Appendix 1

Linear Model: Examples of Results and Software

```
Tariff Results by Peter Lohmander

Solutions without trade:
------------------------
 Country        Price   Production  Consumption
   1.000        7.000        3.000        3.000
   2.000        3.500        2.500        2.500

Country                          1         2
Consumer surplus             4.500     3.125
Pmin1 and Pmin2              4.000     1.000
Producer surplus             4.500     3.125
Total surplus                9.000     6.250

Total surplus World without trade     15.250


Solutions with free trade without tariffs:
------------------------------------------
Free trade Price =  5.25

 Country        Price   Production  Consumption
   1.000        5.250        1.250        4.750
   2.000        5.250        4.250        0.750

Exp1F and Exp2F =     -3.500     3.500

Country                          1         2
Consumer surplus            11.281     0.281
Pmin1 and Pmin2              4.000     1.000
Producer surplus             0.781     9.031
Total surplus               12.063     9.313

Total surplus World with free trade     21.375


Numerical results from discrete tariff optimization:
----------------------------------------------------
    topt      pf   ptopt  ptlopt  resopt
    1.20    5.25    4.65    5.85    1.02
```

```
Analytical results:
-------------------
topt =     1.167
resopt =     1.021

Solution with trade and tariff optimized by Country 1:
------------------------------------------------------
Optimized Tariff =     1.167

dSUR1, dSUR2, dSURW =     1.021   -1.701   -0.681

 Country        Price   Production  Consumption
   1.000        5.833        1.833        4.167
   2.000        4.667        3.667        1.333

Exp1t and Exp2t          -2.333     2.333

Country                       1         2
Consumer surplus          8.681     0.889
Producer surplus          1.681     6.722
Tariff gain               2.722
Total surplus            13.083     7.611
Total surplus World with optimal tariff     20.694
```

```
Rem
Rem smalltariff 250507_2336.bas
Rem Peter Lohmander
Rem

DefDbl A-Z

Screen _NewImage(1800, 2000, 256)

Open "C:\Users\Peter\OneDrive\Desktop\TariffOut\TariffOut.txt" For Output As #2

Print ""
Print "     Tariff Results by Peter Lohmander"
Rem Print ""
Print #2, ""
Print #2, "     Tariff Results by Peter Lohmander"
Print #2, ""

Rem The parmeters of the markets
a11 = 10
b11 = 1
g11 = 4
h11 = 1

a21 = 6
b21 = 1
g21 = 1
h21 = 1

Rem Determination of the solution without trade
Print "     Solutions without trade:"
Print "     ------------------------ "
Print #2, "     Solutions without trade:"
Print #2, "     ------------------------ "

Rem Determination of prices and quantitites in different countries without trade

P1N = (a11 + g11) / (b11 + h11)
Cons1N = a11 - b11 * P1N
Prod1N = -g11 + h11 * P1N

P2N = (a21 + g21) / (b21 + h21)
Cons2N = a21 - b21 * P2N
Prod2N = -g21 + h21 * P2N

Print "     Country       Price   Production  Consumption "
```

```
Print Using "#########.###"; 1; P1N; Prod1N; Cons1N
Print Using "#########.###"; 2; P2N; Prod2N; Cons2N
Print #2, "      Country       Price   Production  Consumption "
Print #2, Using "#########.###"; 1; P1N; Prod1N; Cons1N
Print #2, Using "#########.###"; 2; P2N; Prod2N; Cons2N

Rem Determination of the surpluses without trade

CSURP1N = (a11 - P1N) / 2 * Cons1N
CSURP2N = (a21 - P2N) / 2 * Cons2N
Print ""
Print "     Country                  1        2"; ""
Print "     Consumer surplus     ";
Print Using "######.###"; CSURP1N; CSURP2N
Print #2, ""
Print #2, "     Country                  1        2"; ""
Print #2, "     Consumer surplus     ";
Print #2, Using "######.###"; CSURP1N; CSURP2N

Rem Price when production profitability is zero
Pmin1 = g11 / h11
Pmin2 = g21 / h21

Print "     Pmin1 and Pmin2      ";
Print Using "######.###"; Pmin1; Pmin2
Print #2, "     Pmin1 and Pmin2      ";
Print #2, Using "######.###"; Pmin1; Pmin2


PSURP1N = (P1N - Pmin1) / 2 * Prod1N
PSURP2N = (P2N - Pmin2) / 2 * Prod2N

Print "     Producer surplus     ";
Print Using "######.###"; PSURP1N; PSURP2N
Print #2, "     Producer surplus     ";
Print #2, Using "######.###"; PSURP1N; PSURP2N

TSURP1N = CSURP1N + PSURP1N
TSURP2N = CSURP2N + PSURP2N
TSURPWN = TSURP1N + TSURP2N

Print "     Total surplus        ";
Print Using "######.###"; TSURP1N; TSURP2N
Print ""
Print "     Total surplus World without trade ";
Print Using "######.###"; TSURPWN
Rem Print ""
```

```
Print #2, "     Total surplus        ";
Print #2, Using "######.###"; TSURP1N; TSURP2N
Print #2, ""
Print #2, "     Total surplus World without trade ";
Print #2, Using "######.###"; TSURPWN
Print #2, ""

Rem Determination of the free trade solution without tariffs

Print ""
Print "     Solutions with free trade without tariffs:"
Print "     ----------------------------------------     "
Print #2, ""
Print #2, "     Solutions with free trade without tariffs:"
Print #2, "     ----------------------------------------     "

pf = (a11 + a21 + g11 + g21) / (b11 + b21 + h11 + h21)

Print "     Free trade Price = "; pf
Rem Print ""
Print #2, "     Free trade Price = "; pf
Print #2, ""

Rem Determination of prices and quantitites in different countries with free trade

Cons1F = a11 - b11 * pf
Prod1F = -g11 + h11 * pf

Cons2f = a21 - b21 * pf
Prod2f = -g21 + h21 * pf

Print "     Country       Price   Production  Consumption "
Print Using "##########.###"; 1; pf; Prod1F; Cons1F
Print Using "##########.###"; 2; pf; Prod2f; Cons2f
Print #2, "     Country       Price   Production  Consumption "
Print #2, Using "##########.###"; 1; pf; Prod1F; Cons1F
Print #2, Using "##########.###"; 2; pf; Prod2f; Cons2f

Rem Determination of export levels from the different countries with free trade
Rem (Import is export *(-1) )

Exp1F = Prod1F - Cons1F
Exp2F = Prod2f - Cons2f

Print ""
Print "     Exp1F and Exp2F = ";
Print Using "######.###"; Exp1F; Exp2F
```

```
Print #2, ""
Print #2, "     Exp1F and Exp2F = ";
Print #2, Using "######.###"; Exp1F; Exp2F

Rem Determination of the surpluses with free trade

CSURP1f = (a11 - pf) / 2 * Cons1F
CSURP2f = (a21 - pf) / 2 * Cons2f
Print ""
Print "     Country                  1        2"; ""
Print "     Consumer surplus     ";
Print Using "######.###"; CSURP1f; CSURP2f
Print #2, ""
Print #2, "     Country                  1        2"; ""
Print #2, "     Consumer surplus     ";
Print #2, Using "######.###"; CSURP1f; CSURP2f

Rem Price when production profitability is zero
Pmin1 = g11 / h11
Pmin2 = g21 / h21

Print "     Pmin1 and Pmin2      ";
Print Using "######.###"; Pmin1; Pmin2
Print #2, "     Pmin1 and Pmin2      ";
Print #2, Using "######.###"; Pmin1; Pmin2

PSURP1f = (pf - Pmin1) / 2 * Prod1F
PSURP2f = (pf - Pmin2) / 2 * Prod2f

Print "     Producer surplus     ";
Print Using "######.###"; PSURP1f; PSURP2f
Print #2, "     Producer surplus     ";
Print #2, Using "######.###"; PSURP1f; PSURP2f

TSURP1f = CSURP1f + PSURP1f
TSURP2f = CSURP2f + PSURP2f
TSURPWF = TSURP1f + TSURP2f

Print "     Total surplus        ";
Print Using "######.###"; TSURP1f; TSURP2f
Print ""
Print #2, "     Total surplus        ";
Print #2, Using "######.###"; TSURP1f; TSURP2f
Print #2, ""

Print "     Total surplus World with free trade ";
Print Using "######.###"; TSURPWF
```

```
Print ""

Print #2, "     Total surplus World with free trade ";
Print #2, Using "######.###"; TSURPWF
Print #2, ""
Print #2, ""

Rem Simple discrete numerical optimization of the tariff
resopt = -100
For T = 0 To 5 Step 0.1

   pt = pf - (h11 + b11) / (b11 + b21 + h11 + h21) * T

   ptl = pt + T

   c11f = a11 - b11 * pf
   c11tl = a11 - b11 * ptl

   x11f = -g11 + h11 * pf
   x11tl = -g11 + h11 * ptl

   s11 = x11tl - c11tl

   R = -s11 * (pf - pt)
   y = (ptl - pf) * (c11f - c11tl) / 2
   z = (ptl - pf) * (x11tl - x11f) / 2

   res = R - y - z

   If res < resopt Then GoTo 100
   resopt = res
   topt = T
   ptopt = pt
   ptlopt = ptl
   100 Rem
Next T

Print "     Numerical results from discrete tariff optimization:"
Print "     ----------------------------------------------------  "
Print "        topt      pf   ptopt  ptlopt  resopt "
Print "     ";
Print Using "#####.##"; topt; pf; ptopt;  ptlopt; resopt

Print #2, "     Numerical results from discrete tariff optimization:"
Print #2, "     ---------------------------------------------------- "
Print #2, "        topt      pf   ptopt  ptlopt  resopt "
Print #2, "     ";
```

```
Print #2, Using "#####.##"; topt; pf; ptopt; ptlopt; resopt

Rem Analytical section

N = (b11 + h11) / (b11 + b21 + h11 + h21)

k1 = (a11 + g11 - (b11 + h11) * pf) * N

k2 = (b11 + h11) * (1 - N) * N

k3 = b11 / 2 * (1 - N) ^ 2

k4 = h11 / 2 * (1 - N) ^ 2

topt = k1 / (2 * (k2 + k3 + k4))

ropt = k1 * topt - k2 * topt ^ 2
yopt = k3 * topt ^ 2
zopt = k4 * topt ^ 2

resopt = ropt - yopt - zopt
dSUR1 = resopt

Print ""
Print "     Analytical results:"
Print "     ------------------- "
Print "     topt = ";
Print Using "#####.###"; topt
Print "     resopt = ";
Print Using "#####.###"; resopt
Print #2, ""
Print #2, "     Analytical results:"
Print #2, "     ------------------- "
Print #2, "     topt = ";
Print #2, Using "#####.###"; topt
Print #2, "     resopt = ";
Print #2, Using "#####.###"; resopt

Rem Print ""
Print "     Solution with trade and tariff optimized by Country 1:"
Print "     ----------------------------------------------------  "

Print #2, ""
Print #2, "     Solution with trade and tariff optimized by Country 1:"
Print #2, "     ----------------------------------------------------  "

Rem Determination of prices in different countries with optimized tariff.
```

```
T = topt
Rem Print ""
Print "     Optimized Tariff = ";
Print Using "#####.###"; T
Print ""
Print #2, "     Optimized Tariff = ";
Print #2, Using "#####.###"; T
Print #2, ""

Rem Some effects of the optimized tariff on the exporting country.
Rem p0 is the price if the exported volume is zero.
p0 = (a21 + g21) / (b21 + h21)

Rem pt = Price outside Country 1, where the tariff is used.
pt = pf - N * T
p2 = pt

Rem ptl = Local price in Country 1, where the tariff is used.
ptl = pt + T
p1 = ptl

Rem Special calculation of the change of the surplus

s21f = -(a21 + g21) + (b21 + h21) * pf
s21t = -(a21 + g21) + (b21 + h21) * pt

sur2f = (pf - p0) * s21f / 2
sur2t = (pt - p0) * s21t / 2
dsur2 = sur2t - sur2f
dSURW = dSUR1 + dsur2

Print "     dSUR1, dSUR2, dSURW = ";
Print Using "#####.###"; dSUR1; dsur2; dSURW
Print ""
Print #2, "     dSUR1, dSUR2, dSURW = ";
Print #2, Using "#####.###"; dSUR1; dsur2; dSURW
Print #2, ""

Rem Determination of consumption and production levels with tariffs

Cons1t = a11 - b11 * p1
Prod1t = -g11 + h11 * p1

Cons2t = a21 - b21 * p2
Prod2t = -g21 + h21 * p2

Print "     Country       Price   Production  Consumption "
```

```
Print Using "#########.###"; 1; p1; Prod1t; Cons1t
Print Using "#########.###"; 2; p2; Prod2t; Cons2t
Print #2, "      Country        Price   Production  Consumption "
Print #2, Using "#########.###"; 1; p1; Prod1t; Cons1t
Print #2, Using "#########.###"; 2; p2; Prod2t; Cons2t

Rem Determination of export levels from the different countries with free trade
Rem (Import is export *(-1) )

Exp1t = Prod1t - Cons1t
Exp2t = Prod2t - Cons2t

Print ""
Print "     Exp1t and Exp2t      ";
Print Using "######.###"; Exp1t; Exp2t
Print #2, ""
Print #2, "     Exp1t and Exp2t      ";
Print #2, Using "######.###"; Exp1t; Exp2t

Rem Determination of the surpluses with free trade

CSURP1t = (a11 - p1) / 2 * Cons1t
CSURP2t = (a21 - p2) / 2 * Cons2t
Rem Print ""
Print "     Country                  1        2"; ""
Print "     Consumer surplus     ";
Print Using "######.###"; CSURP1t; CSURP2t

Print #2, ""
Print #2, "     Country                  1        2"; ""
Print #2, "     Consumer surplus     ";
Print #2, Using "######.###"; CSURP1t; CSURP2t

PSURP1t = (p1 - Pmin1) / 2 * Prod1t
PSURP2t = (p2 - Pmin2) / 2 * Prod2t

Print "     Producer surplus     ";
Print Using "######.###"; PSURP1t; PSURP2t
Print #2, "     Producer surplus     ";
Print #2, Using "######.###"; PSURP1t; PSURP2t

GSURP1t = -Exp1t * T
Print "     Tariff gain          ";
Print Using "######.###"; GSURP1t
Print #2, "     Tariff gain          ";
Print #2, Using "######.###"; GSURP1t
TSURP1t = CSURP1t + PSURP1t + GSURP1t
```

```
TSURP2t = CSURP2t + PSURP2t
TSURPWt = TSURP1t + TSURP2t

Print "     Total surplus        ";
Print Using "######.###"; TSURP1t; TSURP2t
Print #2, "     Total surplus        ";
Print #2, Using "######.###"; TSURP1t; TSURP2t
Print "     Total surplus World with optimal tariff ";
Print Using "######.###"; TSURPWt
Print ""
Print #2, "     Total surplus World with optimal tariff ";
Print #2, Using "######.###"; TSURPWt
Print #2, ""

Close #2
End
```

Appendix 2

Nonlinear Model: Examples of Results and Software

Results and software by Peter Lohmander 2025–06-02

Case 0

Tariff (%) = 0

Production possibility frontiers parameters

---

x11max, x12max, x13max, x14max = 2.000 4.000 0.000 0.000
x21max, x22max, x23max, x24max = 0.000 2.000 4.000 0.000
x31max, x32max, x33max, x34max = 0.000 0.000 2.000 4.000

Total sum of squared excess supplies = tesopt = 6.456329543206307D-08

Equilibrium prices = p1, p2, p3, p4 = 100.000 45.449 77.129 95.391

Global Excess Supplies = es1, es2, es3, es4 = 0.00019 -0.00010 -0.00005 -0.00012

Production levels =

x11, x12, x13, x14 = 1.480 2.691 0.000 0.000
x21, x22, x23, x24 = 0.000 0.565 3.837 0.000
x31, x32, x33, x34 = 0.000 0.000 0.750 3.708

Excess supply levels =

s11, s12, s13, s14 = 0.804 1.204 -0.876 -0.708
s21, s22, s23, s24 = -0.804 -1.204 2.794 -0.843
s31, s32, s33, s34 = 0.000 0.000 -1.918 1.551

Consumption levels =

c11, c12, c13, c14 = 0.676 1.487 0.876 0.708
c21, c22, c22, c24 = 0.804 1.769 1.043 0.843
c31, c32, c33, c34 = 0.000 0.000 2.668 2.157

GNP levels = w1, w2 ,w3 = 270.278 321.628 411.565

Utility levels = u1, u2, u3 = 0.889 1.115 2.399

Global GNP = 1003.470
Global Utility = 4.403

Case 0

Tariff (%) = 50

Production possibility frontiers parameters
---
x11max, x12max, x13max, x14max = 2.000 4.000 0.000 0.000
x21max, x22max, x23max, x24max = 0.000 2.000 4.000 0.000
x31max, x32max, x33max, x34max = 0.000 0.000 2.000 4.000

Total sum of squared excess supplies = tesopt = 3.99792725031482D-08

Equilibrium prices = p1, p2, p3, p4 = 100.000 45.176 71.396 82.832

Global Excess Supplies = es1, es2, es3, es4 = 0.00003 0.00003 0.00012 -0.00015

Production levels =

x11, x12, x13, x14 = 1.484 2.682 0.000 0.000
x21, x22, x23, x24 = 0.000 0.603 3.814 0.000
x31, x32, x33, x34 = 0.000 0.000 0.792 3.673

Excess supply levels =

s11, s12, s13, s14 = 0.749 1.054 -1.030 -0.592
s21, s22, s23, s24 = -0.749 -1.054 2.765 -0.904
s31, s32, s33, s34 = 0.000 0.000 -1.735 1.496

Consumption levels =

c11, c12, c13, c14 = 0.735 1.627 1.030 0.592
c21, c22, c22, c24 = 0.749 1.658 1.049 0.904
c31, c32, c33, c34 = 0.000 0.000 2.527 2.178

GNP levels = w1, w2 ,w3 = 294.047 299.538 360.788

Utility levels = u1, u2, u3 = 0.924 1.081 2.346

Global GNP = 954.373
Global Utility = 4.351

Case 0

Tariff (%) = 100

Production possibility frontiers parameters
---
x11max, x12max, x13max, x14max = 2.000 4.000 0.000 0.000
x21max, x22max, x23max, x24max = 0.000 2.000 4.000 0.000
x31max, x32max, x33max, x34max = 0.000 0.000 2.000 4.000

Total sum of squared excess supplies = tesopt = 8.681383082576513D-08

Equilibrium prices = p1, p2, p3, p4 = 100.000 45.000 68.145 75.742

Global Excess Supplies = es1, es2, es3, es4 = 0.00018 -0.00021 -0.00001 -0.00011

Production levels =

x11, x12, x13, x14 = 1.487 2.676 0.000 0.000
x21, x22, x23, x24 = 0.000 0.627 3.798 0.000
x31, x32, x33, x34 = 0.000 0.000 0.820 3.648

Excess supply levels =

s11, s12, s13, s14 = 0.718 0.967 -1.128 -0.507
s21, s22, s23, s24 = -0.718 -0.968 2.745 -0.947
s31, s32, s33, s34 = 0.000 0.000 -1.617 1.455

Consumption levels =

c11, c12, c13, c14 = 0.769 1.708 1.128 0.507
c21, c22, c22, c24 = 0.718 1.595 1.053 0.947
c31, c32, c33, c34 = 0.000 0.000 2.438 2.193

GNP levels = w1, w2 ,w3 = 307.511 287.052 332.212

Utility levels = u1, u2, u3 = 0.931 1.061 2.312

Global GNP = 926.775
Global Utility = 4.305

Case 0

Tariff (%) = 150

Production possibility frontiers parameters
---
x11max, x12max, x13max, x14max = 2.000 4.000 0.000 0.000
x21max, x22max, x23max, x24max = 0.000 2.000 4.000 0.000
x31max, x32max, x33max, x34max = 0.000 0.000 2.000 4.000

Total sum of squared excess supplies = tesopt = 1.04538358161312D-07

Equilibrium prices = p1, p2, p3, p4 = 100.000 44.893 66.074 71.221

Global Excess Supplies = es1, es2, es3, es4 = -0.00020 0.00023 0.00002 0.00011

Production levels =

x11, x12, x13, x14 = 1.488 2.672 0.000 0.000
x21, x22, x23, x24 = 0.000 0.643 3.787 0.000
x31, x32, x33, x34 = 0.000 0.000 0.842 3.629

Excess supply levels =

s11, s12, s13, s14 = 0.698 0.911 -1.196 -0.444
s21, s22, s23, s24 = -0.698 -0.911 2.731 -0.980
s31, s32, s33, s34 = 0.000 0.000 -1.535 1.424

Consumption levels =

c11, c12, c13, c14 = 0.791 1.761 1.196 0.444
c21, c22, c22, c24 = 0.698 1.554 1.056 0.980
c31, c32, c33, c34 = 0.000 0.000 2.376 2.205

GNP levels = w1, w2 ,w3 = 316.218 279.131 314.040

Utility levels = u1, u2, u3 = 0.927 1.049 2.289

Global GNP = 909.390
Global Utility = 4.265

Case 0

Tariff (%) = 200

Production possibility frontiers parameters
---
x11max, x12max, x13max, x14max = 2.000 4.000 0.000 0.000
x21max, x22max, x23max, x24max = 0.000 2.000 4.000 0.000
x31max, x32max, x33max, x34max = 0.000 0.000 2.000 4.000

Total sum of squared excess supplies = tesopt = 4.90099160193288D-08

Equilibrium prices = p1, p2, p3, p4 = 100.000 44.805 64.619 68.057

Global Excess Supplies = es1, es2, es3, es4 = -0.00010 -0.00006 0.00019 -0.00000

Production levels =

x11, x12, x13, x14 = 1.489 2.669 0.000 0.000
x21, x22, x23, x24 = 0.000 0.655 3.779 0.000
x31, x32, x33, x34 = 0.000 0.000 0.858 3.613

Excess supply levels =

s11, s12, s13, s14 = 0.684 0.871 -1.247 -0.395
s21, s22, s23, s24 = -0.684 -0.871 2.721 -1.005
s31, s32, s33, s34 = 0.000 0.000 -1.474 1.400

Consumption levels =

c11, c12, c13, c14 = 0.806 1.798 1.247 0.395
c21, c22, c22, c24 = 0.684 1.526 1.058 1.005
c31, c32, c33, c34 = 0.000 0.000 2.332 2.214

GNP levels = w1, w2 ,w3 = 322.261 273.569 301.347

Utility levels = u1, u2, u3 = 0.919 1.040 2.272

Global GNP = 897.176
Global Utility = 4.231

Case 1 (x22max and x23max have been changed)

Tariff (%) = 0

Production possibility frontiers parameters
---
x11max, x12max, x13max, x14max = 2.000 4.000 0.000 0.000
x21max, x22max, x23max, x24max = 0.000 4.000 2.000 0.000
x31max, x32max, x33max, x34max = 0.000 0.000 2.000 4.000

Total sum of squared excess supplies = tesopt = 1.031094533439831D-08

Equilibrium prices = p1, p2, p3, p4 = 100.000 38.730 173.213 144.229

Global Excess Supplies = es1, es2, es3, es4 = -0.00005 -0.00006 -0.00001 0.00007

Production levels =

x11, x12, x13, x14 = 1.581 2.450 0.000 0.000
x21, x22, x23, x24 = 0.000 1.633 1.826 0.000
x31, x32, x33, x34 = 0.000 0.000 1.030 3.429

Excess supply levels =

s11, s12, s13, s14 = 0.949 0.817 -0.365 -0.439
s21, s22, s23, s24 = -0.949 -0.817 1.278 -0.658
s31, s32, s33, s34 = 0.000 0.000 -0.913 1.096

Consumption levels =

c11, c12, c13, c14 = 0.632 1.633 0.365 0.439
c21, c22, c22, c24 = 0.949 2.450 0.548 0.658
c31, c32, c33, c34 = 0.000 0.000 1.943 2.333

GNP levels = w1, w2 ,w3 = 252.984 379.489 672.934

Utility levels = u1, u2, u3 = 0.638 0.914 2.129

Global GNP = 1305.407
Global Utility = 3.680

Case 1 (x22max and x23max have been changed)

Tariff (%) = 50

Production possibility frontiers parameters
---
x11max, x12max, x13max, x14max = 2.000 4.000 0.000 0.000
x21max, x22max, x23max, x24max = 0.000 4.000 2.000 0.000
x31max, x32max, x33max, x34max = 0.000 0.000 2.000 4.000

Total sum of squared excess supplies = tesopt = 7.515735279736754D-08

Equilibrium prices = p1, p2, p3, p4 = 100.000 38.340 162.744 130.654

Global Excess Supplies = es1, es2, es3, es4 = 0.00024 -0.00008 -0.00010 -0.00003

Production levels =

x11, x12, x13, x14 = 1.587 2.434 0.000 0.000
x21, x22, x23, x24 = 0.000 1.705 1.809 0.000
x31, x32, x33, x34 = 0.000 0.000 1.057 3.395

Excess supply levels =

s11, s12, s13, s14 = 0.900 0.641 -0.422 -0.351
s21, s22, s23, s24 = -0.900 -0.641 1.257 -0.688
s31, s32, s33, s34 = 0.000 0.000 -0.834 1.039

Consumption levels =

c11, c12, c13, c14 = 0.687 1.793 0.422 0.351
c21, c22, c22, c24 = 0.900 2.346 0.553 0.688
c31, c32, c33, c34 = 0.000 0.000 1.892 2.356

GNP levels = w1, w2 ,w3 = 274.942 359.808 615.688

Utility levels = u1, u2, u3 = 0.654 0.896 2.111

Global GNP = 1250.437
Global Utility = 3.661

Case 1 (x22max and x23max have been changed)

Tariff (%) = 100

Production possibility frontiers parameters
---
x11max, x12max, x13max, x14max = 2.000 4.000 0.000 0.000
x21max, x22max, x23max, x24max = 0.000 4.000 2.000 0.000
x31max, x32max, x33max, x34max = 0.000 0.000 2.000 4.000

Total sum of squared excess supplies = tesopt = 1.027059118454071D-08

Equilibrium prices = p1, p2, p3, p4 = 100.000 38.115 156.875 123.027

Global Excess Supplies = es1, es2, es3, es4 = -0.00005 0.00008 -0.00002 0.00004

Production levels =

x11, x12, x13, x14 = 1.591 2.425 0.000 0.000
x21, x22, x23, x24 = 0.000 1.748 1.799 0.000
x31, x32, x33, x34 = 0.000 0.000 1.075 3.373

Excess supply levels =

s11, s12, s13, s14 = 0.872 0.540 -0.458 -0.292
s21, s22, s23, s24 = -0.872 -0.540 1.243 -0.709
s31, s32, s33, s34 = 0.000 0.000 -0.785 1.001

Consumption levels =

c11, c12, c13, c14 = 0.719 1.885 0.458 0.292
c21, c22, c22, c24 = 0.872 2.288 0.556 0.709
c31, c32, c33, c34 = 0.000 0.000 1.860 2.372

GNP levels = w1, w2 ,w3 = 287.411 348.831 583.619

Utility levels = u1, u2, u3 = 0.652 0.886 2.100

Global GNP = 1219.861
Global Utility = 3.639

Case 1 (x22max and x23max have been changed)

Tariff (%) = 150

Production possibility frontiers parameters
---
x11max, x12max, x13max, x14max = 2.000 4.000 0.000 0.000
x21max, x22max, x23max, x24max = 0.000 4.000 2.000 0.000
x31max, x32max, x33max, x34max = 0.000 0.000 2.000 4.000

Total sum of squared excess supplies = tesopt = 4.259192187127558D-08

Equilibrium prices = p1, p2, p3, p4 = 100.000 37.959 153.057 118.086

Global Excess Supplies = es1, es2, es3, es4 = 0.00015 -0.00002 0.00001 -0.00014

Production levels =

x11, x12, x13, x14 = 1.593 2.419 0.000 0.000
x21, x22, x23, x24 = 0.000 1.777 1.792 0.000
x31, x32, x33, x34 = 0.000 0.000 1.088 3.357

Excess supply levels =

s11, s12, s13, s14 = 0.854 0.473 -0.483 -0.250
s21, s22, s23, s24 = -0.854 -0.473 1.234 -0.723
s31, s32, s33, s34 = 0.000 0.000 -0.751 0.973

Consumption levels =

c11, c12, c13, c14 = 0.739 1.946 0.483 0.250
c21, c22, c22, c24 = 0.854 2.250 0.558 0.723
c31, c32, c33, c34 = 0.000 0.000 1.839 2.383

GNP levels = w1, w2 ,w3 = 295.419 341.701 562.862

Utility levels = u1, u2, u3 = 0.645 0.880 2.093

Global GNP = 1199.982
Global Utility = 3.618

Case 1 (x22max and x23max have been changed)

Tariff (%) = 200

Production possibility frontiers parameters
---
x11max, x12max, x13max, x14max = 2.000 4.000 0.000 0.000
x21max, x22max, x23max, x24max = 0.000 4.000 2.000 0.000
x31max, x32max, x33max, x34max = 0.000 0.000 2.000 4.000

Total sum of squared excess supplies = tesopt = 2.059154126645428D-08

Equilibrium prices = p1, p2, p3, p4 = 100.000 37.852 150.420 114.668

Global Excess Supplies = es1, es2, es3, es4 = 0.00008 -0.00001 0.00003 -0.00011

Production levels =

x11, x12, x13, x14 = 1.595 2.414 0.000 0.000
x21, x22, x23, x24 = 0.000 1.798 1.787 0.000
x31, x32, x33, x34 = 0.000 0.000 1.097 3.345

Excess supply levels =

s11, s12, s13, s14 = 0.842 0.426 -0.500 -0.219
s21, s22, s23, s24 = -0.842 -0.426 1.227 -0.734
s31, s32, s33, s34 = 0.000 0.000 -0.726 0.953

Consumption levels =

c11, c12, c13, c14 = 0.753 1.988 0.500 0.219
c21, c22, c22, c24 = 0.842 2.224 0.560 0.734
c31, c32, c33, c34 = 0.000 0.000 1.823 2.392

GNP levels = w1, w2 ,w3 = 301.016 336.791 548.529

Utility levels = u1, u2, u3 = 0.636 0.875 2.088

Global GNP = 1186.336
Global Utility = 3.600

The software

```
Rem
Rem largetariff_Alt_250602_1929.bas
Rem NOTE In this Alt version, alternative
Rem production possibility frontiers are introduced.
Rem Peter Lohmander
Rem

DefDbl A-Z

Screen _NewImage(1800, 2000, 256)

Open "C:\Users\Peter\OneDrive\Desktop\TariffOut\L_TariffOut.txt" For Output As #2

Input "tariff = ", tariff
Print ""
Print "     Tariff (%) = "; tariff
Print #2, ""
Print #2, "     Tariff (%) = "; tariff


Rem Collect parameters
GoSub 200
Rem Print some parameters
GoSub 300

Rem Total squares of excess supplies

Rem initial conditions
xopttot = 1
yopttot = 1
zopttot = 1

tseopt = 99999999
xopt = 0
yopt = 0
zopt = 0
```

```
For tot = 1 To 10

   steg = 0.1 / (2 ^ tot)

   For ix = xopttot - 10 * steg To xopttot + 10 * steg Step steg
      x = ix
      For iy = yopttot - 10 * steg To yopttot + 10 * steg Step steg
         y = iy
         For iz = zopttot - 10 * steg To zopttot + 10 * steg Step steg
            z = iz
            xsub = x
            ysub = y
            zsub = z
            GoSub 100
            tseloc = fsub * fsub + gsub * gsub + hsub * hsub + tsub * tsub
            If tseloc > tseopt Then GoTo 700
            tseopt = tseloc
            xopt = x
            yopt = y
            zopt = z
            700 Rem
         Next iz
      Next iy
   Next ix

   xopttot = xopt
   yopttot = yopt
   zopttot = zopt

Next tot

Print ""
Print "     Total sum of squared excess supplies = tesopt = "; tseopt
Print ""
Print #2, ""
Print #2, "     Total sum of squared excess supplies = tesopt = "; tseopt
Print #2, ""

Rem The optimal results are collected from from the subroutine.

xsub = xopttot
ysub = yopttot
zsub = zopttot
GoSub 100
```

```
Rem The optimal results are printed.

Print ""
Print #2, ""

Print "     Equilibrium prices     = ";
Print "     p1, p2, p3, p4         = ";
Print Using "######.###"; p1; p2; p3; p4
Print ""
Print "     Global Excess Supplies = ";
Print "     es1, es2, es3, es4     = ";
Print Using "####.#####"; es1; es2; es3; es4
Print #2, "     Equilibrium prices     = ";
Print #2, "     p1, p2, p3, p4         = ";
Print #2, Using "######.###"; p1; p2; p3; p4
Print #2, ""
Print #2, "     Global Excess Supplies = ";
Print #2, "     es1, es2, es3, es4     = ";
Print #2, Using "####.#####"; es1; es2; es3; es4

Print ""
Print "     Production levels = "
Print ""
Print "     x11, x12, x13, x14 = ";
Print Using "####.###"; x11; x12, x13, x14
Print "     x21, x22, x23, x24 = ";
Print Using "####.###"; x21, x22; x23, x24
Print "     x31, x32, x33, x34 = ";
Print Using "####.###"; x31, x32, x33; x34
Print " "
Print "     Excess supply levels = "
Print ""
Print "     s11, s12, s13, s14 = ";
Print Using "####.###"; s11; s12; s13; s14
Print "     s21, s22, s23, s24 = ";
Print Using "####.###"; s21; s22; s23; s24
Print "     s31, s32, s33, s34 = ";
Print Using "####.###"; s31; s32; s33; s34
Print #2, ""
Print #2, "     Production levels = "
Print #2, ""
Print #2, "     x11, x12, x13, x14 = ";
Print #2, Using "####.###"; x11; x12, x13, x14
Print #2, "     x21, x22, x23, x24 = ";
Print #2, Using "####.###"; x21, x22; x23, x24
```

```
Print #2, "     x31, x32, x33, x34 = ";
Print #2, Using "####.###"; x31, x32, x33; x34
Print #2, " "
Print #2, "     Excess supply levels = "
Print #2, ""
Print #2, "     s11, s12, s13, s14 = ";
Print #2, Using "####.###"; s11; s12; s13; s14
Print #2, "     s21, s22, s23, s24 = ";
Print #2, Using "####.###"; s21; s22; s23; s24
Print #2, "     s31, s32, s33, s34 = ";
Print #2, Using "####.###"; s31; s32; s33; s34

Print ""
Print "     Consumption levels = "
Print ""
Print "     c11, c12, c13, c14 = ";
Print Using "####.###"; c11; c12; c13; c14
Print "     c21, c22, c22, c24 = ";
Print Using "####.###"; c21; c22; c23; c24
Print "     c31, c32, c33, c34 = ";
Print Using "####.###"; c31; c32; c33; c34
Print ""
Print "     GNP levels     = ";
Print "       w1, w2 ,w3   = ";
Print Using "####.###"; w1; w2; w3
Print ""
Print "     Utility levels = ";
Print "       u1, u2, u3   = ";
Print Using "####.###"; u1; u2; u3
Print ""
Print "     Global GNP     = ";
Print Using "#####.###"; wsum
Print "     Global Utility = ";
Print Using "#####.###"; usum

Print #2, ""
Print #2, "     Consumption levels = "
Print #2, ""
Print #2, "     c11, c12, c13, c14 = ";
Print #2, Using "####.###"; c11; c12; c13; c14
Print #2, "     c21, c22, c22, c24 = ";
Print #2, Using "####.###"; c21; c22; c23; c24
Print #2, "     c31, c32, c33, c34 = ";
Print #2, Using "####.###"; c31; c32; c33; c34
Print #2, ""
Print #2, "     GNP levels     = ";
Print #2, "       w1, w2 ,w3   = ";
```

```
Print #2, Using "####.###"; w1; w2; w3
Print #2, ""
Print #2, "     Utility levels = ";
Print #2, "       u1, u2, u3   = ";
Print #2, Using "####.###"; u1; u2; u3
Print #2, ""
Print #2, "     Global GNP     = ";
Print #2, Using "#####.###"; wsum
Print #2, "     Global Utility = ";
Print #2, Using "#####.###"; usum

End

100 Rem subroutine
p1 = 100
p2 = p1 * xsub
p3 = p1 * ysub
p4 = p1 * zsub

Rem Initial guess of the value of s14
s14 = 0
Rem The following ii loop is used to let s14 be iteratively determined.
Rem In the application, it is found that "ii = 1 to 5" gives the same results (with the
Rem selected precision), as "ii = 1 to 10" or "ii = 1 to 20". To be safe, we use "ii = 1 to 10".
For ii = 1 To 10
   Rem Production, GNP, and consumption in three nations

   Rem N1:
   rat = p1 / p2
   rat2 = rat ^ 2
   x11 = (1 / m11 * (rat2 / (m11 / m12 + rat2))) ^ (1 / 2)
   x12 = (1 / m12 - rat2 / (m11 + m12 * rat2)) ^ (1 / 2)
   x13 = 0
   x14 = 0
   w1 = p1 * x11 + p2 * x12 + p4 * (tariff / 100) * (-s14)
   c11 = a11 * w1 / p1
   c12 = a12 * w1 / p2
   c13 = a13 * w1 / p3
   c14 = a14 * w1 / (p4 * (1 + tariff / 100))
   u1 = c11 ^ a11 * c12 ^ a12 * c13 ^ a13 * c14 ^ a14

   Rem N2:
   x21 = 0
   rat = p2 / p3
   rat2 = rat ^ 2
   x22 = (1 / m22 * (rat2 / (m22 / m23 + rat2))) ^ (1 / 2)
   x23 = (1 / m23 - rat2 / (m22 + m23 * rat2)) ^ (1 / 2)
```

```
x24 = 0
w2 = p2 * x22 + p3 * x23
c21 = a21 * w2 / p1
c22 = a22 * w2 / p2
c23 = a23 * w2 / p3
c24 = a24 * w2 / p4
u2 = c21 ^ a21 * c22 ^ a22 * c23 ^ a23 * c23 ^ a24
Rem N3:
x31 = 0
x32 = 0
rat = p3 / p4
rat2 = rat ^ 2
x33 = (1 / m33 * (rat2 / (m33 / m34 + rat2))) ^ (1 / 2)
x34 = (1 / m34 - rat2 / (m33 + m34 * rat2)) ^ (1 / 2)
w3 = p3 * x33 + p4 * x34
c33 = a33 * w3 / p3
c34 = a34 * w3 / p4
u3 = (c33 ^ a33) * (c34 ^ a34)

wsum = w1 + w2 + w3
usum = u1 + u2 + u3

Rem Export supplies from three nations
Rem N1:
s11 = x11 - c11
s12 = x12 - c12
s13 = -c13
s14 = -c14

Rem N2:
s21 = -c21
s22 = x22 - c22
s23 = x23 - c23
s24 = -c24

Rem N3:
s31 = 0
s32 = 0
s33 = x33 - c33
s34 = x34 - c34

Rem Total export supplies

es1 = s11 + s21 + s31
es2 = s12 + s22 + s32
es3 = s13 + s23 + s33
es4 = s14 + s24 + s34
```

```
    fsub = es1
    gsub = es2
    hsub = es3
    tsub = es4

Next ii

Return

200 Rem Parameters
Rem initial conditions
x = 1
y = 1
z = 1

delta = 0.001

a11 = 1 / 4
a12 = 1 / 4
a13 = 1 / 4
a14 = 1 / 4
a21 = 1 / 4
a22 = 1 / 4
a23 = 1 / 4
a24 = 1 / 4
a33 = 1 / 2
a34 = 1 / 2

x11max = 2
x12max = 4
m11 = x11max ^ (-2)
m12 = x12max ^ (-2)

x22max = 2
x23max = 4
m22 = x22max ^ (-2)
m23 = x23max ^ (-2)

x33max = 2
x34max = 4

m33 = x33max ^ (-2)
m34 = x34max ^ (-2)

Return
```

```
300 Rem Parameter print outs

Print ""
Print "     Production possibility frontiers parameters"
Print "     ----------------------------------------"
Print "     x11max, x12max, x13max, x14max = ";
Print Using "#####.###"; x11max; x12max; 0; 0
Print "     x21max, x22max, x23max, x24max = ";
Print Using "#####.###"; 0; x22max; x23max; 0
Print "     x31max, x32max, x33max, x34max = ";
Print Using "#####.###"; 0; 0; x33max; x34max

Print #2, ""
Print #2, "     Production possibility frontiers parameters"
Print #2, "     ----------------------------------------"
Print #2, "     x11max, x12max, x13max, x14max = ";
Print #2, Using "#####.###"; x11max; x12max; 0; 0
Print #2, "     x21max, x22max, x23max, x24max = ";
Print #2, Using "#####.###"; 0; x22max; x23max; 0
Print #2, "     x31max, x32max, x33max, x34max = ";
Print #2, Using "#####.###"; 0; 0; x33max; x34max

Return
```

Appendix 3

Alternative way to calculate the effects of the tariff based on iterative determination of the tariff revenue and the excess supply functions

The software included below is developed in Smart Basic.

```
rem Tariff_opt_250529
rem Peter Lohmander

a = 1/2

for T = 0 to 2 step 0.2
print ""
print "Tariff = "T

x_opt = 1
abss1_opt = 10
u1_eq = 0
u2_eq = 0
usum_eq = 0

for x = 0.5 to 2.5 step 0.001

Trev = 0

for i = 1 to 100

p1 = 1
p2 = p1*x
p3 = p1*(1+T)

r = p1/p2
z = p3/p2

x11 = 1/2*z
x22 = 1/2*r^(-1)
x12 = 4 - x11^2
x21 = 4 - x22^2

TRev = T*(-s11)

B1 = p3*x11 + p2*x12 + Trev
B2 = p1*x21 + p2*x22

c11 = a*B1/p3
c12 = a*B1/p2
```

```
c21 = a*B2/p1
c22 = a*B2/p2

u1 = (c11^a)*(c12^a)
u2 = (c21^a)*(c22^a)
usum = u1 + u2

s11 = x11 - c11
s12 = x12 - c12
s21 = x21 - c21
s22 = x22 - c22

s1 = s11 + s21
s2 = s12 + s22

next i

abss1 = s1
if s1 < 0 then abss1 = -s1
if abss1 > abss1_opt then goto 100

x_opt = x
u1_eq = u1
u2_eq = u2
usum_eq = usum
abss1_opt = abss1
s11_opt = s11
s12_opt = s12
100 rem
next x

p1 = 100
p2 = p1*x_opt
p3 = p1*(1+T)

print "abss1_opt = "abss1_opt
print "x_opt = "x_opt
print "p1, p2, p3 = " p1;p2;p3
print "s11_opt = "s11_opt
print "s12_opt = "s12_opt
print "u1_eq = "u1_eq
print "u2_eq = "u2_eq
print "usum_eq = "usum_eq

next T
```

Results from the alternative way to calculate the effects of the tariff based on iterative determination of the tariff revenue and the excess supply functions

The following results clearly show that both methods led to almost the same numerical results. You may compare the results for the tariff levels 0 and 100%, reported for both methods.

Tariff = 0

abss1_opt = 1.11022E-15
x_opt = 1
p1, p2, p3 = 100 100 100
s11_opt = -1.625
s12_opt = 1.625
u1_eq = 2.125
u2_eq = 2.125
usum_eq = 4.25

Tariff = 0.2
abss1_opt = 0.00137125
x_opt = 1.098
p1, p2, p3 = 100 109.8 120
s11_opt = -1.54927
s12_opt = 1.41099
u1_eq = 2.1909
u2_eq = 2.05248
usum_eq = 4.24338

Tariff = 0.4
abss1_opt = 0.000335049
x_opt = 1.19
p1, p2, p3 = 100 119 140
s11_opt = -1.46863
s12_opt = 1.23414
u1_eq = 2.23098
u2_eq = 1.99566
usum_eq = 4.22664

Tariff = 0.6
abss1_opt = 0.000809416
x_opt = 1.278
p1, p2, p3 = 100 127.8 160
s11_opt = -1.38833
s12_opt = 1.08633
u1_eq = 2.25382
u2_eq = 1.94974
usum_eq = 4.20357

Tariff = 0.8
abss1_opt = 0.00124471
x_opt = 1.361
p1, p2, p3 = 100 136.1 180
s11_opt = -1.30662
s12_opt = 0.960047
u1_eq = 2.26314
u2_eq = 1.91283
usum_eq = 4.17596

Tariff = 1
abss1_opt = 0.000260009
x_opt = 1.439
p1, p2, p3 = 100 143.9 200
s11_opt = -1.22374
s12_opt = 0.85041
u1_eq = 2.26196
u2_eq = 1.88302
usum_eq = 4.14498

Tariff = 1.2
abss1_opt = 6.37731E-05
x_opt = 1.513
p1, p2, p3 = 100 151.3 220
s11_opt = -1.1415
s12_opt = 0.75446
u1_eq = 2.25316
u2_eq = 1.85859
usum_eq = 4.11175

Tariff = 1.4
abss1_opt = 0.000584442
x_opt = 1.583
p1, p2, p3 = 100 158.3 240
s11_opt = -1.05971
s12_opt = 0.66943
u1_eq = 2.23822
u2_eq = 1.83857
usum_eq = 4.07678

Tariff = 1.6
abss1_opt = 0.00073548
x_opt = 1.65
p1, p2, p3 = 100 165 260
s11_opt = -0.979798
s12_opt = 0.593817
u1_eq = 2.21895
u2_eq = 1.82193
usum_eq = 4.04088

Tariff = 1.8
abss1_opt = 0.000237209
x_opt = 1.713
p1, p2, p3 = 100 171.3 280
s11_opt = -0.899849
s12_opt = 0.525306
u1_eq = 2.19535
u2_eq = 1.80835
usum_eq = 4.0037

Tariff = 2
abss1_opt = 4.67253E-05
x_opt = 1.773
p1, p2, p3 = 100 177.3 300
s11_opt = -0.821223
s12_opt = 0.463183
u1_eq = 2.16873
u2_eq = 1.79712
usum_eq = 3.96586