald and Siegel [25, 26].

Furthermore, if we have Martingale prices, the expected deviation from the expected value increases with the prediction horizon. In fact, it turns out that, thanks to the central limit theorem, the probability distribution of the future state approaches the normal distribution as the number of periods increases irrespective of the shape of the probability distributions of the individual increments. The variance is proportional to the prediction horizon and the standard deviation is

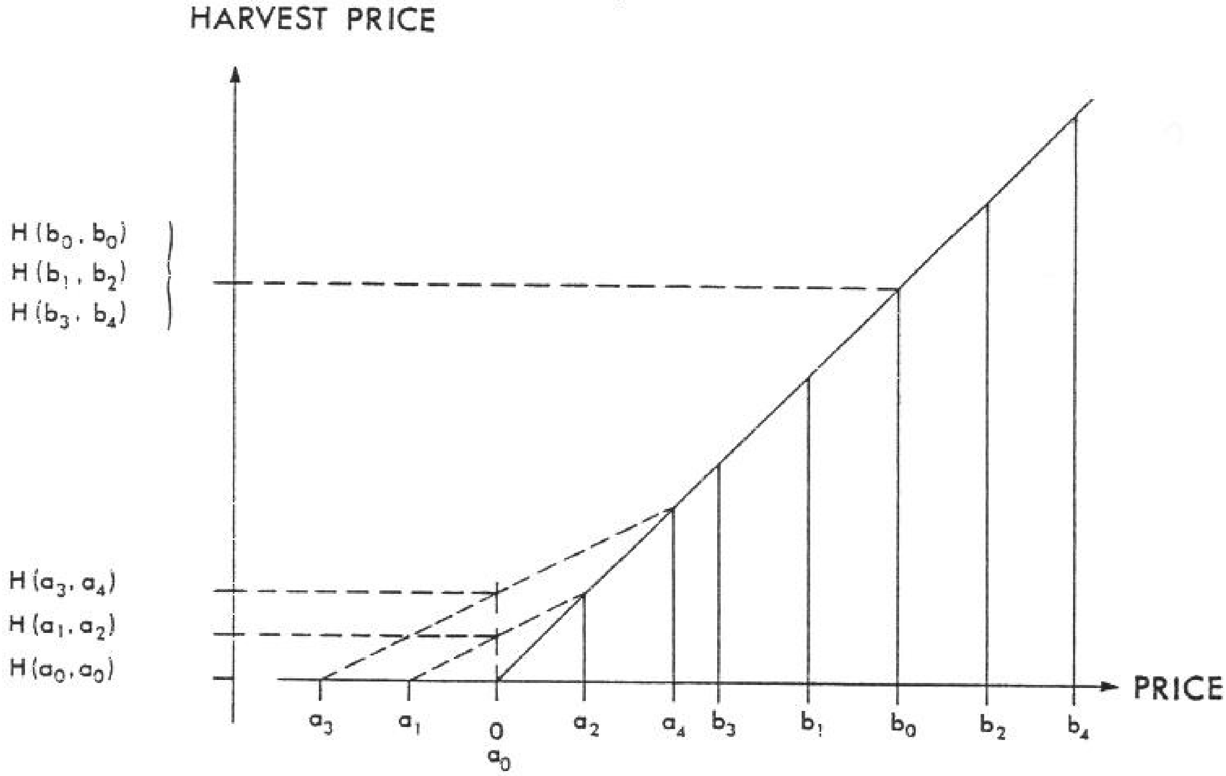

Figure 3 The figure shows how the expected harvest price is affected by increasing risk in the price probability distribution at *T*2 for different levels of the expected price (*a*0 and *b*0). We benefit from increasing prices when the price is above zero. We are not affected by decreasing prices when the price is below zero. $H(x, y)$ is the expected harvest price when the price takes the values x and y with 50% probability each. When the expected price is close to zero, *a*0, then we benefit from increasing risk. When the expected price is high, *b*0, the risk is less important (of no importance in the figure) to the expected harvest price.

proportional to the square root of the prediction horizon. This is shown in Figure 4. Compare the two dimensional case shown in Figure 1.

2.3. The Jensen Inequality Effect

We end up with the following important conclusion, the "Jensen inequality effect": Since the harvest price is a kinked and convex function of price and the risk in the price distribution increases with the prediction horizon, the expected harvest price increases with *T*2. This effect is stronger in cases where the expected price is close to zero than otherwise.

2.4. The role of thinning in the "stiff" problem

We may assume that the initial number of plants per area unit in the stand is reduced by 50% at *T*1 via a thinning. The thinning costs are usually very high compared to the revenues. Hence, we may simply assume that the net thinning profit is zero. This is a common situation at least in Swedish forestry. Nevertheless, it may very well be economically justified to have more stems in the forest in the initial phase of production than in the end because the quality of the

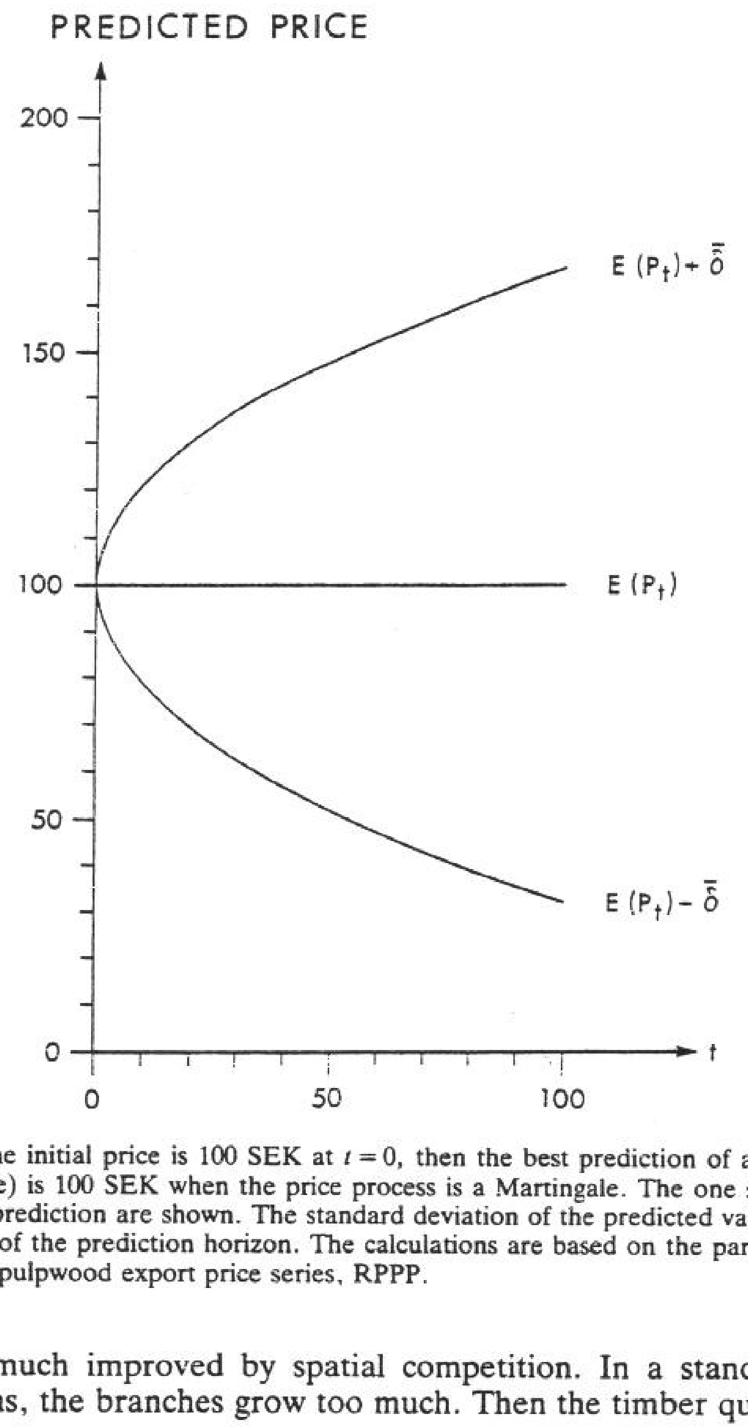

Figure 4 When the initial price is 100 SEK at $t=0$, then the best prediction of a future value (the true expected value) is 100 SEK when the price process is a Martingale. The one standard deviation boundaries of the prediction are shown. The standard deviation of the predicted value is proportional to the square root of the prediction horizon. The calculations are based on the parameters estimated from the real pine pulpwood export price series, RPPP.

stems is very much improved by spatial competition. In a stand with a small number of stems, the branches grow too much. Then the timber quality and price decrease.

2.5. The Flexible Initial Investment and Selective Thinning

Now, we turn to a two stage information and decision problem. At $T0$ we place two different species in the stand, A and B. The number of stems per area unit is the same as in the stiff case before thinning. At $T1$, 50% of the stems are taken away via *selective* (adaptive) thinning. The species chosen for continued production is selected via the rule illustrated in Figure 5a. The idea is the following:

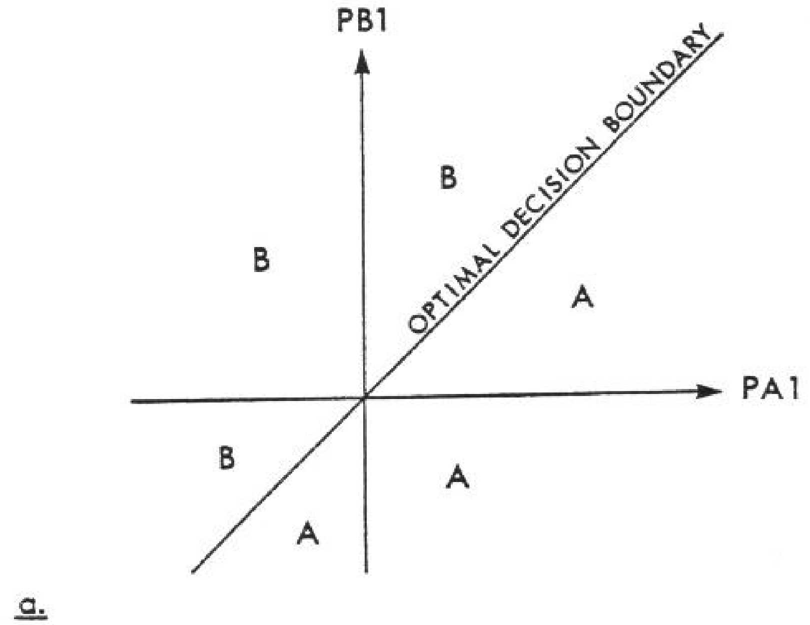

Figure 5a The optimal adaptive decision at $T1$ should be based on the revealed and observed state in the $(PA1, PB1)$ space at $T1$. Species A or species B should be selected for continued production according to the figure.

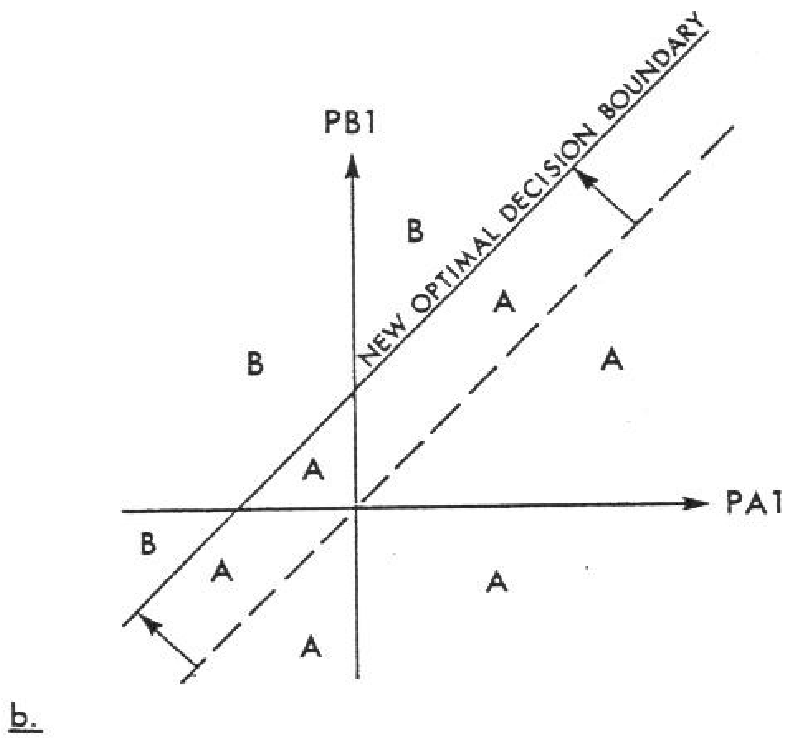

Figure 5b When the parameters of the different species are not the same, then the optimal decision boundary is generally not the one shown in Figure 5a. Compare the main text.

When the state at $T1$ has been revealed and observed at $T1$, the expected profitability of continued production is calculated for each species alternative.

If the species have the same properties except for the prices and if the two stochastic price processes have the same stochastic properties, then the species that happens to have the highest price at $T1$ is the species that should be selected. Of course, these calculations are affected by the "Jensen inequality effects" discussed earlier. The time horizon of (continued and final) prediction is now equal to $T2 - T1$.

However, if species A has a "more stochastic" price process (the variance of the increments is higher) than species B, then the "Jensen inequality effect" improves the expected profitability of species A compared to species B. Figure 5b shows a possible optimal selection rule in this case. Figure 5b is relevant also in case the two species have price processes with the same stochastic properties but the final harvest year of species A occurs later than the corresponding age of species B. This is the case since the prediction horizon increases the expected harvest price thanks to the "Jensen inequality effect".

3. QUANTITATIVE ANALYSIS

Now, we will calculate the expected harvest price that is obtained when the optimal adaptive selective thinning rule is used. We assume that two species are planted at $T0$ in a mixed forest stand. The first step is to investigate the stochastic properties of the prices of two typical forest species in Swedish forestry, namely

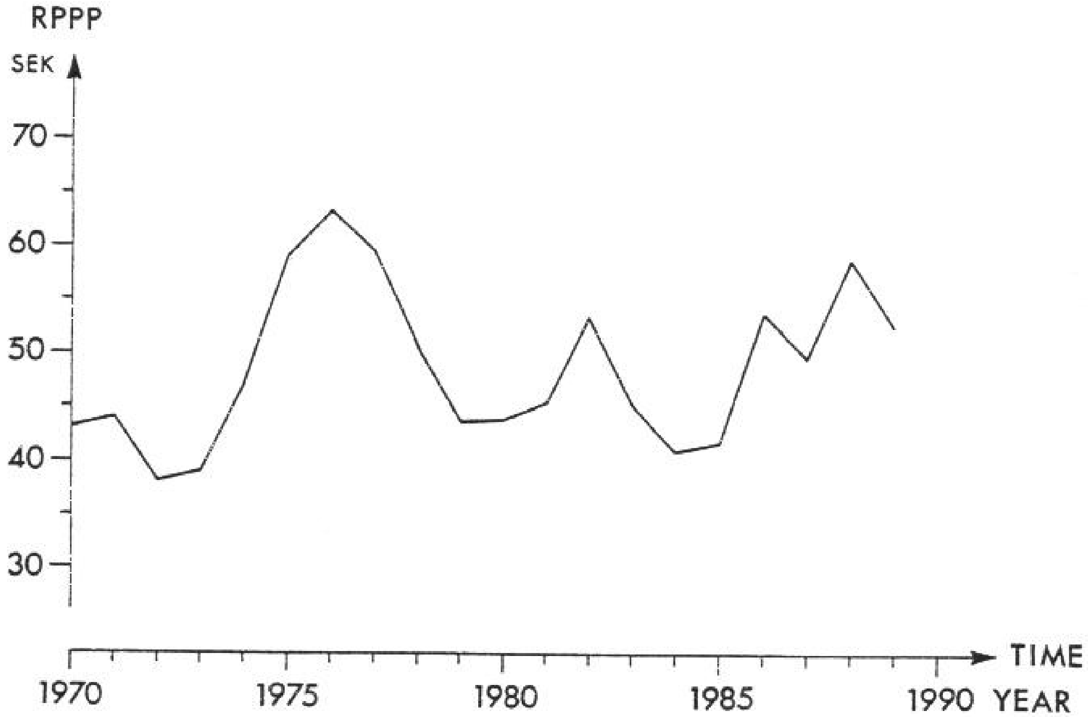

Figure 6 The Swedish real pine pulpwood export price series RPPP. (The export value divided by the export quantity, deflated by the consumer price index.) Source: Skogsstyrelsen. All data are found in the empirical appendix.

pine and birch. These species, and closely related species with similar properties, are common in most forest producing countries in the north. Both species can most often grow on the same land. Moreover, the species are very different with respect to diseases: Normally, a particular disease will only attack one of the species. Hence, a mixed plantation or naturally regenerated forest stand will also give us the important option to survive species specific diseases. Related issues have been discussed by Lohmander (20).

3.1. The Prices of Pulpwood Based on Pine and Birch

Figure 6 shows the time path of RPPP, the real (deflated by consumer price index) export price of pine pulpwood, from 1970 to 1989. The series has been obtained via the official statistics from Skogsstyrelsen. The "raw" data consist of the export value series, the export volume series and the consumer price index series. These can be found in the empirical appendix.

Figure 7 shows the time path of RPHP, the real export price of hardwood (mainly birch) pulpwood. The series has been derived in a way similar to the series RPPP. All raw data are included in the empirical appendix.

If the prices of the two species do not follow each other over time, then it could be interesting to invest in a mixed species stand. Figure 8 illustrates the time path of DIFFPMH, the difference RPPP–RPHP. Obviously, the difference between the prices of the two species changes dramatically over time. In 1977, RPPP was 20 SEK higher than RPHP and in 1985, RPPP was 4 SEK lower than RPHP. We should be aware that the change in the price difference over time is very high compared to the absolute levels of the two prices. Compare Figures 6 and 7.

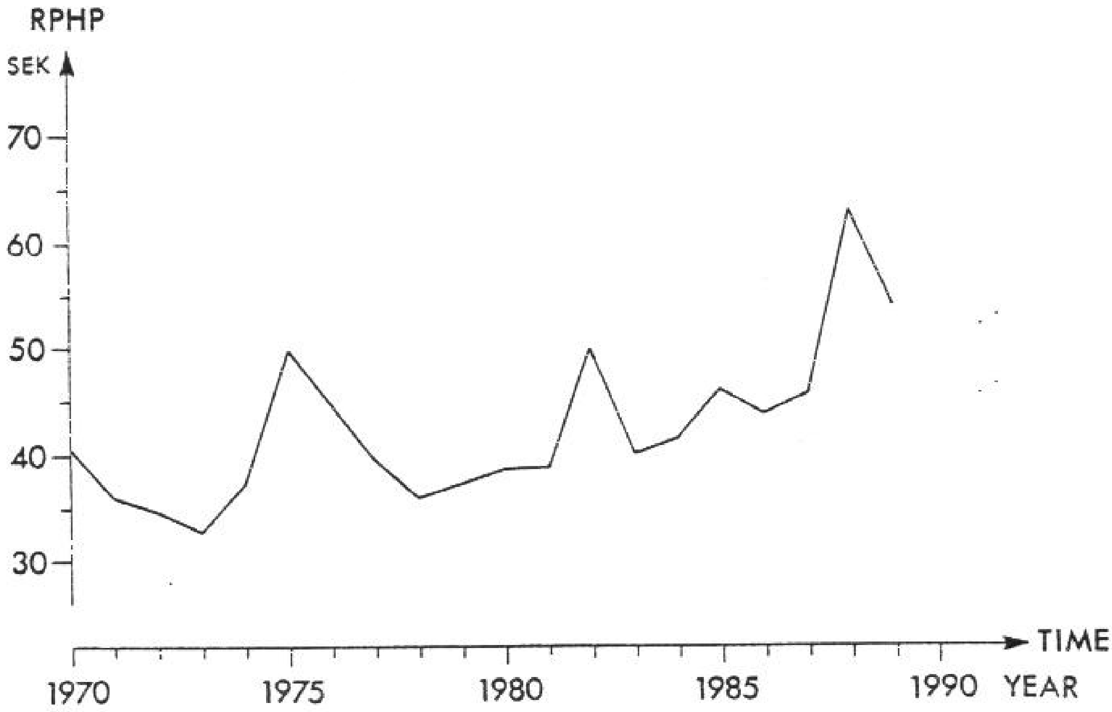

Figure 7 The Swedish real hardwood pulpwood export price series RPHP. (The export value divided by the export quantity, deflated by the consumer price index.) Source: Skogsstyrelsen. All data are found in the empirical appendix.

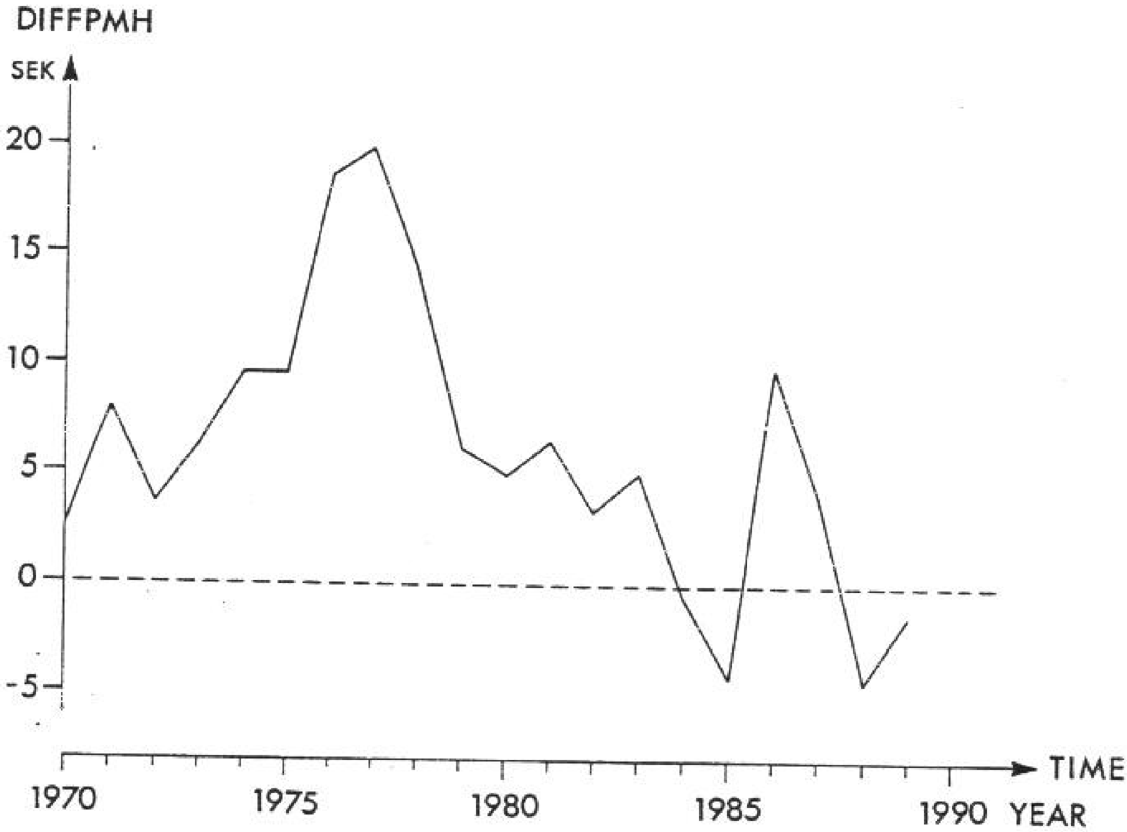

Figure 8 The price difference process DIFFPMH (= RPPP − RPHP). Compare Figure 6 and Figure 7

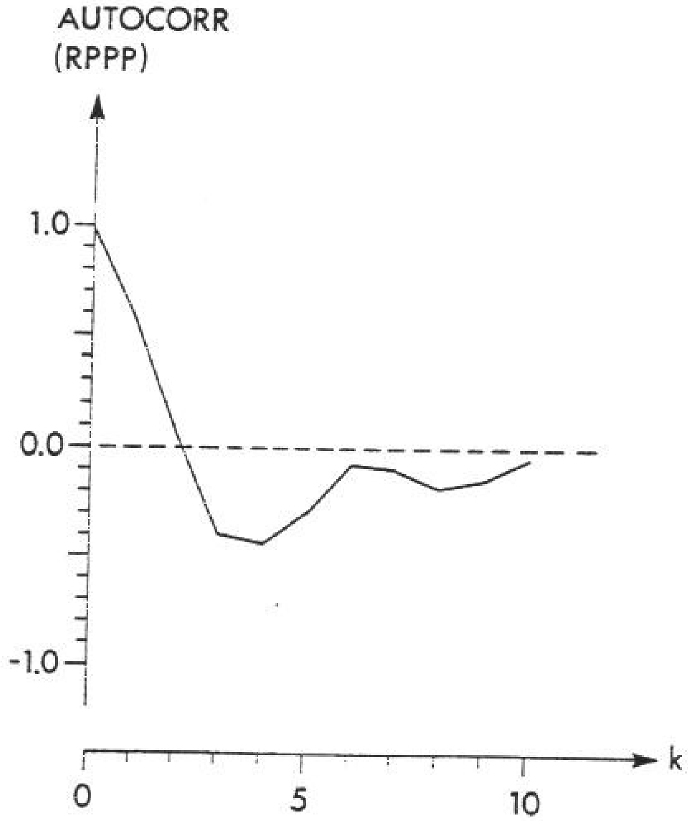

Figure 9 The sample autocorrelation function of RPPP

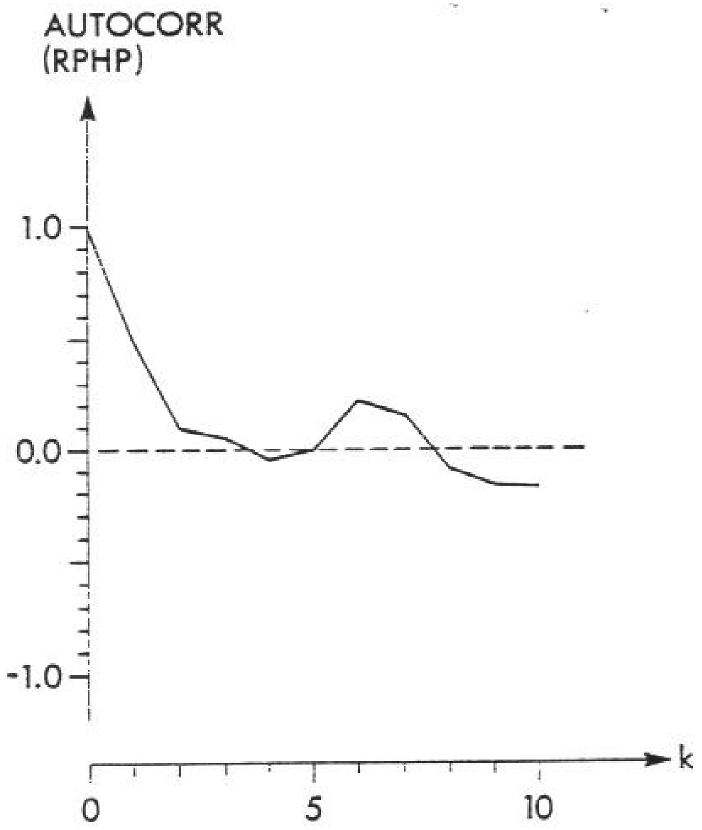

Figure 10 The sample autocorrelation function of RPHP

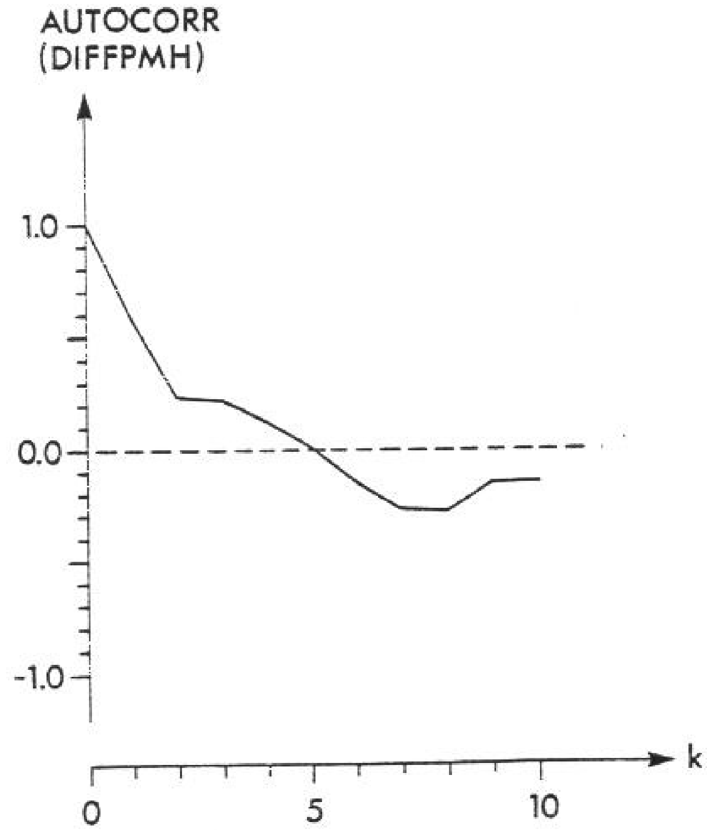

Figure 11 The sample autocorrelation function of DIFFPMH

Furthermore, the prices RPPP and RPHP should be reduced by the variable harvesting and transportation costs when we look at the profitability of harvesting the different species. Clearly, the variability over time in the difference between the prices of the two species is very high compared to the net prices (price-variable costs in harvesting and transportation). This observation is of particular economical importance in forest areas far away from the buyers and where the terrain conditions are difficult from a harvesting point of view.

3.2. *Stochastic Properties of Species Dependent Pulpwood Prices*

Now, it is important to find a model that fits the processes RPPP, RPHP and DIFFPMH. The first and most natural hypothesis, supported by the above discussion, is that the prices are Martingales. A Martingale is a nonstationary process and the autocorrelation does not approach zero as the number of lags approaches infinity. If, on the other hand, the series are stationary, then the autocorrelation should approach zero as the number of lags approaches infinity. Compare Pindyck and Rubinfeld [29].

The Figures 9, 10 and 11 show the sample autocorrelation functions of the three processes RPPP, RPHP and DIFFPMH respectively. Clearly, the sample

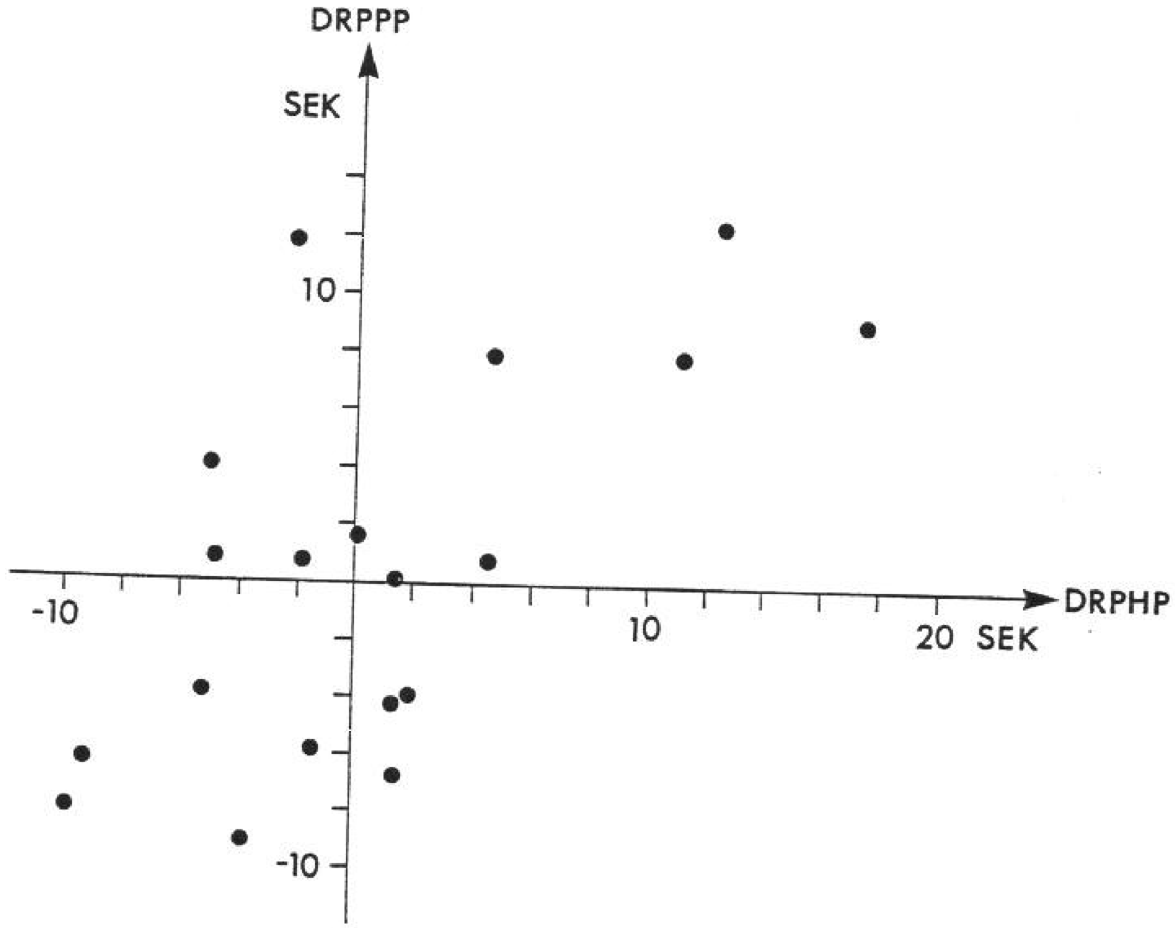

Figure 12 The first difference of RPPP, (=DRPPP), plotted against the first difference of RPHP, (=DRPHP). The variances of DRPPP and DRPHP are 46.37 and 50.08 respectively. DRPPP and DRPHP show positive correlation. The linear correlation coefficient $R \approx 0.643$ and $R2 \approx 0.414$

autocorrelation seems to decrease as the number of lags increases in all cases. However, the number of available observations (years) is low. Hence, we should not reject the hypothesis of Martingale processes on these grounds. In other words, if the number of observations is small, the probability is high that we obtain sample autocorrelation functions that indicate stationarity also when the stochastic processes are in reality Martingales. Since we have no theoretical reason to expect that the processes are not Martingales, we continue to assume Martingale processes. As time passes, in case the same kinds of forest products will be sold in the future (which is not at all clear!), the number of observations increases. Then, more interesting statistical hypothesis tests could be performed. We should be aware, however, that the types of forest products that are sold in the markets today are not the same as the types sold a century ago. Maybe we will never get a statistically "satisfactory" number of observations? Maybe, we will have to use more general product definitions in order to get longer price series? Then, which is the best way to define a forest product?

Some estimations of the stochastic properties of the processes will be needed to support the numerical analysis. Let us denote the increments, or the first differences, of the three stochastic processes RPPP, RPHP and DIFFPMH by DRPPP, DRPHP and DDIFF in the same order. The means of DRPPP, DRPHP and DDIFF are not statistically different from zero and the variances are 46.37, 50.08 and 34.45 respectively. DRPPP and DRPHP are correlated: R2 was found to be 41.4%. This seems reasonable in the light of Figure 12.

3.3. A Numerical Adaptive Selective Thinning Model

In order to estimate the expected harvest price which is obtained via optimal adaptive behaviour in the mixed species stand, a numerical model has been designed. The expected harvest price, $\Gamma(.)$, is a function of the prices at $T0$, $PA0$ and $PB0$, and is calculated from (1).

$$\Gamma(PA0, PB0) = \iint \Omega(PA1, PB1) f(PA1, PB1 \mid PA0, PB0)\, \mathrm{d}PB1\, \mathrm{d}PA1 \quad (1)$$

$f(.)$ is the two dimensional conditional probability density function of the prices at the point in time when the selective thinning takes place, $T1$. $f(.)$ is calculated as an approximation of the two dimensional normal distribution according to the method found in Råde and Westergren [31]. The reference also contains the formulae needed in the derivation of probability density functions of higher dimensionality. This is of particular interest when more than two species are planted in the same stand. $\Omega(.)$ denotes the expected harvest price as a function of the revealed price state at $T1$ and optimal selective choice of species for continued production until $T2$.

$$\Omega(.) = \max\left(\int S(PA2) g(PA2 \mid PA1)\, \mathrm{d}PA2,\ \int S(PB2) h(PB2 \mid PB1)\, \mathrm{d}PB2\right) \quad (2)$$

$g(.)$ and $h(.)$ denote the conditional probability density functions of the prices at the point in time when final harvest takes place, $T2$. $S(.)$ is the "harvest price function", $S(P) = \max(0, P)$, which is also found in Figure 2. (P denotes $PA2$ or $PB2$.)

3.4. *The Expected Value of a Mixed Species Stand*

As the standard deviations of the increments of the price processes increase, the expected value of the harvest price conditional on optimal adaptive species selection increases. This is shown in Figure 13. The standard deviations of the increments are proportional to k in the figure. The results presented are based on the estimated processes when species A represents pine and species B birch. Selective adaptive thinning and final harvest take place 70 years and 120 years

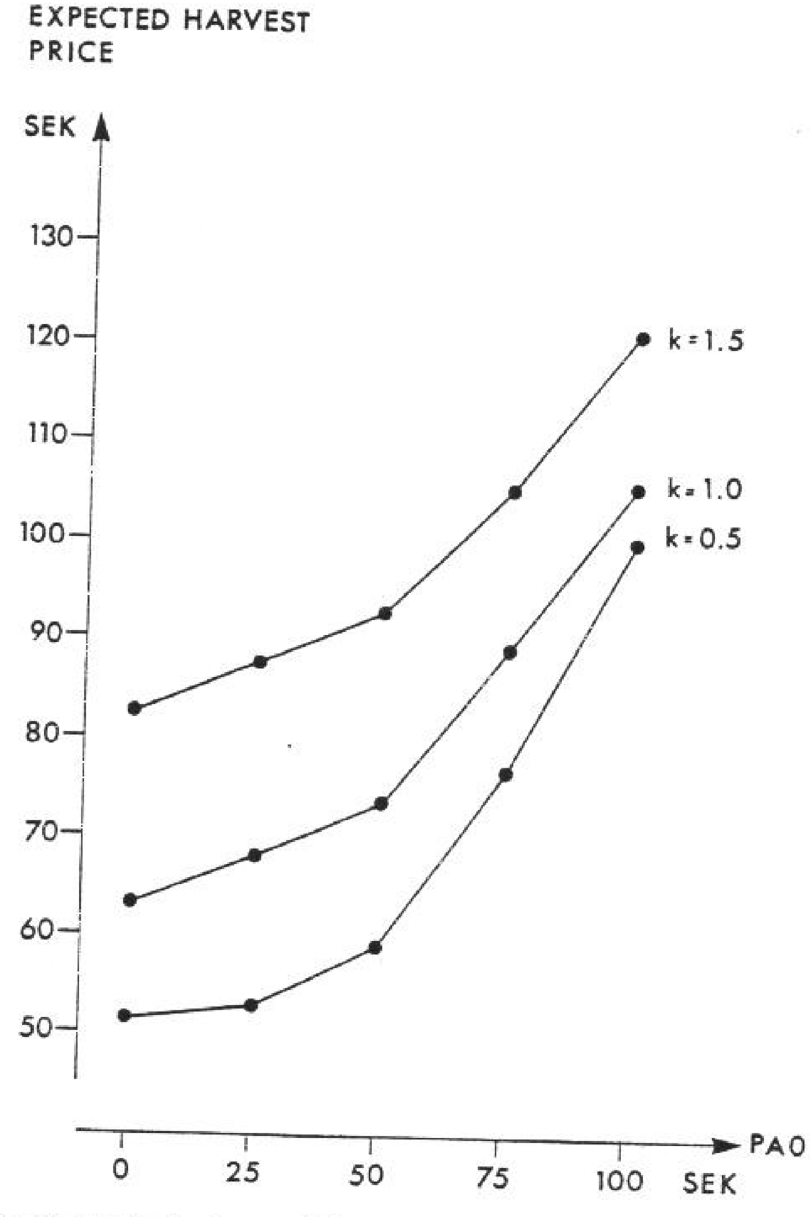

Figure 13 The expected harvest price conditional on optimal adaptive selective thinning as a function of $PA0$ and k. $PA0$ is the price of species A at $T0$ and $PB0$, the price of species B at $T0$, is 50 SEK. The thinning takes place at $T1$, 70 years after plantation, $T0$. The final harvest takes place at $T2$, 120 years after $T0$. It is assumed that the stochastic properties of the prices of species A and B are those reported for RPPP and RPHP respectively when k takes the value 1.0. When k takes the values 0.5 and 1.5, then the standard deviations of the price differences are 50% lower and 50% higher than according to the investigated price series. Note that the expected harvest price is 74 SEK when the initial prices of the two species, $PA0$ and $PB0$, are 50 SEK and the stochastic properties of the prices are those estimated from the empirical data, $k = 1.0$.

after plantation respectively. The price of species *B* is 50 SEK at year 0 and the price of species *A* is found on the *PA*0 axis.

The functions in Figure 13 are convex because of the following reason: When *PA*0 is low compared to *PB*0, the probability is low (but positive) that species *A* will be selected at *T*1. Hence, the expected harvest price is only slightly affected by marginal changes in *PA*0. When *PA*0 is high compared to *PB*0, however, the probability is high that species *A* will be the optimal alternative at *T*1. Then the expected harvest price is of course strongly affected by marginal changes in *PA*0.

The Figures 14 and 15 show how the difference between the initial price at *T*0, *PA*0, and the expected harvest price, is affected by the parameters.

The general lesson is of course that we should select species as late as possible. The later the selection is made, the higher is the relevance of the latest price information and the better is the decision. Here, however, a warning is motivated. Reasons may exist in practical applications why one should not wait too long:

—Maybe the adaptive and selective thinning must be performed when the trees are still rather young. The remaining trees should be able to utilize the released production resources in the neighbourhood and this ability may be more or less age dependent.

—Maybe severe physical damage will result from late thinnings in the forest stand, in particular when heavy harvesting machines are used.

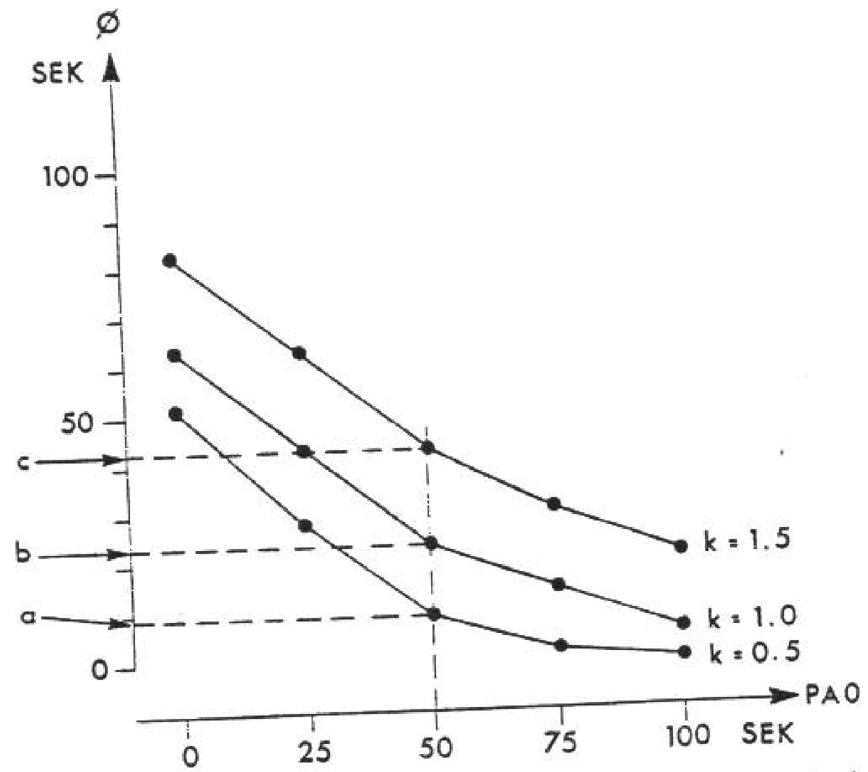

Figure 14 ϕ = The difference (Expected harvest price conditional on optimal adaptive thinning—*PA*0). The assumptions are the same as those reported in connection to Figure 13. The expected profitability of multi species forestry increases with the price difference standard deviation ($a < b < c$). When the stochastic properties of the prices are those found in the empirical material and the initial prices of both species are the same, 50 SEK, ϕ takes the value 24 SEK (b). This is of course a strong reason to invest in a multi species stand.

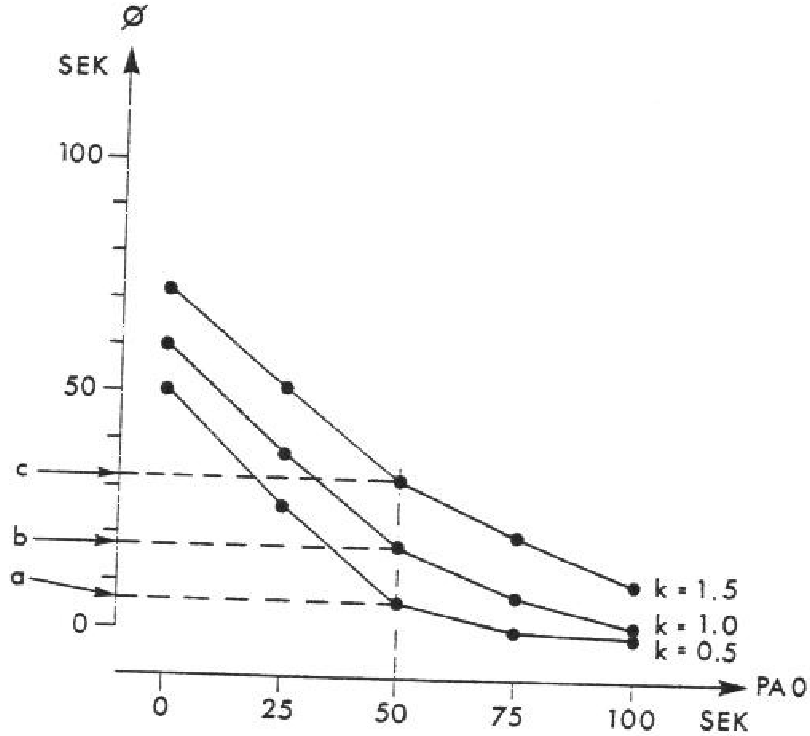

Figure 15 ϕ = The difference (Expected harvest price conditional on optimal adaptive thinning—*PA*0). The assumptions are the same as those in Figure 13 except for that the thinning and final harvest take place earlier: $T1 = 30$ years and $T2 = 100$ years. Note that the flexibility in late stages of production obtained in multi species forestry is more important (ϕ is higher in Figure 14 than in Figure 15 ceteres paribus) in case we can make the thinning and final harvest decisions late. However, also when thinning takes place 30 years after plantation, we gain a lot from the species flexibility: $\phi = 17$ SEK when both initial prices are 50 SEK (b).

These stand and technology specific issues must of course be discussed and analysed with a much more restricted optimization model of less general interest.

4. DISCUSSION

The simple analysis of this paper has shown that considerable economic gains can be expected in the presence of Martingale price processes if we undertake flexible initial investments and adaptive selective thinnings. When we consider multi species forestry, it is tempting to analyse the profitability of this in terms of synergy effects in wood production. This has been one of the main topics in forest production research during many decades. The reported "mixed forest effects" have been small and ambiguous.

The author of this paper is convinced, however, that the expected profit obtainable from multi species forestry has been strongly underestimated.

The economic benefits that are results of flexibility and late selective adaptive decisions make multi species forestry particularly interesting in the rapidly changing and unpredictable modern society.

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EMPIRICAL APPENDIX

This appendix contains all the data that have been used in the analysis. The data consists of six series which are defined below.

YEAR	EXQPP	EXQHP	EXVPP	EXVHP	KPI
70	478501	135753	27871	7466	135
71	606347	93984	38652	4903	145
72	600470	48569	35322	2585	154
73	683335	56614	43854	3055	165
74	211118	144871	17894	9768	181
75	176165	162987	20663	16036	198
76	152107	152044	21099	14875	219
77	16950	46312	2451	4449	243
78	10631	86748	1431	8332	267
79	8355	107240	1043	11484	286
80	33304	153710	4769	19512	327
81	47276	55985	7850	7964	366
82	9646	17095	2042	3393	397
83	35138	76429	6918	13374	435
84	18349	244569	3522	47504	467
85	8268	288838	1741	66896	503
86	24486	170443	6872	39032	523
87	12065	154077	3274	38459	546
88	15821	130864	5385	47760	578
89	17000	94000	5468	31009	612

(*continued*)

(continued from p. 247)

Source: SKOGLIG STATISTIKINFORMATION, SKOGSSTYRELSEN, PROGNOS OCH STABSAVDELNINGEN, NR. 845, 1990-05-07. ISSN 0281-5044

Definitions: YEAR = The year − 1900
EXQPP = Export quantity pine pulpwood (m3)
EXQHP = Export quantity hardwood pulpwood (m3)
EXVPP = Export value pine pulpwood (1000 SEK)
EXVHP = Export value hardwood pulpwood (1000 SEK)
KPI = Consumer price index in Sweden (KPI = 100 in YEAR 63) (The index 612 in 1989 is obtained via extrapolation)

NUMERICAL APPENDIX

This appendix contains the computer code that calculates the expected harvest price conditional on optimal adaptive selective thinning. The expected harvest price is determined for different parameter assumptions. The results are presented on the output screen as in Table 1. All definitions and principles used can be found in the main text.

Table 1 The computer output screen

SELECTIVE ADAPTIVE THINNING AND STOCHASTIC PRICES
PROGRAM SEL.BAS VERSION 90-12-10
LOHMANDER PETER

PARAMETERS				EXPECTED OPTIMAL HARVEST PRICES: PA0				
PB0	K	T2	T1	0	25	50	75	100
50	0.5	100	30	50.9	51.3	56.0	75.0	99.5
50	0.5	100	70	50.9	52.4	58.8	76.6	99.9
50	0.5	120	30	51.5	51.9	56.6	75.5	99.9
50	0.5	120	70	51.6	53.0	59.2	76.8	99.9
50	1.0	100	30	59.9	62.3	67.7	82.7	101.7
50	1.0	100	70	61.2	66.3	72.0	87.7	104.8
50	1.0	120	30	62.2	64.6	69.9	84.6	103.3
50	1.0	120	70	63.1	68.2	73.5	89.1	105.6
50	1.5	100	30	72.0	76.4	81.9	95.2	110.6
50	1.5	100	70	78.2	83.2	88.6	102.3	118.1
50	1.5	120	30	74.9	79.4	84.5	97.4	111.9
50	1.5	120	70	82.6	87.7	92.9	105.4	121.2

```
REM
REM PROGRAM SEL.BAS
REM LOHMANDER PETER 90-12-10 13.54
REM

DEFDBL A-H, O-Z
REM T2 = AGE OF FINAL HARVEST
T2 = 100
PI = 3.1415926535#
SAPROC = (46.37) ^ .5: SBPROC = (50.08) ^ .5
R2 = .414
R = R2 ^ .5
CLS
PRINT " SELECTIVE ADAPTIVE THINNING AND STOCHASTIC PRICES"
PRINT " PROGRAM SEL.BAS VERSION 90-12-10"
PRINT " LOHMANDER PETER "
PRINT " "
PRINT "                              EXPECTED OPTIMAL HARVEST PRICES:"
PRINT "  - PARAMETERS --      ------------ PA0 ---------------"
PRINT "  PB0   K  T2  T1        0     25     50     75    100"
REM ***********************************
REM A MODEL PARAMETER LOOP STARTS HERE
REM ***********************************
FOR K = .5 TO 1.5 STEP .5
PRINT " "
SA = SAPROC * K: SB = SBPROC * K
FOR IT2 = 100 TO 120 STEP 20: T2 = IT2
FOR IT1 = 30 TO 70 STEP 40: T1 = IT1
PB0 = 50
PRINT " "
PRINT USING "#####"; PB0;
PRINT USING "##.#"; K;
PRINT USING "####"; T2; T1;

REM **************************************************************
REM A LOOP WITH DIFFERENT INITIAL PRICES (PA AND PB) STARTS HERE
REM **************************************************************

FOR IPA0 = 0 TO 100 STEP 25: PA0 = IPA0

SAT1 = SA * T1 ^ .5: SBT1 = SB * T1 ^ .5
C = 1 / (2 * PI * SAT1 * SBT1 * (1 - R2) ^ .5)
EVALUE = 0
PROB = 0

FOR IPA1 = (PA0 - SAT1 * 3) TO (PA0 + SAT1 * 3) STEP SAT1: PA1 = IPA1
FOR IPB1 = (PB0 - SBT1 * 3) TO (PB0 + SBT1 * 3) STEP SBT1: PB1 = IPB1

A1 = PA1 - PA0: B1 = PB1 - PB0

G = (A1 / SAT1) ^ 2 - 2 * R * A1 * B1 / (SAT1 * SBT1) + (B1 / SBT1) ^ 2
H = -1 / (2 * (1 - R2)) * G
PRPA1PB1 = C * EXP(H) * SAT1 * SBT1
```

```
REM *******************************************************************
REM THE EXPECTED HARVEST PRICE IF SPECIES A IS SELECTED IS CALCULATED
REM *******************************************************************
SAT2 = SA * (T2 - T1) ^ .5
CA = 1 / ((2 * PI) ^ .5 * SAT2): DA = -1 / (2 * SAT2 * SAT2)

EHARVA = 0: TPROBA = 0


FOR IPA2 = (PA1 - SAT2 * 3) TO (PA1 + SAT2 * 3) STEP SAT2: PA2 = IPA2
A2 = PA2 - PA1
PRPA2 = CA * EXP(DA * A2 * A2) * SAT2
TPROBA = TPROBA + PRPA2
HARVPA = PA2: IF PA2 < 0 THEN HARVPA = 0
EHARVA = EHARVA + PRPA2 * HARVPA
NEXT IPA2
EHARVA = EHARVA / TPROBA

REM *******************************************************************
REM THE EXPECTED HARVEST PRICE IF SPECIES B IS SELECTED IS CALCULATED
REM *******************************************************************
SBT2 = SB * (T2 - T1) ^ .5
CB = 1 / ((2 * PI) ^ .5 * SBT2): DB = -1 / (2 * SBT2 * SBT2)

EHARVB = 0: TPROBB = 0

FOR IPB2 = (PB1 - SBT2 * 3) TO (PB1 + SBT2 * 3) STEP SBT2: PB2 = IPB2
B2 = PB2 - PB1
PRPB2 = CB * EXP(DB * B2 * B2) * SBT2
TPROBB = TPROBB + PRPB2
HARVPB = PB2: IF PB2 < 0 THEN HARVPB = 0
EHARVB = EHARVB + PRPB2 * HARVPB
NEXT IPB2
EHARVB = EHARVB / TPROBB


REM *******************************************************************
REM THE SPECIES WITH THE HIGHEST EXPECTED HARVEST PRICE IS SELECTED
REM *******************************************************************

EHARVOPT = EHARVA: IF EHARVB > EHARVA THEN EHARVOPT = EHARVB
EVALUE = EVALUE + PRPA1PB1 * EHARVOPT
PROB = PROB + PRPA1PB1
NEXT IPB1: NEXT IPA1
EVALUE = EVALUE / PROB
PRINT USING "#####.#"; EVALUE;
NEXT IPA0
NEXT IT1
NEXT IT2
NEXT K
END
```