An Adaptive Optimization Approach To Expansion Of The Forest and Bioenergy Sectors Utilizing Wood From The Boreal Forests Version 170816

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This presentation will show that:

- There are very **large options to increase sustainable continuous cover forestry** in the Borel forests. This way, sustainable flows of bioenergy and all kinds of forest industry raw materials can be obtained and these sectors can be expanded. This is also good when we consider the global warming problem.
- Continuous cover forestry is often the production economically **optimal forestry method**. Hence, the method is not just rational from environmental perspectives.)
- The fundamental understanding of forest dynamics can be improved via **new growth functions and estimations**.
- The **complete system can be optimized with new adaptive methods**.

The boreal forest covers very large areas in Canada, USA, Russia and Scandinavia.

TABLE 2 **Top ten countries by reported forest area in 2015**

	Country	Forest area (thousand ha)	% of land area	% of global forest area
1	Russian Federation	814 931	50	20
2	Brazil	493 538	59	12
3	Canada	347 069	38	9
4	United States of America	310 095	34	8

FAO, Global Forest Resources Assessment 2015
How are the world's forests changing?
Second edition

Table 9. Other naturally regenerat						
	Other naturally regenerated forest (1 000 ha)					
Country/Territory	1990	2000	2005	2010	2015	Tier trend
Russian Federation	554573	535777	536358	522180	522372	
United States of America	214500	208671	204623	207862	208431	
Canada	137057	132098	129641	127265	125361	

Parts of Table 9.

522 Million ha

208 Million ha

125 Million ha

Source: FAO (2015) GLOBAL FOREST RESOURCES ASSESSMENT 2015, Desk reference, http://www.fao.org/3/a-i4808e.pdf

Other Natually Regenerated Forests:

Russian Federation
522 Million ha

USA
208 Million ha

Canada
125 Million ha

Source: FAO (2015)

Large parts of these regions are presently covered by more or less natural forests, often dominated by different species of spruce, pine and larch.

- In large parts of these forests, in particular in Canada and Russia, the **industrial utilization** presently is and historically has been close to **zero**.
- Expanding infrastructure, technological development of harvesters and forwarders, increasing costs and environmental problems associated with fossil **fuel** extraction, a growing interest in **sustainability** and the debate on **climate change**, make it rational to investigate environmentally acceptable harvesting options in the remote and natural boreal forests.
- This presentation suggests a **way to develop** the forest and bioenergy sectors, taking relevant objectives, facts and options into account.

- Industrial expansion of forest utilization is often considered as very negative for the environment.
- It is often assumed that the initially existing natural forests, with trees of many size classes, should be removed and replaced by uniform plantations.
- **However, it is often optimal, also from a production economic point of view, to start harvesting the natural forests using continuous cover methods**.

CAN THIS BE PROVED?

- You should never believe in results from a model with hidden parameters, a black box.
- Here, you find a few simple but completely described examples.
- In these examples, it is not economically optimal to make clear fellings.

First example:

Determination of the optimal initial harvest and the stock level after the initial harvest.

In this first example, set up costs are ignored. In the second example, they are considered.

We start with the stock level V0 (usually higher than the optimal stock level).

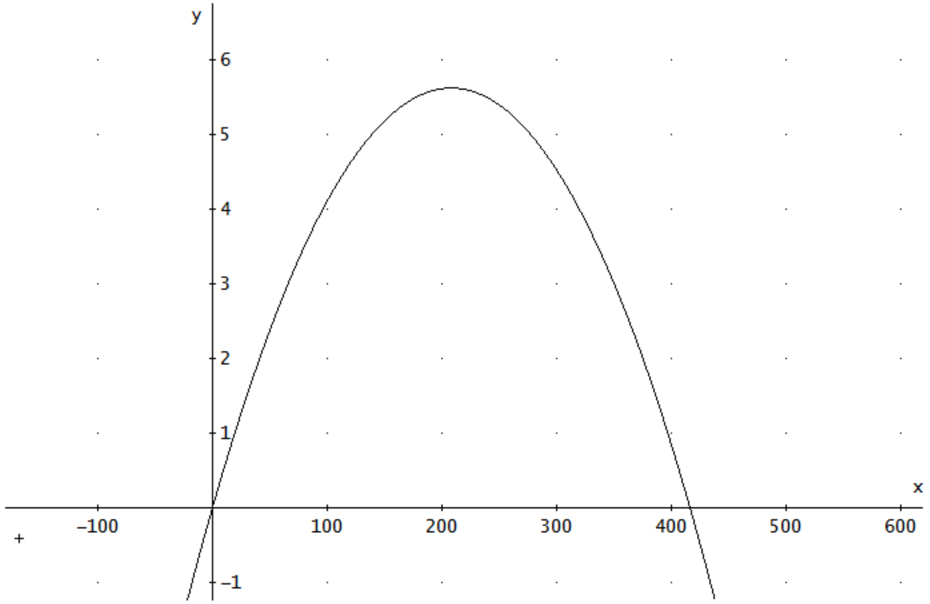
After the initial harvest, h, we have the stock level V1 (= V0-h).

Our objective function is the total present value *(*)*.

*** = The sum of all revenues minus costs, at all points in time, with consideration of the rate of interest in the capital market.**

This is a growth function example:

G(V) = Growth (m3 per hectare and year)



V = Stock level (m3 per hectare)

$$G(V) \approx 0.0540V - 0.000130V^2$$

Maximization of Π , the total present value :

$$\max_{h} \Pi = p_0 h + \int_0^{\infty} p_1 G(V_0 - h) e^{-rt} dt$$

V0 = Initial stock level (m3 per hectare).
p0 = Net price per cubic metre (price minus cost per m3) in the initial harvest.
p1 = Net price per cubic metre (price minus cost per m3) in future harvests.
h = Initial harvest (m3 per hectare).
V0-h = Stock level after the initial harvest (m3 per hectare).
G(V0-h) = Growth (m3/year) after the initial harvest.
(Note that the future harvest level is identical to the future growth.)
r = Rate of interest in the capital market.

$$\max_h \Pi = p_0 h + \int_0^\infty p_1 G(V_0 - h) e^{-rt} dt$$

$$\max_h \Pi = p_0 h + p_1 G(V_0 - h) \int_0^\infty e^{-rt} dt$$

$$\max_h \Pi = p_0 h + p_1 G(V_0 - h) \frac{1}{r}$$

First order optimum condition

$$\frac{d\Pi}{dh} = p_0 - p_1 G'(V_0 - h)\frac{1}{r} = 0$$

Second order maximum condition

$$\frac{d^2\Pi}{dh^2} = p_1 G''(V_0 - h)\frac{1}{r} < 0$$

$$\left(\frac{d\Pi}{dh}=0\right)\Rightarrow\left(p_0=p_1G'(V_0-h)\frac{1}{r}\right)$$

$$G'(V_0-h)=\frac{p_0}{p_1}r$$

The optimal stock level after the initial harvest

$$G'(V_1) = G'(V_0 - h) = \frac{p_0}{p_1} r$$

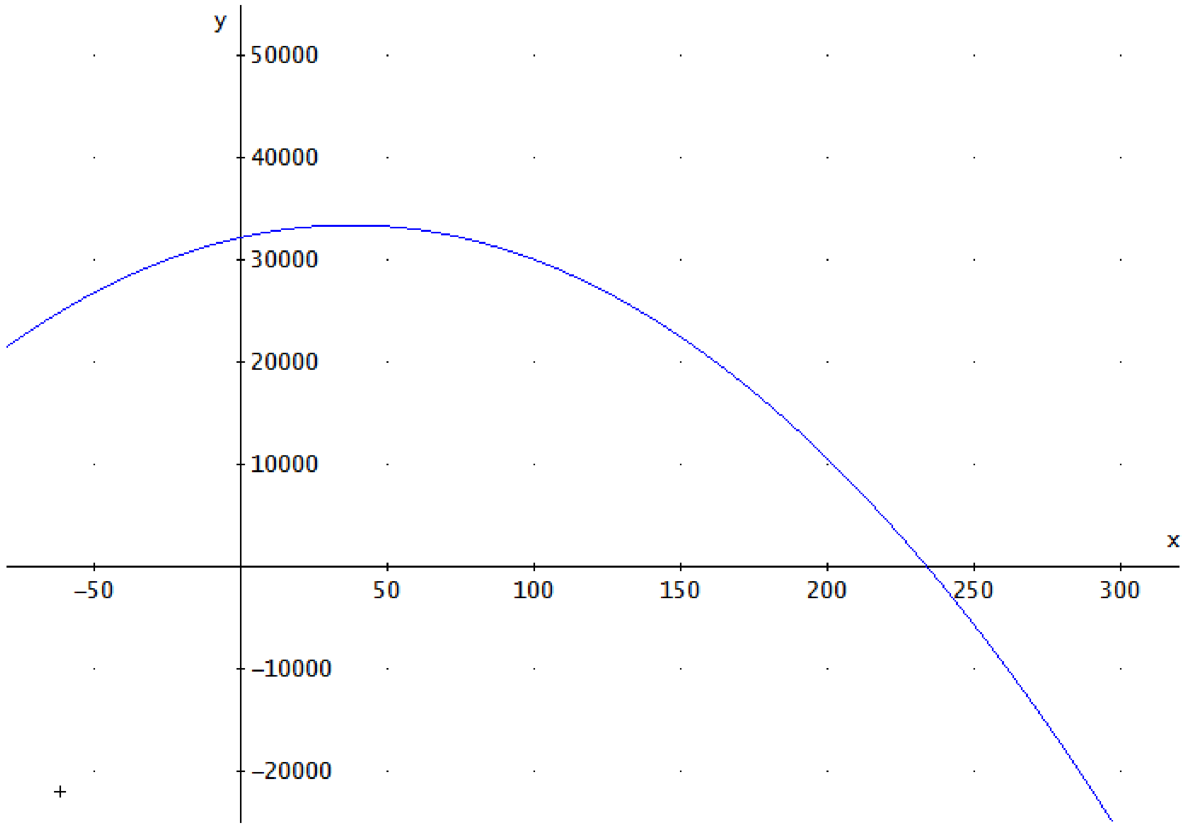
EXAMPLE: ***Optimization with specific parameters and a graphical approach:***

Initial stock level = 130 m3/ha.
p0 = p1 = 200 SEK/m3.
r = 3%.
Estimated growth function: ***G*** = 0.054011123·V - 0.000129731·V·V

$$\max_h \Pi = p_0 h + p_1 G(V_0 - h)\frac{1}{r}$$

$$200 \cdot h + \frac{1}{0.03} \cdot 200 \cdot (0.054011123 \cdot (130 - h) - 0.000129731 \cdot (130 - h) \cdot (130 - h))$$

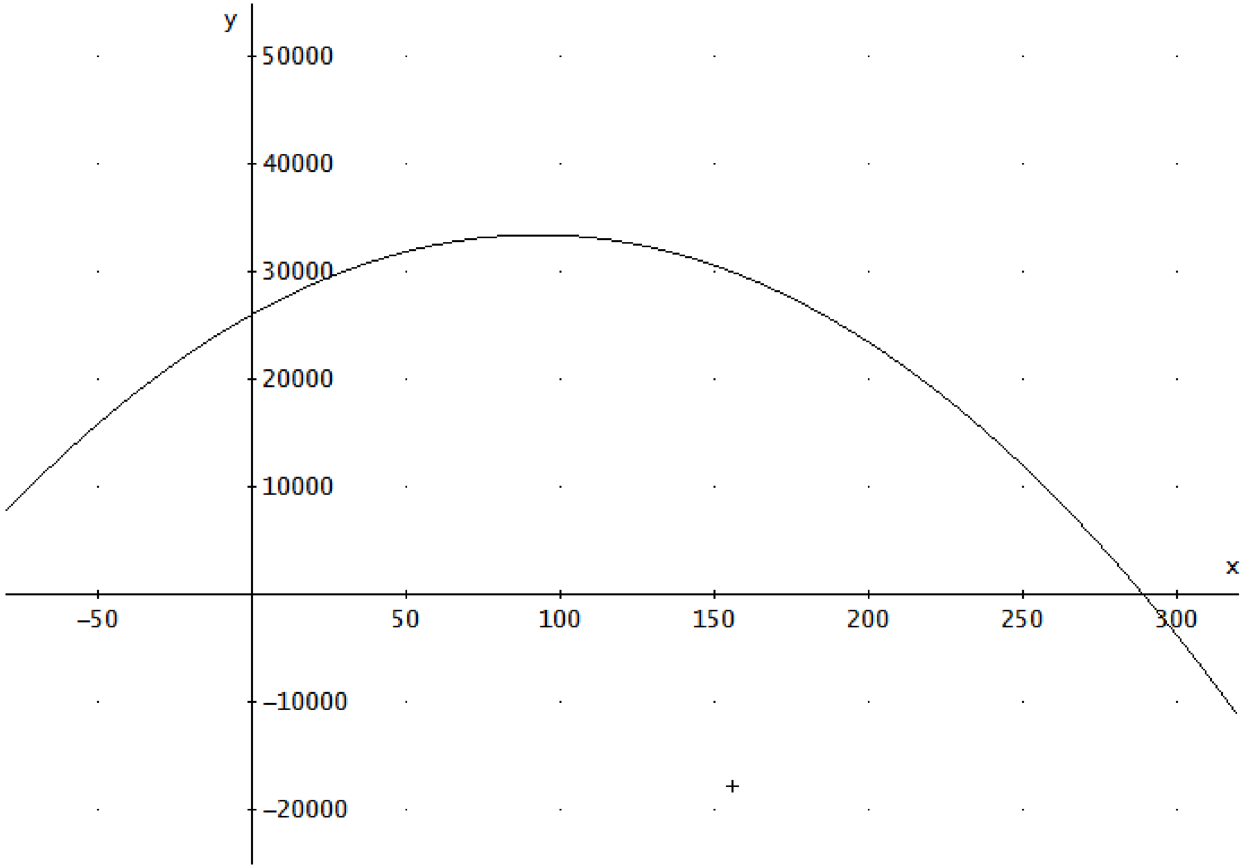
Present value (SEK/ha)



h (m3/ha)

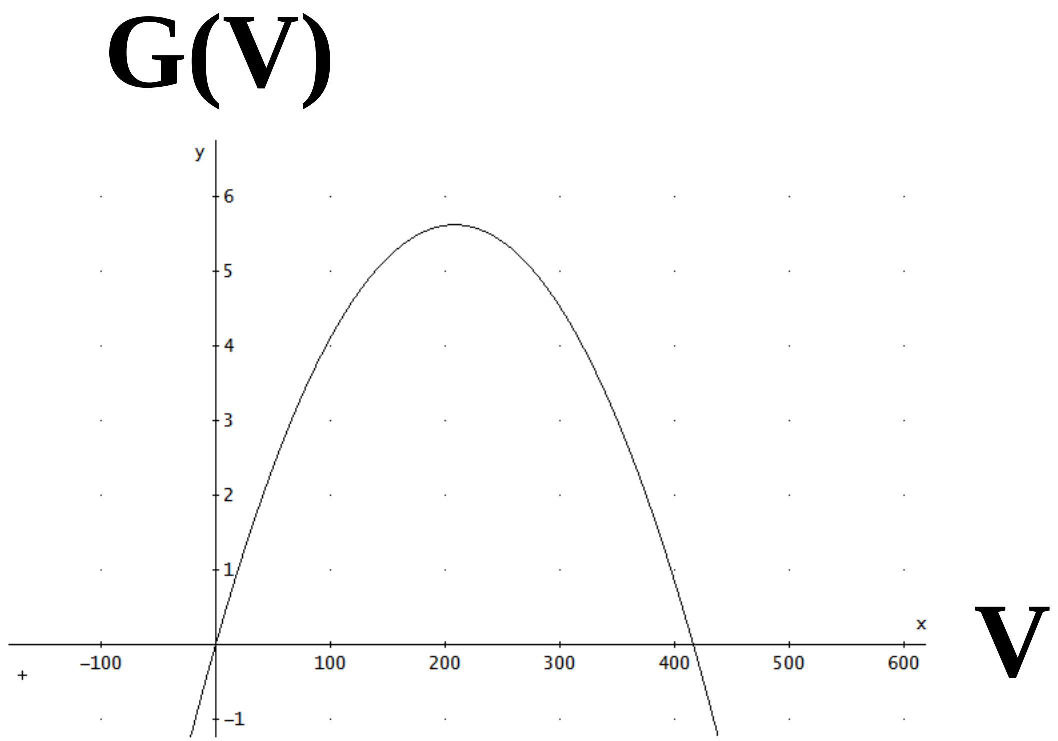
$$200 \cdot h + \frac{1}{0.03} \cdot 200 \cdot (0.054011123 \cdot (130 - h) - 0.000129731 \cdot (130 - h) \cdot (130 - h))$$

Present value (SEK/ha)



V1=V0-h (m3/ha)

$$200\cdot(130 - x) + \frac{1}{0.03}\cdot 200\cdot(0.054011123\cdot x - 0.000129731\cdot x\cdot x)$$



$$G(V) \approx 0.0540V - 0.000130V^2$$

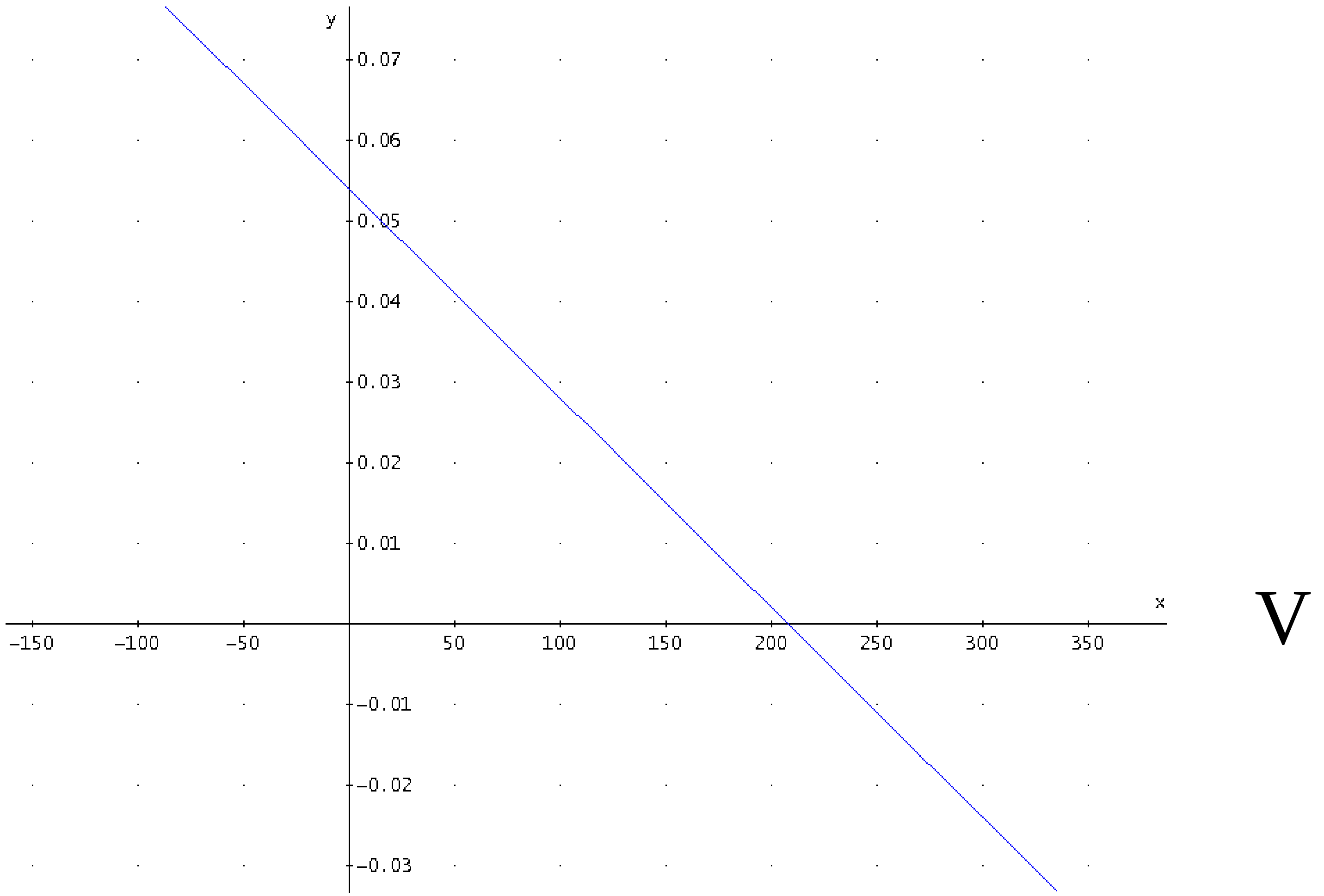
Growth function derivatives:

$$G(V) \approx 0.0540V - 0.000130V^2$$

$$G'(V) \approx 0.0540 - 0.000260V$$

$$G''(V) \approx -0.000260 < 0$$

$$G'(V) \approx 0.0540 - 0.000260V$$

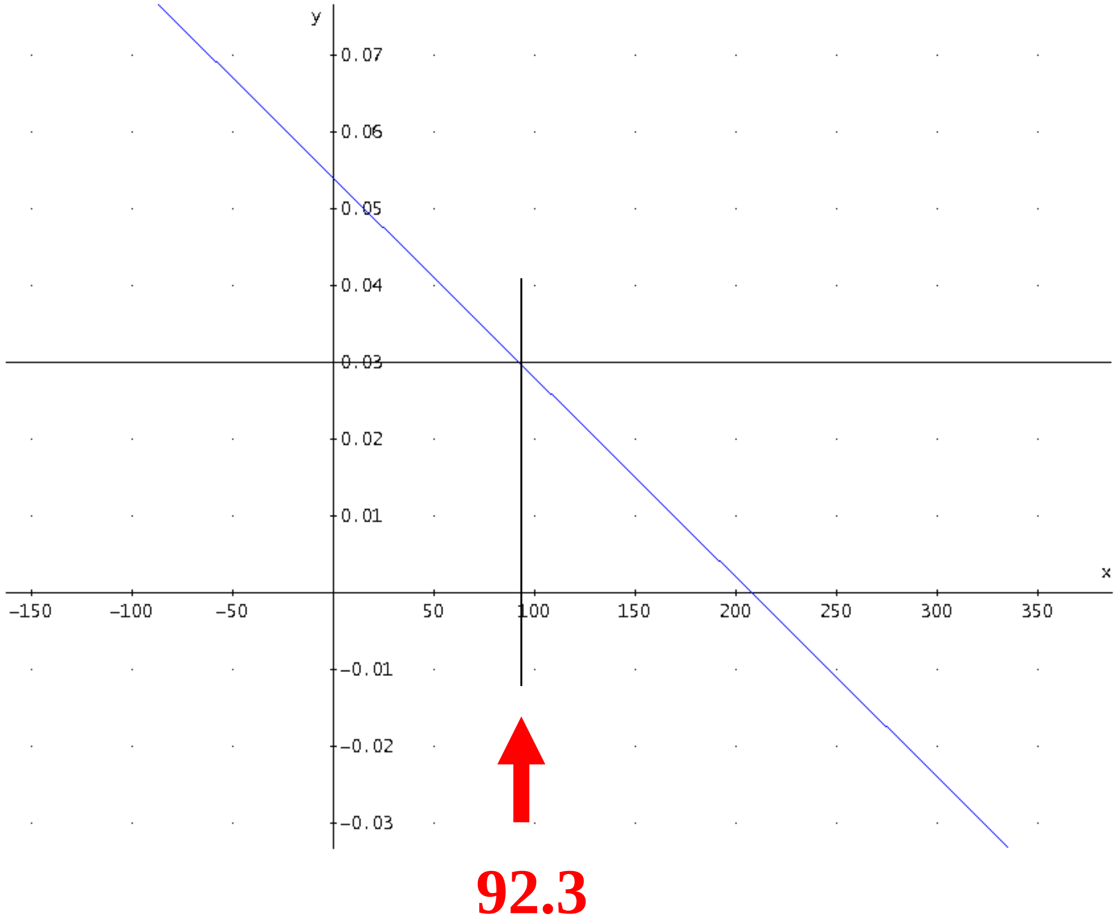


Determinaion of the optimal stock level:

$$G'(V) = \frac{p_0}{p_1} r$$

$$0.0540 - 0.000260V = \frac{p_0}{p_1} r$$

$$G'(V) \approx 0.0540 - 0.000260V$$



$$\boxed{\frac{p_0}{p_1} r = 3\%}$$

V

Conclusion:

The optimal stock level is 92.3 m3 per hectare
if the rate of interst is 3% in this case.

It is not optimal to make a clear felling and reduce the stock level to zero.

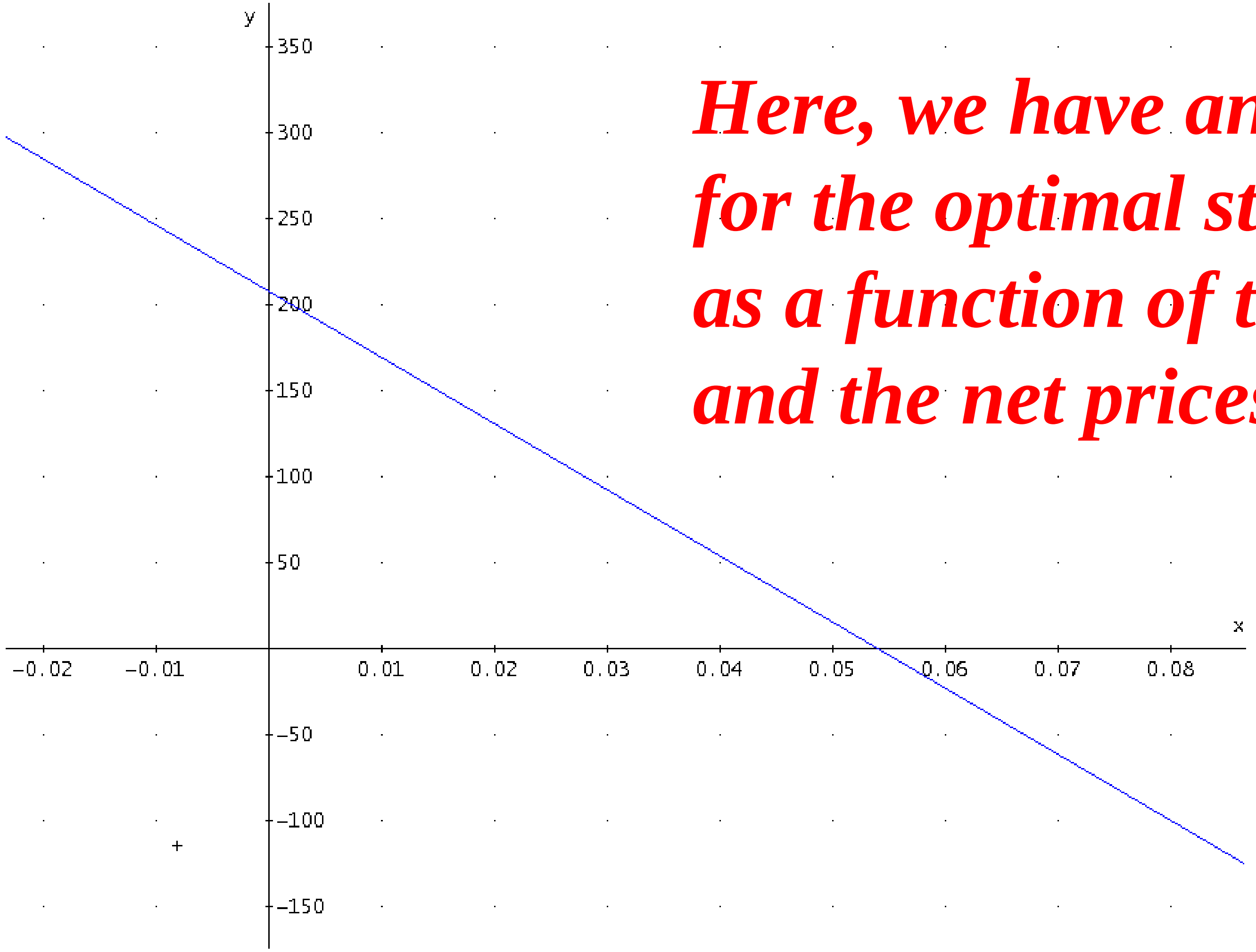
We can also determine an equation for the optimal stock level.

$$0.0540 - 0.000260V = \frac{p_0}{p_1} r$$

$$V = \frac{-0.0540 + \frac{p_0}{p_1} r}{-0.000260}$$

$$V \approx 207.7 - 3846 \frac{p_0}{p_1} r$$

$$V \approx 207.7 - 3846 \frac{p_0}{p_1} r$$



$$\frac{p_0}{p_1} r$$

Here, we have an expression for the optimal stock level as a function of the rate of interest and the net prices.

General principles of rational continuous cover forestry derived from the simple analytical method:

1. The optimal stock level is a decreasing function of the rate of interest.

2. The optimal stock level decreases if the net price per cubic metre in the initial harvest increases in relation to the net price per cubic metre in future harvests.

3. The optimal stock level increases if the net price per cubic metre in the initial harvest decreases in relation to the net price per cubic metre in future harvests.

4. It is optimal to let the stock level be equal to the stock level that maximizes the average growth (MSY) only if the rate of interest in the capital market is zero.

Second example:

Determination of the optimal harvest levels, the stock level after harvest and the harvest intervals.

In this second example, set up costs are considered

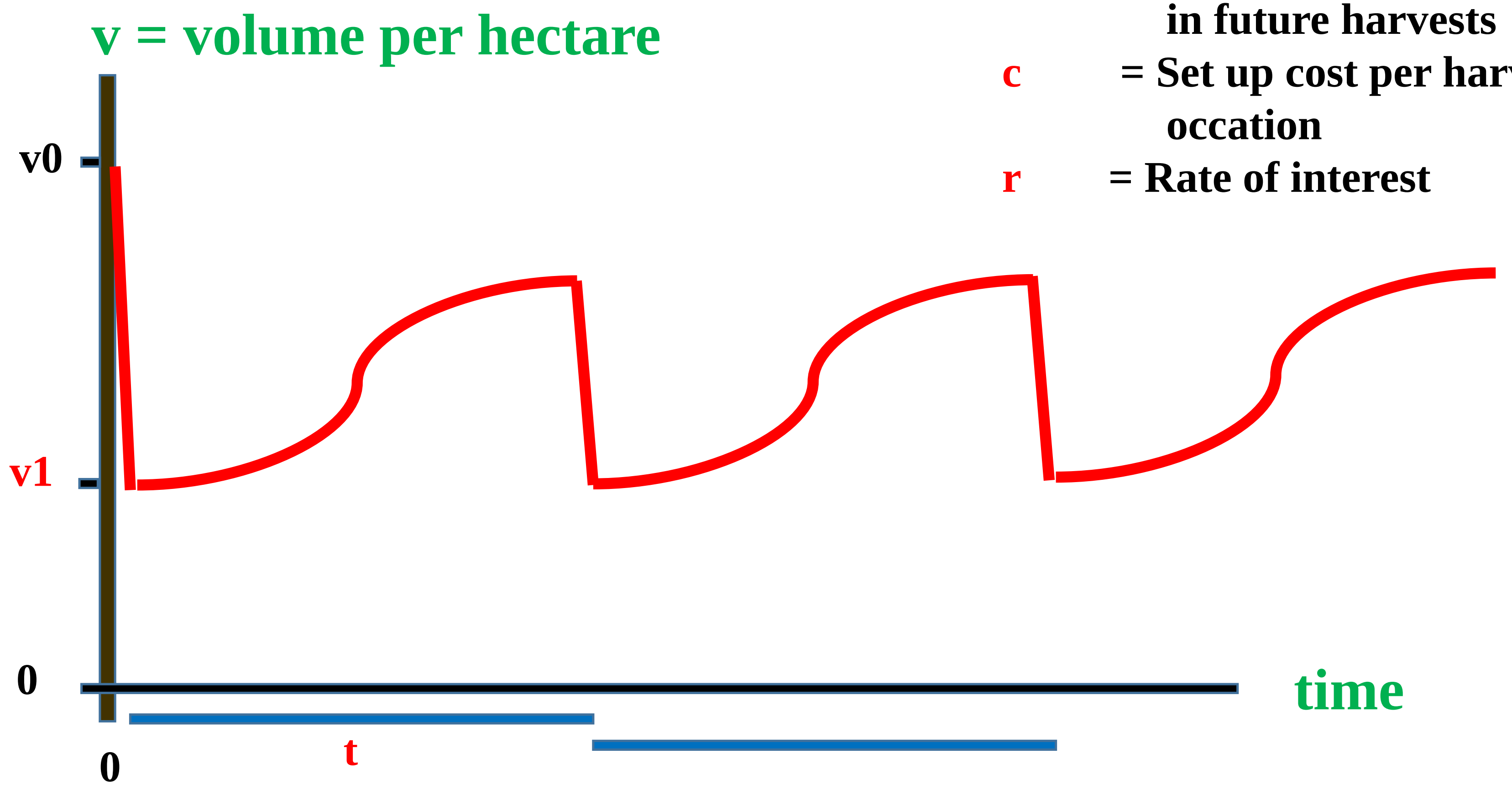
Present value

$$\max \pi = R(h) + \frac{P(v_1,t)Q(v_1,t) - c}{e^{rt} - 1}$$

$s.t.$

$$h = v_0 - v_1$$

h = The first harvest volume
R(h) = Profit from the first harvest
P(.) = Price per cubic metre (reduced by variable cost per cubic metre) in future harvests
Q(.) = Harvest volume per hectare in future harvests
c = Set up cost per harvest occation
r = Rate of interest



In Finland, continuous cover forest management can be optimized without constraints.

In Sweden, there are several constraints in the forest act. For instance, the volume always has to stay above a specified lower limit. If the volume is below the limit, you have to make a clearcut.

WITH Swedish constraints, forestry with clearcuts often is the economically optimal choice.
WITHOUT Swedish constraints, continuous cover forestry is very often the economically optimal choice.

$$\max \pi = R(h) + \frac{P(v_1, t)Q(v_1, t) - c}{e^{rt} - 1}$$

$$s.t.$$

$$h = v_0 - v_1$$

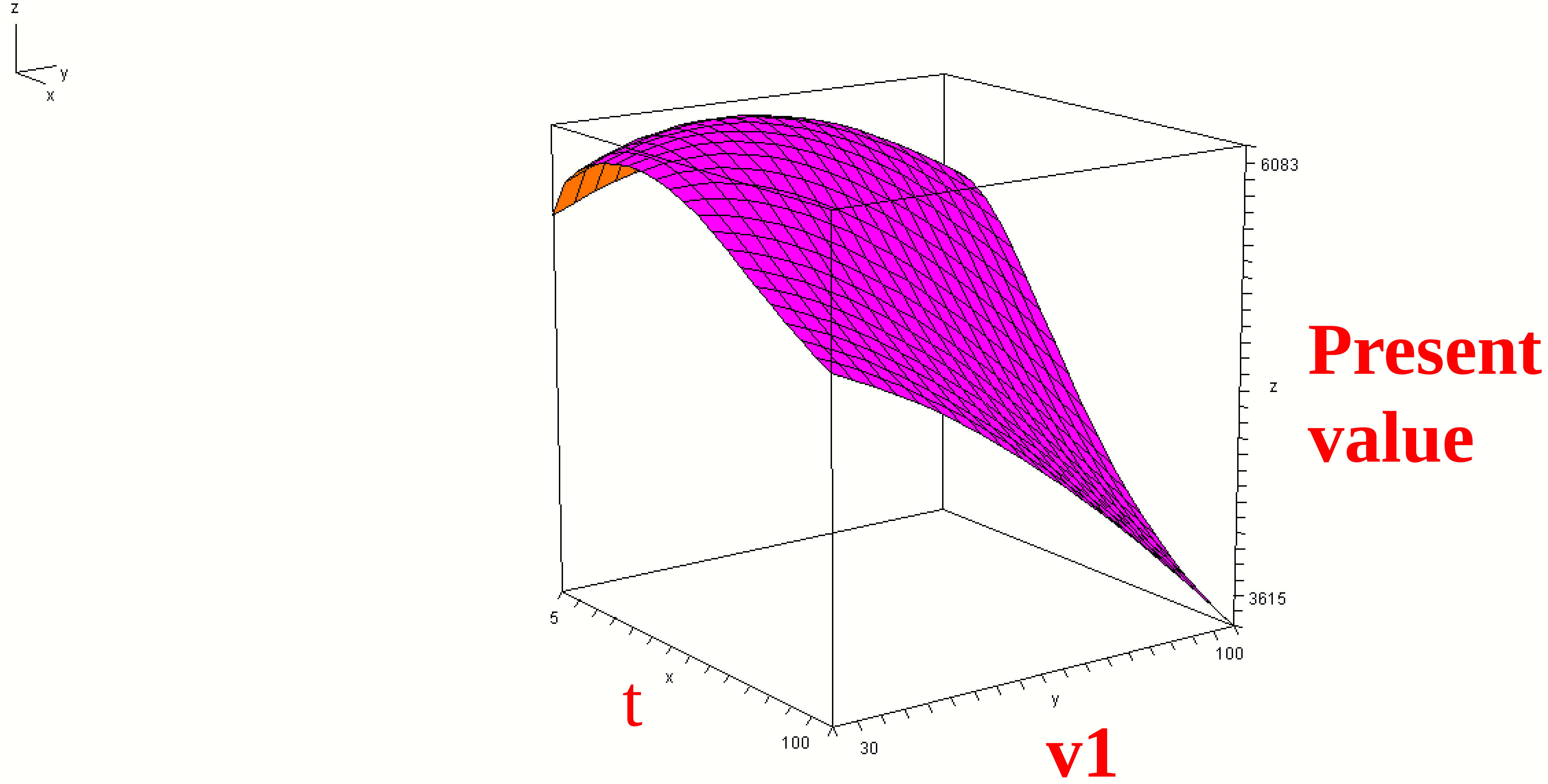
Example:

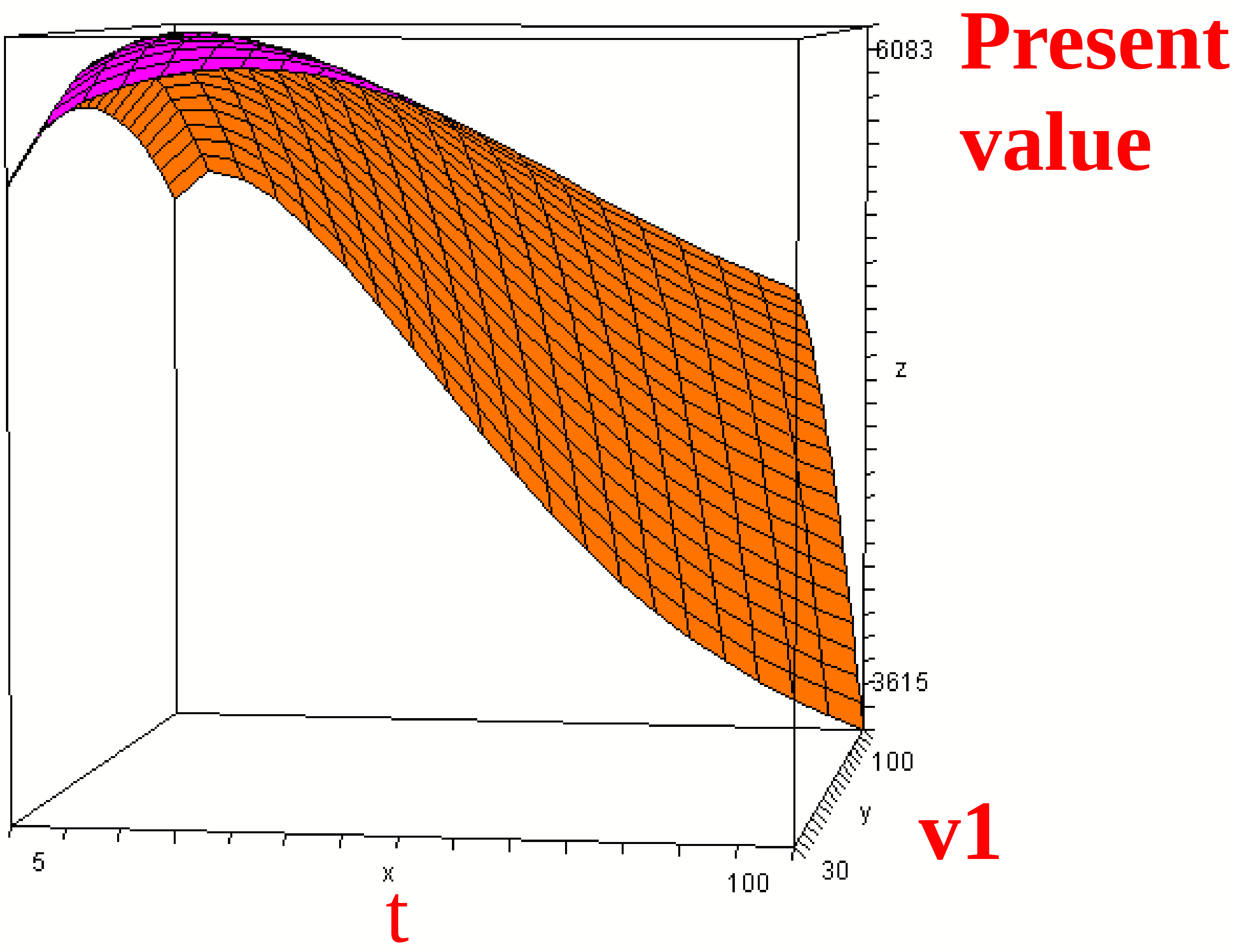
Graphical illustrations based on specified functions and parameters

$$\left(30\cdot(200 - v) + \frac{0.1833333}{2}\cdot(200 - v)\cdot(200 - v) - \right.$$

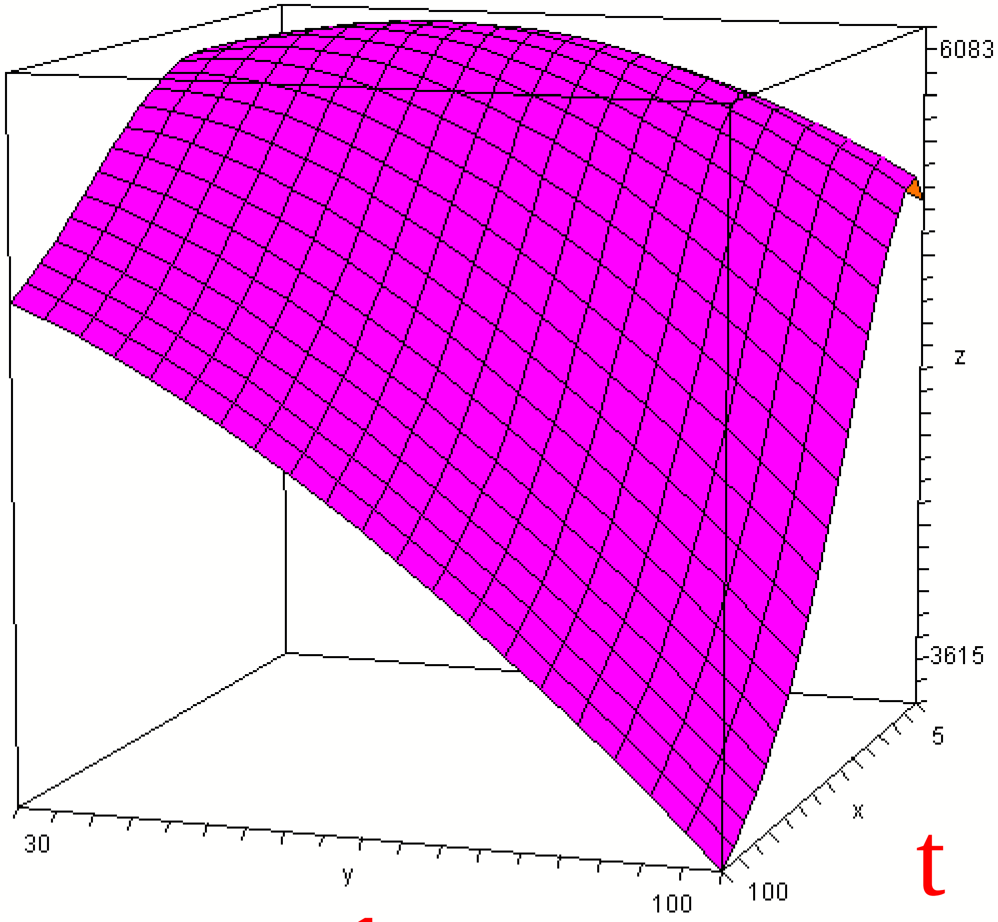
$$\left.\frac{0.001666667}{3}\cdot(200 - v)\cdot(200 - v)\cdot(200 - v) - 50\right) +$$

$$\frac{20\cdot\left(\dfrac{1}{\dfrac{1}{400} + \left(\dfrac{1}{v} - \dfrac{1}{400}\right)\cdot EXP(-\ 0.05\cdot t)} - v\right) - 50}{EXP(0.03\cdot t) - 1}$$


z
y
x
6083
3615
z
5
x
100
30
y
100
t
v1
Present value



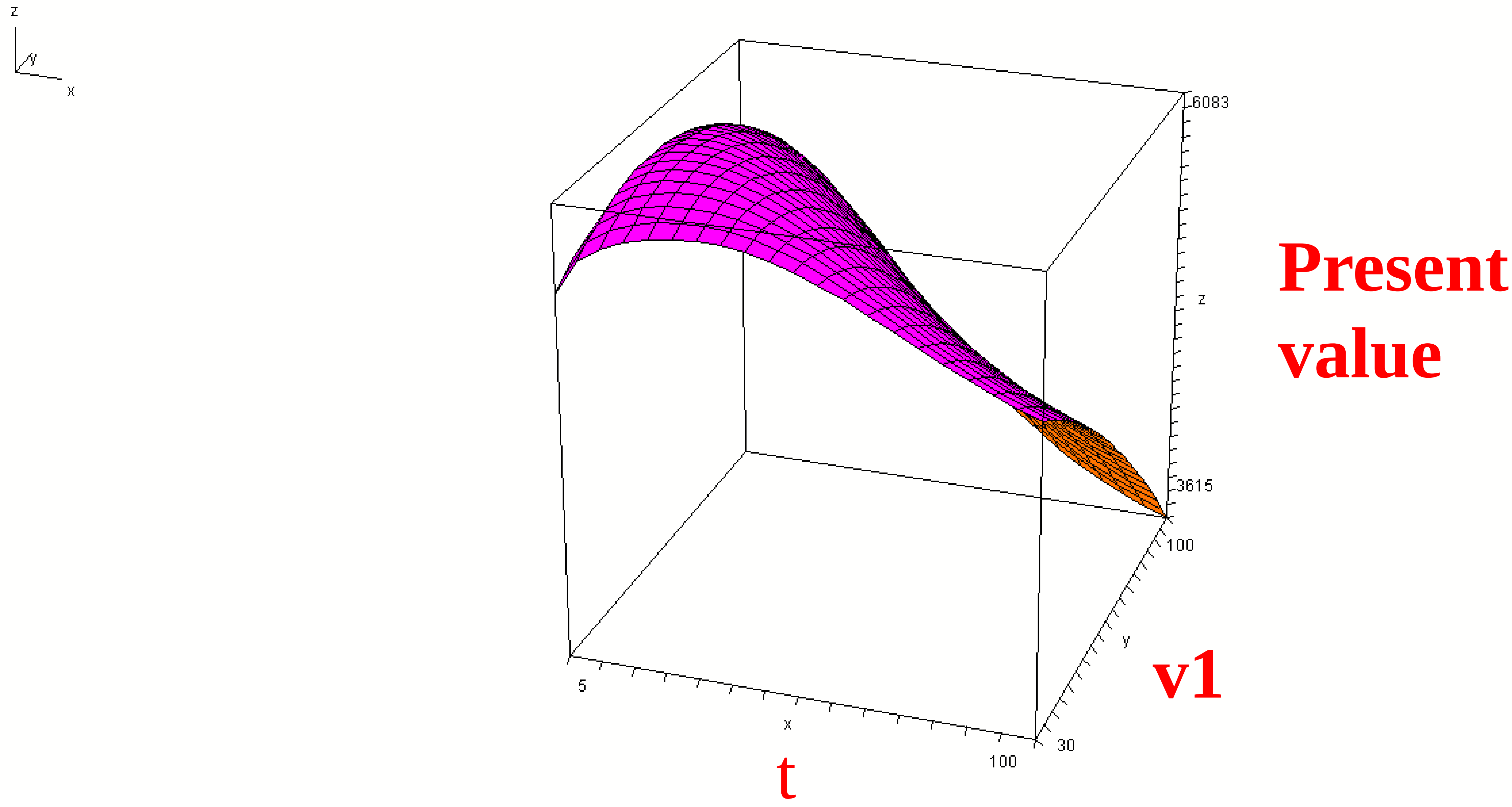
6083
3615
z
5
x
100
100
y
30
Present value
v1
t



Present value

t

v1


z
y
x
6083
z
3615
100
y
30
5
x
100
Present value
v1
t

Numerical Analysis

Peter Lohmander 150812

Case 0:

```
! OPT CCF 150812;
! Peter Lohmander;

v0 = 200;
p = 20;
c = 50;
r = 0.03;
m0 = 30;
c0 = 50;

max = Y;
Y = R0 + (p*Q-c)/(@exp(r*t)-1);
h = v0-v1;
h < v0;
h > 1;

mp = m0 - a*h-b*h*h;
R0 = m0*h-a/2*h*h-b/3*h*h*h-c0;
Q = 1/(1/400+(1/v1-1/400)*@exp(-0.05*t))-v1;

! Derivation of initial marginal price function;
150*a+(150)^2*b= 10;
200*a+(200)^2*b = 30;
@free(a);
@free(b);
```

```
Local optimal solution found.
  Objective value:                              6084.286
  Infeasibilities:                              0.000000
  Total solver iterations:                            34
```

Variable	Value	Reduced Cost
V0	200.0000	0.000000
P	20.00000	0.000000
C	50.00000	0.000000
R	0.3000000E-01	0.000000
M0	30.00000	0.000000
C0	50.00000	0.000000
Y	6084.286	0.000000
R0	4651.524	0.000000
Q	71.17267	0.000000
T	22.40775	0.1339671E-08
H	150.7051	0.000000
V1	49.29487	0.000000
MP	19.77588	0.000000
A	-0.1833333	0.000000
B	0.1666667E-02	0.000000

Maximum present value

Optimal harvest interval

The optimal value of V1 is far below the Swedish constraint.

Conclusions:

- **The two examples showed that it was not economically optimal to make clear fellings.**
- **Considerable numbers of small trees should be left for continued production.**

Continuous cover forest harvesting In the Swedish mountains December 2014

Sometimes, we may want to consider the forest at the individual tree level or at the size class level.

- In order to optimize the utilization of the natural forests, it is necessary to understand the **dynamics of growth and the intertemporal harvesting options** in the already existing forests.
- This study contains new **approaches to nonlinear estimations of diameter increment and mortality functions** for trees of different size classes, under the influence of competition within forests with many size classes.

A general dynamic function for the basal area of individual trees derived from a production theoretically motivated autonomous differential equation

Peter Lohmander

Abstract

A general dynamic function for the basal area of individual trees has been derived from a production theoretically motivated autonomous differential equation. The differential equation is:

$$\frac{dx}{dt} = a\sqrt{x}\left(1 - cx\right) \quad , \quad a > 0, c > 0, 0 < x < c^{-1}$$

and the general dynamic function is: $$x(t) = \frac{\left(\left(\frac{\sqrt{x_0}\sqrt{c}+1}{\sqrt{x_0}\sqrt{c}-1}\right)e^{a\sqrt{c}t}+1\right)^2}{c\left(\left(\frac{\sqrt{x_0}\sqrt{c}+1}{\sqrt{x_0}\sqrt{c}-1}\right)e^{a\sqrt{c}t}-1\right)^2}$$

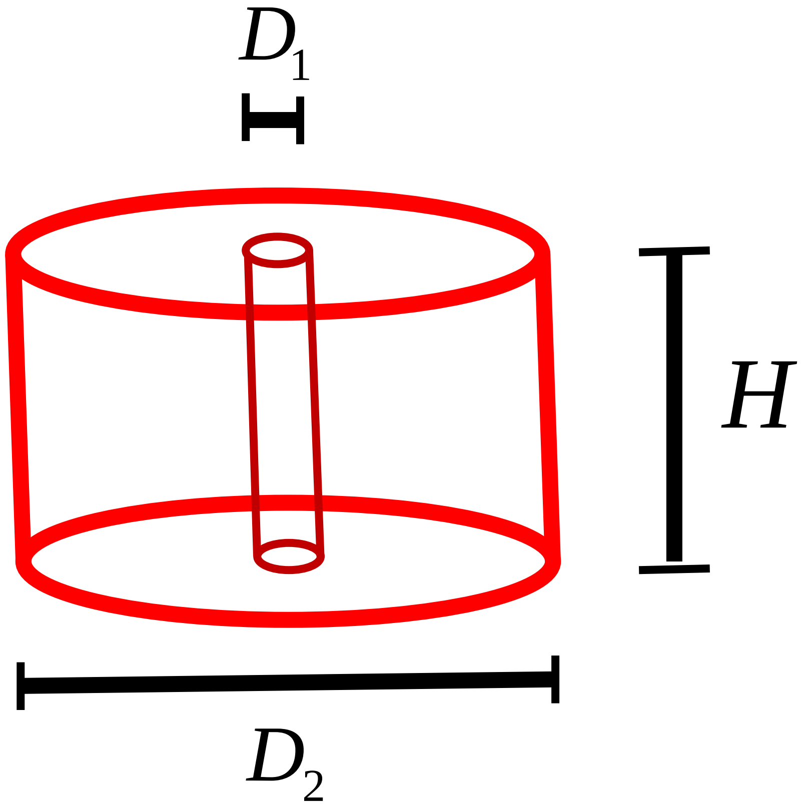
Keywords: Dynamic function, differential equation, basal area, forest growth

Consider a stem segment, of height H

of the tree. The stem segment is cylindrical with diameter D_1

The leaves cover a cylinder with diameter D_2

$$D_2 = \gamma D_1, \gamma > 1$$



The sun light reaches the tree from the side.

V **is the volume of the stem segment.**

x **is the basal area.**

$$x = (\pi / 4) D_1^2$$

$$V = Hx$$

Volume increment is proportional to the photo synthesis level, P

which in turn is proportional to the sun light projection area on the leaves, A

$$A = HD_2$$

We may conclude that:

$$\frac{dV}{dt} = H\frac{dx}{dt} \propto P \propto A \propto D_2 \propto D_1 \propto \sqrt{x}$$

Hence,

$$\frac{dx}{dt} \propto \sqrt{x}$$

or

$$\frac{dx}{dt} = a\sqrt{x},\, a > 0$$

As the size of the tree increases, the production efficiency declines.

Furthermore, the value of γ

is often lower for large trees than for small trees.

A relevant function considering this is:

$$\frac{dx}{dt} = a\sqrt{x}\left(1 - cx\right) \quad a > 0, c > 0, 0 < x < c^{-1}$$

Mathematical model development and analysis

$$\frac{dx}{dt} = a\sqrt{x}\left(1 - cx\right) \quad , \quad c = \frac{b}{a} > 0, 0 < x < c^{-1}$$

The parameters can be estimated via this linear reformulation:

$$\frac{dx}{dt} = ax^{0.5} - bx^{1.5} \qquad a > 0, b > 0, 0 < x < \frac{a}{b}$$

$$\frac{dx}{dt} = a\sqrt{x}\left(1 - cx\right) \quad , \quad c = \frac{b}{a} > 0, 0 < x < c^{-1}$$

$$\frac{1}{\sqrt{x}\left(1 - cx\right)} dx = a\, dt$$

Integration gives

$$\int \frac{1}{\sqrt{x}\left(1 - cx\right)} dx = \int a\, dt + k_0$$

$$\int \frac{1}{\sqrt{x}\left(1-cx\right)}\,dx = \int a\,dt + k_0$$

$$\frac{\ln\left(\sqrt{c}\sqrt{x}+1\right)-\ln\left(\sqrt{c}\sqrt{x}-1\right)}{\sqrt{c}} = at + k_0$$

$$\frac{\ln\left(\sqrt{c}\sqrt{x}+1\right)-\ln\left(\sqrt{c}\sqrt{x}-1\right)}{\sqrt{c}}=at+k_0$$

Let us investigate the left hand side, called Z

$$\frac{dZ}{dx}=\frac{1}{\sqrt{c}\left(\sqrt{c}\sqrt{x}+1\right)}\left(\frac{\sqrt{c}}{2\sqrt{x}}\right)-\frac{1}{\sqrt{c}\left(\sqrt{c}\sqrt{x}-1\right)}\left(\frac{\sqrt{c}}{2\sqrt{x}}\right)$$

$$\frac{dZ}{dx}=\frac{1}{2\sqrt{x}\left(\sqrt{c}\sqrt{x}+1\right)}-\frac{1}{2\sqrt{x}\left(\sqrt{c}\sqrt{x}-1\right)}$$

$$\frac{dZ}{dx}=\frac{\left(\sqrt{c}\sqrt{x}-1\right)-\left(\sqrt{c}\sqrt{x}+1\right)}{2\sqrt{x}\left(\sqrt{c}\sqrt{x}+1\right)\left(\sqrt{c}\sqrt{x}-1\right)}$$

$$\frac{dZ}{dx}=\frac{1}{\sqrt{x}\left(1-cx\right)}$$

This confirmes that the integration was correct.

$$\frac{\ln\left(\dfrac{\sqrt{c}\sqrt{x}+1}{\sqrt{c}\sqrt{x}-1}\right)}{\sqrt{c}} = at + k_0$$

which leads to

$$\ln\left(\frac{\sqrt{c}\sqrt{x}+1}{\sqrt{c}\sqrt{x}-1}\right) = \sqrt{c}at + k_1 \qquad k_1 = \sqrt{c}k_0$$

Let

$$y = \sqrt{x}, g = \sqrt{c}, h = ga$$

Then

$$\ln\left(\frac{gy+1}{gy-1}\right) = ht + k_1$$

Let

$$K = e^{k_1}$$

We get the simplified expression:

$$\frac{gy+1}{gy-1} = Ke^{ht}$$

which can be transformed to:

$$gy+1 = Ke^{ht}\left(gy-1\right)$$

or

$$g\left(1-Ke^{ht}\right)y = -1-Ke^{ht}$$

$$y = \frac{-1 - Ke^{ht}}{g\left(1 - Ke^{ht}\right)}$$

$$y = \sqrt{x} = \frac{Ke^{ht} + 1}{g\left(Ke^{ht} - 1\right)}$$

which gives the desired equation

$$\boxed{x(t) = \frac{\left(Ke^{ht} + 1\right)^2}{c\left(Ke^{ht} - 1\right)^2}}$$

Let us determine K

We utilize the initial condition: $x_0 = x(0)$

$$\sqrt{x_0} = \frac{\left(Ke^0 + 1\right)}{\sqrt{c}\left(Ke^0 - 1\right)}$$

which leads to $\sqrt{x_0}\sqrt{c}\left(K - 1\right) = K + 1$

Hence, $\left(\sqrt{x_0}\sqrt{c} - 1\right)K = \sqrt{x_0}\sqrt{c} + 1$ and finally

$$\boxed{K = \frac{\sqrt{x_0}\sqrt{c} + 1}{\sqrt{x_0}\sqrt{c} - 1}}$$

Now, we know how to determine K .

Later, the sign and magnitude of K

will be needed in the analysis. Do we know the sign of K ?

$$\left(\sqrt{x_0} > 0 \wedge \sqrt{c} > 0\right) \Rightarrow \left(\sqrt{x}\sqrt{c} + 1 > 0\right)$$

Let us investigate the sign of $\sqrt{x_0}\sqrt{c} - 1$

We assume that the value of x_0

makes sure that the increment is strictly positive.

$$\frac{dx}{dt} = a\sqrt{x}\left(1 - cx\right)$$

Then, we know that:

$$1 - cx_0 > 0 \quad \Rightarrow \quad cx_0 < 1 \quad \Rightarrow \quad \sqrt{c}\sqrt{x_0} < \sqrt{1} \quad \Rightarrow \quad \boxed{\sqrt{x_0}\sqrt{c} - 1 < 0}$$

As a result, we know that $\boxed{K < 0}$

Do we know the something about $|K|$?

$$K = \frac{\phi+1}{\phi-1} \qquad 0 < \phi = \sqrt{x_0}\sqrt{c} < 1$$

$$K(\phi=\varepsilon) = \frac{\varepsilon+1}{\varepsilon-1} \quad \Rightarrow \quad \lim_{\varepsilon\to 0} K(\phi=\varepsilon) = -1$$

$$\frac{dK}{d\phi} = \frac{(\phi-1)-(\phi+1)}{(\phi-1)^2} \quad \Rightarrow \quad \frac{dK}{d\phi} = \frac{-2}{(\phi-1)^2} < 0$$

With this information, we know that $\boxed{K < -1}$

Now, we may determine $x(t)$ as an explicit function of x_0 and the parameters.

$$x(t)=\frac{\left(Ke^{ht}+1\right)^2}{c\left(Ke^{ht}-1\right)^2}\quad\wedge\quad K=\frac{\sqrt{x_0}\sqrt{c}+1}{\sqrt{x_0}\sqrt{c}-1}\quad\wedge\quad h=a\sqrt{c}\quad\Rightarrow$$

$$\boxed{x(t)=\frac{\left(\left(\frac{\sqrt{x_0}\sqrt{c}+1}{\sqrt{x_0}\sqrt{c}-1}\right)e^{a\sqrt{c}t}+1\right)^2}{c\left(\left(\frac{\sqrt{x_0}\sqrt{c}+1}{\sqrt{x_0}\sqrt{c}-1}\right)e^{a\sqrt{c}t}-1\right)^2}}$$

Now, the dynamic properties of

$$x(t)=\frac{\left(Ke^{ht}+1\right)^{2}}{c\left(Ke^{ht}-1\right)^{2}}$$

will be determined.

$$\frac{dx}{dt}=\left(\frac{2Khc}{c^{2}\left(Ke^{ht}-1\right)^{4}}\right)\left(\left(Ke^{ht}+1\right)\left(Ke^{ht}-1\right)^{2}-\left(Ke^{ht}+1\right)^{2}\left(Ke^{ht}-1\right)\right)$$

$$\frac{dx}{dt} = \left(\frac{2Khc\left(Ke^{ht}+1\right)\left(Ke^{ht}-1\right)}{c^2\left(Ke^{ht}-1\right)^4} \right)\left(\left(Ke^{ht}-1\right)-\left(Ke^{ht}+1\right)\right)$$

$$\frac{dx}{dt} = \left(\frac{-4Khc\left(\left(Ke^{ht}\right)^2-1\right)}{c^2\left(Ke^{ht}-1\right)^4} \right)$$

We already know that $K < -1$ **. Hence,** $\left(\left(Ke^{ht}\right)^2-1\right) > 0$

As a result, we find that $\boxed{\frac{dx}{dt} > 0}$

$$\lim_{\substack{t\to\infty \\ h>0 \\ K<-1}} x(t) = \frac{\left(\dfrac{d\left(Ke^{ht}+1\right)^2}{dt}\right)}{\left(\dfrac{dc\left(Ke^{ht}-1\right)^2}{dt}\right)} = \frac{2\left(Ke^{ht}+1\right)hK}{2c\left(Ke^{ht}-1\right)hK} = \frac{\left(1+\dfrac{1}{Ke^{ht}}\right)}{c\left(1-\dfrac{1}{Ke^{ht}}\right)} = \frac{1}{c}$$

Hence, we know that, as $t\to\infty$ **,** $\boxed{x(t)\ \textbf{monotonically converges to}\ c^{-1}}$

How does the function work?

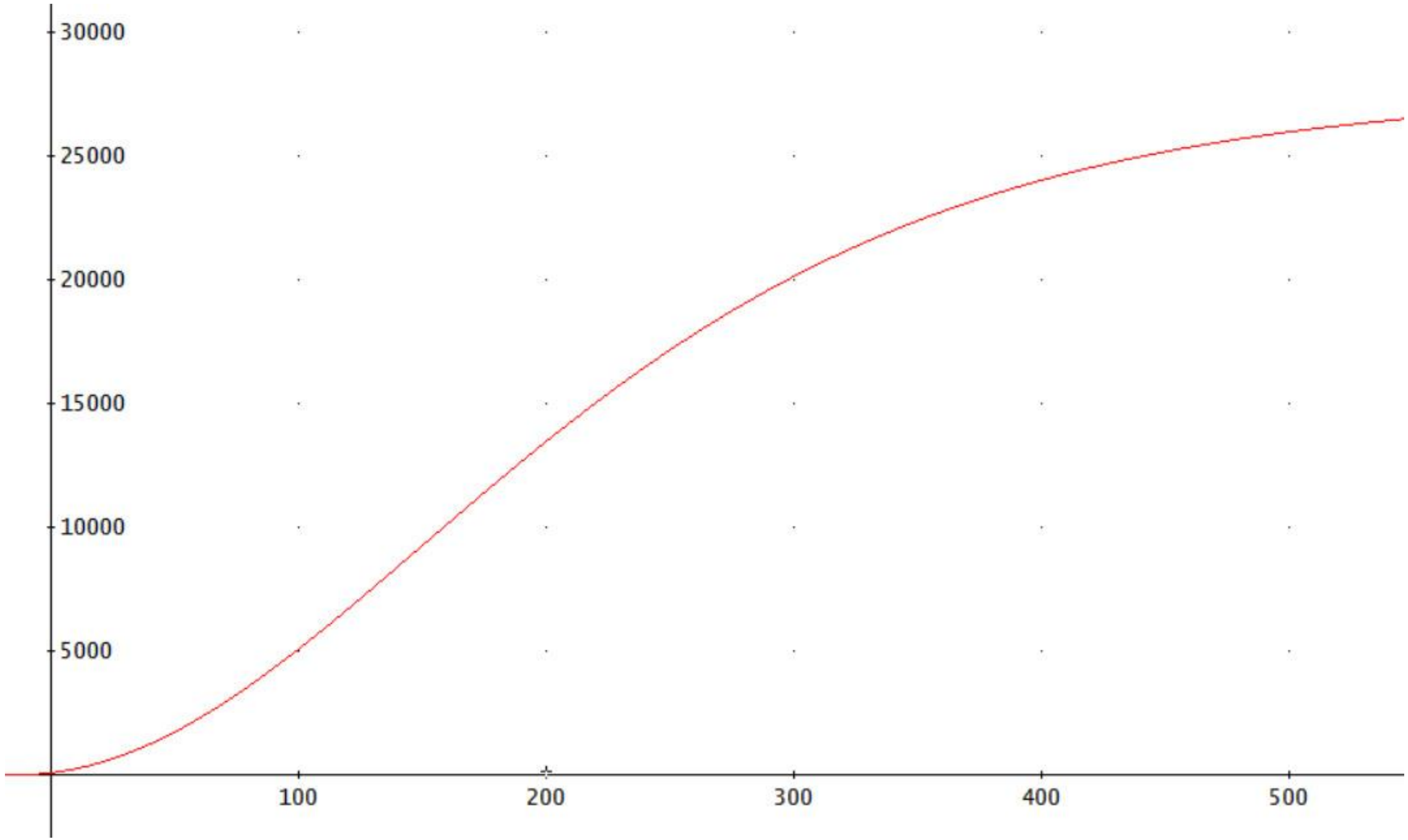
Below, parameter values representing the following case are used.

- Species: Maple (*Acer velutinum)*
- *Initial diameter: 10 cm.*
- *Source: Hatami, Lohmander, Moayeri and Mohammadi Limaei (2017)*
- *Competition: No.*

Basal Area (Square Centimeter) as a function of time, t (Years)

$$x = \frac{\left(\dfrac{\sqrt{\left(\dfrac{\pi}{4}\cdot 100\right)}\cdot\sqrt{(3.615978615\cdot 10^{-5})}+1}{\sqrt{\left(\dfrac{\pi}{4}\cdot 100\right)}\cdot\sqrt{(3.615978615\cdot 10^{-5})}-1}\cdot \mathrm{EXP}((1.3468\cdot\sqrt{(3.615978615\cdot 10^{-5})})\cdot t)+1\right)^2}{(3.615978615\cdot 10^{-5})\cdot\left(\dfrac{\sqrt{\left(\dfrac{\pi}{4}\cdot 100\right)}\cdot\sqrt{(3.615978615\cdot 10^{-5})}+1}{\sqrt{\left(\dfrac{\pi}{4}\cdot 100\right)}\cdot\sqrt{(3.615978615\cdot 10^{-5})}-1}\cdot \mathrm{EXP}((1.3468\cdot\sqrt{(3.615978615\cdot 10^{-5})})\cdot t)-1\right)^2}$$

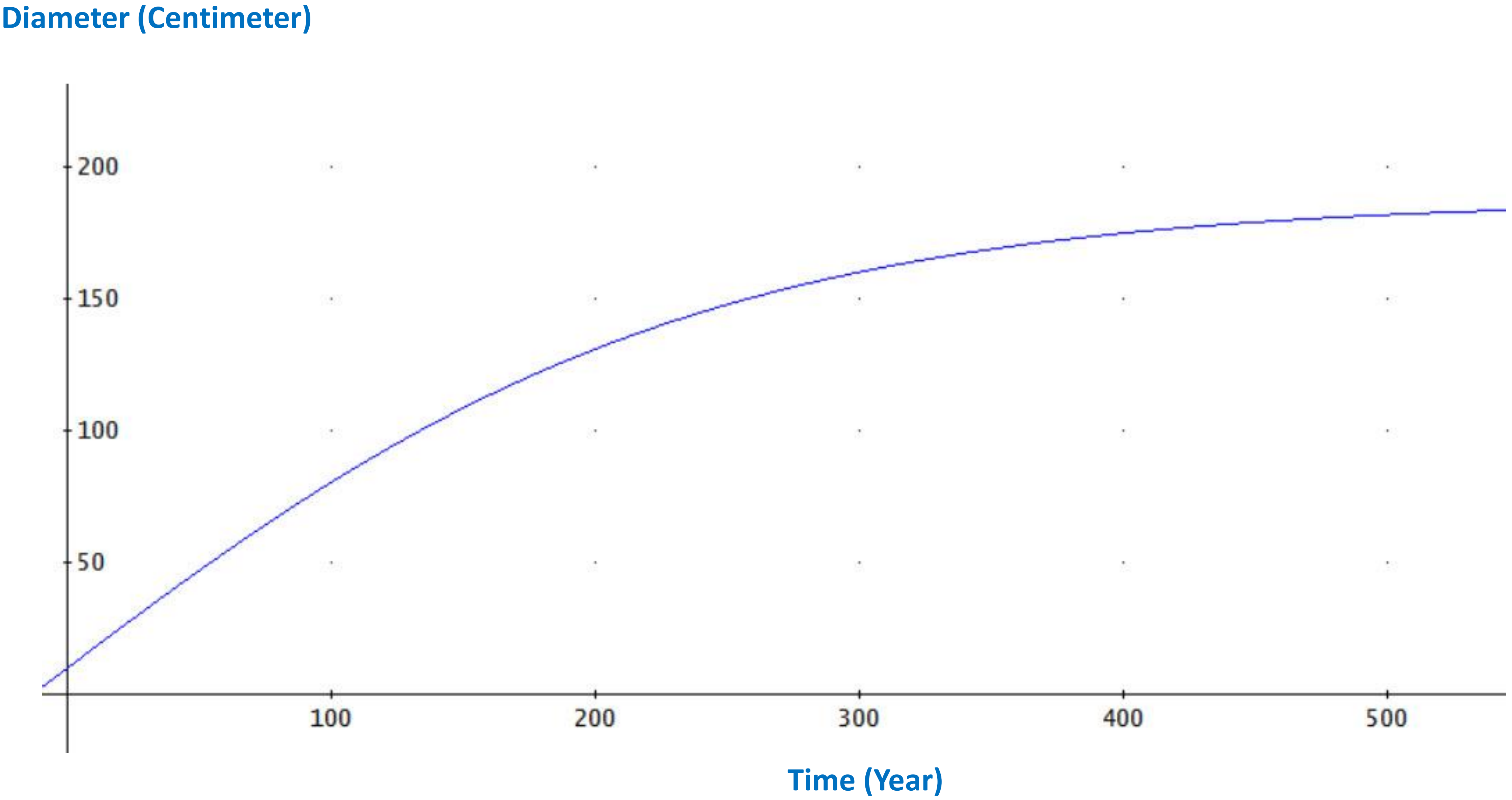
Basal Area (Square Centimeter)



Time (Year)

Diameter (Centimeter) as a function of time, t (Years)

$$d = \sqrt{\left(\frac{4 \cdot \left[\frac{\sqrt{\left(\frac{\pi}{4} \cdot 100\right)} \cdot \sqrt{(3.615978615 \cdot 10^{-5})} + 1}{\sqrt{\left(\frac{\pi}{4} \cdot 100\right)} \cdot \sqrt{(3.615978615 \cdot 10^{-5})} - 1} \cdot \mathrm{EXP}((1.3468 \cdot \sqrt{(3.615978615 \cdot 10^{-5})}) \cdot t) + 1 \right]^2}{(3.615978615 \cdot 10^{-5}) \cdot \left[\frac{\sqrt{\left(\frac{\pi}{4} \cdot 100\right)} \cdot \sqrt{(3.615978615 \cdot 10^{-5})} + 1}{\sqrt{\left(\frac{\pi}{4} \cdot 100\right)} \cdot \sqrt{(3.615978615 \cdot 10^{-5})} - 1} \cdot \mathrm{EXP}((1.3468 \cdot \sqrt{(3.615978615 \cdot 10^{-5})}) \cdot t) - 1 \right]^2} \right) \Big/ \pi}$$


Diameter (Centimeter)
200
150
100
50
100
200
300
400
500
Time (Year)

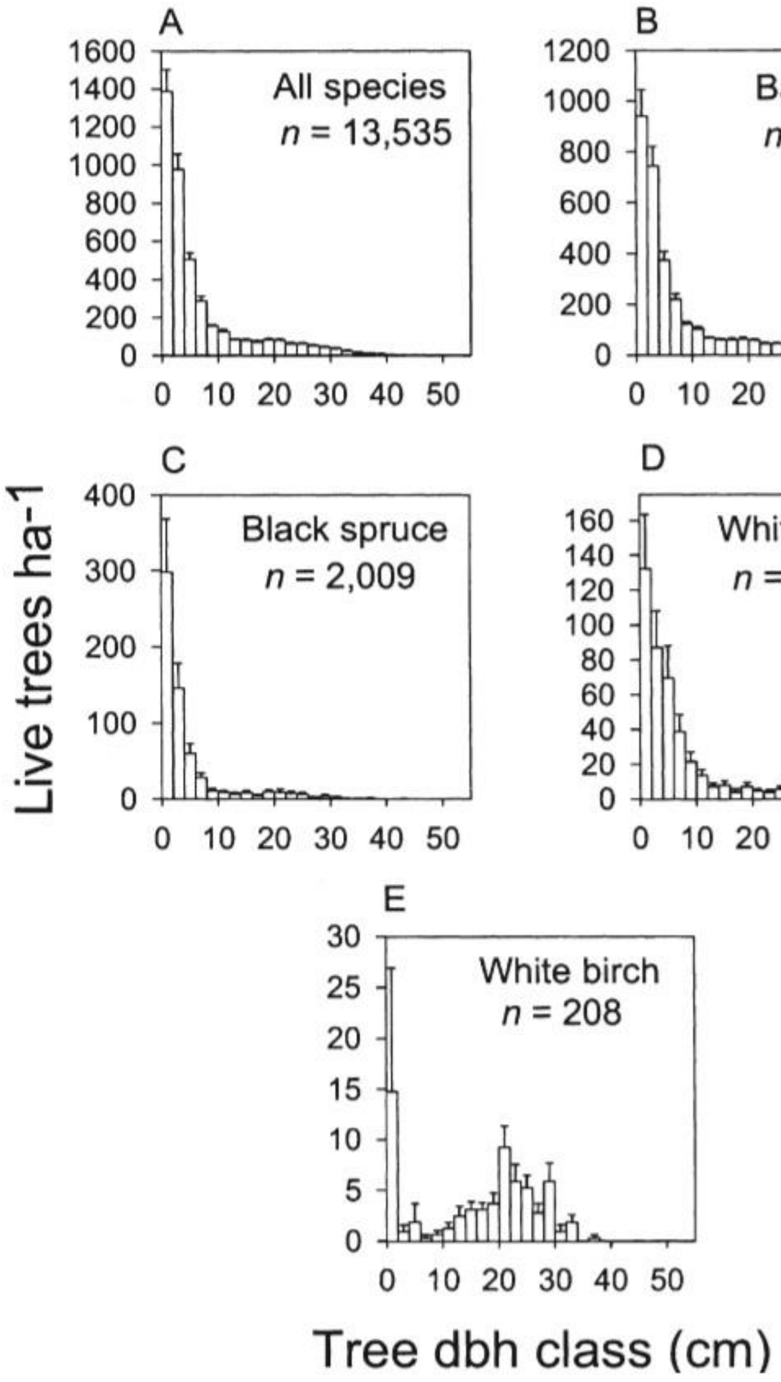
Theoretical Growth Function Results

A **general dynamic function for the basal area** of individual trees has been derived from a production theoretically motivated autonomous differential equation.

The **dynamic properties** have been determined and monotone convergence has been proved.

Several version of the growth function exist.

- Some of these take competition into account, via individual tree information or via size class information.
- They have been statistically estimated with forest data from Sweden and Iran.
- The ***estimations can also be based on already published diameter frequency distributions*** from natural forests in dynamic equilibria, in Canada, Russia and Scandinavia.



NORTH AMERICA

Age and Size Structure of Gap-Dynamic, Old-Growth Boreal Forest Stands in Newfoundland

John W. McCarthy and Gordon Weetman

McCarthy, J.W. & Weetman, G. 2006. Age and size structure of gap-dynamic, old-growth boreal forest stands in Newfoundland. Silva Fennica 40(2): 209–230.

RUSSIA

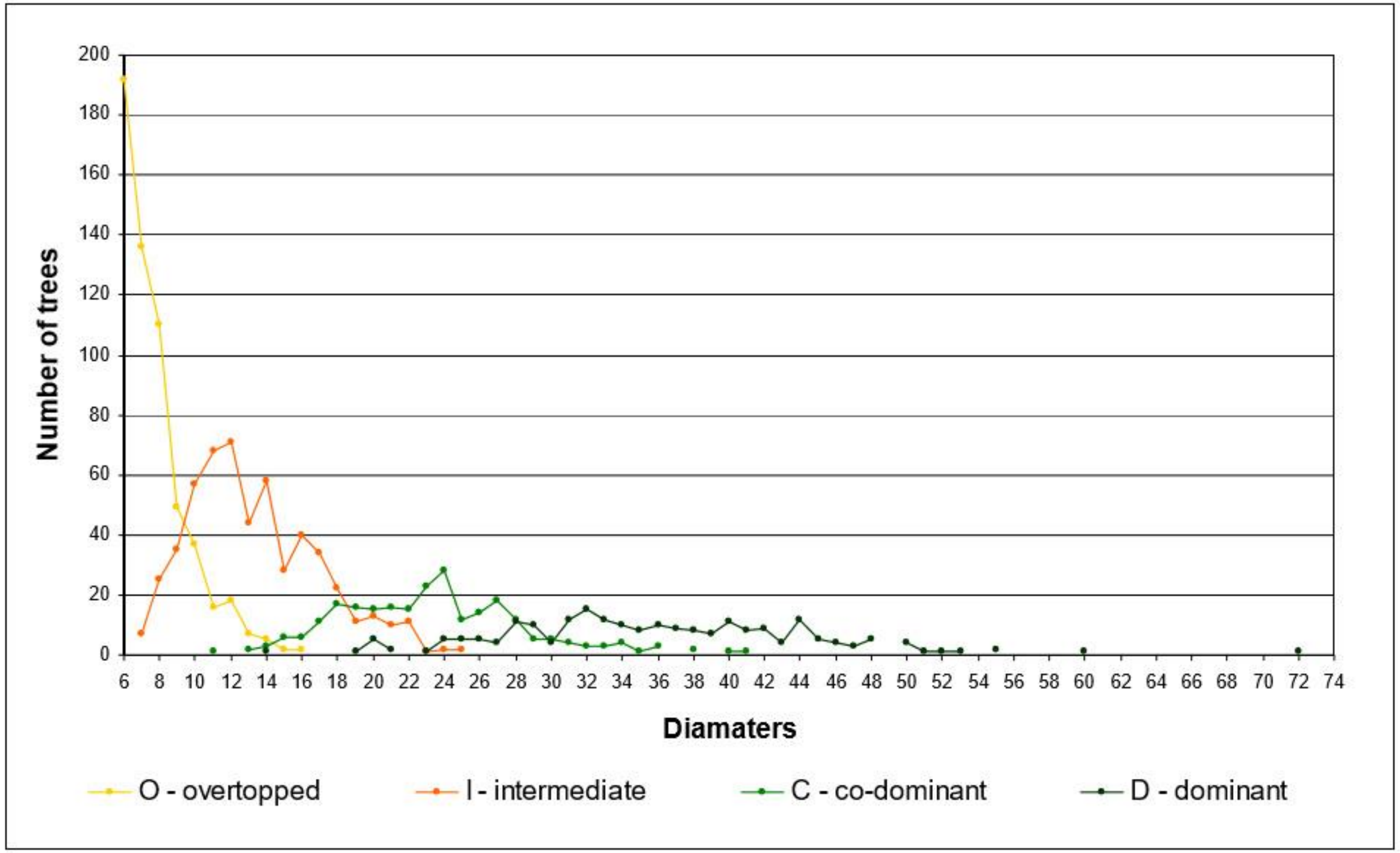


Figure 4 Diameters distribution of spruce in relation to four different canopy positions (D, C, I, O)

250 Years of Disturbance Dynamics in a Pristine Old-growth Picea abies Forest in Arkhangelsk Region, North-Western Russia: a Dendrochronological Reconstruction

Tatiana Khakimulina
Supervisors: Igor Drobyshev
Mats Niklasson

Swedish University of Agricultural Sciences
Master Thesis no. 163
Southern Swedish Forest Research Centre
Alnarp 2010

Forests **2015**, *6*, 2261-2280; doi:10.3390/f6062261

CHINA

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Article

Nonlinear Simultaneous Equations for Individual-Tree Diameter Growth and Mortality Model of Natural Mongolian Oak Forests in Northeast China

Wu Ma [1,2] and Xiangdong Lei [1,*]

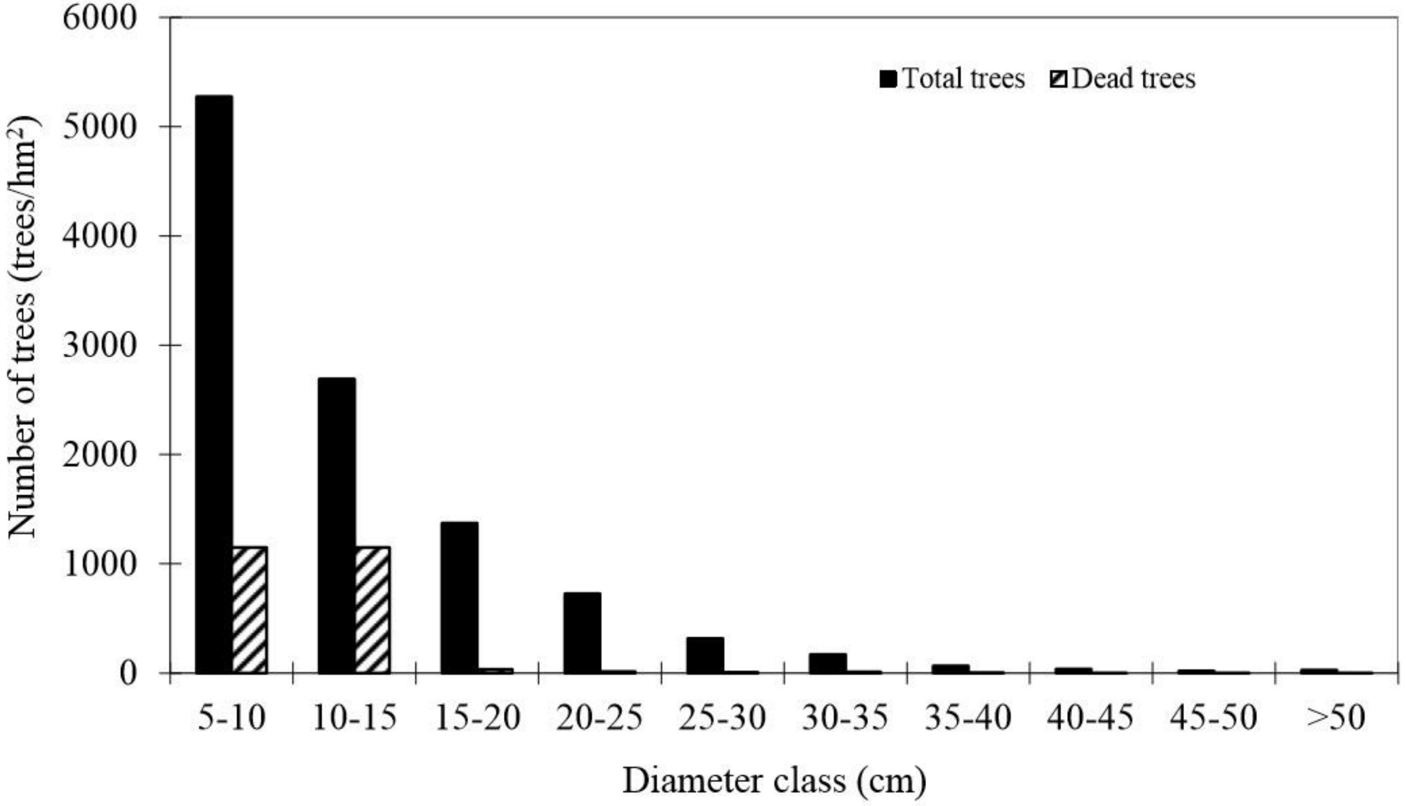


Figure 2. Distribution of total trees and dead trees by diameter classes of Mongolian oak.

Size class dynamics

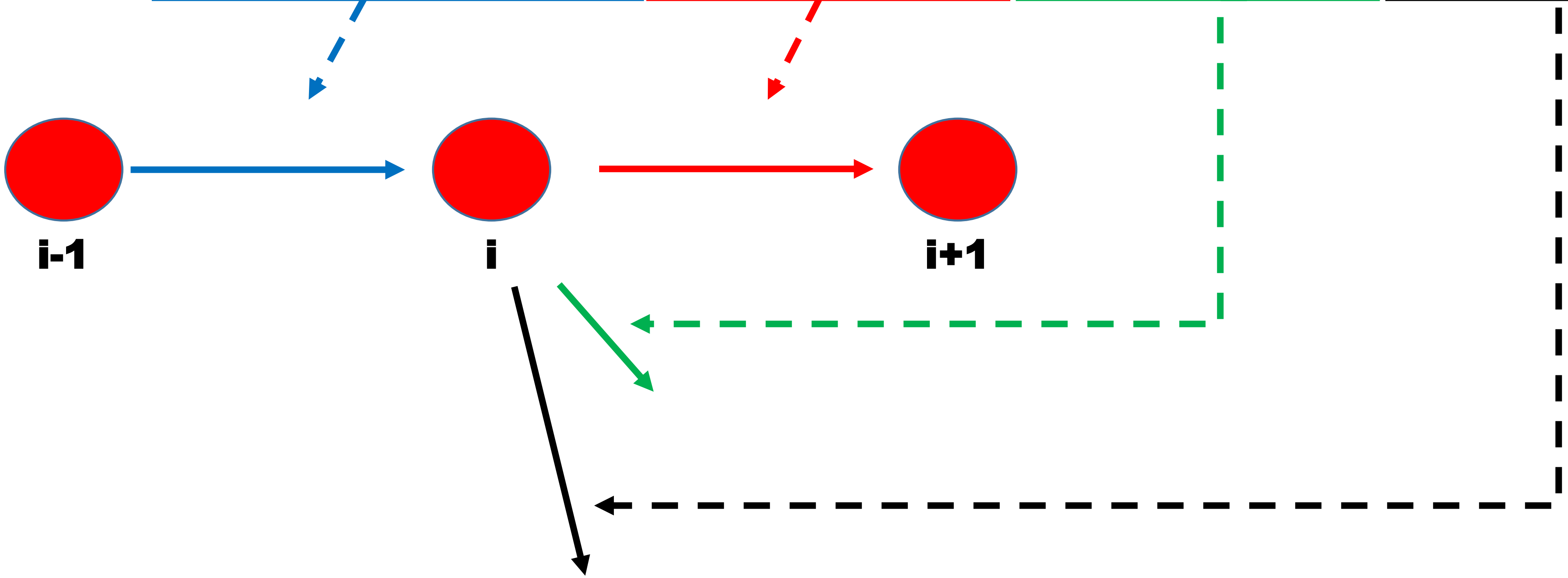
Number of individuals in size class i

Probability to move up

Relative harvest

Relative mortality

$$n_{t+1}(i) = n_t(i) + \boxed{P_t(i-1)n_t(i-1)}\boxed{-P_t(i)n_t(i)}\boxed{-h_t(i)n_t(i)}\boxed{-m_t(i)n_t(i)}$$



Observation:

- Of course, the probabilities to move up are usually affected by competition. We may sometimes want to consider these things explicitly.

$$P_t(i) = P_t(i, n(0), \ldots, n(N); .)$$

- In other cases, we may be interested in equilibrium probabilities (when competition is considered fixed).

$$n_{t+1}(i) = n_t(i) + P_t(i-1)n_t(i-1) \; - P_t(i)n_t(i) \; - h_t(i)n_t(i) \; - m_t(i)n_t(i)$$

In dynamic equilibrium:

$$n_{t+1}(i) = n_t(i)$$

$$0 = P_t(i-1)n_t(i-1) \; - P_t(i)n_t(i) \; - h_t(i)n_t(i) \; - m_t(i)n_t(i)$$

In dynamic equilibrium, without harvesting:

$$0 = P_t(i-1)n_t(i-1) \; - P_t(i)n_t(i) \quad - m_t(i)n_t(i)$$

$$0 = P_t(i-1)n_t(i-1) \quad - P_t(i)n_t(i) \quad - m_t(i)n_t(i)$$

$$P_t(i)n_t(i) + m_t(i)n_t(i) = P_t(i-1)n_t(i-1)$$

$$\left(P_t(i) + m_t(i)\right)n_t(i) = P_t(i-1)n_t(i-1)$$

$$\frac{n_t(i)}{n_t(i-1)} = \frac{P_t(i-1)}{P_t(i) + m_t(i)}$$

$$\frac{n_t(i)}{n_t(i-1)} = \frac{P_t(i-1)}{P_t(i) + m_t(i)}$$

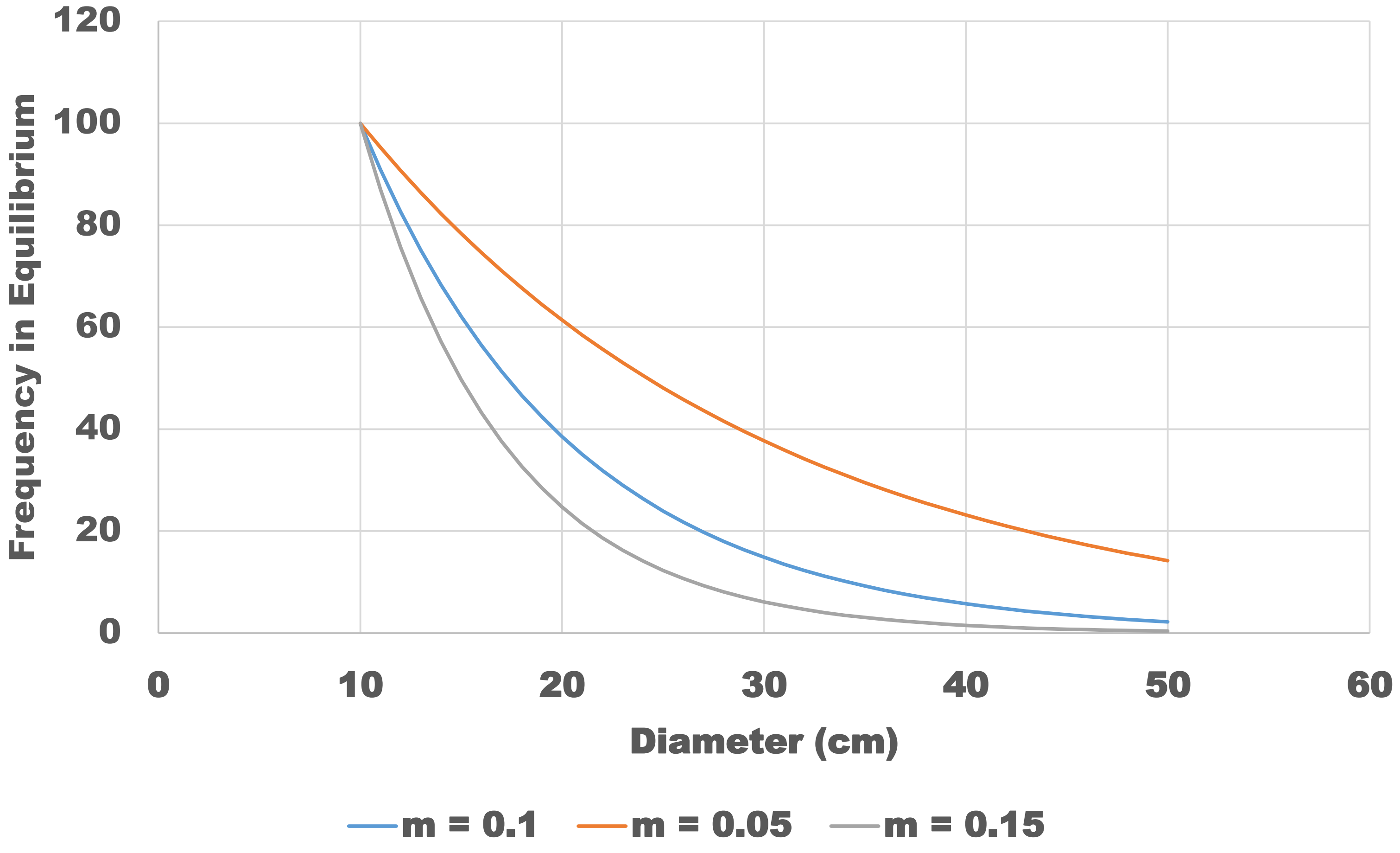
Special case:

P = > 0, P is constant (=1), m > 0, m is constant.

If i represents diameter class, then if P constant, this implies constant diameter increment, which means that the basal area increment is proportional to the square root of the basal area. Then, if m = constant:

$$\frac{n_t(i)}{n_t(i-1)} = \frac{P_t(i-1)}{P_t(i)+m_t(i)} = \frac{P}{P+m} = \frac{1}{1+m}$$

$$0 < \frac{P}{P+m} < 1$$



With realistic growth function:

$$\boxed{\frac{n(i+1)}{n(i)} = \frac{P(i)}{P(i+1)+m}}$$

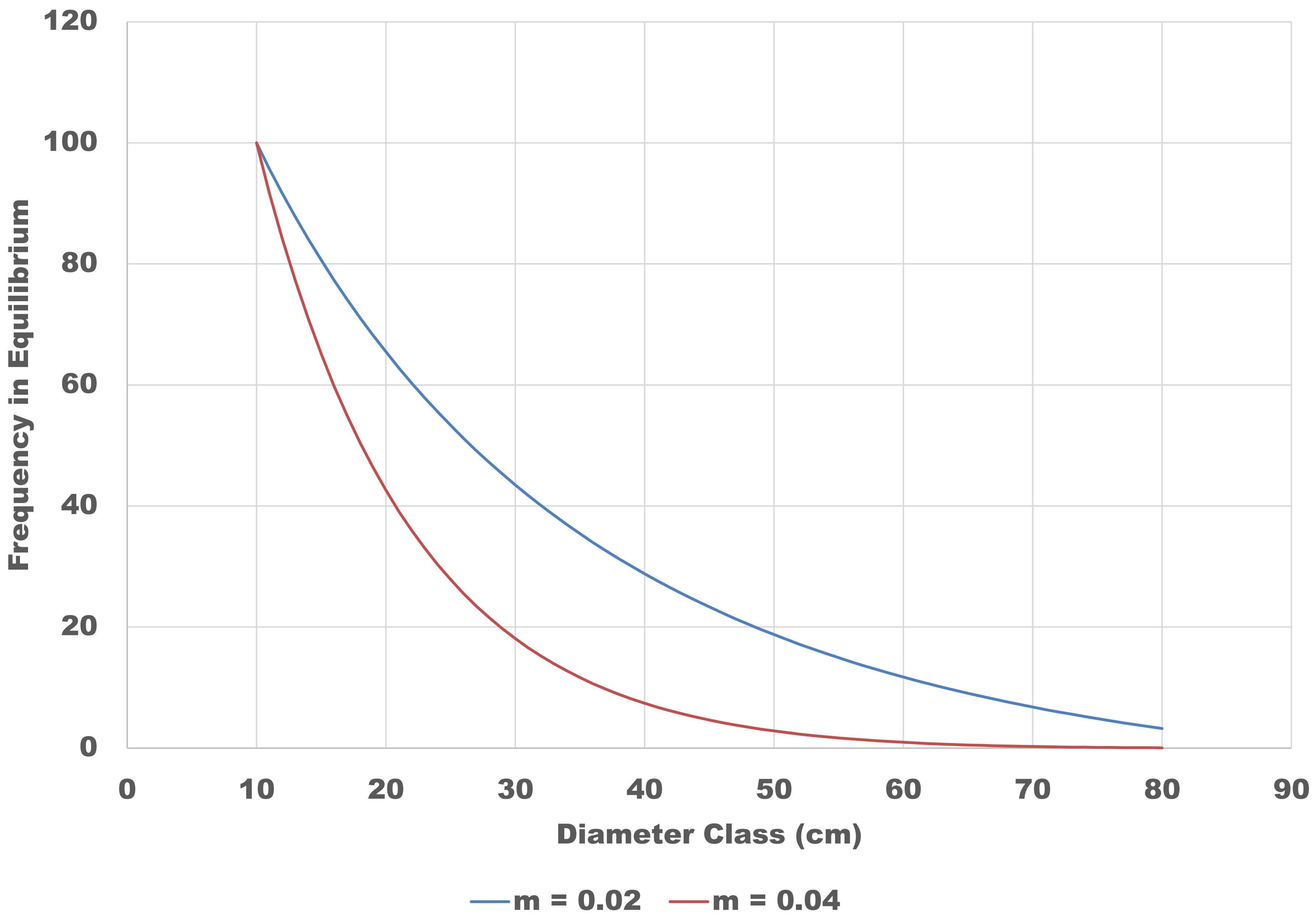
x = basal area $\left(10^{-4}m^2\right)$

$$\boxed{\frac{dx}{dt} = a\sqrt{x}\left(1-bx\right)}$$

$0 < x < b^{-1}$,

$a = 80.765$

$b = 120.02$



IN GENERAL:

With observations of n(i) for sufficiently many values of i, it is possible to simultaneously determine the parameters of the increment function, the competition function and the relative mortality function.

$$\frac{n(i+1)}{n(i)} = \frac{P(i)}{P(i+1)+m(m_1,\ldots,m_k,i)}$$

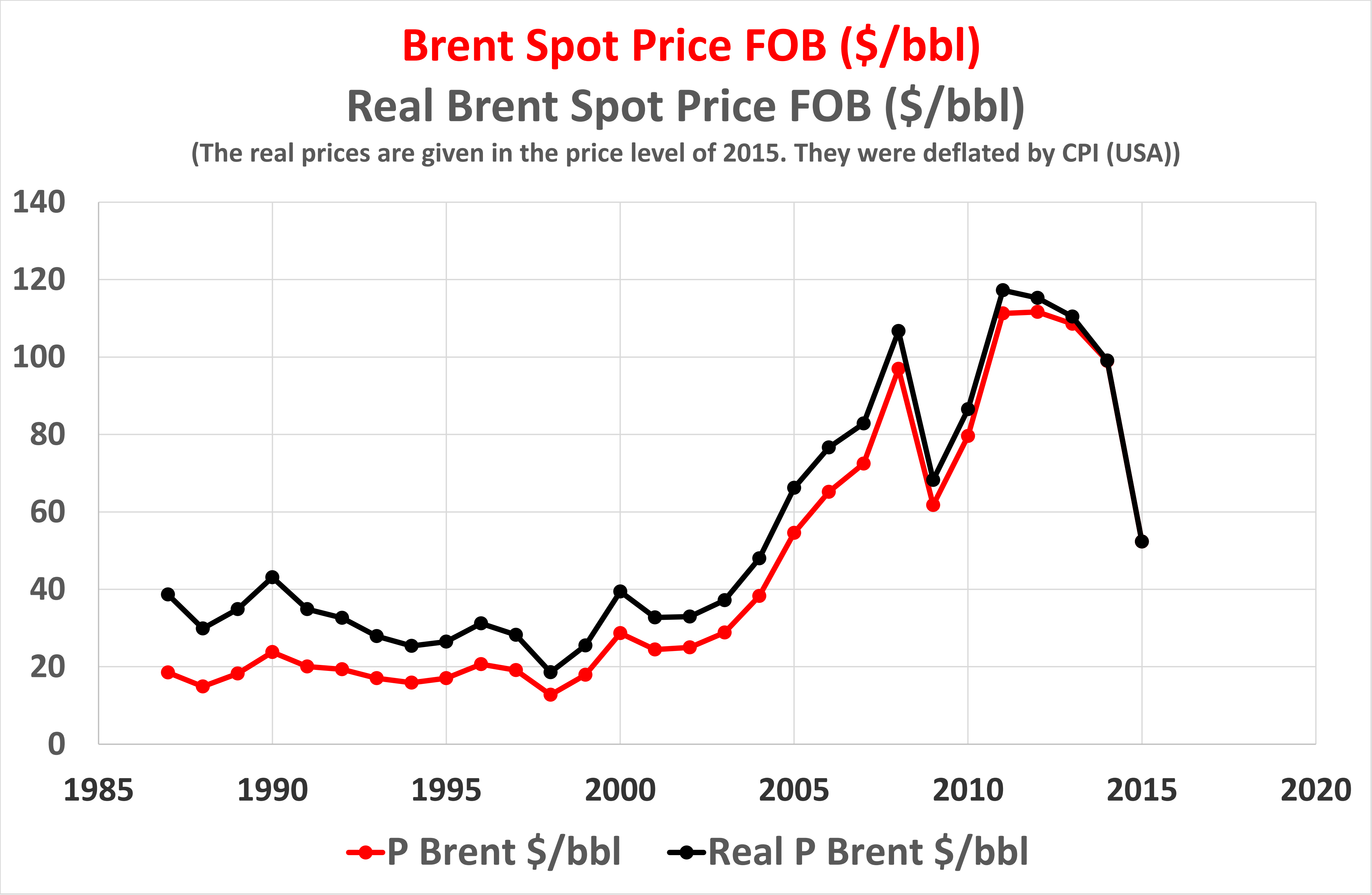
$$P(i) = P(i;a,b) = F\left(a\sqrt{x(i)}\,\big(1-bx(i)\big),C(i)\right)$$

$$C(i) = G(i,n(0),\ldots,n(N);c_1,\ldots,c_L)$$

The master model:

Stochastic dynamic programming with detailed sub problems

$$f(t,s,m) = \max_{u \in U(t,s,m)} \left(\left(\max \pi(x_1,\ldots,x_K; u,t,s,m) \right)_{\substack{s.t. \\ \alpha_{11}x_1+\ldots+\alpha_{1K}x_K \le C_1 \\ \ldots \\ \alpha_{L1}x_1+\ldots+\alpha_{LK}x_K \le C_L}} + \sum_n \tau(n|m)\, f(t+1, s_{t+1}(s,u), n) \right) \quad \forall (t,s,m) \,|\, (0 \le t \le T)$$


Brent Spot Price FOB ($/bbl)
Real Brent Spot Price FOB ($/bbl)
(The real prices are given in the price level of 2015. They were deflated by CPI (USA))
140
120
100
80
60
40
20
0
1985
1990
1995
2000
2005
2010
2015
2020
P Brent $/bbl
Real P Brent $/bbl

Nasdaq

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End of day Commodity Futures Price Quotes for Lumber

Select Timeframe: 10 Years Refresh

Lumber Related ETFs: CUT

Stochastic dynamic programming with detailed sub problems:

$$\max_{u_t \in U(t,s_t,m_t)} \left(\begin{pmatrix} \max \pi(x_1, ..., x_K; u_t, t, s_t, m_t) \\ s.t.\ \ \alpha_{11}x_1+...+\alpha_{1K}x_K \le C_1 \ ...\ \alpha_{L1}x_1+...+\alpha_{LK}x_K \le C_L \end{pmatrix} \atop + \beta \sum_{m_{t+1}} \tau(m_{t+1} | m_t) f(t+1, s_{t+1}(s_t, u_t), m_{t+1}) \right), \forall\ (t, s_t, m_t) | (0 \le t \le T)$$

Fuzzy Inf. Eng. (2017) 9: 269–270

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BULLETIN

ICMDS 2016 Conference Report

Peter Lohmander

For instance, it should be possible to combine the multi-period and adaptive decision making structure of stochastic dynamic programming with the fuzzy representation of the many single period optimization problems. This way, we may handle real world problems in an even more realistic and relevant way. $f(t, s_t, m_t) =$

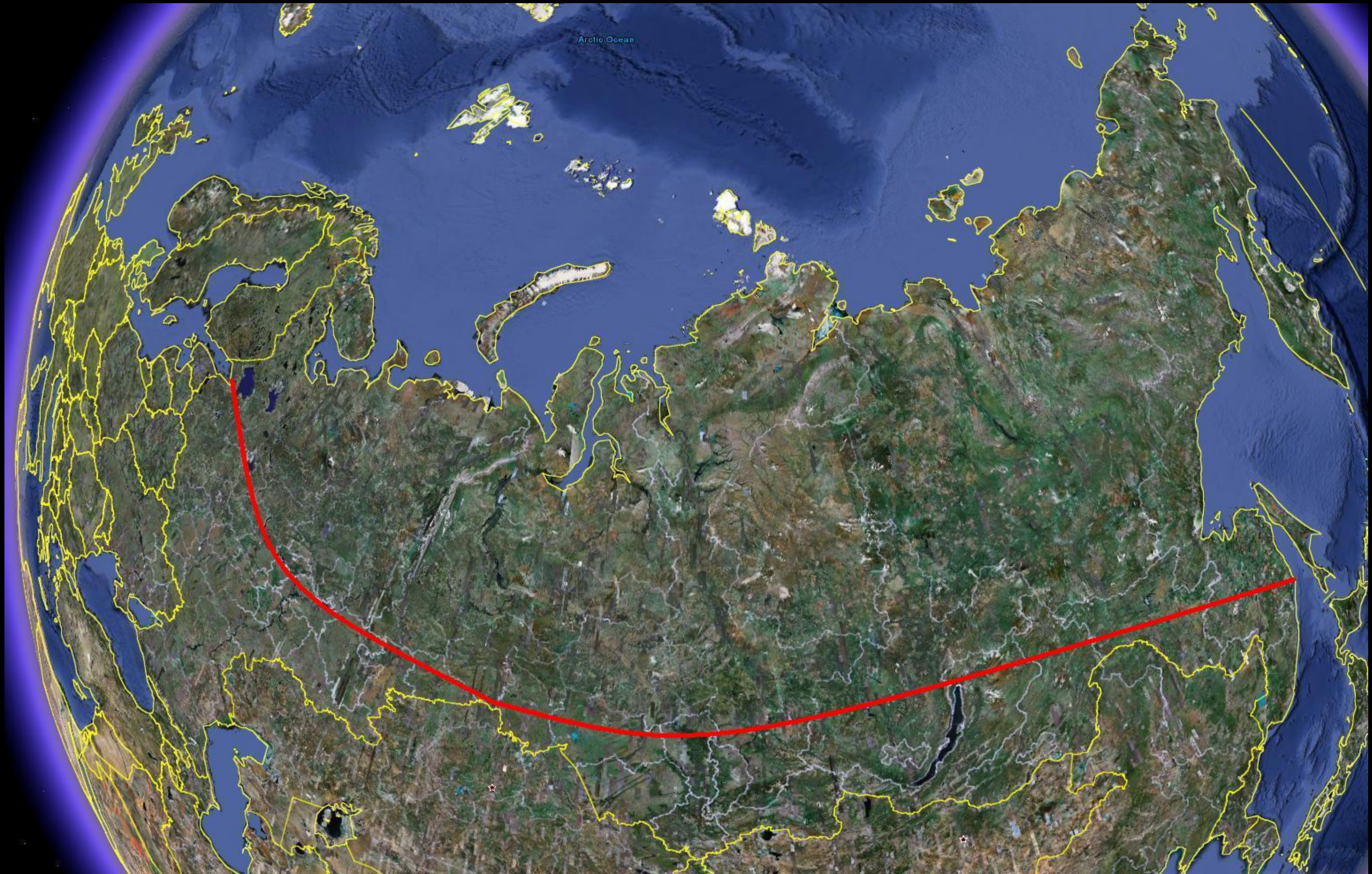
$$\max_{u_t \in U(t,s_t,m_t)} \left(\begin{pmatrix} \max \pi(x_1, ..., x_K; u_t, t, s_t, m_t) \\ s.t.\ \ \alpha_{11}x_1 + ... + \alpha_{1K}x_K \leq C_1 \ ...\ \alpha_{L1}x_1 + ... + \alpha_{LK}x_K \leq C_L \end{pmatrix} \atop + \beta \sum_{m_{t+1}} \tau(m_{t+1} | m_t) f(t+1, s_{t+1}(s_t, u_t), m_{t+1}) \right), \forall\ (t, s_t, m_t) | (0 \leq t \leq T)$$

is the expected present value of an industrial sector as a function of time, state of the sector and state of the markets, giving that all future decisions are optimally taken, conditional on future information. $U(\cdot)$ is the set of feasible controls; controls that are exogenous to the period and state specific fuzzy maximization problems. $\tau(m_{t+1} | m_t)$ is the market state transition probability; the probability that the markets move from state m_t in period t to state m_{t+1} in the next period. β denotes the one period discounting factor. T is the time horizon.

- The **forest resource dynamics sub model** of the industrial model, is based on an **approximation** of the intertemporal forest production function, derived via the estimated dynamic model of a part of the boreal forest.

- The **market price transition probability matrix**, used in the stochastic dynamic programming model, is derived via **world market price series**.

- In each period, the **production levels of bioenergy, sawn wood and fiber products, are optimized, based on the revealed market prices, the state of the natural resource and all other parameters**. The model also determines the expected shadow prices.


Arctic Ocean


Arctic Ocean


Arctic Ocean


Arctic Ocean

Alternative model structure:

A multidimensional loop with different adaptive control function parameter value combinations is created.

For each parameter combination:

10 stochastic simulations of the complete system during 200 years with estimation of the expected present value.

A table with expected present values for different adaptive control function parameter value combinations is created.

A multidimensional nonlinear approximation of the expected present value, as a function of the adaptive control function parameters, is derived via regression analysis.

The adaptive control function parameters are analytically determined via the first order optimum conditions and the second order maximum conditions.

The maximum expected present value is determined via the determined control function parameter values.

FINAL CONCLUSIONS:

- There are very **large options to increase sustainable continuous cover forestry** in the Borel forests. This way, sustainable flows of bioenergy and all kinds of forest industry raw materials can be obtained and these sectors can be expanded. This is also good when we consider the global warming problem.
- Continuous cover forestry is often the production economically **optimal forestry method**. Hence, the method is not just rational from environmental perspectives.)
- The fundamental understanding of forest dynamics can be improved via **new growth functions and estimations**.
- The **complete system can be optimized with new adaptive methods**.

QUESTIONS?

An Adaptive Optimization Approach To Expansion Of The Forest and Bioenergy Sectors Utilizing Wood From The Boreal Forests Version 170816

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