**Appendix 1**

Linear Model: Examples of Results and Software

 Tariff Results by Peter Lohmander

 Solutions without trade:

 ------------------------

 Country Price Production Consumption

 1.000 7.000 3.000 3.000

 2.000 3.500 2.500 2.500

 Country 1 2

 Consumer surplus 4.500 3.125

 Pmin1 and Pmin2 4.000 1.000

 Producer surplus 4.500 3.125

 Total surplus 9.000 6.250

 Total surplus World without trade 15.250

 Solutions with free trade without tariffs:

 ------------------------------------------

 Free trade Price = 5.25

 Country Price Production Consumption

 1.000 5.250 1.250 4.750

 2.000 5.250 4.250 0.750

 Exp1F and Exp2F = -3.500 3.500

 Country 1 2

 Consumer surplus 11.281 0.281

 Pmin1 and Pmin2 4.000 1.000

 Producer surplus 0.781 9.031

 Total surplus 12.063 9.313

 Total surplus World with free trade 21.375

 Numerical results from discrete tariff optimization:

 ----------------------------------------------------

 topt pf ptopt ptlopt resopt

 1.20 5.25 4.65 5.85 1.02

 Analytical results:

 -------------------

 topt = 1.167

 resopt = 1.021

 Solution with trade and tariff optimized by Country 1:

 ------------------------------------------------------

 Optimized Tariff = 1.167

 dSUR1, dSUR2, dSURW = 1.021 -1.701 -0.681

 Country Price Production Consumption

 1.000 5.833 1.833 4.167

 2.000 4.667 3.667 1.333

 Exp1t and Exp2t -2.333 2.333

 Country 1 2

 Consumer surplus 8.681 0.889

 Producer surplus 1.681 6.722

 Tariff gain 2.722

 Total surplus 13.083 7.611

 Total surplus World with optimal tariff 20.694

Rem

Rem smalltariff 250507\_2336.bas

Rem Peter Lohmander

Rem

DefDbl A-Z

Screen \_NewImage(1800, 2000, 256)

Open "C:\Users\Peter\OneDrive\Desktop\TariffOut\TariffOut.txt" For Output As #2

Print ""

Print " Tariff Results by Peter Lohmander"

Rem Print ""

Print #2, ""

Print #2, " Tariff Results by Peter Lohmander"

Print #2, ""

Rem The parmeters of the markets

a11 = 10

b11 = 1

g11 = 4

h11 = 1

a21 = 6

b21 = 1

g21 = 1

h21 = 1

Rem Determination of the solution without trade

Print " Solutions without trade:"

Print " ------------------------ "

Print #2, " Solutions without trade:"

Print #2, " ------------------------ "

Rem Determination of prices and quantitites in different countries without trade

P1N = (a11 + g11) / (b11 + h11)

Cons1N = a11 - b11 \* P1N

Prod1N = -g11 + h11 \* P1N

P2N = (a21 + g21) / (b21 + h21)

Cons2N = a21 - b21 \* P2N

Prod2N = -g21 + h21 \* P2N

Print " Country Price Production Consumption "

Print Using "#########.###"; 1; P1N; Prod1N; Cons1N

Print Using "#########.###"; 2; P2N; Prod2N; Cons2N

Print #2, " Country Price Production Consumption "

Print #2, Using "#########.###"; 1; P1N; Prod1N; Cons1N

Print #2, Using "#########.###"; 2; P2N; Prod2N; Cons2N

Rem Determination of the surpluses without trade

CSURP1N = (a11 - P1N) / 2 \* Cons1N

CSURP2N = (a21 - P2N) / 2 \* Cons2N

Print ""

Print " Country 1 2"; ""

Print " Consumer surplus ";

Print Using "######.###"; CSURP1N; CSURP2N

Print #2, ""

Print #2, " Country 1 2"; ""

Print #2, " Consumer surplus ";

Print #2, Using "######.###"; CSURP1N; CSURP2N

Rem Price when production profitability is zero

Pmin1 = g11 / h11

Pmin2 = g21 / h21

Print " Pmin1 and Pmin2 ";

Print Using "######.###"; Pmin1; Pmin2

Print #2, " Pmin1 and Pmin2 ";

Print #2, Using "######.###"; Pmin1; Pmin2

PSURP1N = (P1N - Pmin1) / 2 \* Prod1N

PSURP2N = (P2N - Pmin2) / 2 \* Prod2N

Print " Producer surplus ";

Print Using "######.###"; PSURP1N; PSURP2N

Print #2, " Producer surplus ";

Print #2, Using "######.###"; PSURP1N; PSURP2N

TSURP1N = CSURP1N + PSURP1N

TSURP2N = CSURP2N + PSURP2N

TSURPWN = TSURP1N + TSURP2N

Print " Total surplus ";

Print Using "######.###"; TSURP1N; TSURP2N

Print ""

Print " Total surplus World without trade ";

Print Using "######.###"; TSURPWN

Rem Print ""

Print #2, " Total surplus ";

Print #2, Using "######.###"; TSURP1N; TSURP2N

Print #2, ""

Print #2, " Total surplus World without trade ";

Print #2, Using "######.###"; TSURPWN

Print #2, ""

Rem Determination of the free trade solution without tariffs

Print ""

Print " Solutions with free trade without tariffs:"

Print " ------------------------------------------ "

Print #2, ""

Print #2, " Solutions with free trade without tariffs:"

Print #2, " ------------------------------------------ "

pf = (a11 + a21 + g11 + g21) / (b11 + b21 + h11 + h21)

Print " Free trade Price = "; pf

Rem Print ""

Print #2, " Free trade Price = "; pf

Print #2, ""

Rem Determination of prices and quantitites in different countries with free trade

Cons1F = a11 - b11 \* pf

Prod1F = -g11 + h11 \* pf

Cons2f = a21 - b21 \* pf

Prod2f = -g21 + h21 \* pf

Print " Country Price Production Consumption "

Print Using "#########.###"; 1; pf; Prod1F; Cons1F

Print Using "#########.###"; 2; pf; Prod2f; Cons2f

Print #2, " Country Price Production Consumption "

Print #2, Using "#########.###"; 1; pf; Prod1F; Cons1F

Print #2, Using "#########.###"; 2; pf; Prod2f; Cons2f

Rem Determination of export levels from the different countries with free trade

Rem (Import is export \*(-1) )

Exp1F = Prod1F - Cons1F

Exp2F = Prod2f - Cons2f

Print ""

Print " Exp1F and Exp2F = ";

Print Using "######.###"; Exp1F; Exp2F

Print #2, ""

Print #2, " Exp1F and Exp2F = ";

Print #2, Using "######.###"; Exp1F; Exp2F

Rem Determination of the surpluses with free trade

CSURP1f = (a11 - pf) / 2 \* Cons1F

CSURP2f = (a21 - pf) / 2 \* Cons2f

Print ""

Print " Country 1 2"; ""

Print " Consumer surplus ";

Print Using "######.###"; CSURP1f; CSURP2f

Print #2, ""

Print #2, " Country 1 2"; ""

Print #2, " Consumer surplus ";

Print #2, Using "######.###"; CSURP1f; CSURP2f

Rem Price when production profitability is zero

Pmin1 = g11 / h11

Pmin2 = g21 / h21

Print " Pmin1 and Pmin2 ";

Print Using "######.###"; Pmin1; Pmin2

Print #2, " Pmin1 and Pmin2 ";

Print #2, Using "######.###"; Pmin1; Pmin2

PSURP1f = (pf - Pmin1) / 2 \* Prod1F

PSURP2f = (pf - Pmin2) / 2 \* Prod2f

Print " Producer surplus ";

Print Using "######.###"; PSURP1f; PSURP2f

Print #2, " Producer surplus ";

Print #2, Using "######.###"; PSURP1f; PSURP2f

TSURP1f = CSURP1f + PSURP1f

TSURP2f = CSURP2f + PSURP2f

TSURPWF = TSURP1f + TSURP2f

Print " Total surplus ";

Print Using "######.###"; TSURP1f; TSURP2f

Print ""

Print #2, " Total surplus ";

Print #2, Using "######.###"; TSURP1f; TSURP2f

Print #2, ""

Print " Total surplus World with free trade ";

Print Using "######.###"; TSURPWF

Print ""

Print #2, " Total surplus World with free trade ";

Print #2, Using "######.###"; TSURPWF

Print #2, ""

Print #2, ""

Rem Simple discrete numerical optimization of the tariff

resopt = -100

For T = 0 To 5 Step 0.1

 pt = pf - (h11 + b11) / (b11 + b21 + h11 + h21) \* T

 ptl = pt + T

 c11f = a11 - b11 \* pf

 c11tl = a11 - b11 \* ptl

 x11f = -g11 + h11 \* pf

 x11tl = -g11 + h11 \* ptl

 s11 = x11tl - c11tl

 R = -s11 \* (pf - pt)

 y = (ptl - pf) \* (c11f - c11tl) / 2

 z = (ptl - pf) \* (x11tl - x11f) / 2

 res = R - y - z

 If res < resopt Then GoTo 100

 resopt = res

 topt = T

 ptopt = pt

 ptlopt = ptl

 100 Rem

Next T

Print " Numerical results from discrete tariff optimization:"

Print " ---------------------------------------------------- "

Print " topt pf ptopt ptlopt resopt "

Print " ";

Print Using "#####.##"; topt; pf; ptopt; ptlopt; resopt

Print #2, " Numerical results from discrete tariff optimization:"

Print #2, " ---------------------------------------------------- "

Print #2, " topt pf ptopt ptlopt resopt "

Print #2, " ";

Print #2, Using "#####.##"; topt; pf; ptopt; ptlopt; resopt

Rem Analytical section

N = (b11 + h11) / (b11 + b21 + h11 + h21)

k1 = (a11 + g11 - (b11 + h11) \* pf) \* N

k2 = (b11 + h11) \* (1 - N) \* N

k3 = b11 / 2 \* (1 - N) ^ 2

k4 = h11 / 2 \* (1 - N) ^ 2

topt = k1 / (2 \* (k2 + k3 + k4))

ropt = k1 \* topt - k2 \* topt ^ 2

yopt = k3 \* topt ^ 2

zopt = k4 \* topt ^ 2

resopt = ropt - yopt - zopt

dSUR1 = resopt

Print ""

Print " Analytical results:"

Print " ------------------- "

Print " topt = ";

Print Using "#####.###"; topt

Print " resopt = ";

Print Using "#####.###"; resopt

Print #2, ""

Print #2, " Analytical results:"

Print #2, " ------------------- "

Print #2, " topt = ";

Print #2, Using "#####.###"; topt

Print #2, " resopt = ";

Print #2, Using "#####.###"; resopt

Rem Print ""

Print " Solution with trade and tariff optimized by Country 1:"

Print " ------------------------------------------------------ "

Print #2, ""

Print #2, " Solution with trade and tariff optimized by Country 1:"

Print #2, " ------------------------------------------------------ "

Rem Determination of prices in different countries with optimized tariff.

T = topt

Rem Print ""

Print " Optimized Tariff = ";

Print Using "#####.###"; T

Print ""

Print #2, " Optimized Tariff = ";

Print #2, Using "#####.###"; T

Print #2, ""

Rem Some effects of the optimized tariff on the exporting country.

Rem p0 is the price if the exported volume is zero.

p0 = (a21 + g21) / (b21 + h21)

Rem pt = Price outside Country 1, where the tariff is used.

pt = pf - N \* T

p2 = pt

Rem ptl = Local price in Country 1, where the tariff is used.

ptl = pt + T

p1 = ptl

Rem Special calculation of the change of the surplus

s21f = -(a21 + g21) + (b21 + h21) \* pf

s21t = -(a21 + g21) + (b21 + h21) \* pt

sur2f = (pf - p0) \* s21f / 2

sur2t = (pt - p0) \* s21t / 2

dsur2 = sur2t - sur2f

dSURW = dSUR1 + dsur2

Print " dSUR1, dSUR2, dSURW = ";

Print Using "#####.###"; dSUR1; dsur2; dSURW

Print ""

Print #2, " dSUR1, dSUR2, dSURW = ";

Print #2, Using "#####.###"; dSUR1; dsur2; dSURW

Print #2, ""

Rem Determination of consumption and production levels with tariffs

Cons1t = a11 - b11 \* p1

Prod1t = -g11 + h11 \* p1

Cons2t = a21 - b21 \* p2

Prod2t = -g21 + h21 \* p2

Print " Country Price Production Consumption "

Print Using "#########.###"; 1; p1; Prod1t; Cons1t

Print Using "#########.###"; 2; p2; Prod2t; Cons2t

Print #2, " Country Price Production Consumption "

Print #2, Using "#########.###"; 1; p1; Prod1t; Cons1t

Print #2, Using "#########.###"; 2; p2; Prod2t; Cons2t

Rem Determination of export levels from the different countries with free trade

Rem (Import is export \*(-1) )

Exp1t = Prod1t - Cons1t

Exp2t = Prod2t - Cons2t

Print ""

Print " Exp1t and Exp2t ";

Print Using "######.###"; Exp1t; Exp2t

Print #2, ""

Print #2, " Exp1t and Exp2t ";

Print #2, Using "######.###"; Exp1t; Exp2t

Rem Determination of the surpluses with free trade

CSURP1t = (a11 - p1) / 2 \* Cons1t

CSURP2t = (a21 - p2) / 2 \* Cons2t

Rem Print ""

Print " Country 1 2"; ""

Print " Consumer surplus ";

Print Using "######.###"; CSURP1t; CSURP2t

Print #2, ""

Print #2, " Country 1 2"; ""

Print #2, " Consumer surplus ";

Print #2, Using "######.###"; CSURP1t; CSURP2t

PSURP1t = (p1 - Pmin1) / 2 \* Prod1t

PSURP2t = (p2 - Pmin2) / 2 \* Prod2t

Print " Producer surplus ";

Print Using "######.###"; PSURP1t; PSURP2t

Print #2, " Producer surplus ";

Print #2, Using "######.###"; PSURP1t; PSURP2t

GSURP1t = -Exp1t \* T

Print " Tariff gain ";

Print Using "######.###"; GSURP1t

Print #2, " Tariff gain ";

Print #2, Using "######.###"; GSURP1t

TSURP1t = CSURP1t + PSURP1t + GSURP1t

TSURP2t = CSURP2t + PSURP2t

TSURPWt = TSURP1t + TSURP2t

Print " Total surplus ";

Print Using "######.###"; TSURP1t; TSURP2t

Print #2, " Total surplus ";

Print #2, Using "######.###"; TSURP1t; TSURP2t

Print " Total surplus World with optimal tariff ";

Print Using "######.###"; TSURPWt

Print ""

Print #2, " Total surplus World with optimal tariff ";

Print #2, Using "######.###"; TSURPWt

Print #2, ""

Close #2

End

**Appendix 2**

**Nonlinear Model: Examples of Results and Software**

***Results and software by Peter Lohmander 2025–06-02***

**Case 0**

 Tariff (%) = 0

 Production possibility frontiers parameters

 ------------------------------------------

 x11max, x12max, x13max, x14max = 2.000 4.000 0.000 0.000

 x21max, x22max, x23max, x24max = 0.000 2.000 4.000 0.000

 x31max, x32max, x33max, x34max = 0.000 0.000 2.000 4.000

 Total sum of squared excess supplies = tesopt = 6.456329543206307D-08

 Equilibrium prices = p1, p2, p3, p4 = 100.000 45.449 77.129 95.391

 Global Excess Supplies = es1, es2, es3, es4 = 0.00019 -0.00010 -0.00005 -0.00012

 Production levels =

 x11, x12, x13, x14 = 1.480 2.691 0.000 0.000

 x21, x22, x23, x24 = 0.000 0.565 3.837 0.000

 x31, x32, x33, x34 = 0.000 0.000 0.750 3.708

 Excess supply levels =

 s11, s12, s13, s14 = 0.804 1.204 -0.876 -0.708

 s21, s22, s23, s24 = -0.804 -1.204 2.794 -0.843

 s31, s32, s33, s34 = 0.000 0.000 -1.918 1.551

 Consumption levels =

 c11, c12, c13, c14 = 0.676 1.487 0.876 0.708

 c21, c22, c22, c24 = 0.804 1.769 1.043 0.843

 c31, c32, c33, c34 = 0.000 0.000 2.668 2.157

 GNP levels = w1, w2 ,w3 = 270.278 321.628 411.565

 Utility levels = u1, u2, u3 = 0.889 1.115 2.399

 Global GNP = 1003.470

 Global Utility = 4.403

**Case 0**

 Tariff (%) = 50

 Production possibility frontiers parameters

 ------------------------------------------

 x11max, x12max, x13max, x14max = 2.000 4.000 0.000 0.000

 x21max, x22max, x23max, x24max = 0.000 2.000 4.000 0.000

 x31max, x32max, x33max, x34max = 0.000 0.000 2.000 4.000

 Total sum of squared excess supplies = tesopt = 3.99792725031482D-08

 Equilibrium prices = p1, p2, p3, p4 = 100.000 45.176 71.396 82.832

 Global Excess Supplies = es1, es2, es3, es4 = 0.00003 0.00003 0.00012 -0.00015

 Production levels =

 x11, x12, x13, x14 = 1.484 2.682 0.000 0.000

 x21, x22, x23, x24 = 0.000 0.603 3.814 0.000

 x31, x32, x33, x34 = 0.000 0.000 0.792 3.673

 Excess supply levels =

 s11, s12, s13, s14 = 0.749 1.054 -1.030 -0.592

 s21, s22, s23, s24 = -0.749 -1.054 2.765 -0.904

 s31, s32, s33, s34 = 0.000 0.000 -1.735 1.496

 Consumption levels =

 c11, c12, c13, c14 = 0.735 1.627 1.030 0.592

 c21, c22, c22, c24 = 0.749 1.658 1.049 0.904

 c31, c32, c33, c34 = 0.000 0.000 2.527 2.178

 GNP levels = w1, w2 ,w3 = 294.047 299.538 360.788

 Utility levels = u1, u2, u3 = 0.924 1.081 2.346

 Global GNP = 954.373

 Global Utility = 4.351

**Case 0**

 Tariff (%) = 100

 Production possibility frontiers parameters

 ------------------------------------------

 x11max, x12max, x13max, x14max = 2.000 4.000 0.000 0.000

 x21max, x22max, x23max, x24max = 0.000 2.000 4.000 0.000

 x31max, x32max, x33max, x34max = 0.000 0.000 2.000 4.000

 Total sum of squared excess supplies = tesopt = 8.681383082576513D-08

 Equilibrium prices = p1, p2, p3, p4 = 100.000 45.000 68.145 75.742

 Global Excess Supplies = es1, es2, es3, es4 = 0.00018 -0.00021 -0.00001 -0.00011

 Production levels =

 x11, x12, x13, x14 = 1.487 2.676 0.000 0.000

 x21, x22, x23, x24 = 0.000 0.627 3.798 0.000

 x31, x32, x33, x34 = 0.000 0.000 0.820 3.648

 Excess supply levels =

 s11, s12, s13, s14 = 0.718 0.967 -1.128 -0.507

 s21, s22, s23, s24 = -0.718 -0.968 2.745 -0.947

 s31, s32, s33, s34 = 0.000 0.000 -1.617 1.455

 Consumption levels =

 c11, c12, c13, c14 = 0.769 1.708 1.128 0.507

 c21, c22, c22, c24 = 0.718 1.595 1.053 0.947

 c31, c32, c33, c34 = 0.000 0.000 2.438 2.193

 GNP levels = w1, w2 ,w3 = 307.511 287.052 332.212

 Utility levels = u1, u2, u3 = 0.931 1.061 2.312

 Global GNP = 926.775

 Global Utility = 4.305

**Case 0**

 Tariff (%) = 150

 Production possibility frontiers parameters

 ------------------------------------------

 x11max, x12max, x13max, x14max = 2.000 4.000 0.000 0.000

 x21max, x22max, x23max, x24max = 0.000 2.000 4.000 0.000

 x31max, x32max, x33max, x34max = 0.000 0.000 2.000 4.000

 Total sum of squared excess supplies = tesopt = 1.04538358161312D-07

 Equilibrium prices = p1, p2, p3, p4 = 100.000 44.893 66.074 71.221

 Global Excess Supplies = es1, es2, es3, es4 = -0.00020 0.00023 0.00002 0.00011

 Production levels =

 x11, x12, x13, x14 = 1.488 2.672 0.000 0.000

 x21, x22, x23, x24 = 0.000 0.643 3.787 0.000

 x31, x32, x33, x34 = 0.000 0.000 0.842 3.629

 Excess supply levels =

 s11, s12, s13, s14 = 0.698 0.911 -1.196 -0.444

 s21, s22, s23, s24 = -0.698 -0.911 2.731 -0.980

 s31, s32, s33, s34 = 0.000 0.000 -1.535 1.424

 Consumption levels =

 c11, c12, c13, c14 = 0.791 1.761 1.196 0.444

 c21, c22, c22, c24 = 0.698 1.554 1.056 0.980

 c31, c32, c33, c34 = 0.000 0.000 2.376 2.205

 GNP levels = w1, w2 ,w3 = 316.218 279.131 314.040

 Utility levels = u1, u2, u3 = 0.927 1.049 2.289

 Global GNP = 909.390

 Global Utility = 4.265

**Case 0**

 Tariff (%) = 200

 Production possibility frontiers parameters

 ------------------------------------------

 x11max, x12max, x13max, x14max = 2.000 4.000 0.000 0.000

 x21max, x22max, x23max, x24max = 0.000 2.000 4.000 0.000

 x31max, x32max, x33max, x34max = 0.000 0.000 2.000 4.000

 Total sum of squared excess supplies = tesopt = 4.90099160193288D-08

 Equilibrium prices = p1, p2, p3, p4 = 100.000 44.805 64.619 68.057

 Global Excess Supplies = es1, es2, es3, es4 = -0.00010 -0.00006 0.00019 -0.00000

 Production levels =

 x11, x12, x13, x14 = 1.489 2.669 0.000 0.000

 x21, x22, x23, x24 = 0.000 0.655 3.779 0.000

 x31, x32, x33, x34 = 0.000 0.000 0.858 3.613

 Excess supply levels =

 s11, s12, s13, s14 = 0.684 0.871 -1.247 -0.395

 s21, s22, s23, s24 = -0.684 -0.871 2.721 -1.005

 s31, s32, s33, s34 = 0.000 0.000 -1.474 1.400

 Consumption levels =

 c11, c12, c13, c14 = 0.806 1.798 1.247 0.395

 c21, c22, c22, c24 = 0.684 1.526 1.058 1.005

 c31, c32, c33, c34 = 0.000 0.000 2.332 2.214

 GNP levels = w1, w2 ,w3 = 322.261 273.569 301.347

 Utility levels = u1, u2, u3 = 0.919 1.040 2.272

 Global GNP = 897.176

 Global Utility = 4.231

**Case 1 (x22max and x23max have been changed)**

 Tariff (%) = 0

 Production possibility frontiers parameters

 ------------------------------------------

 x11max, x12max, x13max, x14max = 2.000 4.000 0.000 0.000

 x21max, x22max, x23max, x24max = 0.000 4.000 2.000 0.000

 x31max, x32max, x33max, x34max = 0.000 0.000 2.000 4.000

 Total sum of squared excess supplies = tesopt = 1.031094533439831D-08

 Equilibrium prices = p1, p2, p3, p4 = 100.000 38.730 173.213 144.229

 Global Excess Supplies = es1, es2, es3, es4 = -0.00005 -0.00006 -0.00001 0.00007

 Production levels =

 x11, x12, x13, x14 = 1.581 2.450 0.000 0.000

 x21, x22, x23, x24 = 0.000 1.633 1.826 0.000

 x31, x32, x33, x34 = 0.000 0.000 1.030 3.429

 Excess supply levels =

 s11, s12, s13, s14 = 0.949 0.817 -0.365 -0.439

 s21, s22, s23, s24 = -0.949 -0.817 1.278 -0.658

 s31, s32, s33, s34 = 0.000 0.000 -0.913 1.096

 Consumption levels =

 c11, c12, c13, c14 = 0.632 1.633 0.365 0.439

 c21, c22, c22, c24 = 0.949 2.450 0.548 0.658

 c31, c32, c33, c34 = 0.000 0.000 1.943 2.333

 GNP levels = w1, w2 ,w3 = 252.984 379.489 672.934

 Utility levels = u1, u2, u3 = 0.638 0.914 2.129

 Global GNP = 1305.407

 Global Utility = 3.680

**Case 1 (x22max and x23max have been changed)**

 Tariff (%) = 50

 Production possibility frontiers parameters

 ------------------------------------------

 x11max, x12max, x13max, x14max = 2.000 4.000 0.000 0.000

 x21max, x22max, x23max, x24max = 0.000 4.000 2.000 0.000

 x31max, x32max, x33max, x34max = 0.000 0.000 2.000 4.000

 Total sum of squared excess supplies = tesopt = 7.515735279736754D-08

 Equilibrium prices = p1, p2, p3, p4 = 100.000 38.340 162.744 130.654

 Global Excess Supplies = es1, es2, es3, es4 = 0.00024 -0.00008 -0.00010 -0.00003

 Production levels =

 x11, x12, x13, x14 = 1.587 2.434 0.000 0.000

 x21, x22, x23, x24 = 0.000 1.705 1.809 0.000

 x31, x32, x33, x34 = 0.000 0.000 1.057 3.395

 Excess supply levels =

 s11, s12, s13, s14 = 0.900 0.641 -0.422 -0.351

 s21, s22, s23, s24 = -0.900 -0.641 1.257 -0.688

 s31, s32, s33, s34 = 0.000 0.000 -0.834 1.039

 Consumption levels =

 c11, c12, c13, c14 = 0.687 1.793 0.422 0.351

 c21, c22, c22, c24 = 0.900 2.346 0.553 0.688

 c31, c32, c33, c34 = 0.000 0.000 1.892 2.356

 GNP levels = w1, w2 ,w3 = 274.942 359.808 615.688

 Utility levels = u1, u2, u3 = 0.654 0.896 2.111

 Global GNP = 1250.437

 Global Utility = 3.661

**Case 1 (x22max and x23max have been changed)**

 Tariff (%) = 100

 Production possibility frontiers parameters

 ------------------------------------------

 x11max, x12max, x13max, x14max = 2.000 4.000 0.000 0.000

 x21max, x22max, x23max, x24max = 0.000 4.000 2.000 0.000

 x31max, x32max, x33max, x34max = 0.000 0.000 2.000 4.000

 Total sum of squared excess supplies = tesopt = 1.027059118454071D-08

 Equilibrium prices = p1, p2, p3, p4 = 100.000 38.115 156.875 123.027

 Global Excess Supplies = es1, es2, es3, es4 = -0.00005 0.00008 -0.00002 0.00004

 Production levels =

 x11, x12, x13, x14 = 1.591 2.425 0.000 0.000

 x21, x22, x23, x24 = 0.000 1.748 1.799 0.000

 x31, x32, x33, x34 = 0.000 0.000 1.075 3.373

 Excess supply levels =

 s11, s12, s13, s14 = 0.872 0.540 -0.458 -0.292

 s21, s22, s23, s24 = -0.872 -0.540 1.243 -0.709

 s31, s32, s33, s34 = 0.000 0.000 -0.785 1.001

 Consumption levels =

 c11, c12, c13, c14 = 0.719 1.885 0.458 0.292

 c21, c22, c22, c24 = 0.872 2.288 0.556 0.709

 c31, c32, c33, c34 = 0.000 0.000 1.860 2.372

 GNP levels = w1, w2 ,w3 = 287.411 348.831 583.619

 Utility levels = u1, u2, u3 = 0.652 0.886 2.100

 Global GNP = 1219.861

 Global Utility = 3.639

**Case 1 (x22max and x23max have been changed)**

 Tariff (%) = 150

 Production possibility frontiers parameters

 ------------------------------------------

 x11max, x12max, x13max, x14max = 2.000 4.000 0.000 0.000

 x21max, x22max, x23max, x24max = 0.000 4.000 2.000 0.000

 x31max, x32max, x33max, x34max = 0.000 0.000 2.000 4.000

 Total sum of squared excess supplies = tesopt = 4.259192187127558D-08

 Equilibrium prices = p1, p2, p3, p4 = 100.000 37.959 153.057 118.086

 Global Excess Supplies = es1, es2, es3, es4 = 0.00015 -0.00002 0.00001 -0.00014

 Production levels =

 x11, x12, x13, x14 = 1.593 2.419 0.000 0.000

 x21, x22, x23, x24 = 0.000 1.777 1.792 0.000

 x31, x32, x33, x34 = 0.000 0.000 1.088 3.357

 Excess supply levels =

 s11, s12, s13, s14 = 0.854 0.473 -0.483 -0.250

 s21, s22, s23, s24 = -0.854 -0.473 1.234 -0.723

 s31, s32, s33, s34 = 0.000 0.000 -0.751 0.973

 Consumption levels =

 c11, c12, c13, c14 = 0.739 1.946 0.483 0.250

 c21, c22, c22, c24 = 0.854 2.250 0.558 0.723

 c31, c32, c33, c34 = 0.000 0.000 1.839 2.383

 GNP levels = w1, w2 ,w3 = 295.419 341.701 562.862

 Utility levels = u1, u2, u3 = 0.645 0.880 2.093

 Global GNP = 1199.982

 Global Utility = 3.618

**Case 1 (x22max and x23max have been changed)**

 Tariff (%) = 200

 Production possibility frontiers parameters

 ------------------------------------------

 x11max, x12max, x13max, x14max = 2.000 4.000 0.000 0.000

 x21max, x22max, x23max, x24max = 0.000 4.000 2.000 0.000

 x31max, x32max, x33max, x34max = 0.000 0.000 2.000 4.000

 Total sum of squared excess supplies = tesopt = 2.059154126645428D-08

 Equilibrium prices = p1, p2, p3, p4 = 100.000 37.852 150.420 114.668

 Global Excess Supplies = es1, es2, es3, es4 = 0.00008 -0.00001 0.00003 -0.00011

 Production levels =

 x11, x12, x13, x14 = 1.595 2.414 0.000 0.000

 x21, x22, x23, x24 = 0.000 1.798 1.787 0.000

 x31, x32, x33, x34 = 0.000 0.000 1.097 3.345

 Excess supply levels =

 s11, s12, s13, s14 = 0.842 0.426 -0.500 -0.219

 s21, s22, s23, s24 = -0.842 -0.426 1.227 -0.734

 s31, s32, s33, s34 = 0.000 0.000 -0.726 0.953

 Consumption levels =

 c11, c12, c13, c14 = 0.753 1.988 0.500 0.219

 c21, c22, c22, c24 = 0.842 2.224 0.560 0.734

 c31, c32, c33, c34 = 0.000 0.000 1.823 2.392

 GNP levels = w1, w2 ,w3 = 301.016 336.791 548.529

 Utility levels = u1, u2, u3 = 0.636 0.875 2.088

 Global GNP = 1186.336

 Global Utility = 3.600

**The software**

Rem

Rem largetariff\_Alt\_250602\_1929.bas

Rem NOTE In this Alt version, alternative

Rem production possibility frontiers are introduced.

Rem Peter Lohmander

Rem

DefDbl A-Z

Screen \_NewImage(1800, 2000, 256)

Open "C:\Users\Peter\OneDrive\Desktop\TariffOut\L\_TariffOut.txt" For Output As #2

Input "tariff = ", tariff

Print ""

Print " Tariff (%) = "; tariff

Print #2, ""

Print #2, " Tariff (%) = "; tariff

Rem Collect parameters

GoSub 200

Rem Print some parameters

GoSub 300

Rem Total squares of excess supplies

Rem initial conditions

xopttot = 1

yopttot = 1

zopttot = 1

tseopt = 99999999

xopt = 0

yopt = 0

zopt = 0

For tot = 1 To 10

 steg = 0.1 / (2 ^ tot)

 For ix = xopttot - 10 \* steg To xopttot + 10 \* steg Step steg

 x = ix

 For iy = yopttot - 10 \* steg To yopttot + 10 \* steg Step steg

 y = iy

 For iz = zopttot - 10 \* steg To zopttot + 10 \* steg Step steg

 z = iz

 xsub = x

 ysub = y

 zsub = z

 GoSub 100

 tseloc = fsub \* fsub + gsub \* gsub + hsub \* hsub + tsub \* tsub

 If tseloc > tseopt Then GoTo 700

 tseopt = tseloc

 xopt = x

 yopt = y

 zopt = z

 700 Rem

 Next iz

 Next iy

 Next ix

 xopttot = xopt

 yopttot = yopt

 zopttot = zopt

Next tot

Print ""

Print " Total sum of squared excess supplies = tesopt = "; tseopt

Print ""

Print #2, ""

Print #2, " Total sum of squared excess supplies = tesopt = "; tseopt

Print #2, ""

Rem The optimal results are collected from from the subroutine.

xsub = xopttot

ysub = yopttot

zsub = zopttot

GoSub 100

Rem The optimal results are printed.

Print ""

Print #2, ""

Print " Equilibrium prices = ";

Print " p1, p2, p3, p4 = ";

Print Using "######.###"; p1; p2; p3; p4

Print ""

Print " Global Excess Supplies = ";

Print " es1, es2, es3, es4 = ";

Print Using "####.#####"; es1; es2; es3; es4

Print #2, " Equilibrium prices = ";

Print #2, " p1, p2, p3, p4 = ";

Print #2, Using "######.###"; p1; p2; p3; p4

Print #2, ""

Print #2, " Global Excess Supplies = ";

Print #2, " es1, es2, es3, es4 = ";

Print #2, Using "####.#####"; es1; es2; es3; es4

Print ""

Print " Production levels = "

Print ""

Print " x11, x12, x13, x14 = ";

Print Using "####.###"; x11; x12, x13, x14

Print " x21, x22, x23, x24 = ";

Print Using "####.###"; x21, x22; x23, x24

Print " x31, x32, x33, x34 = ";

Print Using "####.###"; x31, x32, x33; x34

Print " "

Print " Excess supply levels = "

Print ""

Print " s11, s12, s13, s14 = ";

Print Using "####.###"; s11; s12; s13; s14

Print " s21, s22, s23, s24 = ";

Print Using "####.###"; s21; s22; s23; s24

Print " s31, s32, s33, s34 = ";

Print Using "####.###"; s31; s32; s33; s34

Print #2, ""

Print #2, " Production levels = "

Print #2, ""

Print #2, " x11, x12, x13, x14 = ";

Print #2, Using "####.###"; x11; x12, x13, x14

Print #2, " x21, x22, x23, x24 = ";

Print #2, Using "####.###"; x21, x22; x23, x24

Print #2, " x31, x32, x33, x34 = ";

Print #2, Using "####.###"; x31, x32, x33; x34

Print #2, " "

Print #2, " Excess supply levels = "

Print #2, ""

Print #2, " s11, s12, s13, s14 = ";

Print #2, Using "####.###"; s11; s12; s13; s14

Print #2, " s21, s22, s23, s24 = ";

Print #2, Using "####.###"; s21; s22; s23; s24

Print #2, " s31, s32, s33, s34 = ";

Print #2, Using "####.###"; s31; s32; s33; s34

Print ""

Print " Consumption levels = "

Print ""

Print " c11, c12, c13, c14 = ";

Print Using "####.###"; c11; c12; c13; c14

Print " c21, c22, c22, c24 = ";

Print Using "####.###"; c21; c22; c23; c24

Print " c31, c32, c33, c34 = ";

Print Using "####.###"; c31; c32; c33; c34

Print ""

Print " GNP levels = ";

Print " w1, w2 ,w3 = ";

Print Using "####.###"; w1; w2; w3

Print ""

Print " Utility levels = ";

Print " u1, u2, u3 = ";

Print Using "####.###"; u1; u2; u3

Print ""

Print " Global GNP = ";

Print Using "#####.###"; wsum

Print " Global Utility = ";

Print Using "#####.###"; usum

Print #2, ""

Print #2, " Consumption levels = "

Print #2, ""

Print #2, " c11, c12, c13, c14 = ";

Print #2, Using "####.###"; c11; c12; c13; c14

Print #2, " c21, c22, c22, c24 = ";

Print #2, Using "####.###"; c21; c22; c23; c24

Print #2, " c31, c32, c33, c34 = ";

Print #2, Using "####.###"; c31; c32; c33; c34

Print #2, ""

Print #2, " GNP levels = ";

Print #2, " w1, w2 ,w3 = ";

Print #2, Using "####.###"; w1; w2; w3

Print #2, ""

Print #2, " Utility levels = ";

Print #2, " u1, u2, u3 = ";

Print #2, Using "####.###"; u1; u2; u3

Print #2, ""

Print #2, " Global GNP = ";

Print #2, Using "#####.###"; wsum

Print #2, " Global Utility = ";

Print #2, Using "#####.###"; usum

End

100 Rem subroutine

p1 = 100

p2 = p1 \* xsub

p3 = p1 \* ysub

p4 = p1 \* zsub

Rem Initial guess of the value of s14

s14 = 0

Rem The following ii loop is used to let s14 be iteratively determined.

Rem In the application, it is found that "ii = 1 to 5" gives the same results (with the

Rem selected precision), as "ii = 1 to 10" or "ii = 1 to 20". To be safe, we use "ii = 1 to 10".

For ii = 1 To 10

 Rem Production, GNP, and consumption in three nations

 Rem N1:

 rat = p1 / p2

 rat2 = rat ^ 2

 x11 = (1 / m11 \* (rat2 / (m11 / m12 + rat2))) ^ (1 / 2)

 x12 = (1 / m12 - rat2 / (m11 + m12 \* rat2)) ^ (1 / 2)

 x13 = 0

 x14 = 0

 w1 = p1 \* x11 + p2 \* x12 + p4 \* (tariff / 100) \* (-s14)

 c11 = a11 \* w1 / p1

 c12 = a12 \* w1 / p2

 c13 = a13 \* w1 / p3

 c14 = a14 \* w1 / (p4 \* (1 + tariff / 100))

 u1 = c11 ^ a11 \* c12 ^ a12 \* c13 ^ a13 \* c14 ^ a14

 Rem N2:

 x21 = 0

 rat = p2 / p3

 rat2 = rat ^ 2

 x22 = (1 / m22 \* (rat2 / (m22 / m23 + rat2))) ^ (1 / 2)

 x23 = (1 / m23 - rat2 / (m22 + m23 \* rat2)) ^ (1 / 2)

 x24 = 0

 w2 = p2 \* x22 + p3 \* x23

 c21 = a21 \* w2 / p1

 c22 = a22 \* w2 / p2

 c23 = a23 \* w2 / p3

 c24 = a24 \* w2 / p4

 u2 = c21 ^ a21 \* c22 ^ a22 \* c23 ^ a23 \* c23 ^ a24

 Rem N3:

 x31 = 0

 x32 = 0

 rat = p3 / p4

 rat2 = rat ^ 2

 x33 = (1 / m33 \* (rat2 / (m33 / m34 + rat2))) ^ (1 / 2)

 x34 = (1 / m34 - rat2 / (m33 + m34 \* rat2)) ^ (1 / 2)

 w3 = p3 \* x33 + p4 \* x34

 c33 = a33 \* w3 / p3

 c34 = a34 \* w3 / p4

 u3 = (c33 ^ a33) \* (c34 ^ a34)

 wsum = w1 + w2 + w3

 usum = u1 + u2 + u3

 Rem Export supplies from three nations

 Rem N1:

 s11 = x11 - c11

 s12 = x12 - c12

 s13 = -c13

 s14 = -c14

 Rem N2:

 s21 = -c21

 s22 = x22 - c22

 s23 = x23 - c23

 s24 = -c24

 Rem N3:

 s31 = 0

 s32 = 0

 s33 = x33 - c33

 s34 = x34 - c34

 Rem Total export supplies

 es1 = s11 + s21 + s31

 es2 = s12 + s22 + s32

 es3 = s13 + s23 + s33

 es4 = s14 + s24 + s34

 fsub = es1

 gsub = es2

 hsub = es3

 tsub = es4

Next ii

Return

200 Rem Parameters

Rem initial conditions

x = 1

y = 1

z = 1

delta = 0.001

a11 = 1 / 4

a12 = 1 / 4

a13 = 1 / 4

a14 = 1 / 4

a21 = 1 / 4

a22 = 1 / 4

a23 = 1 / 4

a24 = 1 / 4

a33 = 1 / 2

a34 = 1 / 2

x11max = 2

x12max = 4

m11 = x11max ^ (-2)

m12 = x12max ^ (-2)

x22max = 2

x23max = 4

m22 = x22max ^ (-2)

m23 = x23max ^ (-2)

x33max = 2

x34max = 4

m33 = x33max ^ (-2)

m34 = x34max ^ (-2)

Return

300 Rem Parameter print outs

Print ""

Print " Production possibility frontiers parameters"

Print " ------------------------------------------"

Print " x11max, x12max, x13max, x14max = ";

Print Using "#####.###"; x11max; x12max; 0; 0

Print " x21max, x22max, x23max, x24max = ";

Print Using "#####.###"; 0; x22max; x23max; 0

Print " x31max, x32max, x33max, x34max = ";

Print Using "#####.###"; 0; 0; x33max; x34max

Print #2, ""

Print #2, " Production possibility frontiers parameters"

Print #2, " ------------------------------------------"

Print #2, " x11max, x12max, x13max, x14max = ";

Print #2, Using "#####.###"; x11max; x12max; 0; 0

Print #2, " x21max, x22max, x23max, x24max = ";

Print #2, Using "#####.###"; 0; x22max; x23max; 0

Print #2, " x31max, x32max, x33max, x34max = ";

Print #2, Using "#####.###"; 0; 0; x33max; x34max

Return

**Appendix 3**

***Alternative way to calculate the effects of the tariff based on iterative determination of the tariff revenue and the excess supply functions***

The software included below is developed in Smart Basic.

rem Tariff\_opt\_250529

rem Peter Lohmander

a = 1/2

for T = 0 to 2 step 0.2

print ""

print "Tariff = "T

x\_opt = 1

abss1\_opt = 10

u1\_eq = 0

u2\_eq = 0

usum\_eq = 0

for x = 0.5 to 2.5 step 0.001

Trev = 0

for i = 1 to 100

p1 = 1

p2 = p1\*x

p3 = p1\*(1+T)

r = p1/p2

z = p3/p2

x11 = 1/2\*z

x22 = 1/2\*r^(-1)

x12 = 4 - x11^2

x21 = 4 - x22^2

TRev = T\*(-s11)

B1 = p3\*x11 + p2\*x12 + Trev

B2 = p1\*x21 + p2\*x22

c11 = a\*B1/p3

c12 = a\*B1/p2

c21 = a\*B2/p1

c22 = a\*B2/p2

u1 = (c11^a)\*(c12^a)

u2 = (c21^a)\*(c22^a)

usum = u1 + u2

s11 = x11 - c11

s12 = x12 - c12

s21 = x21 - c21

s22 = x22 - c22

s1 = s11 + s21

s2 = s12 + s22

next i

abss1 = s1

if s1 < 0 then abss1 = -s1

if abss1 > abss1\_opt then goto 100

x\_opt = x

u1\_eq = u1

u2\_eq = u2

usum\_eq = usum

abss1\_opt = abss1

s11\_opt = s11

s12\_opt = s12

100 rem

next x

p1 = 100

p2 = p1\*x\_opt

p3 = p1\*(1+T)

print "abss1\_opt = "abss1\_opt

print "x\_opt = "x\_opt

print "p1, p2, p3 = " p1;p2;p3

print "s11\_opt = "s11\_opt

print "s12\_opt = "s12\_opt

print "u1\_eq = "u1\_eq

print "u2\_eq = "u2\_eq

print "usum\_eq = "usum\_eq

next T

***Results from the alternative way to calculate the effects of the tariff based on iterative determination of the tariff revenue and the excess supply functions***

The following results clearly show that both methods led to almost the same numerical results. You may compare the results for the tariff levels 0 and 100%, reported for both methods.

Tariff = 0

abss1\_opt = 1.11022E-15

x\_opt = 1

p1, p2, p3 = 100 100 100

s11\_opt = -1.625

s12\_opt = 1.625

u1\_eq = 2.125

u2\_eq = 2.125

usum\_eq = 4.25

Tariff = 0.2

abss1\_opt = 0.00137125

x\_opt = 1.098

p1, p2, p3 = 100 109.8 120

s11\_opt = -1.54927

s12\_opt = 1.41099

u1\_eq = 2.1909

u2\_eq = 2.05248

usum\_eq = 4.24338

Tariff = 0.4

abss1\_opt = 0.000335049

x\_opt = 1.19

p1, p2, p3 = 100 119 140

s11\_opt = -1.46863

s12\_opt = 1.23414

u1\_eq = 2.23098

u2\_eq = 1.99566

usum\_eq = 4.22664

Tariff = 0.6

abss1\_opt = 0.000809416

x\_opt = 1.278

p1, p2, p3 = 100 127.8 160

s11\_opt = -1.38833

s12\_opt = 1.08633

u1\_eq = 2.25382

u2\_eq = 1.94974

usum\_eq = 4.20357

Tariff = 0.8

abss1\_opt = 0.00124471

x\_opt = 1.361

p1, p2, p3 = 100 136.1 180

s11\_opt = -1.30662

s12\_opt = 0.960047

u1\_eq = 2.26314

u2\_eq = 1.91283

usum\_eq = 4.17596

Tariff = 1

abss1\_opt = 0.000260009

x\_opt = 1.439

p1, p2, p3 = 100 143.9 200

s11\_opt = -1.22374

s12\_opt = 0.85041

u1\_eq = 2.26196

u2\_eq = 1.88302

usum\_eq = 4.14498

Tariff = 1.2

abss1\_opt = 6.37731E-05

x\_opt = 1.513

p1, p2, p3 = 100 151.3 220

s11\_opt = -1.1415

s12\_opt = 0.75446

u1\_eq = 2.25316

u2\_eq = 1.85859

usum\_eq = 4.11175

Tariff = 1.4

abss1\_opt = 0.000584442

x\_opt = 1.583

p1, p2, p3 = 100 158.3 240

s11\_opt = -1.05971

s12\_opt = 0.66943

u1\_eq = 2.23822

u2\_eq = 1.83857

usum\_eq = 4.07678

Tariff = 1.6

abss1\_opt = 0.00073548

x\_opt = 1.65

p1, p2, p3 = 100 165 260

s11\_opt = -0.979798

s12\_opt = 0.593817

u1\_eq = 2.21895

u2\_eq = 1.82193

usum\_eq = 4.04088

Tariff = 1.8

abss1\_opt = 0.000237209

x\_opt = 1.713

p1, p2, p3 = 100 171.3 280

s11\_opt = -0.899849

s12\_opt = 0.525306

u1\_eq = 2.19535

u2\_eq = 1.80835

usum\_eq = 4.0037

Tariff = 2

abss1\_opt = 4.67253E-05

x\_opt = 1.773

p1, p2, p3 = 100 177.3 300

s11\_opt = -0.821223

s12\_opt = 0.463183

u1\_eq = 2.16873

u2\_eq = 1.79712

usum\_eq = 3.96586