Fundamental Principles of Optimal Continuous Cover Forestry

Linnaeus University Research Seminar (RESEM)

Tuesday 2017-05-23, 10.15-11.00, Lecture Hall: Karlavagnen

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Fundamental Principles of Optimal Continuous Cover Forestry

- **Introduction**
- Simple analytical method to optimize the stock level in continuous cover forestry.
- More advanced analytical method to simultaneously optimize the stock level after harvest and the harvest interval. General aspects on optimal CCF, forest regulations and environmental issues.
- Advanced method to optimize CCF at the individual tree level, considering spatial competition, stochastic prices and timber quality variations.


CCF in Neuchatel, Switzerland

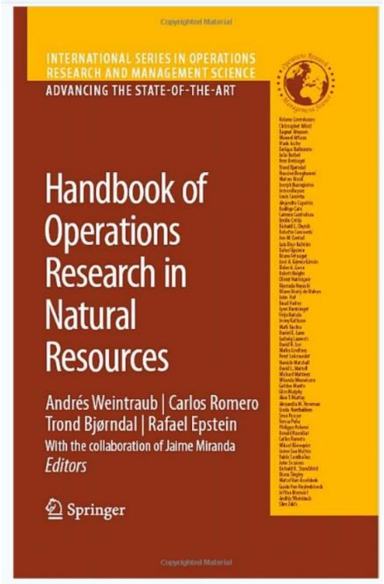
CCF in Neuchatel, Switzerland and Professor J.P. Shütz, ETH.

Rotation forestry with clear cuts in Sweden (10 km S Umeå).

Lohmander (2000) and (2007) shows how **dynamic and stochastic management decisions can be optimized with different methods, including different versions of stochastic dynamic programming.**

Lohmander, P., **Optimal sequential forestry decisions under risk**, Annals of Operations Research, Vol. 95, pp. 217-228, 2000

Lohmander, P., **Adaptive Optimization of Forest Management in a Stochastic World**, in Weintraub A. et al (Editors), Handbook of Operations Research in Natural Resources, Springer, Springer Science, International Series in Operations Research and Management Science, New York, USA, pp 525-544, 2007



Lohmander, P., Mohammadi, S., Optimal Continuous Cover Forest Management in an Uneven-Aged Forest in the North of Iran, Journal of Applied Sciences 8(11), 2008

Schütz, J-P., Modelling the demographic sustainability of pure beech plenter forests in Eastern Germany, Ann. For. Sci. 63 (2006) 93–100

Fundamental Principles of Optimal Continuous Cover Forestry

- Introduction
- **Simple analytical method to optimize the stock level in continuous cover forestry.**
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Simple analytical method to optimize the stock level in continuous cover forestry:

Determination of the optimal initial harvest and the stock level after the initial harvest.

We start with the stock level V0.

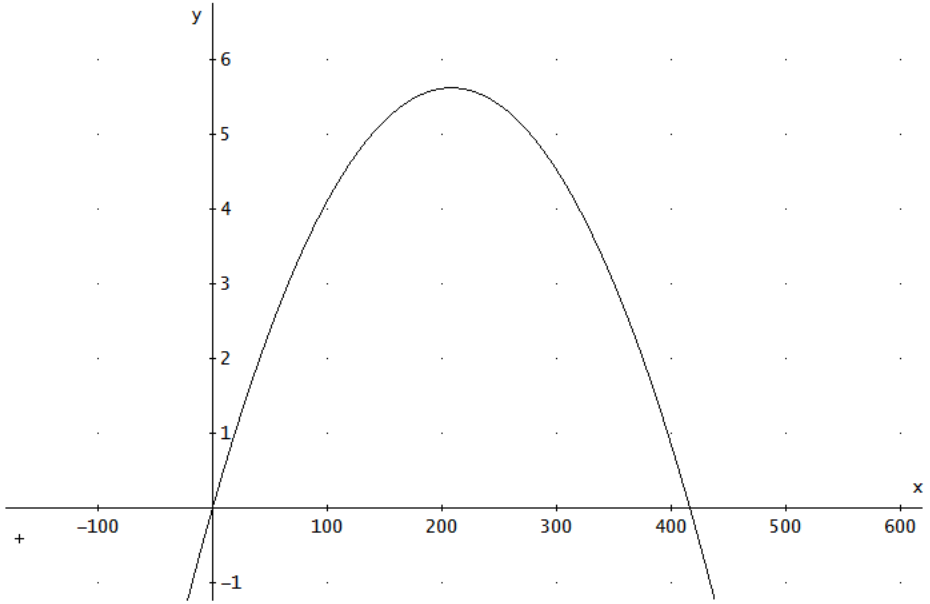
After the initial harvest, h, we have the stock level V1 (= V0-h).

Our objective function is the total present value *(*)*.

*** = The sum of all revenues minus costs, at all points in time, with consideration of the rate of interest in the capital market.**

This is a growth function example:

G(V) = Growth (m3 per hectare and year)



V = Stock level (m3 per hectare)

$$G(V) \approx 0.0540V - 0.000130V^2$$

Maximization of Π , the total present value :

$$\max_h \Pi = p_0 h + \int_0^{\infty} p_1 G(V_0 - h) e^{-rt} dt$$

V0 = Initial stock level (m3 per hectare).
p0 = Net price per cubic metre (price minus cost per m3) in the initial harvest.
p1 = Net price per cubic metre (price minus cost per m3) in future harvests.
h = Initial harvest (m3 per hectare).
V0-h = Stock level after the initial harvest (m3 per hectare).
G(V0-h) = Growth (m3/year) after the initial harvest.
(Note that the future harvest level is identical to the future growth.)
r = Rate of interest in the capital market.

$$\max_{h} \Pi = p_0 h + \int_0^{\infty} p_1 G(V_0 - h) e^{-rt} dt$$

$$\max_{h} \Pi = p_0 h + p_1 G(V_0 - h) \int_0^{\infty} e^{-rt} dt$$

$$\max_{h} \Pi = p_0 h + p_1 G(V_0 - h) \frac{1}{r}$$

Optimization of the initial harvest, h, (and indirectly of the optimal stock level after the initial harvest), V0-h, via derivatives:

$$\max_{h} \Pi = p_0 h + p_1 G(V_0 - h)\frac{1}{r}$$

First order optimum condition

$$\frac{d\Pi}{dh} = p_0 - p_1 G'(V_0 - h)\frac{1}{r} = 0$$

Second order maximum condition

$$\frac{d^2\Pi}{dh^2} = p_1 G''(V_0 - h)\frac{1}{r} < 0$$

$$\left(\frac{d\Pi}{dh} = 0 \right) \Rightarrow \left(p_0 = p_1 G'(V_0 - h) \frac{1}{r} \right)$$

$$G'(V_0 - h) = \frac{p_0}{p_1} r$$

The optimal stock level after the initial harvest, V1, can be determined from this equation.

$$G'(V_1) = G'(V_0 - h) = \frac{p_0}{p_1} r$$

(Then, the optimal initial harvest level, h, is determined from V1=V0-h. h=V0-V1.)

EXAMPLE: ***Optimization with specific parameters and a graphical approach:***

Initial stock level = 130 m3/ha.

p0 = p1 = 200 SEK/m3.

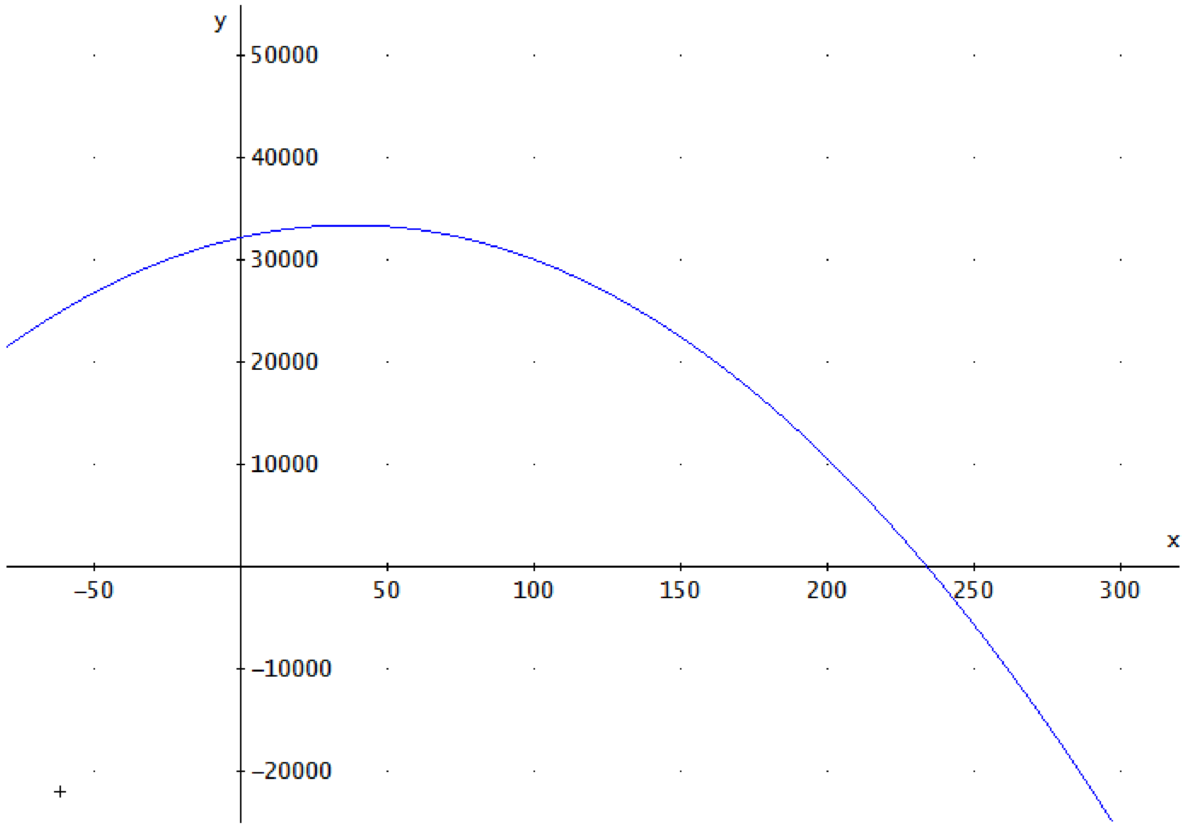
r = 3%.

Estimated growth function: ***G*** = 0.054011123·V - 0.000129731·V·V

$$\max_{h} \Pi = p_0 h + p_1 G(V_0 - h)\frac{1}{r}$$

$$200 \cdot h + \frac{1}{0.03} \cdot 200 \cdot (0.054011123 \cdot (130 - h) - 0.000129731 \cdot (130 - h) \cdot (130 - h))$$

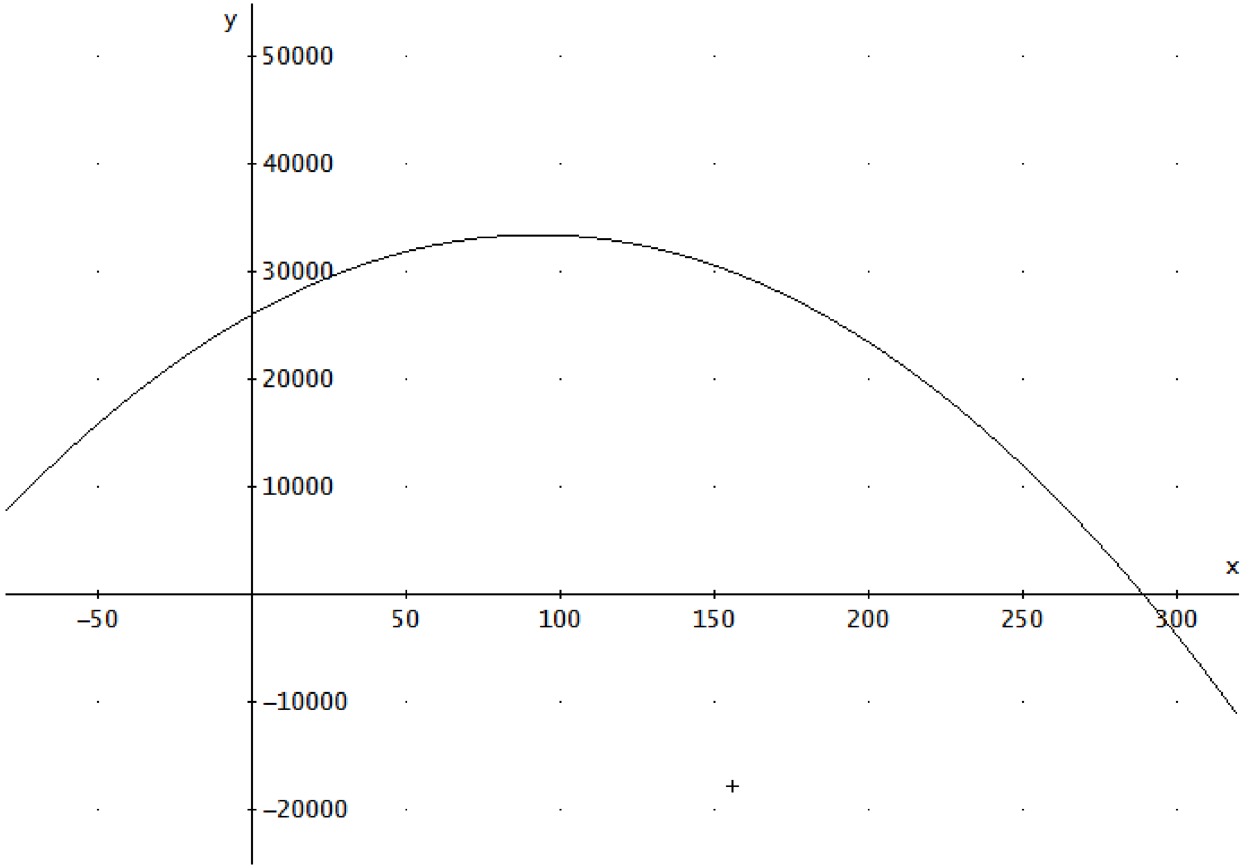
Present value (SEK/ha)



h (m3/ha)

$$200 \cdot h + \frac{1}{0.03} \cdot 200 \cdot (0.054011123 \cdot (130 - h) - 0.000129731 \cdot (130 - h) \cdot (130 - h))$$

Present value (SEK/ha)

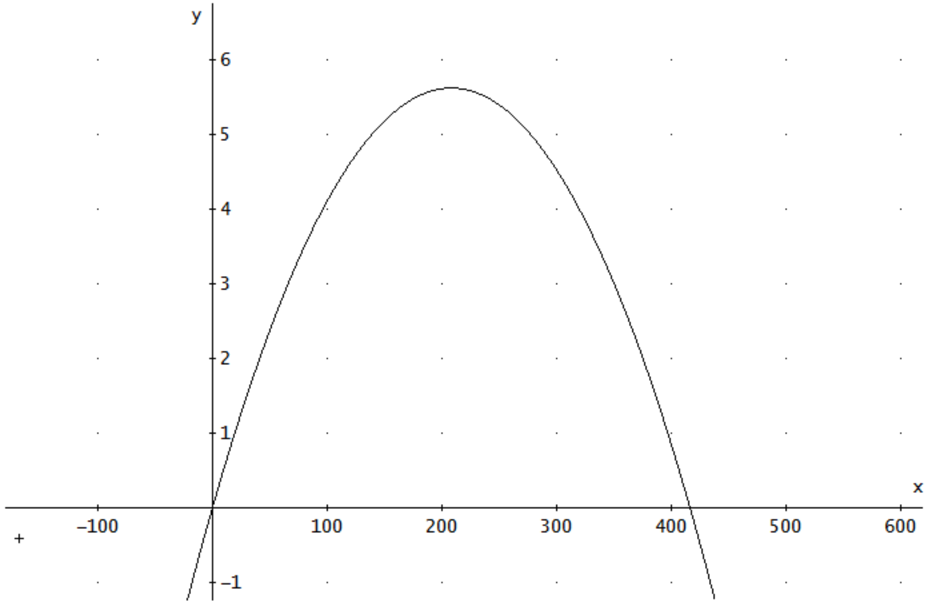


V1=V0-h (m3/ha)

$$200 \cdot (130 - x) + \frac{1}{0.03} \cdot 200 \cdot (0.054011123 \cdot x - 0.000129731 \cdot x \cdot x)$$

Illustration via an estimated growth function:

G(V)



V

$$G(V) \approx 0.0540V - 0.000130V^2$$

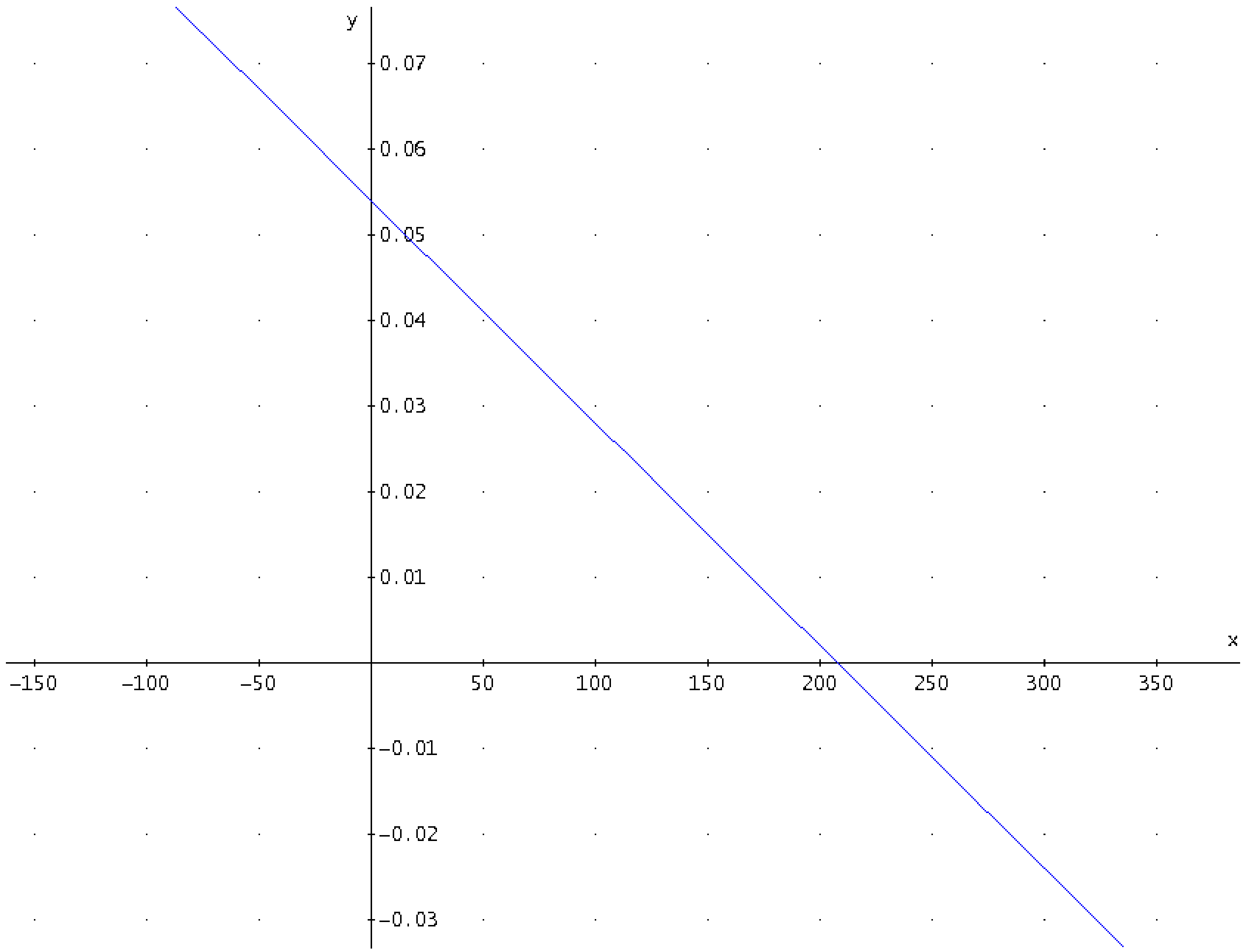
Growth function derivatives:

$$G(V) \approx 0.0540V - 0.000130V^2$$

$$G'(V) \approx 0.0540 - 0.000260V$$

$$G''(V) \approx -0.000260 < 0$$

$$G'(V) \approx 0.0540 - 0.000260V$$



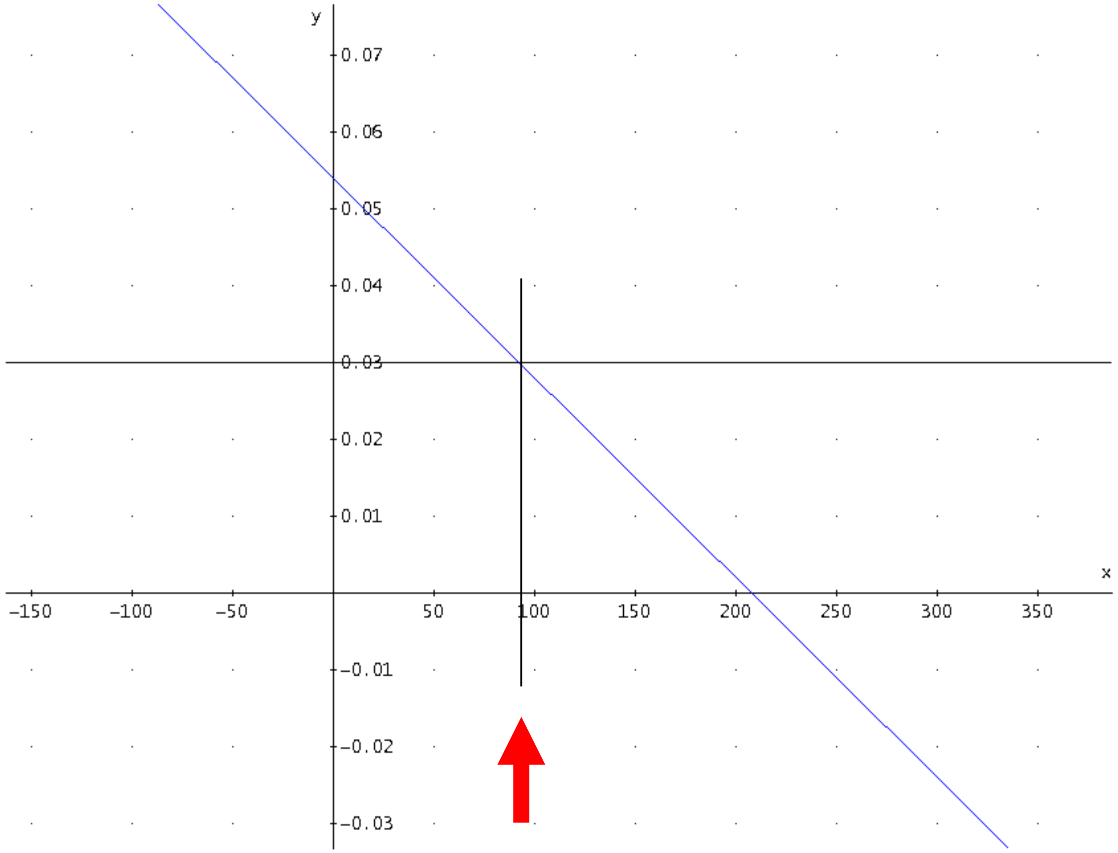
V

Determinaion of the optimal stock level:

$$G'(V) = \frac{p_0}{p_1} r$$

$$0.0540 - 0.000260V = \frac{p_0}{p_1} r$$

$$G'(V) \approx 0.0540 - 0.000260V$$



$$\boxed{\frac{p_0}{p_1} r = 3\%}$$

V

92.3

Conclusion:

The optimal stock level is 92.3 m3 per hectare if the rate of interst is 3% in this case.

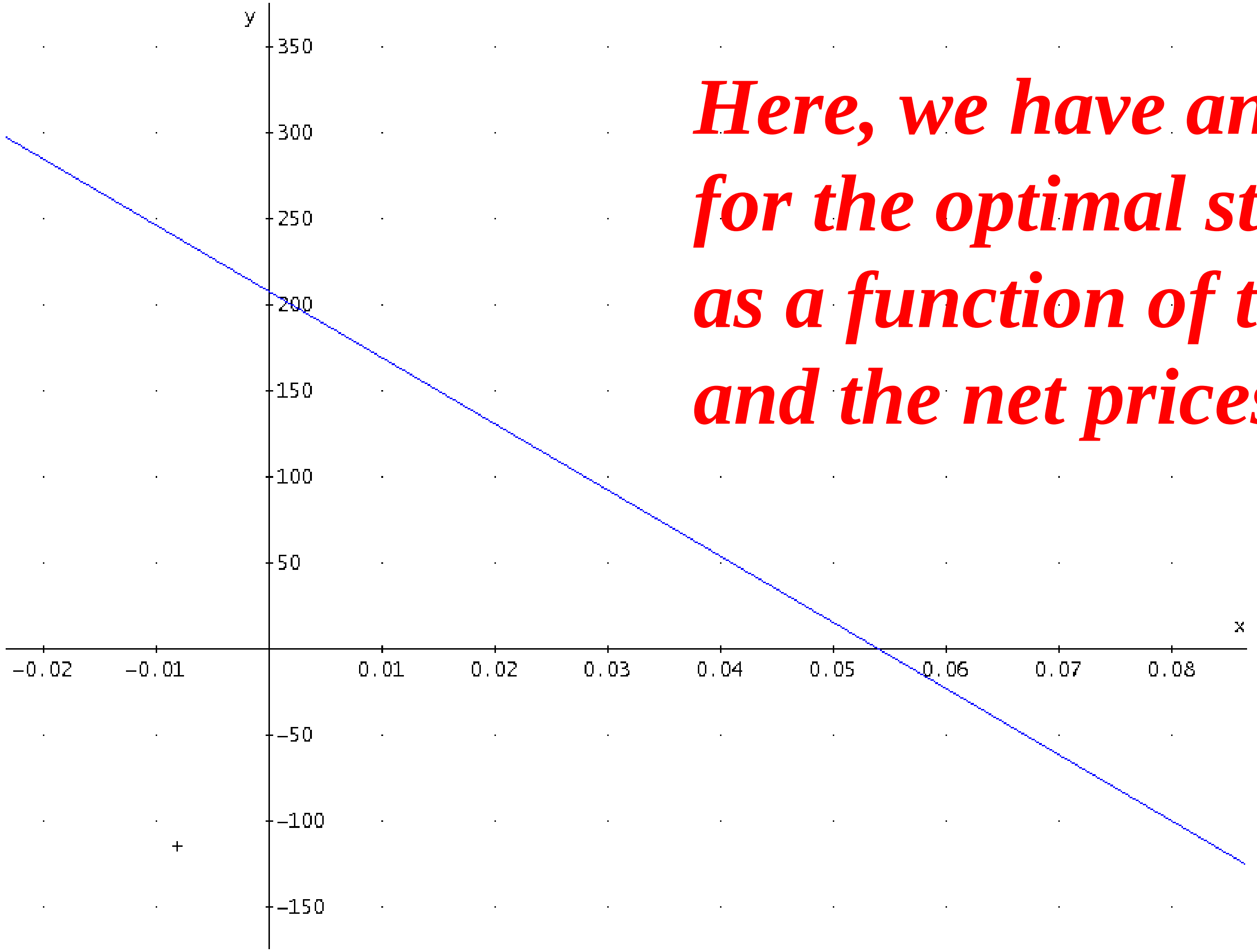
We can also determine an equation for the optimal stock level.

$$0.0540 - 0.000260V = \frac{p_0}{p_1} r$$

$$V = \frac{-0.0540 + \frac{p_0}{p_1} r}{-0.000260}$$

$$V \approx 207.7 - 3846 \frac{p_0}{p_1} r$$

$$V \approx 207.7 - 3846 \frac{p_0}{p_1} r$$



Here, we have an expression for the optimal stock level as a function of the rate of interest and the net prices.

$$\frac{p_0}{p_1} r$$

General principles of rational continuous cover forestry derived from the simple analytical method:

1. The optimal stock level is a decreasing function of the rate of interest.

2. The optimal stock level decreases if the net price per cubic metre in the initial harvest increases in relation to the net price per cubic metre in future harvests.

3. The optimal stock level increases if the net price per cubic metre in the initial harvest decreases in relation to the net price per cubic metre in future harvests.

4. It is optimal to let the stock level be equal to the stock level that maximizes the average growth (MSY) only if the rate of interest in the capital market is zero.

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Optimal continuous cover forest management:
- Economic and environmental effects and legal considerations

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BIT's 5th Low Carbon Earth Summit

(LCES 2015 & ICE-2015)

Theme: "Take Actions for Rebuilding a Clean World"

September 24-26, 2015

Venue: Xi'an, China

Optimal forest management

Forest management can be performed in many different ways.

Decisions in forestry affect economic results, the flow of bioenergy raw materials, the CO_2 balance of the world, species diversity, recreation options for humans and much more.

Some of the fundamental decision problems concern:

Continuous Cover Forestry (CCF) or Plantation Forestry (PF),

the Stand Density (SD),

the Harvest Interval (HI),

Single Species Forestry (SSF) or Multi Species Forestry (MSF).

With present prices, costs, technology and initial forest conditions in many dominating forest countries, CCF is often a better choice than PF when we optimize the economic present values.

CCF is also a better choice than PF from several environmental perspectives.

The optimal levels of SD and HI are affected by all parameters.

MSF can give environmental benefits in relation to SSF.

MSF can also give economically valuable options to sequentially adjust forest production to future market changes.

MSF is less sensitive to species specific damages and is more flexible to changing environmental conditions.

Therefore, the expected present value of MSF is often higher than the expected present value of SSF.

Similar countries
with very different forest laws

The forest laws in different countries, also neighbour countries such as Finland and Sweden, with almost the same prices, costs, technology and forest conditions, are very different with respect to the fundamental decisions: CCF or PF, SD, HI and SSF or MSF.

The economic and environmental development of the world would benefit from more rational forest management.

Several forest laws need to be adjusted in order to make rational decisions legal.

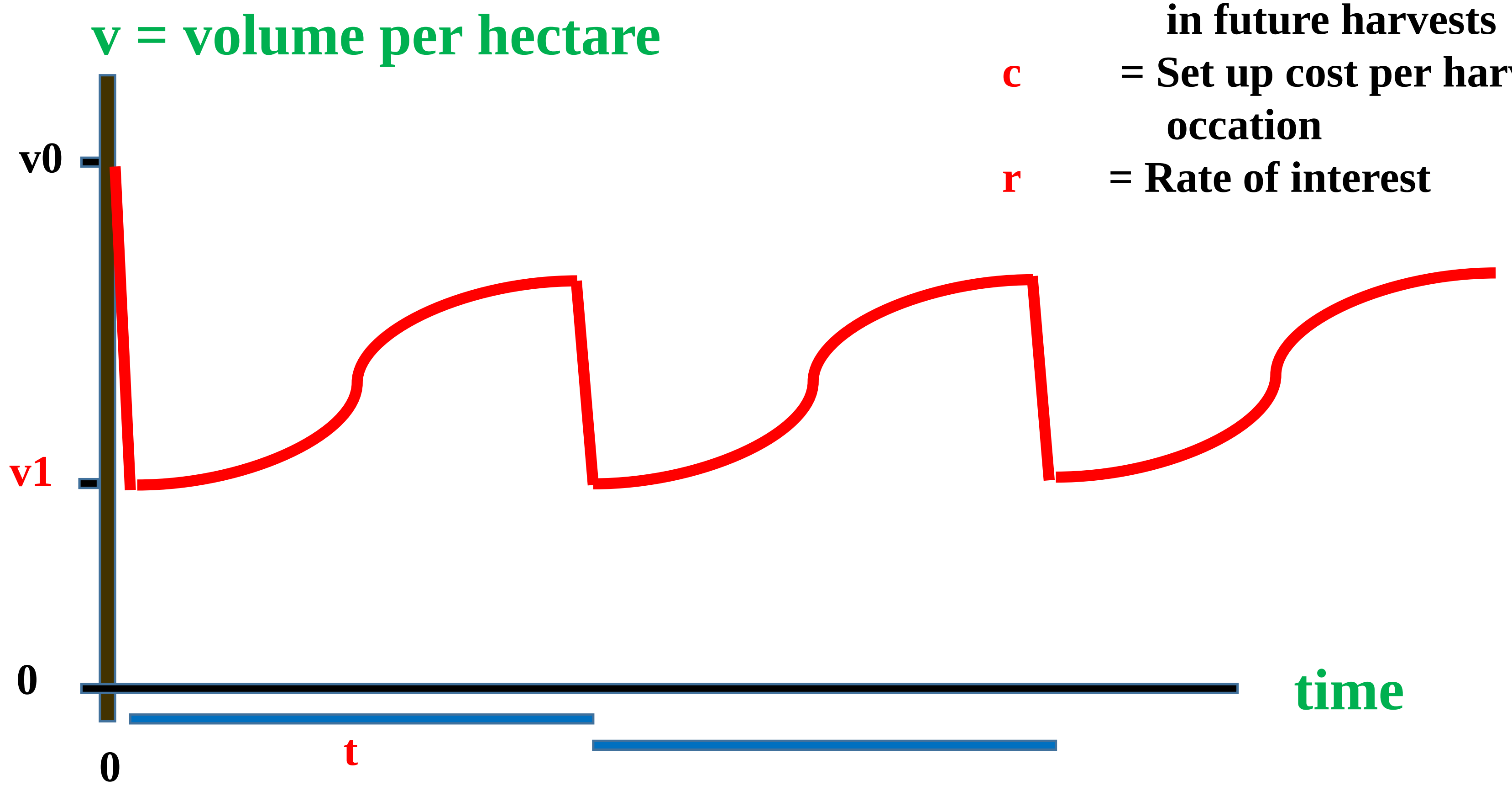
Present value

$$\max \pi = R(h) + \frac{P(v_1,t)Q(v_1,t) - c}{e^{rt} - 1}$$

$$s.t.$$

$$h = v_0 - v_1$$

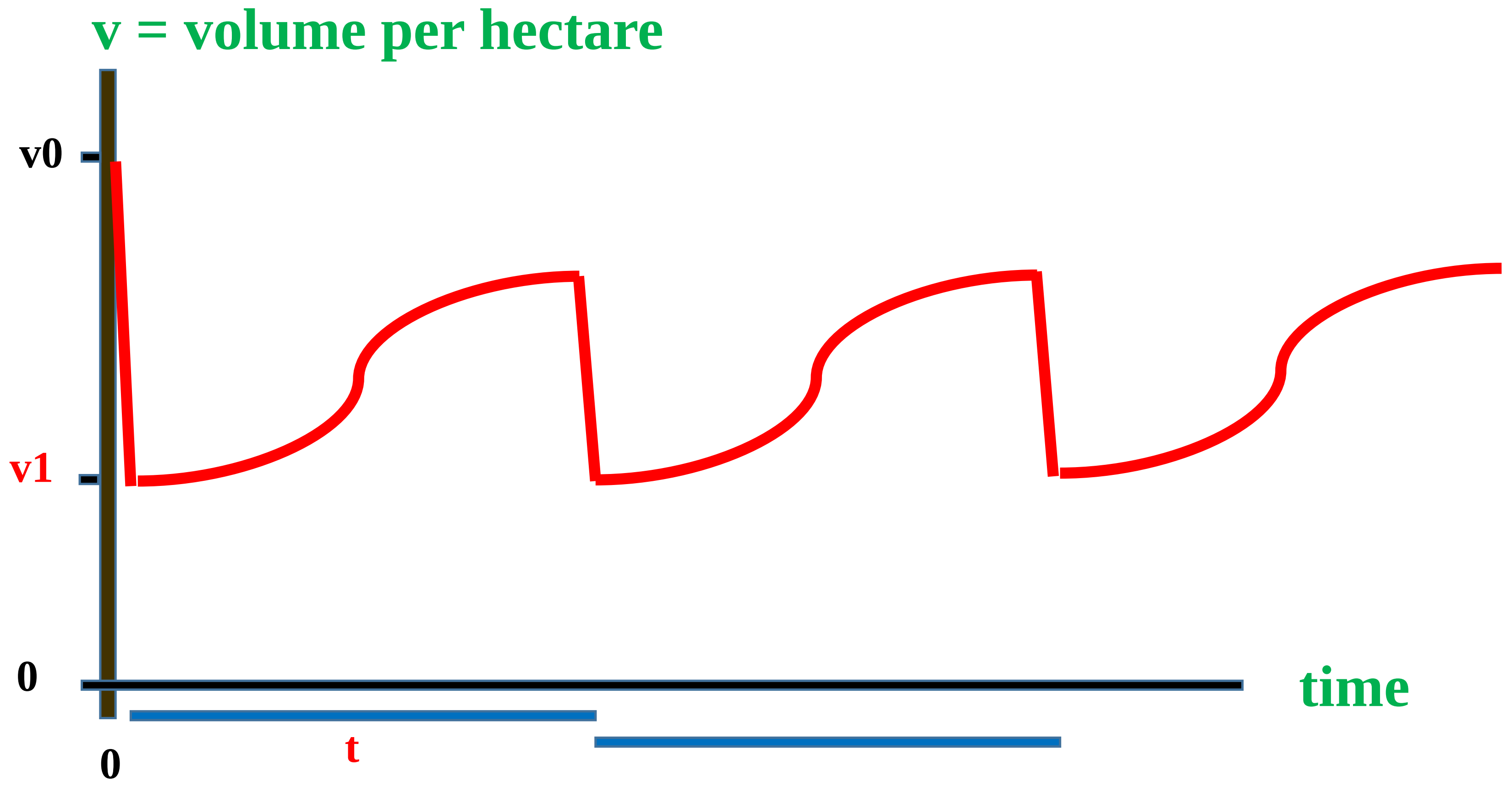
h = The first harvest volume
R(h) = Profit from the first harvest
P(.) = Price per cubic metre (reduced by variable cost per cubic metre) in future harvests
Q(.) = Harvest volume per hectare in future harvests
c = Set up cost per harvest occation
r = Rate of interest



In Finland, continuous cover forest management can be optimized without constraints.

In Sweden, there are several constraints in the forest act. For instance, the volume always has to stay above a specified lower limit. If the volume is below the limit, you have to make a clearcut.

WITH Swedish constraints, forestry with clearcuts often is the economically optimal choice.
WITHOUT Swedish constraints, continuous cover forestry is very often the economically optimal choice.



$$\max \pi = R(h) + \frac{P(v_1, t)Q(v_1, t) - c}{e^{rt} - 1}$$

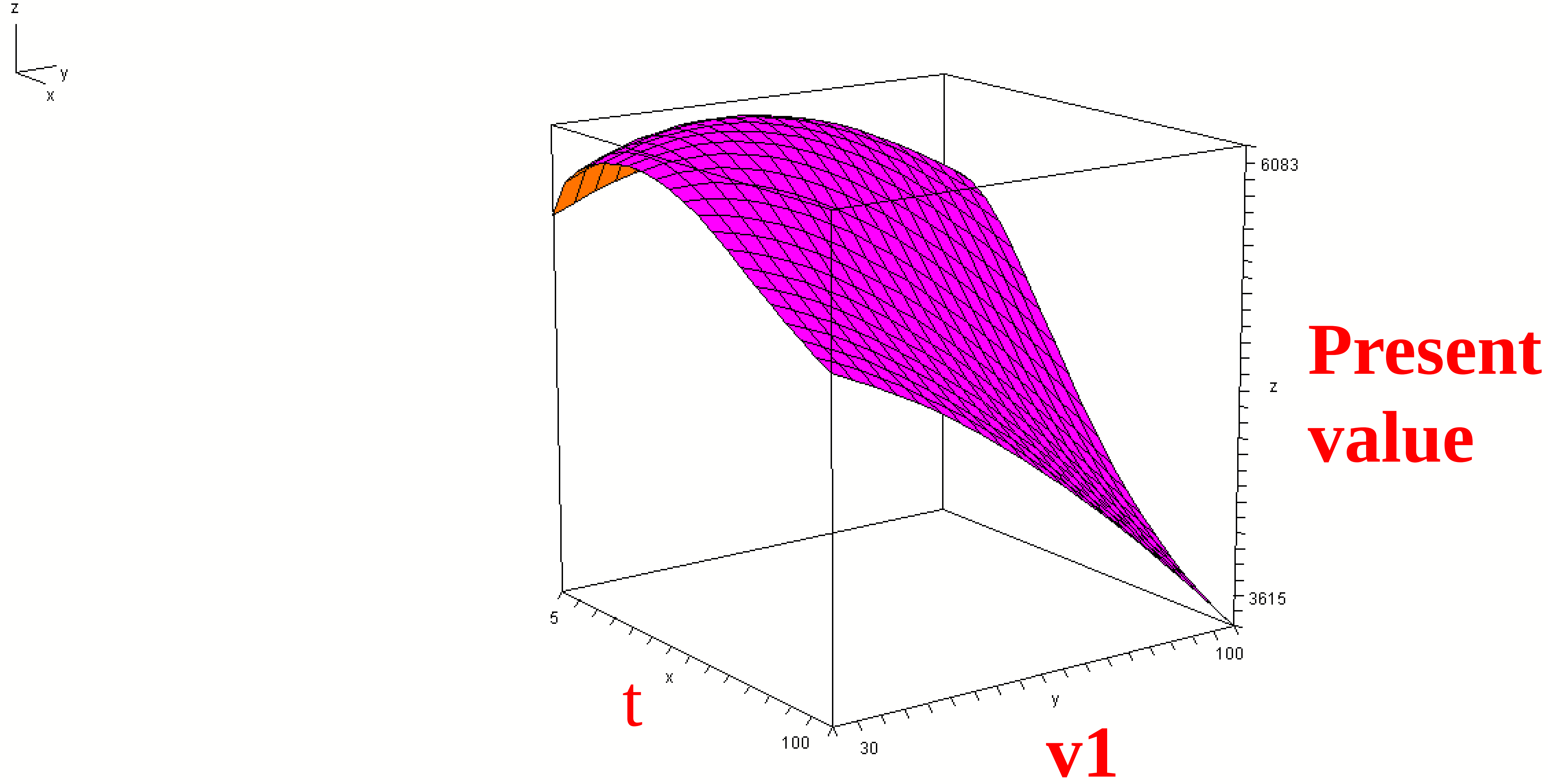
$$s.t.$$

$$h = v_0 - v_1$$

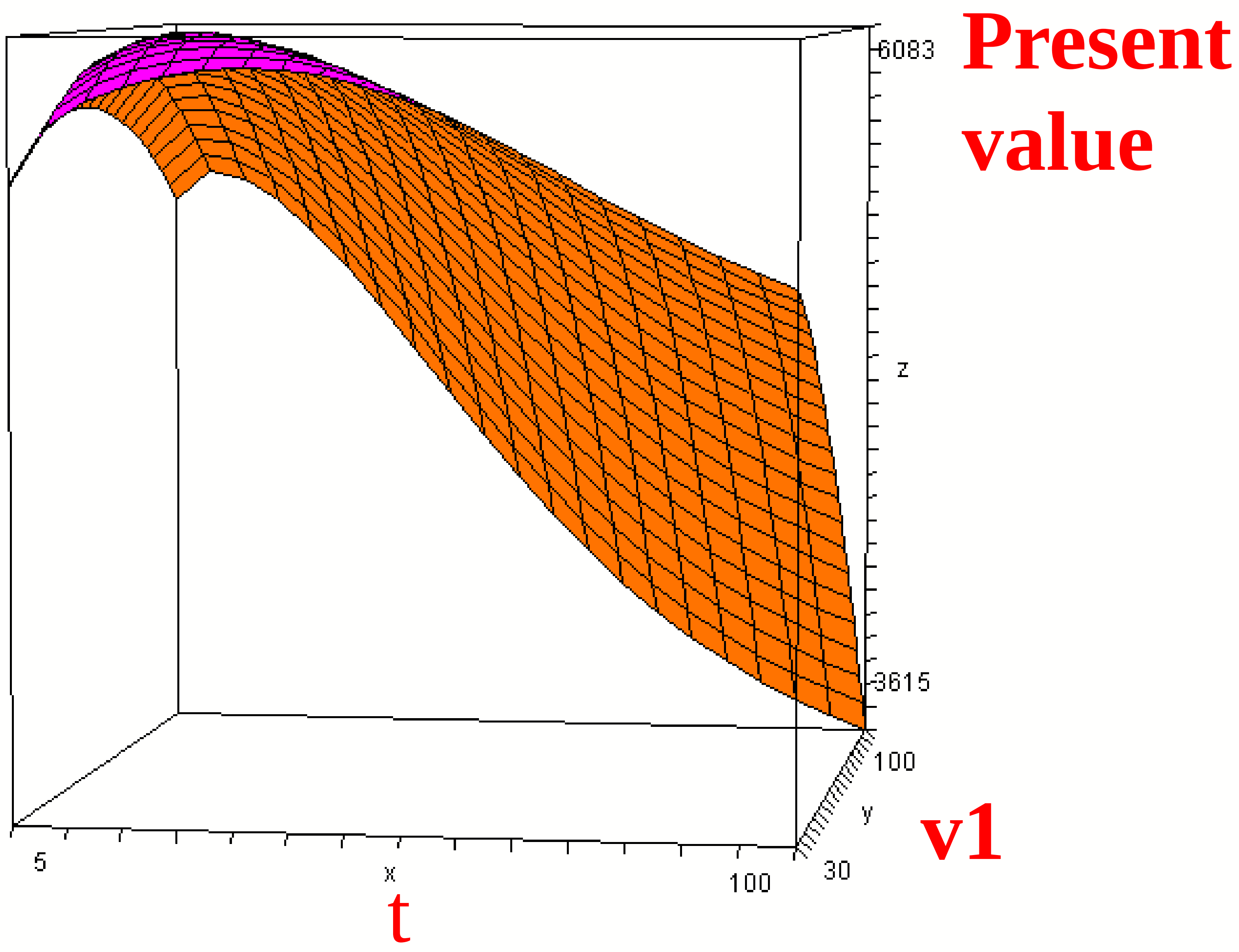
Example:

Graphical illustrations based on specified functions and parameters

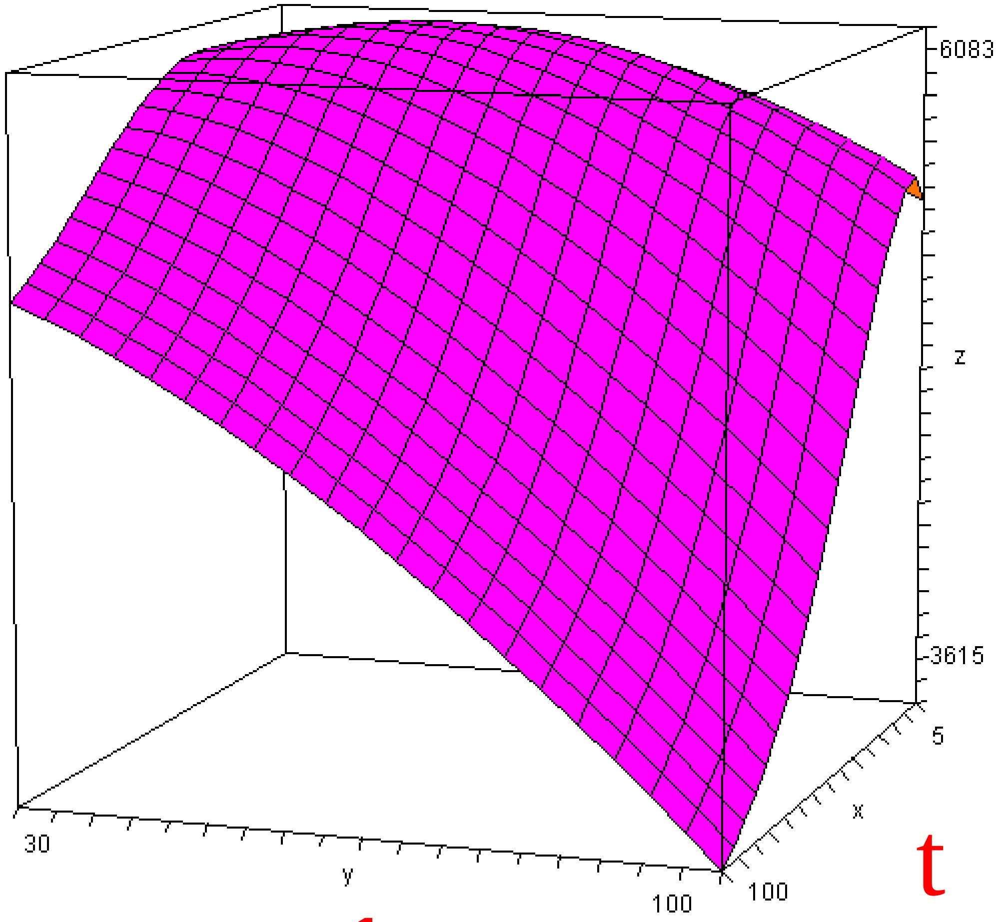
$$\left(30\cdot(200-v)+\frac{0.1833333}{2}\cdot(200-v)\cdot(200-v)-\frac{0.001666667}{3}\cdot(200-v)\cdot(200-v)\cdot(200-v)-50\right)+\frac{20\cdot\left(\dfrac{1}{\dfrac{1}{400}+\left(\dfrac{1}{v}-\dfrac{1}{400}\right)\cdot \mathrm{EXP}(-0.05\cdot t)}-v\right)-50}{\mathrm{EXP}(0.03\cdot t)-1}$$



z
y
x
6083
z
3615
5
x
100
30
y
100
Present value
t
v1



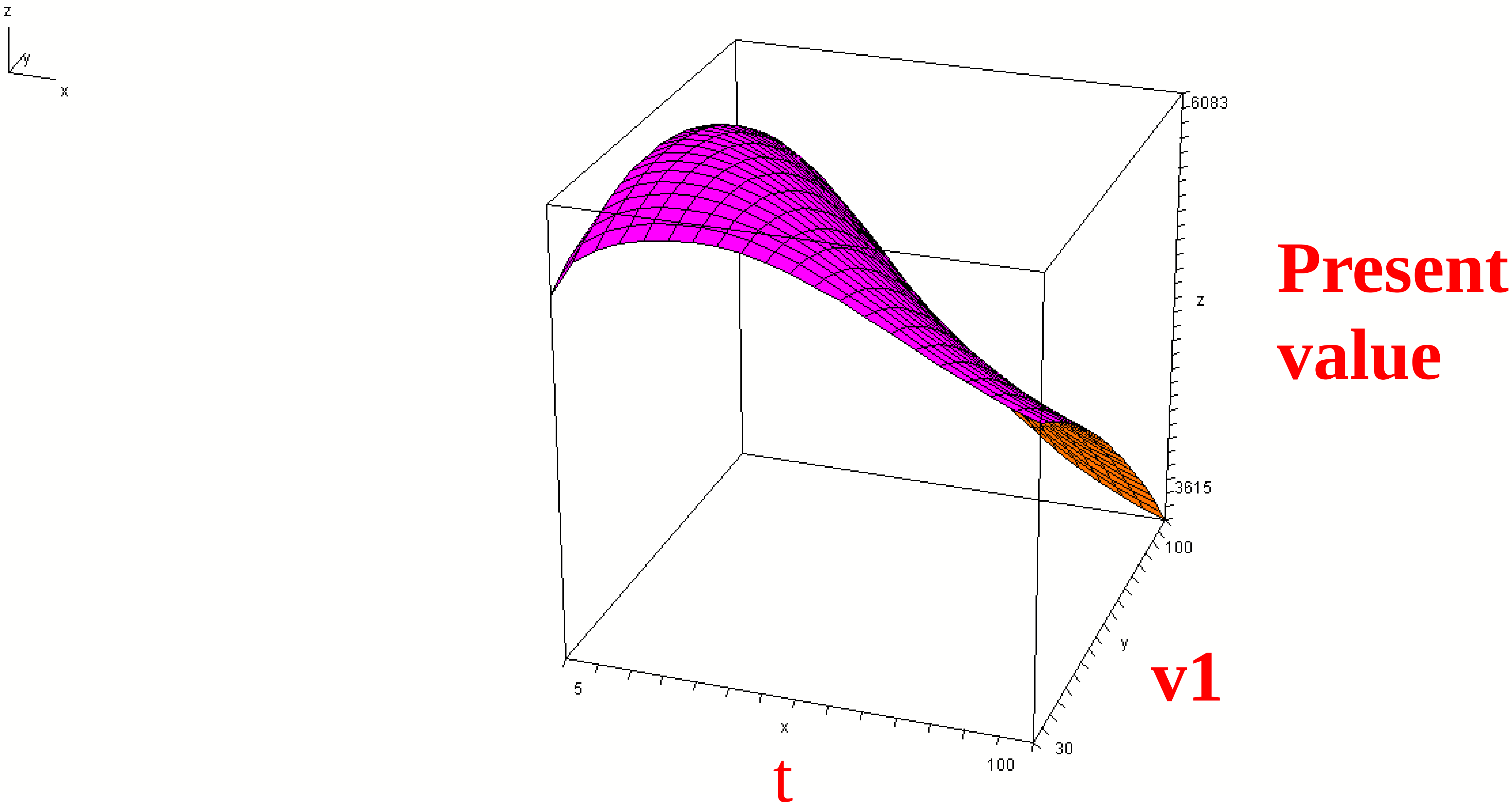
Present
value
6083
z
3615
100
y
30
v1
5
x
100
t



Present value

t

v1


z
y
x
6083
z
3615
100
y
30
5
x
100
Present value
v1
t

Numerical Analysis
Peter Lohmander 150812

Case 0:

```
! OPT CCF 150812;
! Peter Lohmander;

v0 = 200;
p = 20;
c = 50;
r = 0.03;
m0 = 30;
c0 = 50;

max = Y;
Y = R0 + (p*Q-c)/(@exp(r*t)-1);
h = v0-v1;
h < v0;
h > 1;

mp = m0 - a*h-b*h*h;
R0 = m0*h-a/2*h*h-b/3*h*h*h-c0;
Q = 1/(1/400+(1/v1-1/400)*@exp(-0.05*t))-v1;

! Derivation of initial marginal price function;
150*a+(150)^2*b= 10;
200*a+(200)^2*b = 30;
@free(a);
@free(b);
```

```
Local optimal solution found.
  Objective value:                          6084.286
  Infeasibilities:                          0.000000
  Total solver iterations:                        34
```

Variable	Value	Reduced Cost
V0	200.0000	0.000000
P	20.00000	0.000000
C	50.00000	0.000000
R	0.3000000E-01	0.000000
M0	30.00000	0.000000
C0	50.00000	0.000000
Y	6084.286	0.000000
R0	4651.524	0.000000
Q	71.17267	0.000000
T	22.40775	0.1339671E-08
H	150.7051	0.000000
V1	49.29487	0.000000
MP	19.77588	0.000000
A	-0.1833333	0.000000
B	0.1666667E-02	0.000000

Maximum present value

Optimal harvest interval

The optimal value of V1 is far below the Swedish constraint.

Optimal continuous cover forest management

First, we study:

One dimensional optimization in the time interval dimension (of relevance when the stock level after harvest is determined by law or can not be determined for some other reason)

$$\max \pi = R(h) + \frac{P(v_1, t)Q(v_1, t) - c}{e^{rt} - 1}$$

$$s.t.$$

$$h = v_0 - v_1$$

$$\frac{d\pi}{dt} = \frac{e^{rt}}{\left(e^{rt}-1\right)^2}\left(\left(\frac{dP}{dt}Q + P\frac{dQ}{dt}\right)\left(1-e^{-rt}\right) - \left(PQ-c\right)r\right) = 0$$

$$\left(\frac{dP}{dt}Q + P\frac{dQ}{dt}\right)\left(1-e^{-rt}\right) - \left(PQ-c\right)r = 0$$

$\left(M(.)=0\right) \Rightarrow$

Optimal principle in the time dimension:

$$\frac{\dfrac{dP}{dt}Q + P\dfrac{dQ}{dt}}{PQ + \dfrac{\left(-c + e^{-rt}PQ\right)}{1 - e^{-rt}}} = r$$

How is the optimal time interval affected if the parameter c marginally increases (ceteres paribus)?

$$\frac{d\pi}{dt} = 0$$

$$d\left(\frac{d\pi}{dt}\right) = \frac{d^2\pi}{dt^2} dt^* + \frac{d^2\pi}{dtdc} dc = 0$$

$$\frac{dt^*}{dc} = \frac{-\left(\frac{d^2\pi}{dtdc}\right)}{\left(\frac{d^2\pi}{dt^2}\right)}$$

A unique maximum is assumed in the time interval dimension

$$\frac{d^2\pi}{dt^2} < 0$$

$$\frac{d^2\pi}{dtdc} = k(.)r > 0$$

$$\frac{dt^*}{dc} = \frac{-\left(\frac{d^2\pi}{dtdc}\right)}{\left(\frac{d^2\pi}{dt^2}\right)} > 0$$

Conclusion:

The optimal time interval is a strictly increasing function of c.

How is the optimal time interval affected if the parameter r marginally increases (ceteres paribus)?

$$\frac{dt^*}{dr} = \frac{-\left(\frac{d^2\pi}{dtdr}\right)}{\left(\frac{d^2\pi}{dt^2}\right)} < 0$$

Conclusion:

The optimal time interval is a strictly decreasing function of the parameter r.

How is the optimal time interval affected if the future prices marginally increase (ceteres paribus)?

$$\max \pi = R(h) + \frac{pQ(v_1,t) - c}{e^{rt} - 1}$$

$$\frac{dt^*}{dp} = \frac{-\left(\dfrac{d^2\pi}{dtdp}\right)}{\left(\dfrac{d^2\pi}{dt^2}\right)} < 0$$

Conclusion:

The optimal time interval is a strictly decreasing function of the parameter p.

Optimal continuous cover forest management

Now, we study:

One dimensional optimization in the volume dimension (of relevance when the time interval is determined by law or can not be determined for some other reason)

$$\max \pi = R(h) + \frac{P(v_1, t)Q(v_1, t) - c}{e^{rt} - 1}$$

$$s.t.$$

$$h = v_0 - v_1$$

$$\frac{d\pi}{dv_1} = \frac{dR}{dh}\frac{dh}{dv_1} + \frac{1}{e^{rt}-1}\left(\frac{dP}{dv_1}Q(.) + P\frac{dQ}{dv_1}\right) = 0$$

$$\frac{dh}{dv_1} = -1$$

Optimal principle in the volume dimension:

$$\frac{dR}{dh} = \frac{1}{e^{rt}-1}\left(\frac{dP}{dv_1}Q(.) + P\frac{dQ}{dv_1}\right)$$

How is the optimal volume affected if one parameter marginally increases (ceteres paribus)?

- **The optimal volume is not affected by changes of c.**

- **The optimal volume is a strictly decreasing function of r.**

- **The optimal volume is a strictly increasing function of p.**

Observation:

- If the volume is constrained (by law or something else), we may study the effects of the volume constraint on the optimal time interval, via one dimensional optimization.

$$\frac{d^2\pi}{dv_1 dt} < 0$$

$$\frac{dt^*}{dv_1} = \frac{-\left(\dfrac{d^2\pi}{dt dv_1}\right)}{\left(\dfrac{d^2\pi}{dt^2}\right)} < 0$$

Observation (extended):

- If the time interval is constrained (by law or something else), we may study the effects of the time interval constraint on the volume, via one dimensional optimization.

$$\frac{d^2\pi}{dv_1 dt} < 0$$

$$\frac{dv_1^*}{dt} = \frac{-\left(\dfrac{d^2\pi}{dv_1 dt}\right)}{\left(\dfrac{d^2\pi}{dv_1^2}\right)} < 0$$

Optimal continuous cover forest management

Now, we study:

Two dimensional optimization in the volume AND time interval dimensions

$$\max \pi = R(h) + \frac{pQ(v_1, t) - c}{e^{rt} - 1}$$

$$s.t.$$

$$h = v_0 - v_1$$

The first order optimum conditions:

$$\begin{cases} \dfrac{d\pi}{dv_1} = 0 \\ \dfrac{d\pi}{dt} = 0 \end{cases}$$

$$\begin{cases} \dfrac{dR}{dh}\dfrac{dh}{dv_1} + \left(\dfrac{1}{e^{rt}-1}\right) p \dfrac{dQ}{dv_1} = 0 \\ \dfrac{e^{rt}}{\left(e^{rt}-1\right)^2}\left(p\dfrac{dQ}{dt}\left(1-e^{-rt}\right) - \left(pQ-c\right)r \right) = 0 \end{cases}$$

Assumption: A unique maximum exists. The following conditions hold:

$$\left|\frac{d^2\pi}{dv_1{}^2}\right| < 0, \qquad \left|\frac{d^2\pi}{dt^2}\right| < 0, \qquad \begin{vmatrix} \dfrac{d^2\pi}{dv_1{}^2} & \dfrac{d^2\pi}{dv_1 dt} \\ \dfrac{d^2\pi}{dt dv_1} & \dfrac{d^2\pi}{dt^2} \end{vmatrix} > 0$$

$$\left(|D| = \begin{vmatrix} \frac{d^2\pi}{dv_1{}^2} & \frac{d^2\pi}{dv_1 dt} \\ \frac{d^2\pi}{dt dv_1} & \frac{d^2\pi}{dt^2} \end{vmatrix} > 0 \right) \Rightarrow \frac{d^2\pi}{dv_1{}^2}\frac{d^2\pi}{dt^2} - \frac{d^2\pi}{dt dv_1}\frac{d^2\pi}{dv_1 dt} > 0$$

$$\frac{d^2\pi}{dv_1{}^2}\frac{d^2\pi}{dt^2} > \frac{d^2\pi}{dt dv_1}\frac{d^2\pi}{dv_1 dt} = \left(\frac{d^2\pi}{dv_1 dt} \right)^2$$

Comparative statics analysis based on two dimensional optimization:

$$\begin{bmatrix} \dfrac{d^2\pi}{dv_1^{\,2}} & \dfrac{d^2\pi}{dv_1 dt} \\[2ex] \dfrac{d^2\pi}{dt dv_1} & \dfrac{d^2\pi}{dt^2} \end{bmatrix} \begin{bmatrix} dv_1^{\,*} \\ dt^* \end{bmatrix} = \begin{bmatrix} -\dfrac{d^2\pi}{dv_1 dc}dc - \dfrac{d^2\pi}{dv_1 dp}dp - \dfrac{d^2\pi}{dv_1 dr}dr \\[2ex] -\dfrac{d^2\pi}{dt dc}dc - \dfrac{d^2\pi}{dt dp}dp - \dfrac{d^2\pi}{dt dr}dr \end{bmatrix}$$

$$\begin{bmatrix} \dfrac{d^2\pi}{dv_1{}^2} & \dfrac{d^2\pi}{dv_1 dt} \\ \dfrac{d^2\pi}{dt dv_1} & \dfrac{d^2\pi}{dt^2} \end{bmatrix} \begin{bmatrix} dv_1{}^* \\ dt^* \end{bmatrix} = \begin{bmatrix} -\dfrac{d^2\pi}{dv_1 dc} dc \\ -\dfrac{d^2\pi}{dt dc} dc \end{bmatrix}$$

$$\frac{dv_1^{\ *}}{dc} = \frac{\begin{vmatrix} 0 & \dfrac{d^2\pi}{dv_1 dt} \\ -\dfrac{d^2\pi}{dtdc} & \dfrac{d^2\pi}{dt^2} \end{vmatrix}}{|D|} = \frac{\dfrac{d^2\pi}{dtdc}\dfrac{d^2\pi}{dv_1 dt}}{|D|} < 0$$

$$\frac{dt^*}{dc} = \frac{\begin{vmatrix} \frac{d^2\pi}{dv_1^2} & -\frac{d^2\pi}{dv_1 dc} \\ \frac{d^2\pi}{dtdv_1} & -\frac{d^2\pi}{dtdc} \end{vmatrix}}{\begin{vmatrix} \frac{d^2\pi}{dv_1^2} & \frac{d^2\pi}{dv_1 dt} \\ \frac{d^2\pi}{dtdv_1} & \frac{d^2\pi}{dt^2} \end{vmatrix}} = \frac{\begin{vmatrix} \frac{d^2\pi}{dv_1^2} & 0 \\ \frac{d^2\pi}{dtdv_1} & -\frac{d^2\pi}{dtdc} \end{vmatrix}}{|D|} = \frac{-\frac{d^2\pi}{dv_1^2}\frac{d^2\pi}{dtdc}}{|D|} > 0$$

$$\frac{dv_1^{\ *}}{dp} = \frac{\begin{vmatrix} -\dfrac{d^2\pi}{dv_1 dp} & \dfrac{d^2\pi}{dv_1 dt} \\ -\dfrac{d^2\pi}{dt dp} & \dfrac{d^2\pi}{dt^2} \end{vmatrix}}{|D|} = \frac{-\dfrac{d^2\pi}{dv_1 dp}\dfrac{d^2\pi}{dt^2} + \dfrac{d^2\pi}{dt dp}\dfrac{d^2\pi}{dv_1 dt}}{|D|} > 0$$

$$\frac{dt^*}{dp} = \frac{\begin{vmatrix} \dfrac{d^2\pi}{dv_1{}^2} & -\dfrac{d^2\pi}{dv_1 dp} \\ \dfrac{d^2\pi}{dtdv_1} & -\dfrac{d^2\pi}{dtdp} \end{vmatrix}}{|D|} = \frac{-\dfrac{d^2\pi}{dv_1{}^2}\dfrac{d^2\pi}{dtdp} + \dfrac{d^2\pi}{dtdv_1}\dfrac{d^2\pi}{dv_1 dp}}{|D|} < 0$$

$$\frac{dv_1^*}{dr} = \frac{\begin{vmatrix} -\dfrac{d^2\pi}{dv_1 dr} & \dfrac{d^2\pi}{dv_1 dt} \\ -\dfrac{d^2\pi}{dt dr} & \dfrac{d^2\pi}{dt^2} \end{vmatrix}}{|D|} = \frac{-\dfrac{d^2\pi}{dv_1 dr}\dfrac{d^2\pi}{dt^2} + \dfrac{d^2\pi}{dt dr}\dfrac{d^2\pi}{dv_1 dt}}{|D|} < 0$$

$$\frac{dt^*}{dr} = \frac{\begin{vmatrix} \dfrac{d^2\pi}{dv_1{}^2} & -\dfrac{d^2\pi}{dv_1 dr} \\ \dfrac{d^2\pi}{dtdv_1} & -\dfrac{d^2\pi}{dtdr} \end{vmatrix}}{\begin{vmatrix} \dfrac{d^2\pi}{dv_1{}^2} & \dfrac{d^2\pi}{dv_1 dt} \\ \dfrac{d^2\pi}{dtdv_1} & \dfrac{d^2\pi}{dt^2} \end{vmatrix}} = \frac{-\dfrac{d^2\pi}{dv_1{}^2}\dfrac{d^2\pi}{dtdr} + \dfrac{d^2\pi}{dtdv_1}\dfrac{d^2\pi}{dv_1 dr}}{|D|} < 0$$

Conclusions:

The forest laws in different countries, also neighbour countries such as Finland and Sweden, with almost the same prices, costs, technology and forest conditions, are very different.

If constraints that make continuous cover forest management less profitable than clear cut forestry are removed, we can expect better economic results and environmental improvements.

The analyses have shown how optimal decisions in forestry can be determined and how these optimal decisions are affected by parameter changes.

Several laws need to be adjusted in order to make rational forestry decisions legal.

The economic and environmental development of the world would benefit from more rational forest management.

There is a mathematical appendix available that contains all of the derivations.

Fundamental Principles of Optimal Continuous Cover Forestry

- Introduction
- Simple analytical method to optimize the stock level in continuous cover forestry.
- More advanced analytical method to simultaneously optimize the stock level after harvest and the harvest interval. General aspects on optimal CCF, forest regulations and environmental issues.
- **Advanced method to optimize CCF at the individual tree level, considering spatial competition, stochastic prices and timber quality variations.**

OPTIMAL STOCHASTIC DYNAMIC CONTROL OF SPATIALLY DISTRIBUTED INTERDEPENDENT PRODUCTION UNITS

Version 160901

Peter Lohmander

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International Conference on Mathematics and Decision Sciences

September 12-15, 2016, Guangzhou, China

Best Paper Award

The International Conference Of Mathematics and
Decition Sciences, ICMDS, 2016
September 12-15, 2016
Guangzhou, China

Author: Professor Peter Lohmander

Title: Optimal stochastic dynamic control of spatially distributed interdependent production units

Bing-yuan Cao
Chair of Conference ICMDS, 2016

hadi nasseri
Scientific Chair of Cnference, ICMDS 2016

Guangzhou, Sep. 15, 2016

The ambition of this study is to develop

a general method for optimization of stochastic and dynamic decision problems

with spatial dependencies that cannot be neglected and

where the need to use a multidimensional state space in high resolution

makes it computationally and economically impossible to apply the otherwise relevant method stochastic dynamic programming.

Applications can be found in most sectors of the economies.

One of the most obvious cases, where useful and statistically estimated functions already exist, is the forest sector.

We start with a forest area with 1000 trees of different sizes, as shown in Figure 1.

The initial locations and sizes of the trees are simulated.

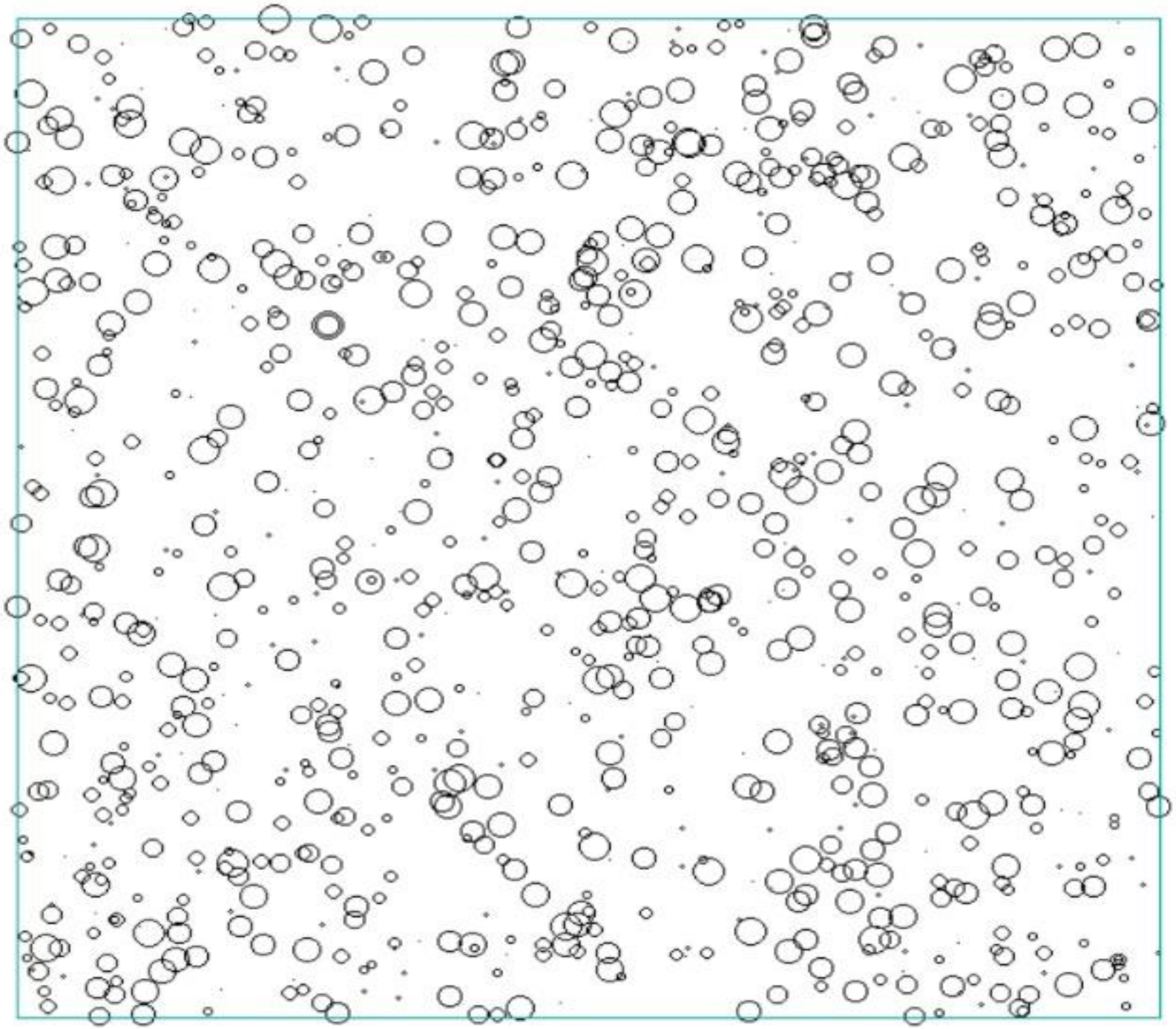


Figure 1:

Spatial map of initial conditions at t = -1

(years from the present time).

The locations of the circle centers are the locations of the trees.

The circle diameters are proportional to the tree diameters.

The square represents one hectare (100m*100m).

The problem is to determine an adaptive control function to be used in this forest,

giving the maximum of the total expected present value of all activities over time.

The annual increment of each tree is a function of tree size and competition from neighbor trees.

The different trees have different wood qualities, initially randomly assigned to the individuals.

The market value of a tree is a function of size, quality and stochastic price variations.

The variable harvesting cost of a tree is size dependent.

Every five years, the trees in the forest are inspected.

Then, depending on market prices, tree sizes, competition, quality etc., it is possible that some or many trees are harvested.

The optimized control function is used to make all of these decisions.

Figure 2 shows the structure of the forest directly after optimal harvesting at t = 0.

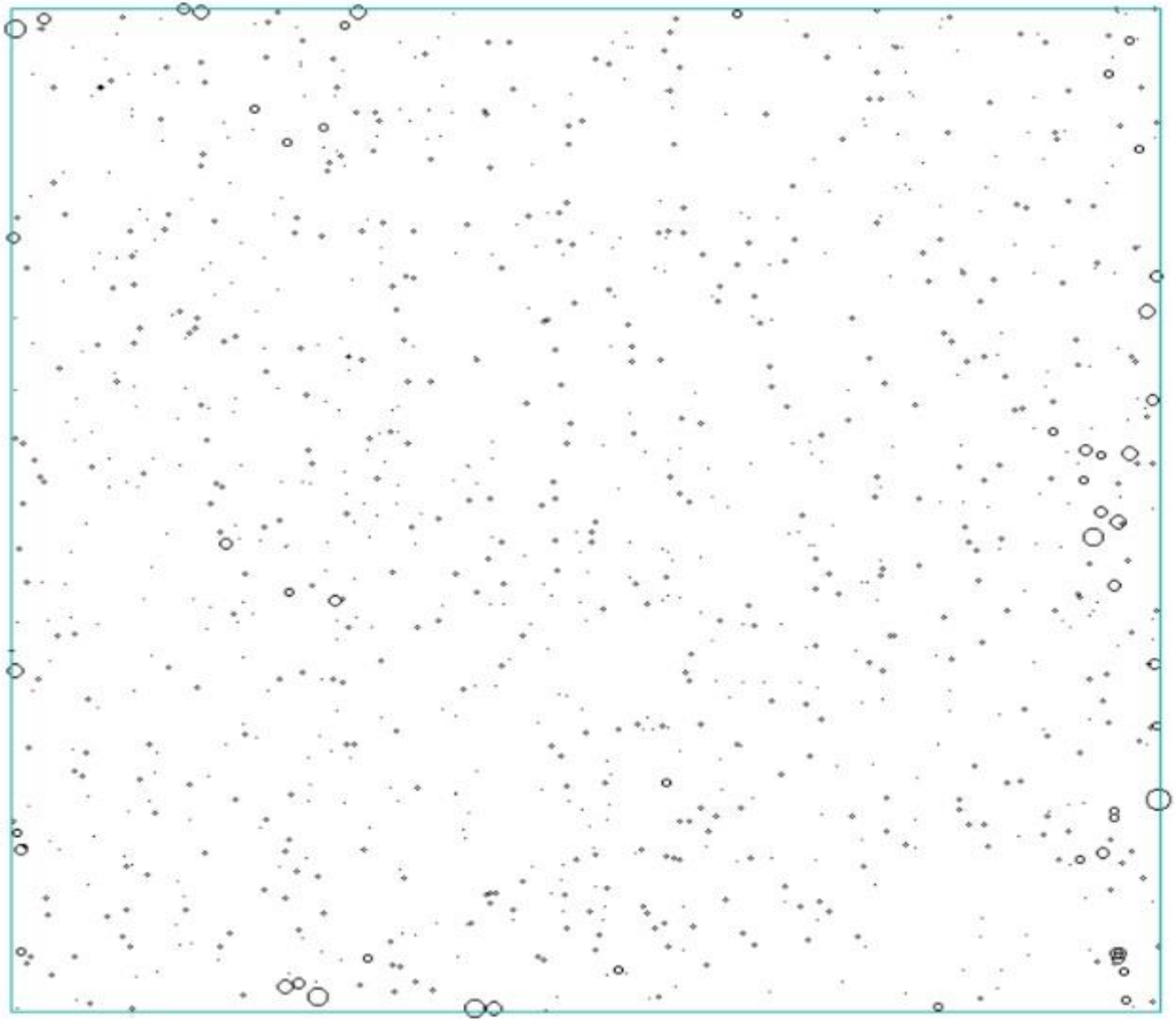


Figure 2:

The state after the first application of the optimized control function at t = 0.

Most of the largest trees have been removed.

Obviously, a considerable number of large trees have been removed.

Many new seedlings are however found on the land, in random positions.

The trees continue to grow and Figure 3 illustrates the situation 35 years later.

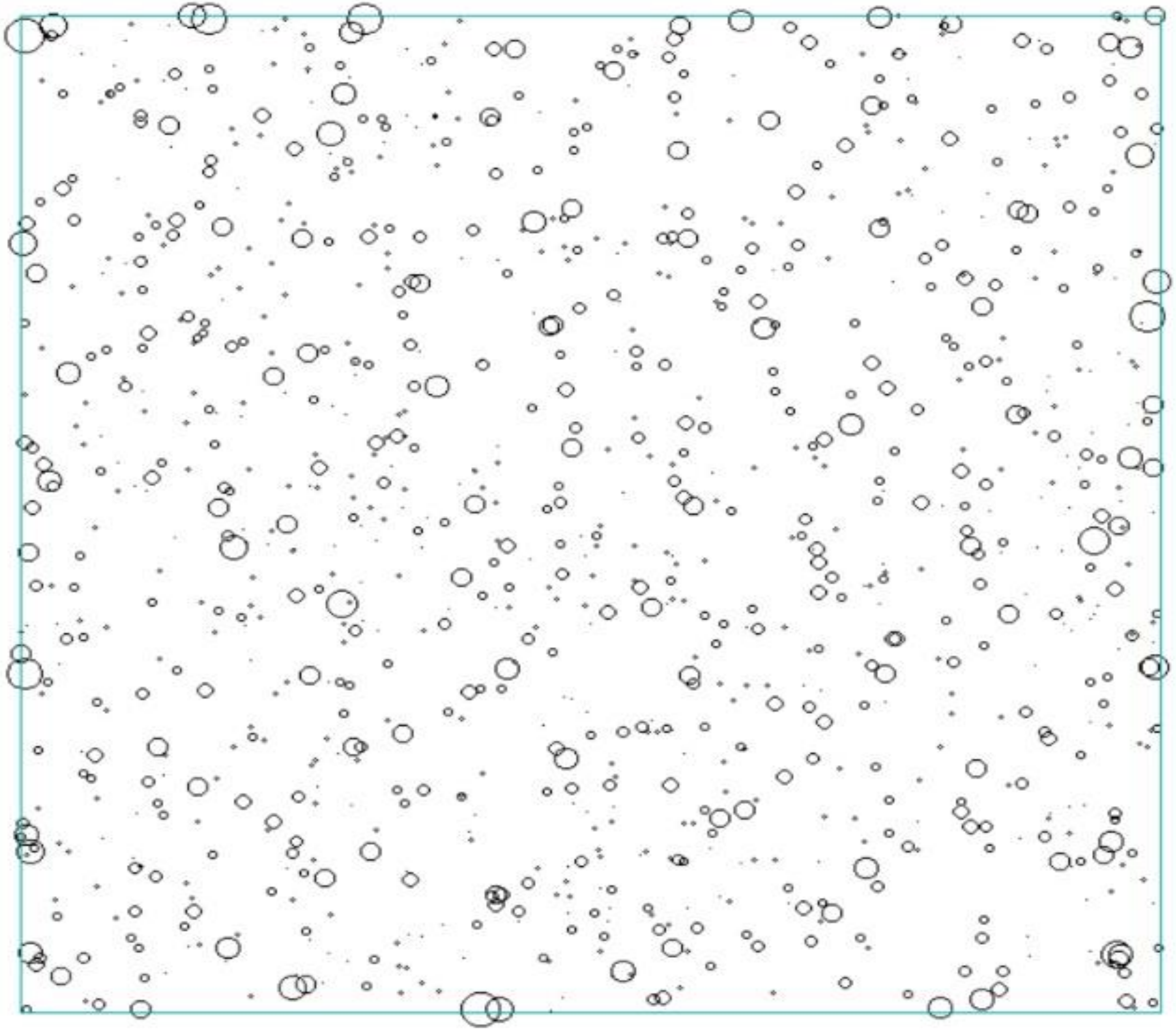
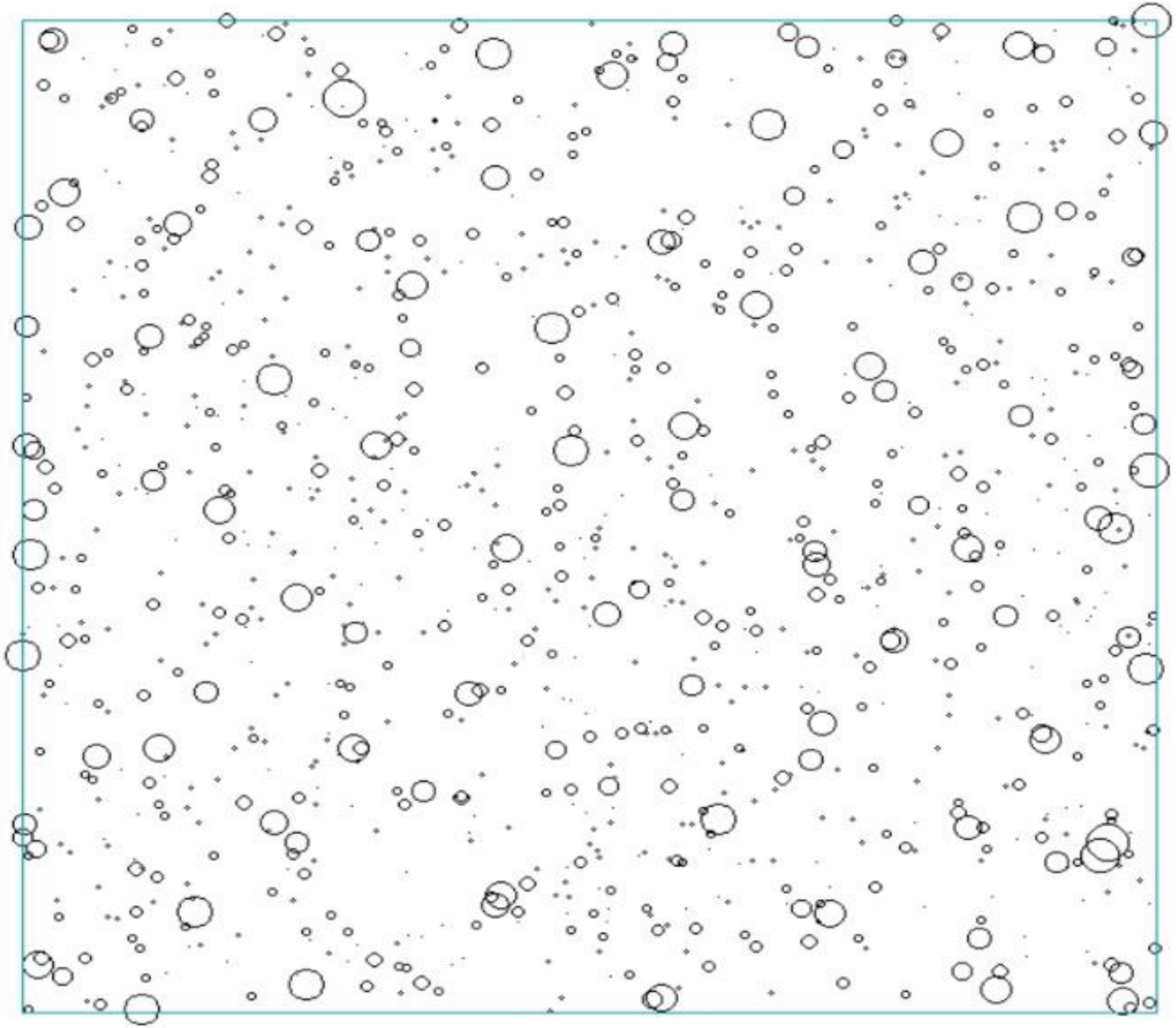


Figure 3:

The forest at t = 35.

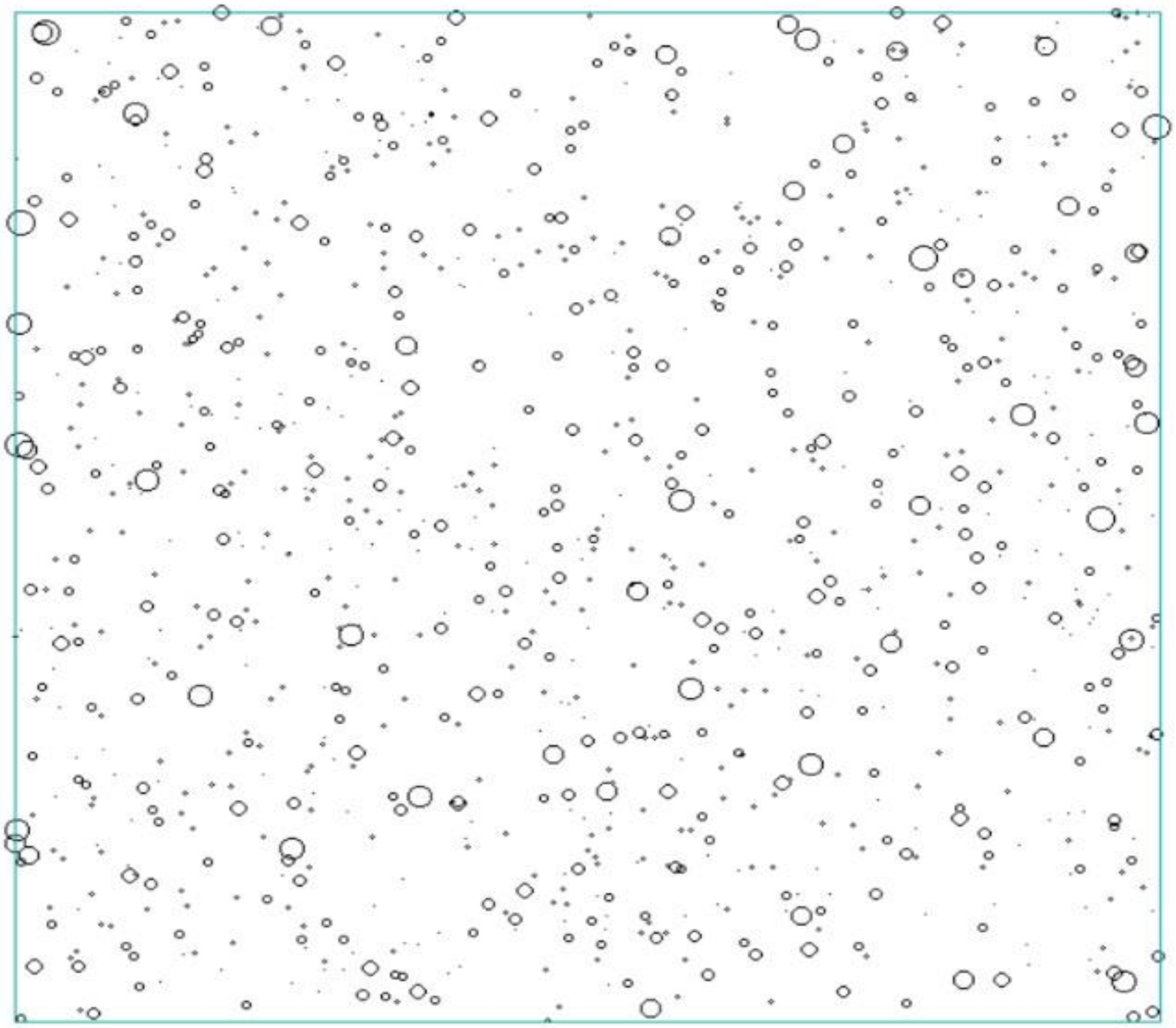


69 years after the first harvest, trees of considerable sizes exist (Figure 4).

The total number of large trees in year 69 is however much lower than before the harvest during year 0.

Figure 4:

The forest at t = 69.



Several large trees are harvested in year 70
(Figure 5).

Figure 5:
The forest at t = 70.

This type of stochastic dynamic and spatial forest development is sustainable. Furthermore, there are always trees in the forest. We have a system of "optimal continuous cover forestry".

Lohmander [1] describes several alternative methods to optimize forest management decisions at higher levels.

[1] Lohmander, P., **Adaptive Optimization of Forest Management in a Stochastic World**, in Weintraub A. et al (Editors), Handbook of Operations Research in Natural Resources, Springer, Springer Science, International Series in Operations Research and Management Science, New York, USA, pp 525-544, 2007

Lohmander and Mohammadi [2] determine optimal harvest levels in beech forests in Iran, using stochastic dynamic programming.

Then, however, the tree selection decisions were never analyzed.

[2] Lohmander, P., Mohammadi, S., **Optimal Continuous Cover Forest Management in an Uneven-Aged Forest in the North of Iran**, Journal of Applied Sciences 8(11), 2008

2 Analysis

The optimal decisions for each tree, i, at time t, is determined by the diameter limit function $d_L(i,t)$. If the diameter is larger than the diameter limit, then the tree should be harvested. Otherwise, it should be left for continued production.

$$d_L(i,t) = d_0 + d_c C(i) + d_q Q(i) + d_p \Delta P(t)$$

The parameters

$$\left(d_0, d_c, d_q, d_p\right)$$

are optimized in this study. In the graphs and software, they are denoted

(dlim_0, dlim_c, dlim_q and dlim_p).

$$\left(C(i), Q(i), \Delta P(t)\right)$$

denote competition index for tree i,
quality of tree i and
the stochastic deviation of the market price from the expected price, at time t.

The stochastic price deviations are i.i.d. and have uniform pdf on the interval -10 to +10 EURO/cubic metre.

Lohmander, P., Reservation price models in forest management: Errors in the estimation of probability density function parameters and optimal adjustment of the biasfree point estimates, Management Systems for a Global Forest Economy with Global Resource Concerns, Society of American Foresters, Asilomar, California, September 1994, Brodie, D. & Sessions, J., (Editors), College of Forestry, Oregon State University, Corvallis, Oregon, USA, 439-456, 1995
http://www.Lohmander.com/Lohmander_SAF_1995.pdf

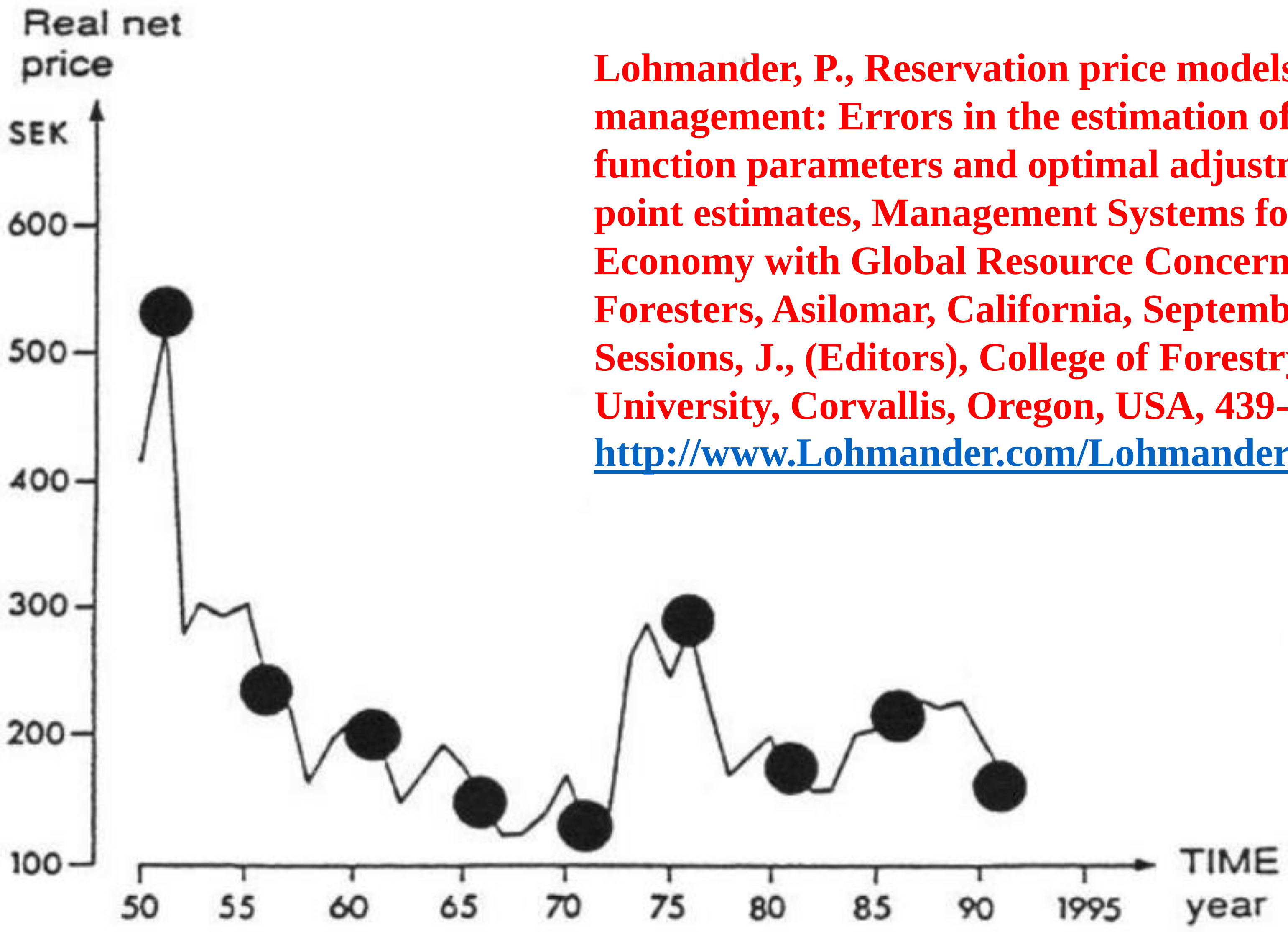


Figure 1. Real stumpage price (price - harvest cost) per cubic metre, Sweden. Source: National Board of Forestry (1993).

The objective function is the
total expected present value of all revenues minus all costs from year 0 until year 200.

The real rate of interest is set to 3%.

The computer model includes functions for
tree height as a function of diameter,
functions used in tree volume calculations,
set up costs,
tree size dependent revenues and variable harvesting costs etc.

The trees grow according to a modified version of the function reported by Schütz [3].

The modification is that in [3], competition is assumed to come from all parts of the forest area, also far away from the individual tree.

In the function applied in this new analysis, **only competition from trees at distances ten meters or closer**, is considered.

[3] Schütz, J-P., Modelling the demographic sustainability of pure beech plenter forests in Eastern Germany, Ann. For. Sci. 63 (2006) 93–100

Furthermore, in the Schütz function, each tree is only affected by competition from trees with larger diameters.

In the present study, also competition from trees with smaller diameters is considered.

However, it is probably the case that trees with smaller diameter give a lower degree of competition.

The motivation for the new function, used here, is that **competition for light, water and nutrients, obviously is stronger from neighbor trees than from trees far away**. Furthermore, **also smaller trees use some of the available light, water and nutrients**.

$$I(i) = b_0 + b_1 LN(d(i)) + b_2 \left(C(i)\right)^3$$

$I(i)$ is the diameter increment of tree i and

$d(i)$ is the diameter.

(b_0, b_1, b_2) is a set of empirically estimated parameters, published by Schütz [3], for beech in Germany.

$C(i)$ is now expressed as the basal area per hectare of larger competing trees plus the basal area of smaller competing trees divided by 2 (all within the 10 meter radius circle). In future studies, the competition function should be estimated with locally relevant data.

The **optimization of the total expected present value**, **via the parameters of the adaptive control function**, contained the following steps:

A software code was constructed and tested in QB64.

The objective function was estimated for a set of combinations of the control function parameters

$$\left(d_0, d_c, d_q, d_p\right)$$

For this purpose, a **four dimensional loop** with alternative parameter values was run.

Preliminary iterative studies were first made to determine interesting parameter intervals.

Then, a **3*3*3*4 loop** was used, which gave **108 parameter combinations**.

For each parameter combination, the **total expected present value during 200 years was estimated for 10 different forest areas of one hectare, each with 1000 initial random trees.**

That analysis took approximately 8 hours on an Acer Aspire V personal computer with an Intel Core i5 processor.

Next, the parameter values of $\left(d_c, d_q\right)$ determined in the “108-loop”, were considered optimal and fixed. A more detailed analysis, with higher resolution, of the parameters $\left(d_0, d_p\right)$ was made.

3 Main Results

The adaptive control function parameters $\left(d_0, d_c, d_q, d_p\right)$

were determined in a general loop.

108 combinations were evaluated.

This is the adaptive control function:

$$d_{L,a}(i,t) = 0.60 - 0.0030\,C(i) + 0.020\,Q(i) - 0.020\,\Delta P(t)$$

This is the **optimal adaptive control function**:

$$d_{L,a}(i,t) = 0.60 - 0.0030\,C(i) + 0.020\,Q(i) - 0.020\,\Delta P(t)$$

The optimal objective function value was estimated to 2571 EURO/hectare.

Next, the parameter values of $\left(d_c, d_q\right) = \left(-0.003, 0.020\right)$

determined in the “108-loop”, were considered optimal and fixed.

A more detailed analysis, with higher resolution, of the parameters

$\left(d_0, d_p\right)$ = (dlim_0, dlim_p) was made.

Figure 6 shows the objective function and in Figure 7, the objective function level curves are given.

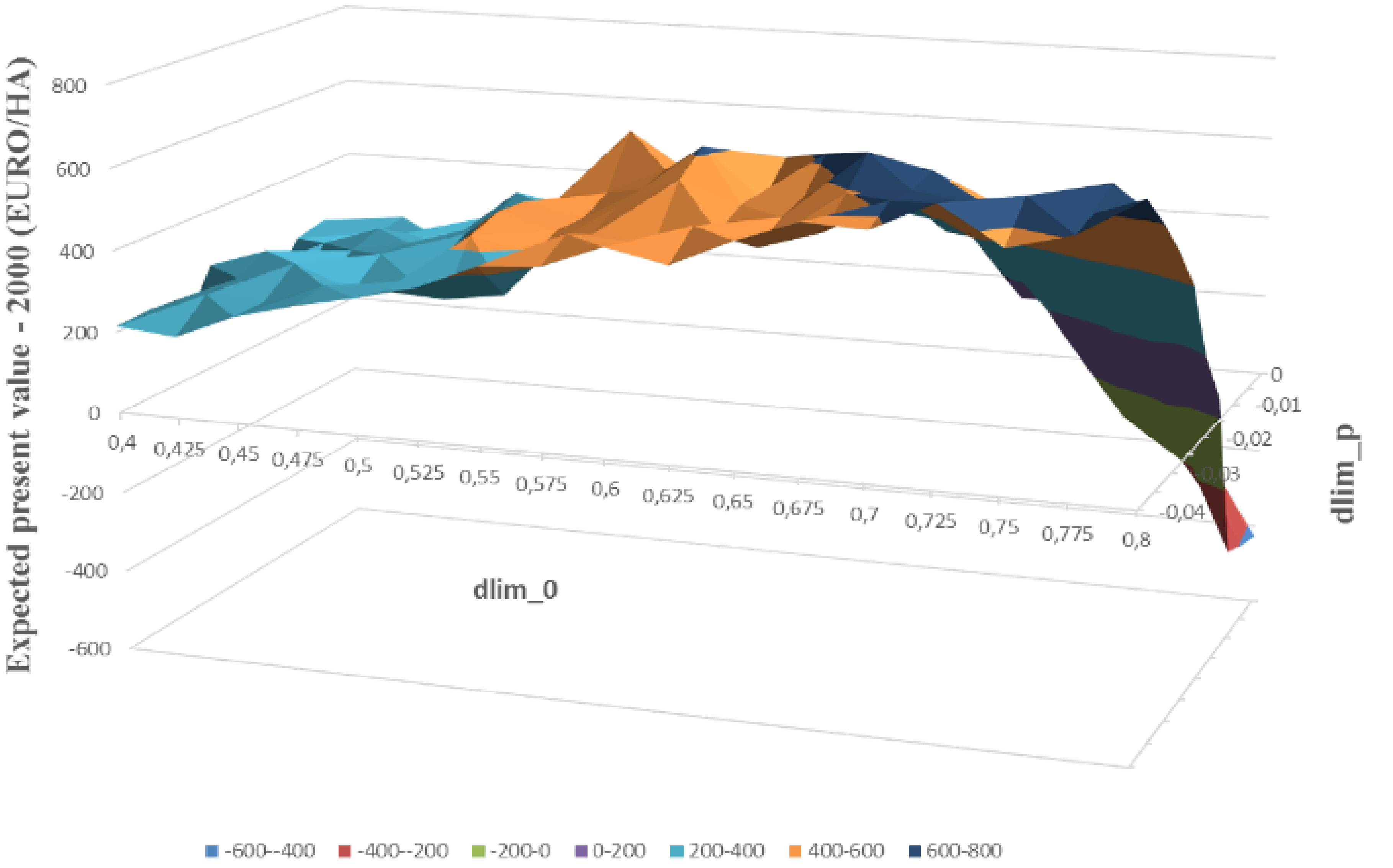


Figure 6:

The objective function reduced by a constant

as a function of the parameters

dlim_0 and dlim_p,

for optimal values of the other parameters, namely dlim_c = -0.003 and dlim_q = 0.02.

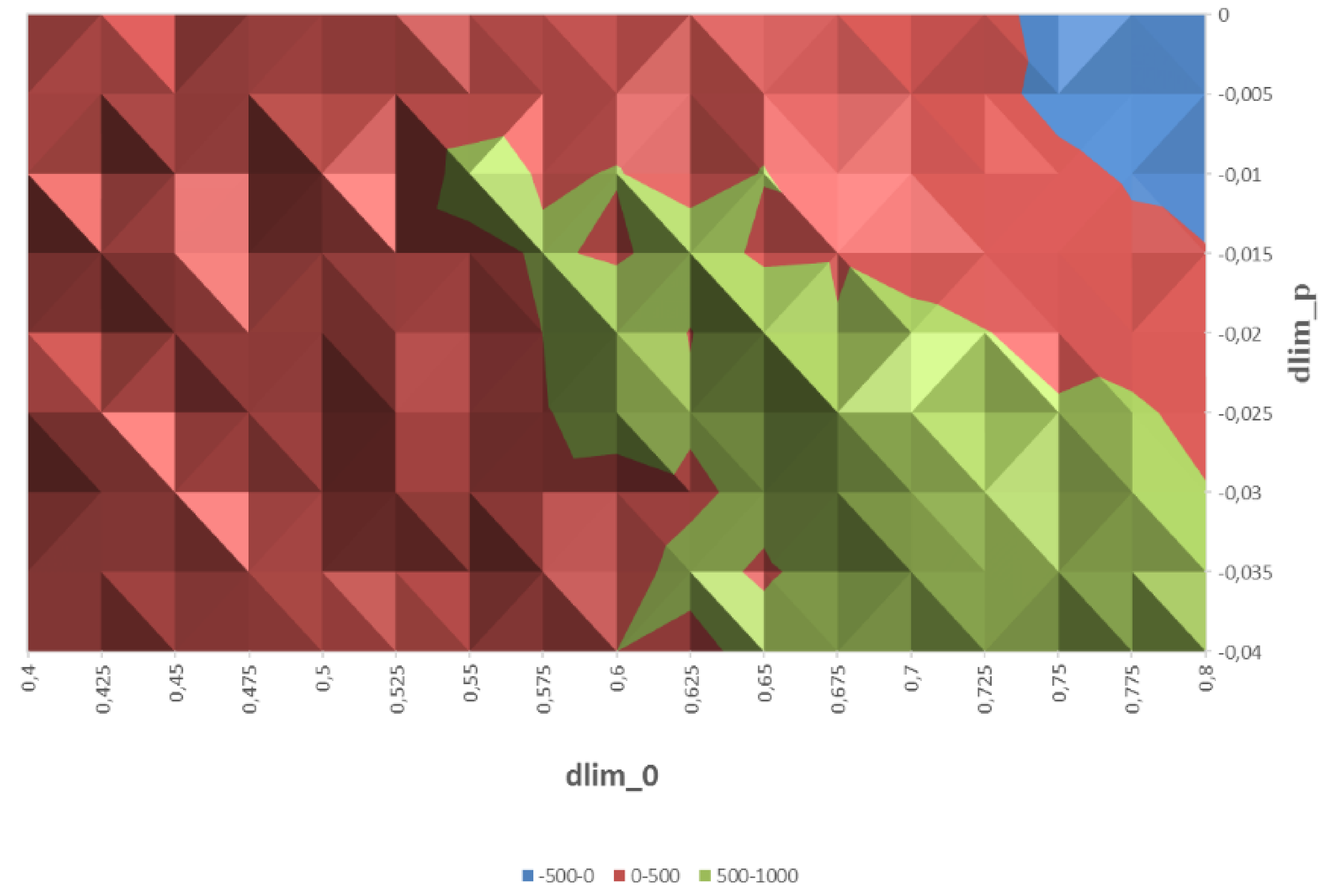


Figure 7:

The **level curves of the objective function as a function of the parameters dlim_0 and dlim_p**,

when the **other parameters were held constant** at their optimal values.

Multiple regression analysis
and the data presented in Figure 6 were used to estimate a

quadratic approximation of the objective function, Z.

Let (x, y) = $\left(d_0, d_p\right)$

$$Z = 8694x + 22248y - 8170x^2 - 235019y^2 - 65389xy$$

The R2 value of the regression was 0.999 and all coefficients were statistically significant, with p-values below 0.00003.

Approximation of the objective function, Z

$$Z = 8694x + 22248y - 8170x^2 - 235019y^2 - 65389xy$$

The first order optimum conditions are:

$$\frac{dZ}{dx} = -16340x - 65389y + 8694 = 0$$

$$\frac{dZ}{dy} = -65389x - 470038y + 22248 = 0$$

The equation system

$$\begin{bmatrix} -16340 & -65389 \\ -65389 & -470038 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -8694 \\ -22248 \end{bmatrix}$$

gives this unique solution: $(x, y) \approx (0.773, -0.0602)$

Now, the objective function value is 2690 EURO/hectare.

The derived optimum is a **unique maximum**, which is confirmed by:

$$\left|-16340\right| = -16340 < 0$$

and

$$\begin{vmatrix} -16340 & -65389 \\ -65389 & -470038 \end{vmatrix} \approx 3.405 \cdot 10^9 > 0$$

The **quadratic approximation** gave this **optimal control function**:

$$d_{L,b}(i,t) = 0.773 - 0.0030\,C(i) + 0.020\,Q(i) - 0.0602\,\Delta P(t)$$

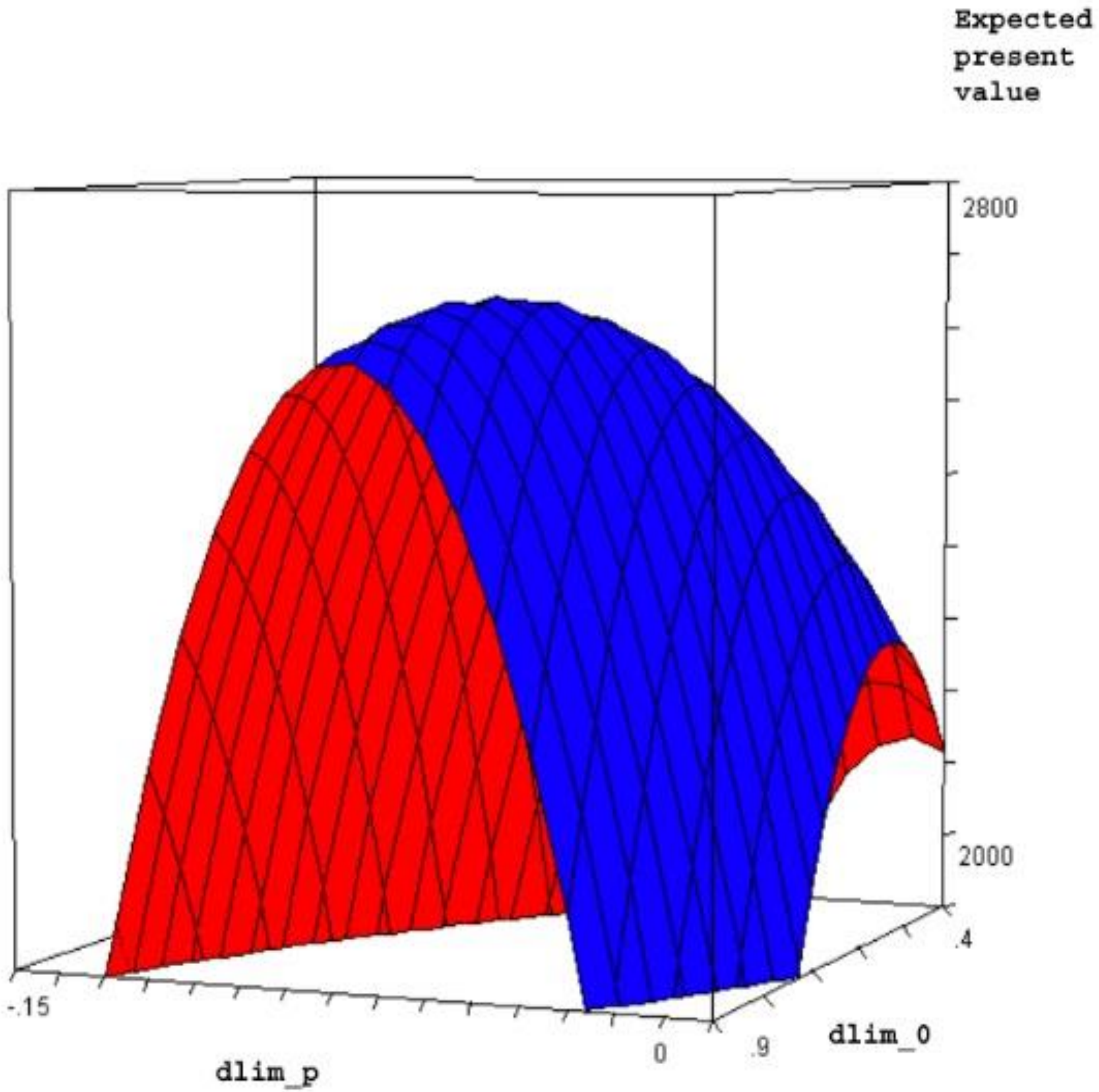


Figure 8:

The **objective function as a function of the parameters dlim_0 and dlim_p**, **according to the quadratic approximation**, when the other parameters were held constant at their optimal values.

The quadratic approximation gave this optimal control function:

$$d_{L,b}(i,t) = 0.773 - 0.0030\,C(i) + 0.020\,Q(i) - 0.0602\,\Delta P(t)$$

Three General Forest Management Conclusions:

A tree should be harvested at a smaller tree diameter, in case the local competition from other trees increases.

A tree should be harvested at a larger tree diameter, in case the wood quality of the tree increases.

A tree should be harvested at a smaller tree diameter, in case the market net price (price – harvesting cost) for wood increases.

FINAL CONCLUSIONS

- **The general principles of optimal continuous cover forestry have been derived and presented.**
- **General analytical solutions have been obtained for the optimal stock level and the optimal harvest time interval, via stand level optimization.**
- **Optimal CCF rules for decisions concerning individual trees have been derived via numerical optimization of adaptive control functions. These rules handle tree dimension, timber quality, spatial distribution, local competition and stochastic prices.**

Fundamental Principles of Optimal Continuous Cover Forestry

Linnaeus University Research Seminar (RESEM)

Tuesday 2017-05-23, 10.15-11.00, Lecture Hall: Karlavagnen

Version 170518

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