Optimization of Multi Species Continuous Cover Forest Management with Stochastic Prices via Determination of the Adaptive Harvest Control Function

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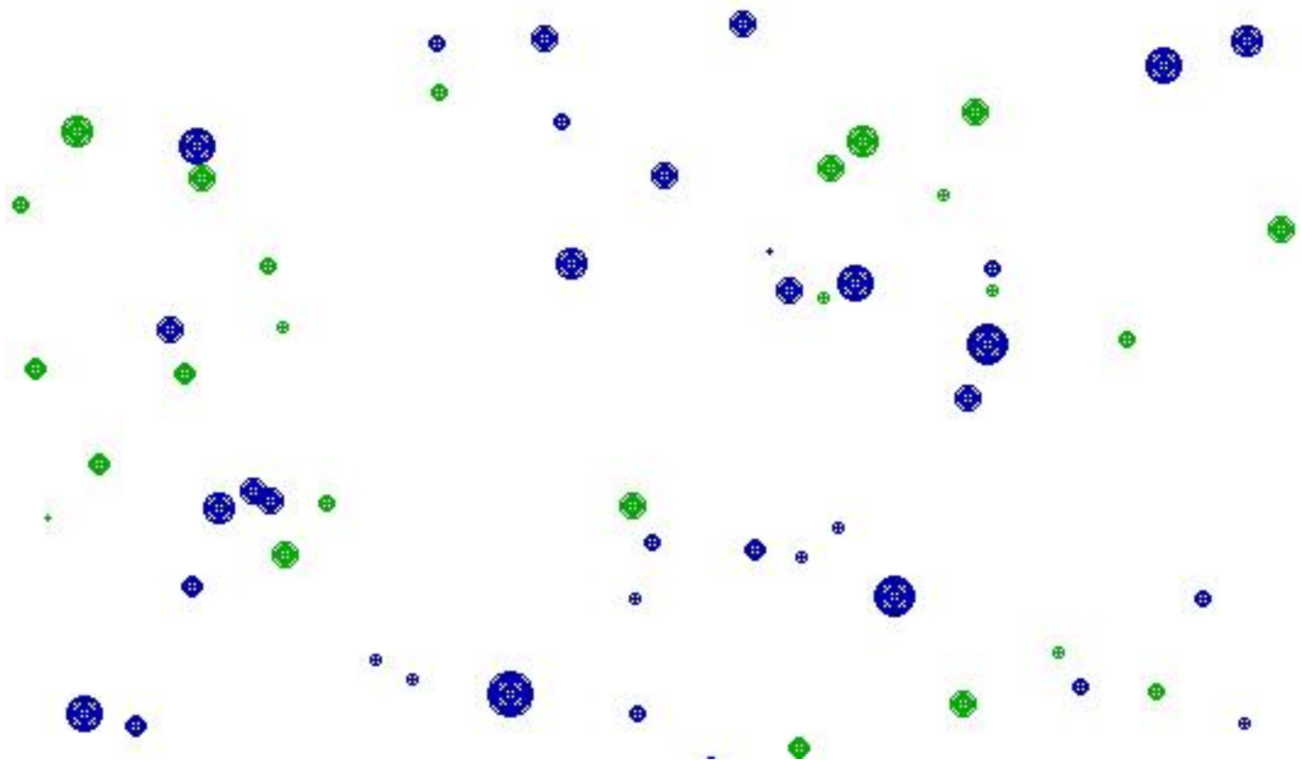

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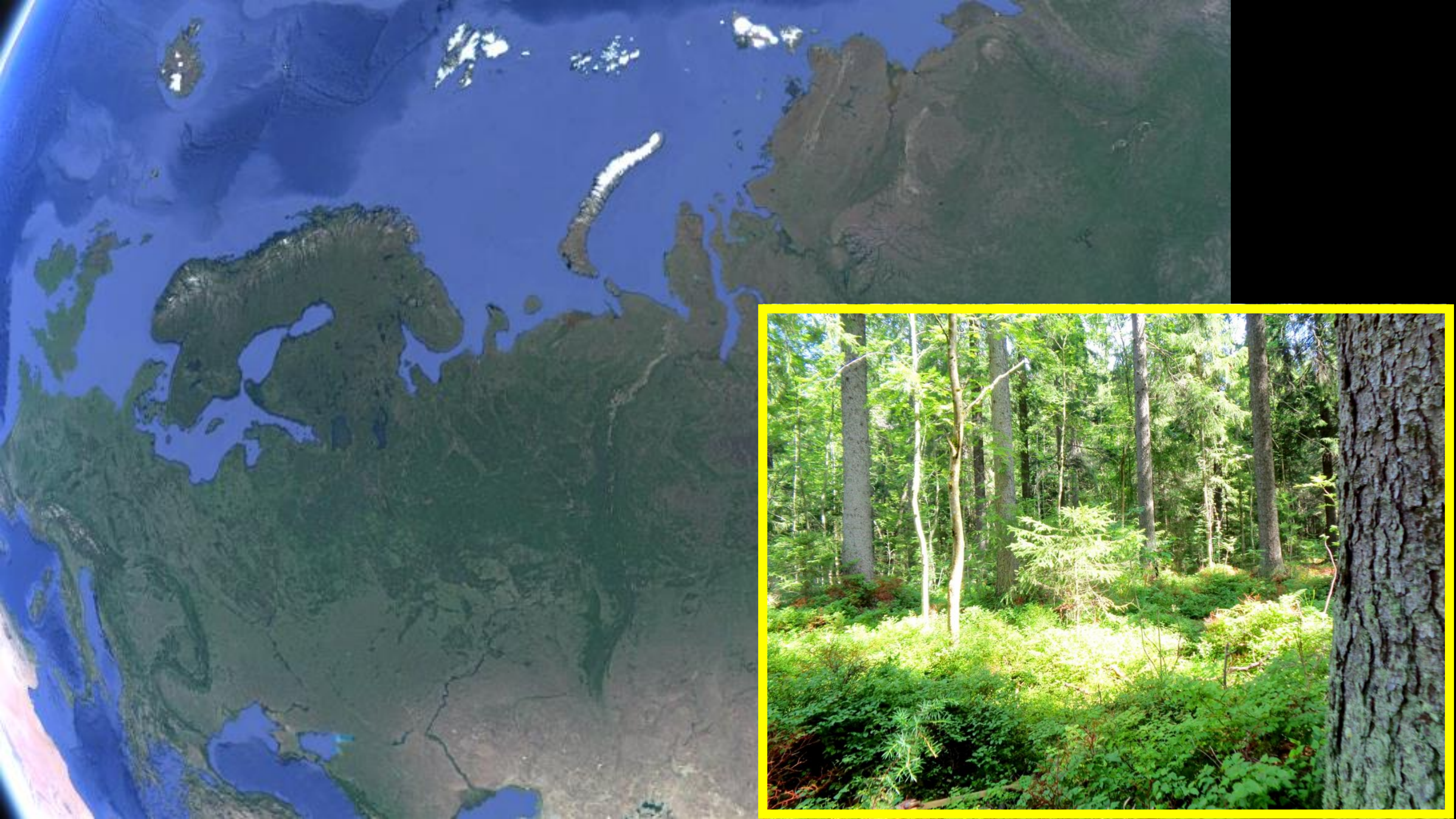
Our world contains very large areas of mixed species forests with trees of different sizes.



- These forests are important to several industries and to the environment.

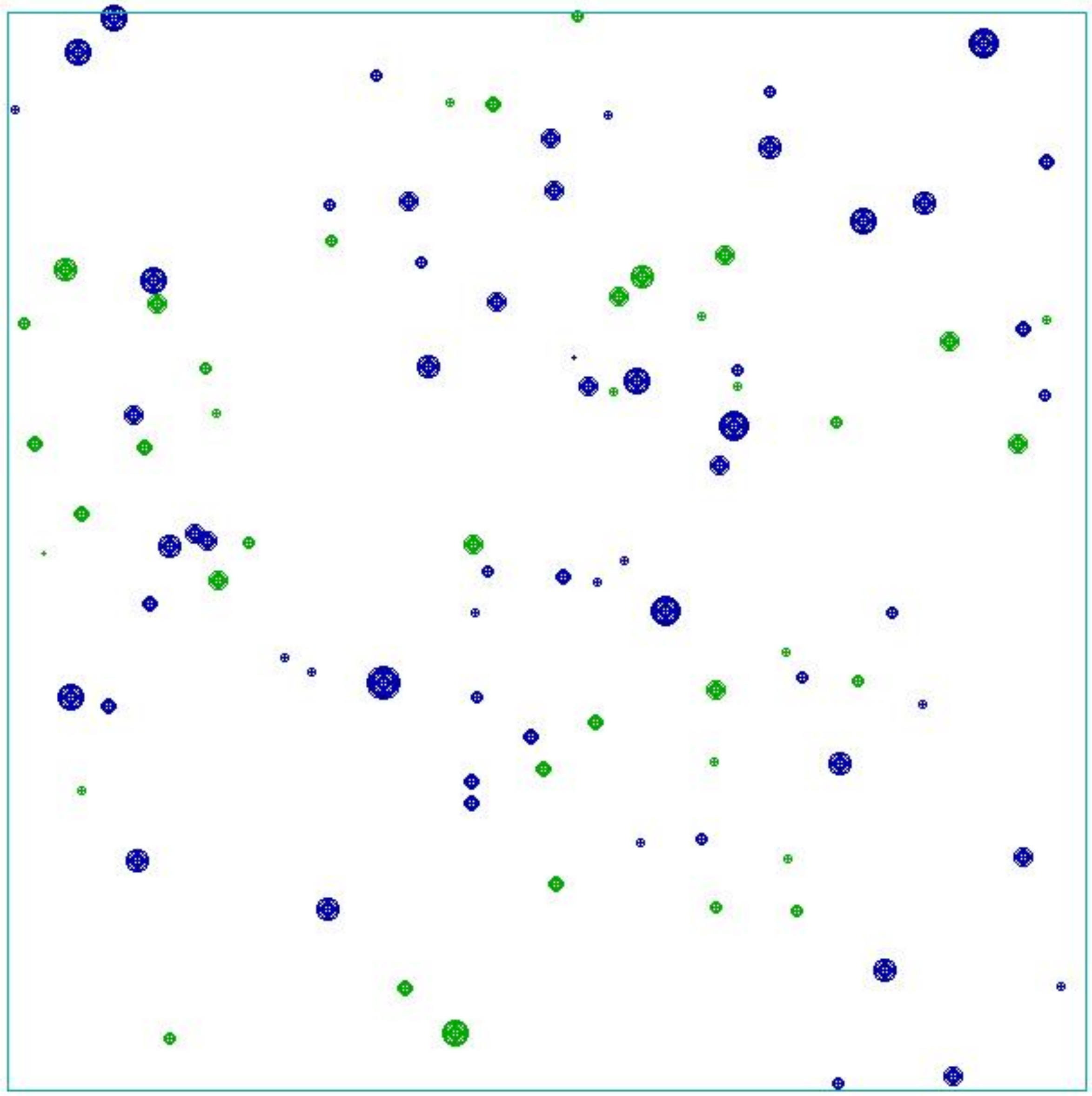
- The prices for wood from different species of trees are stochastic.

- This paper presents a new adaptive method for sustainable and economically optimal management of these forests.

Example of a multi species continuous cover forest:

Different species have different colors (blue and green).

Trees have different sizes.



- In principle, this problem could be correctly solved via stochastic dynamic programming, SDP.

- However, SDP can not in reasonable time handle the extremely large number of state dimensions.

- The method used here includes optimization of adaptive control function parameters via repeated full system stochastic simulations, objective function approximation and analytical parameter optimization.

- A computer model was constructed for this purpose.

- The expected present value of a mixed species continuous cover forest is maximized with consideration of correlated stochastic roundwood market prices, dynamically changing tree sizes and local competition.

Method references:

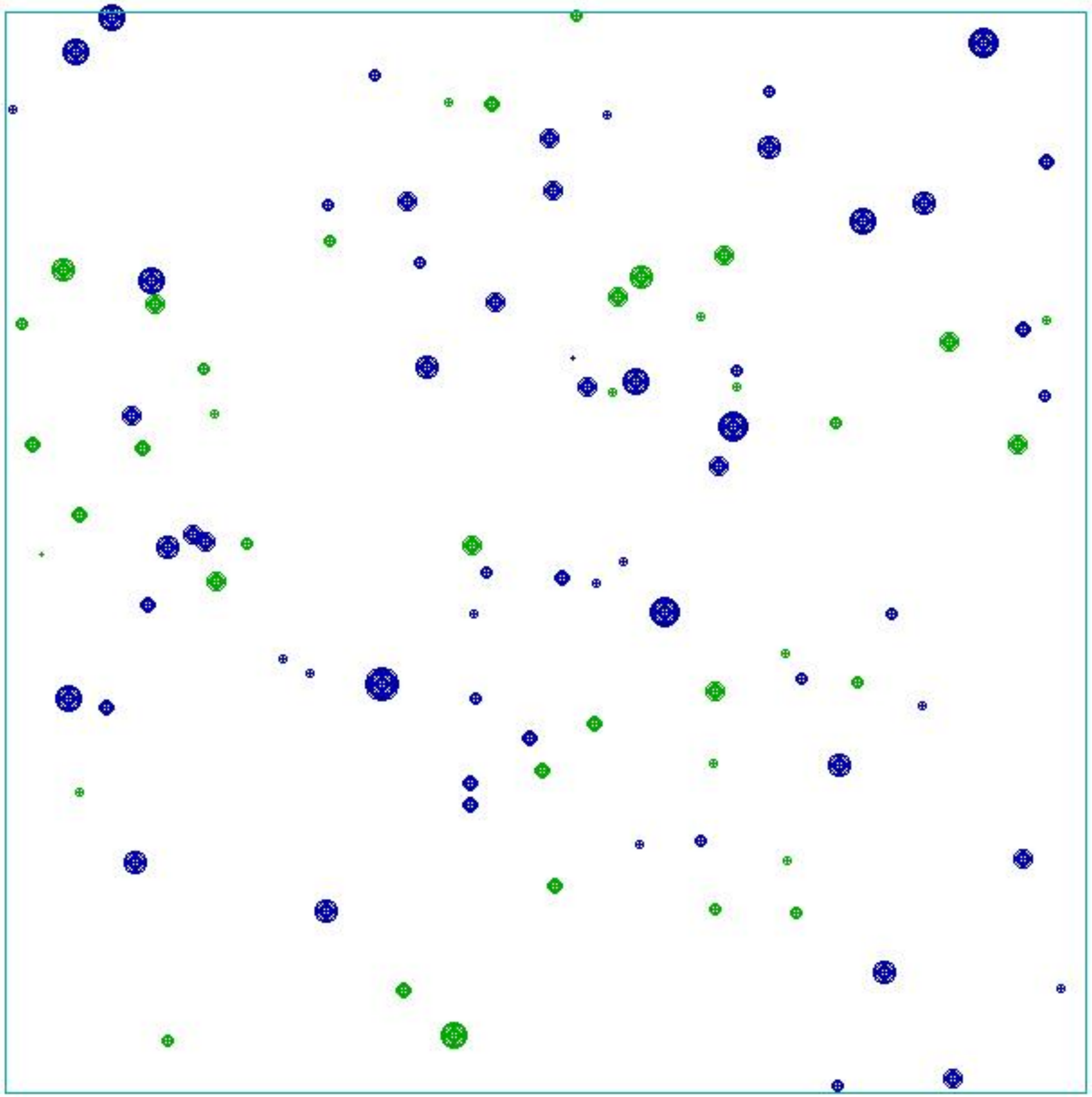
Optimization of adaptive control function parameters via repeated full system stochastic simulations, objective function approximation and analytical parameter optimization

- **Lohmander, P., Two Approaches to Optimal Adaptive Control under Large Dimensionality, INTERNATIONAL ROBOTICS AND AUTOMATION JOURNAL, Volume 3, Issue 4, 2017, DOI:10.15406/iratj.2017.03.00062**
  http://medcraveonline.com/IRATJ/IRATJ-03-00062.php
  http://www.Lohmander.com/PL_171204a.pdf
  http://www.Lohmander.com/PL_171204aORIG.pdf
  http://www.Lohmander.com/PL_171204aORIG.docx

- **Lohmander, P., Optimal Stochastic Dynamic Control of Spatially Distributed Interdependent Production Units. In: Cao BY. (ed) Fuzzy Information and Engineering and Decision. IWDS 2016.**
  Advances in Intelligent Systems and Computing, vol 646. Springer, Cham, 2018
  Print ISBN 978-3-319-66513-9, Online ISBN 978-3-319-66514-6, eBook Package: Engineering, LOSDCSDI
  https://doi.org/10.1007/978-3-319-66514-6_13

The initial conditions:

A forest with spatially distributed trees of different species, initial diameters and spatial coordinates.



Softwood trees are affected by local competition from other softwood trees.

Hardwood trees are affected by local competition from other hardwood trees.

Trees are affected by local competition from other trees of different species.

A complete numerical forest model has been developed with individual tree growth functions that are sensitive to local competition from neighbour trees.

<u>References to related papers on tree growth functions:</u>

- Lohmander, P., A General Dynamic Function for the Basal Area of Individual Trees Derived from a Production Theoretically Motivated Autonomous Differential Equation,
Iranian Journal of Management Studies (IJMS), Vol. 10, No. 4, Autumn 2017, pp. 917-928,
https://ijms.ut.ac.ir/article_64225_61b32fe374f9df8bca512abbe3b5c379.pdf
https://ijms.ut.ac.ir/article_64225.html
http://www.Lohmander.com/PL_IJMS_2017.pdf

- Hatami, N., Lohmander, P., Moayeri, M.H., Mohammadi Limaei, S., A basal area increment model for individual trees in mixed continuous cover forests in Iranian Caspian forests,
Journal of Forestry Research, 2018. pp 1-8. Springer Link.
HLMM
https://doi.org/10.1007/s11676-018-0862-8
https://link.springer.com/article/10.1007%2Fs11676-018-0862-8
Link for reading

- Mohammadi, Z., Mohammadi Limaei, S., Lohmander, P., Olsson, L., Estimation of a basal area growth model for individual trees in uneven-aged Caspian mixed species forests,
JOURNAL OF FORESTRY RESEARCH, November 30, 2017
DOI: https://doi.org/10.1007/s11676-017-0556-7
https://link.springer.com/article/10.1007%2Fs11676-017-0556-7
http://www.Lohmander.com/PL_171204f.pdf

References to related papers on tree growth functions (continued):

- Mohammadi Limaei, S., Lohmander, P., Olsson, L.,
  Dynamic growth models for continuous cover multi-species forestry
  in Iranian Caspian forests,
  JOURNAL OF FOREST SCIENCE, 63, 2017 (11): 519-529,
  doi: 10.17221/32/2017-JFS
  http://www.agriculturejournals.cz/publicFiles/232535.pdf
  http://www.Lohmander.com/PL_171204d.pdf
- Lohmander, P., Optimal Stochastic Dynamic Control of Spatially Distributed Interdependent Production Units. In: Cao BY. (ed) Fuzzy Information and Engineering and Decision. IWDS 2016. Advances in Intelligent Systems and Computing, vol 646. Springer, Cham, 2018
  Print ISBN 978-3-319-66513-9, Online ISBN 978-3-319-66514-6, eBook Package: Engineering,
  LOSDCSDI
  https://doi.org/10.1007/978-3-319-66514-6_13
- Lohmander, P., Olsson, J.O., Fagerberg, N., Bergh, J., Adamopoulos, S.,
  High resolution adaptive optimization of continuous cover
  spruce forest management in southern Sweden,
  SSAFR 2017, Symposium on Systems Analysis in Forest Resources,
  Clearwater Resort, Suquamish, Washington, (near Seattle), August 27-30, 2017
  http://www.Lohmander.com/SSAFR_2017_Lohmander_et_al.pptx
  http://www.Lohmander.com/SSAFR_2017_Lohmander_et_al.pdf
  http://www.Lohmander.com/SSAFR_2017_Lohmander_Soft.txt
  SSAFR 2017

- Every five year period, a harvesting team visits the forest.

- **Optimal adaptive harvest decisions** are taken, based on the **prices** of the different species, the **local competition** conditions in the forest, **harvest cost and revenue functions** and **interest rate** in the capital market.

Continuous cover forest harvesting in the Swedish mountains, December 2014

- The **optimal adaptive control** of the system works this way:

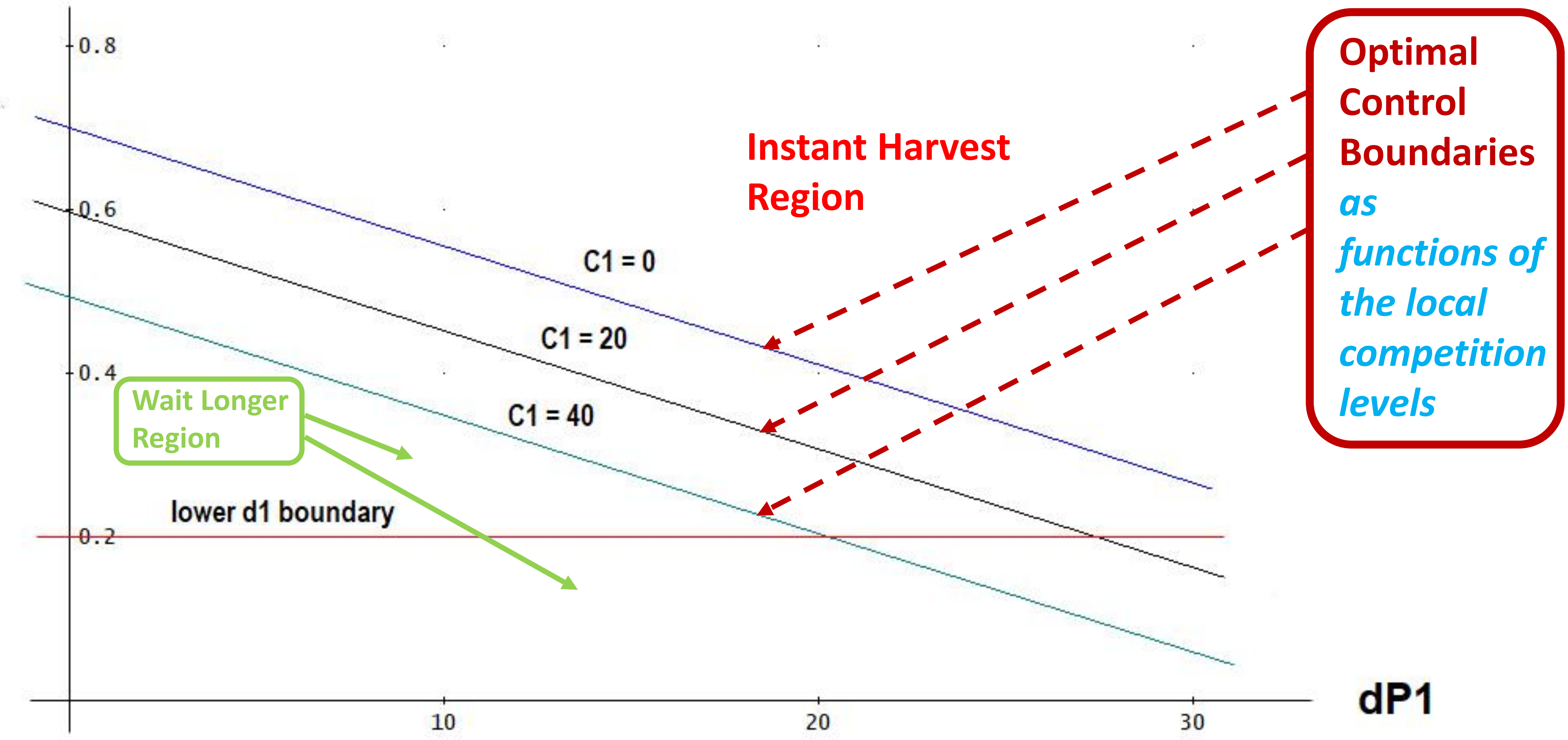
- **A limit diameter function value, DL**, is calculated.

- The value of the DL is derived **for each tree, in every period**. If the diameter of the tree exceeds the DL, then the tree is instantly harvested.

- Otherwise, the tree is left to continue to grow at least one more period.

- The **parameters of the DL are optimized** via a large number of stochastic full system simulations, expected objective function approximation and analytical control function parameter optimization.

DL1 = Optimal Limit Diameter of Species 1



Deviation of the price of species 1 from the expected price level

Example with particular parameters and particular stochastic price developments

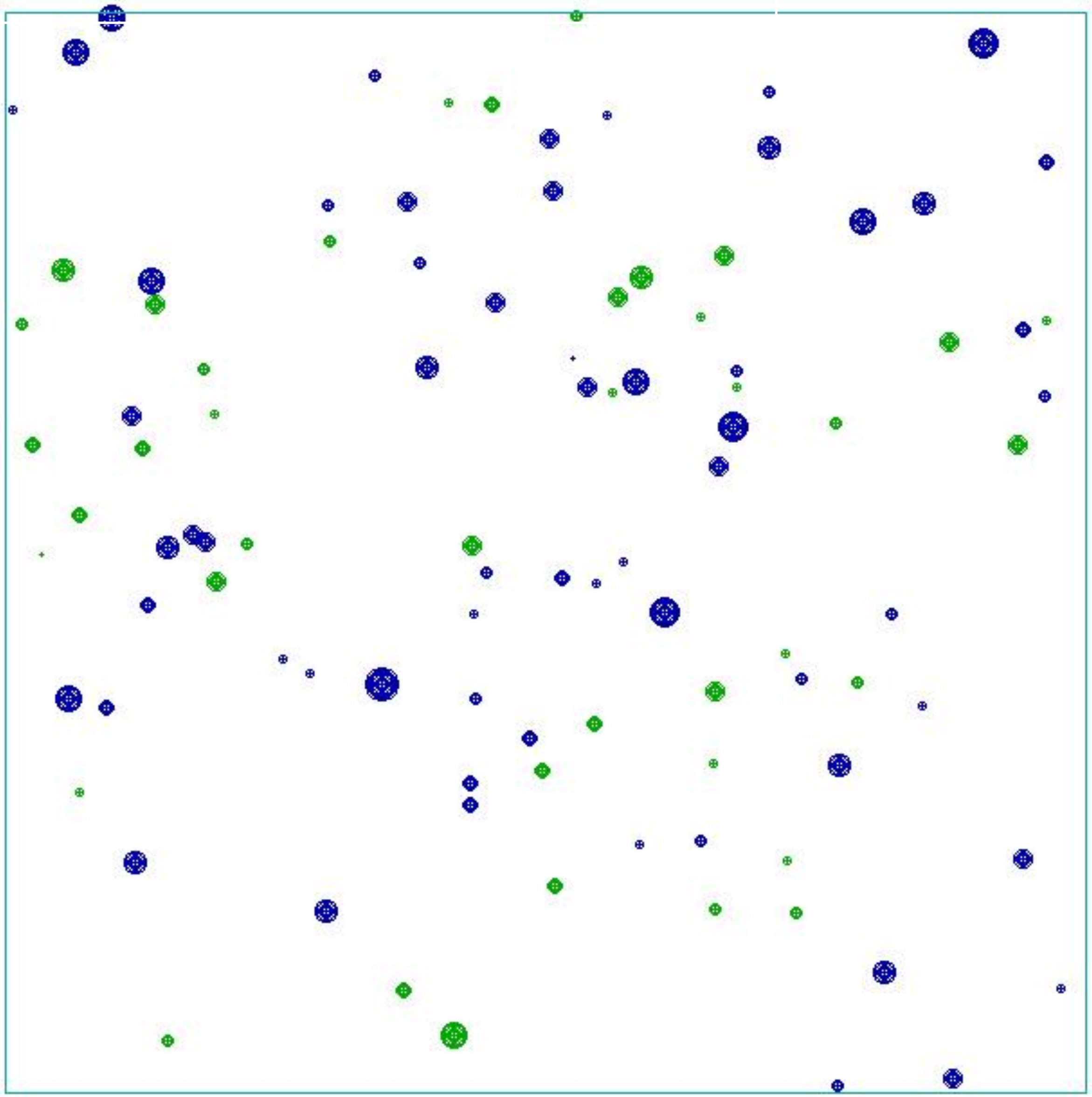
Period = 0

Year = 0

P1 = 52.831

P2 = 54.309

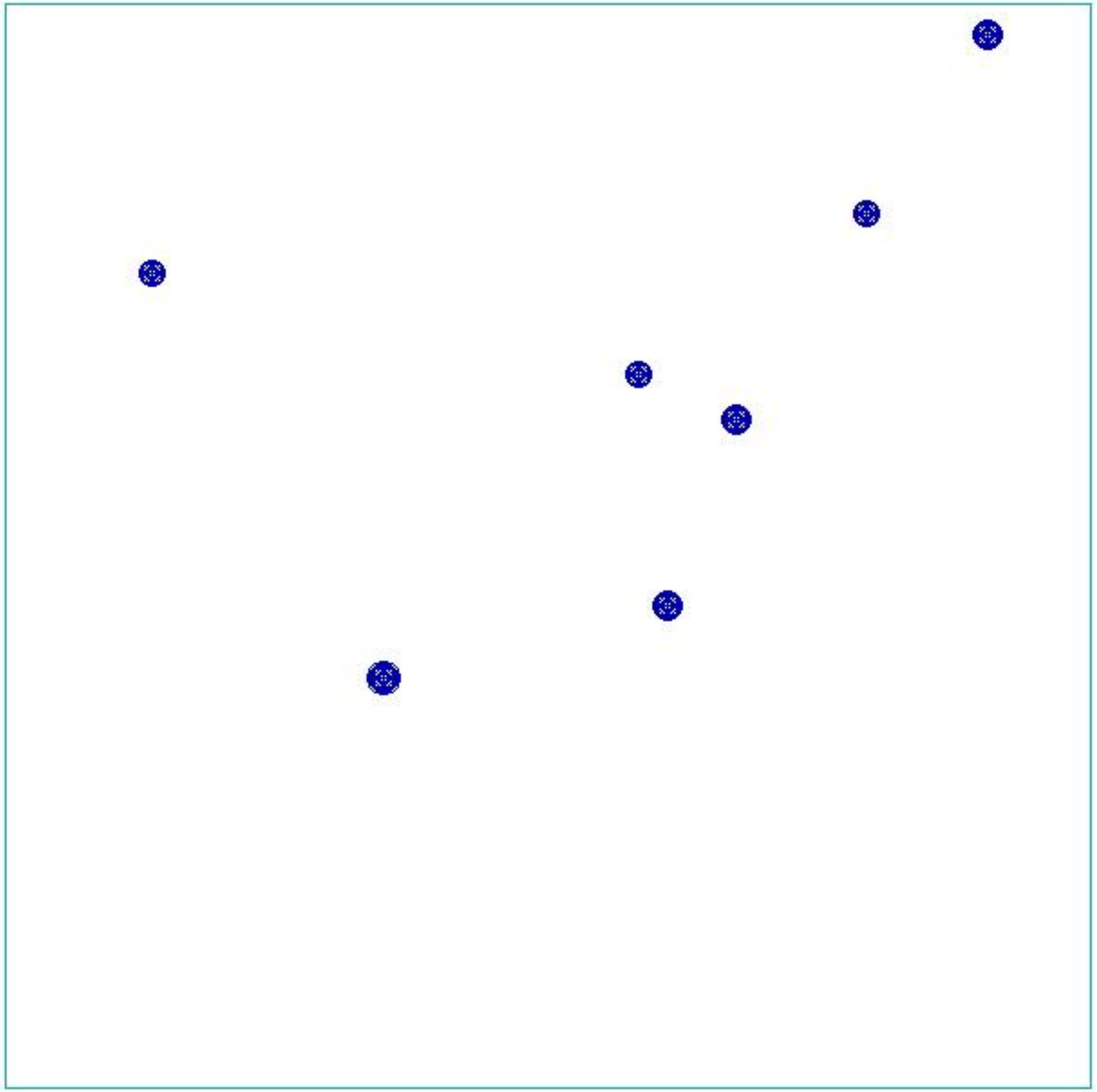
Map after growth and before harvest



Period = 0
Year = 0

P1 = 52.831
P2 = 54.309

Map of harvested trees



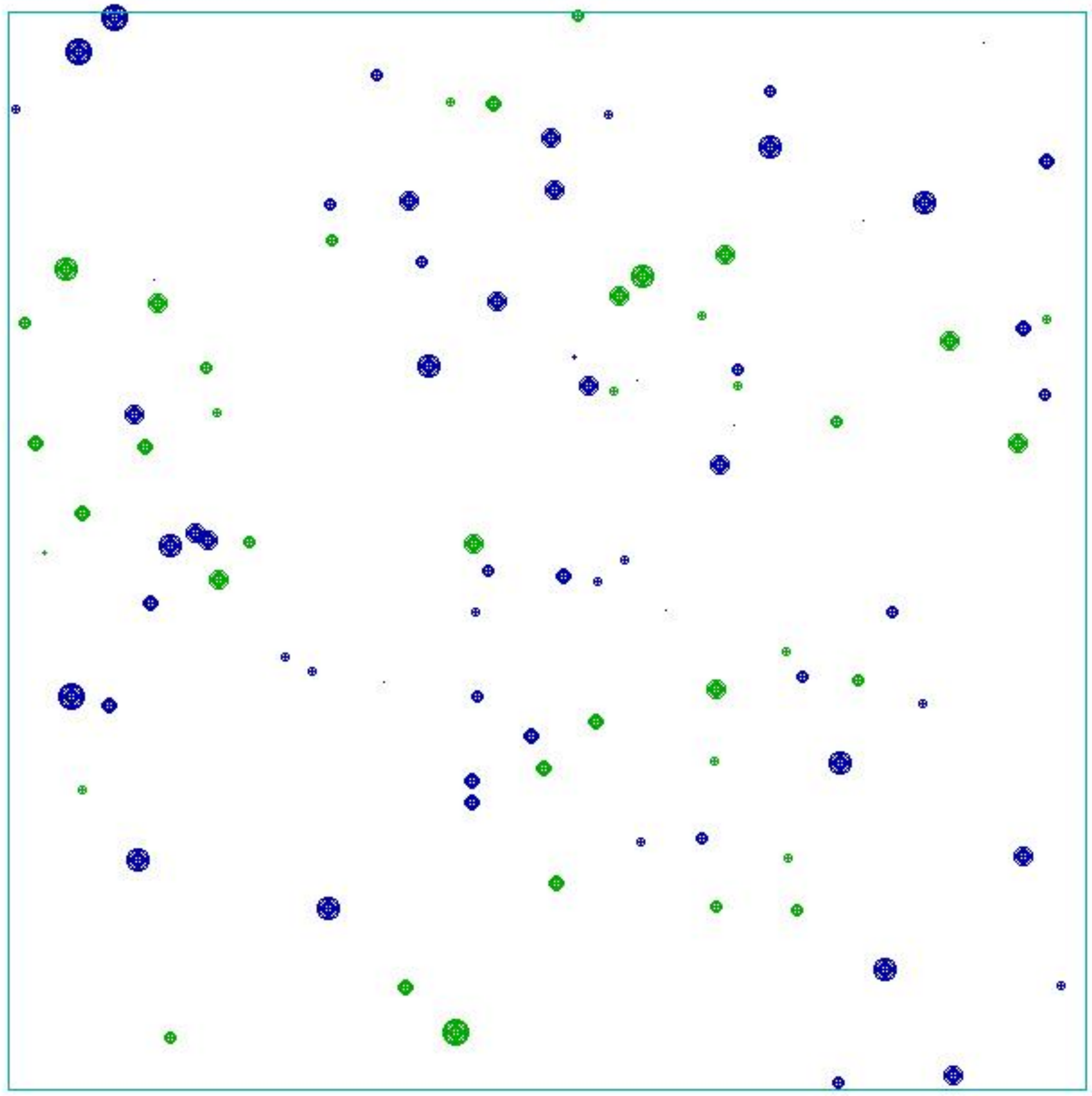
Period = 0

Year = 0

P1 = 52.831

P2 = 54.309

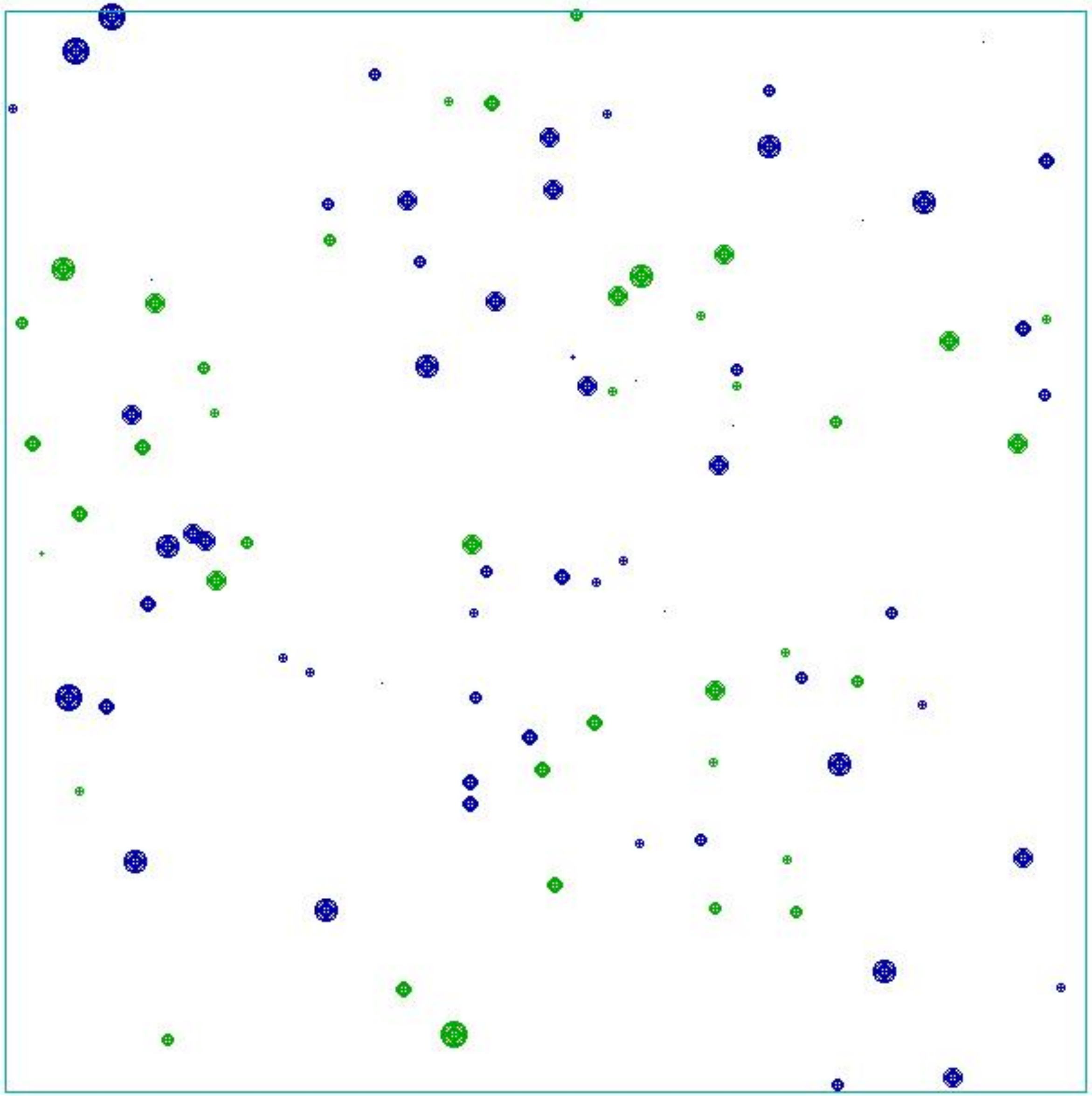
Map after harvest



Period = 1
Year = 5

P1 = 48.111
P2 = 57.984

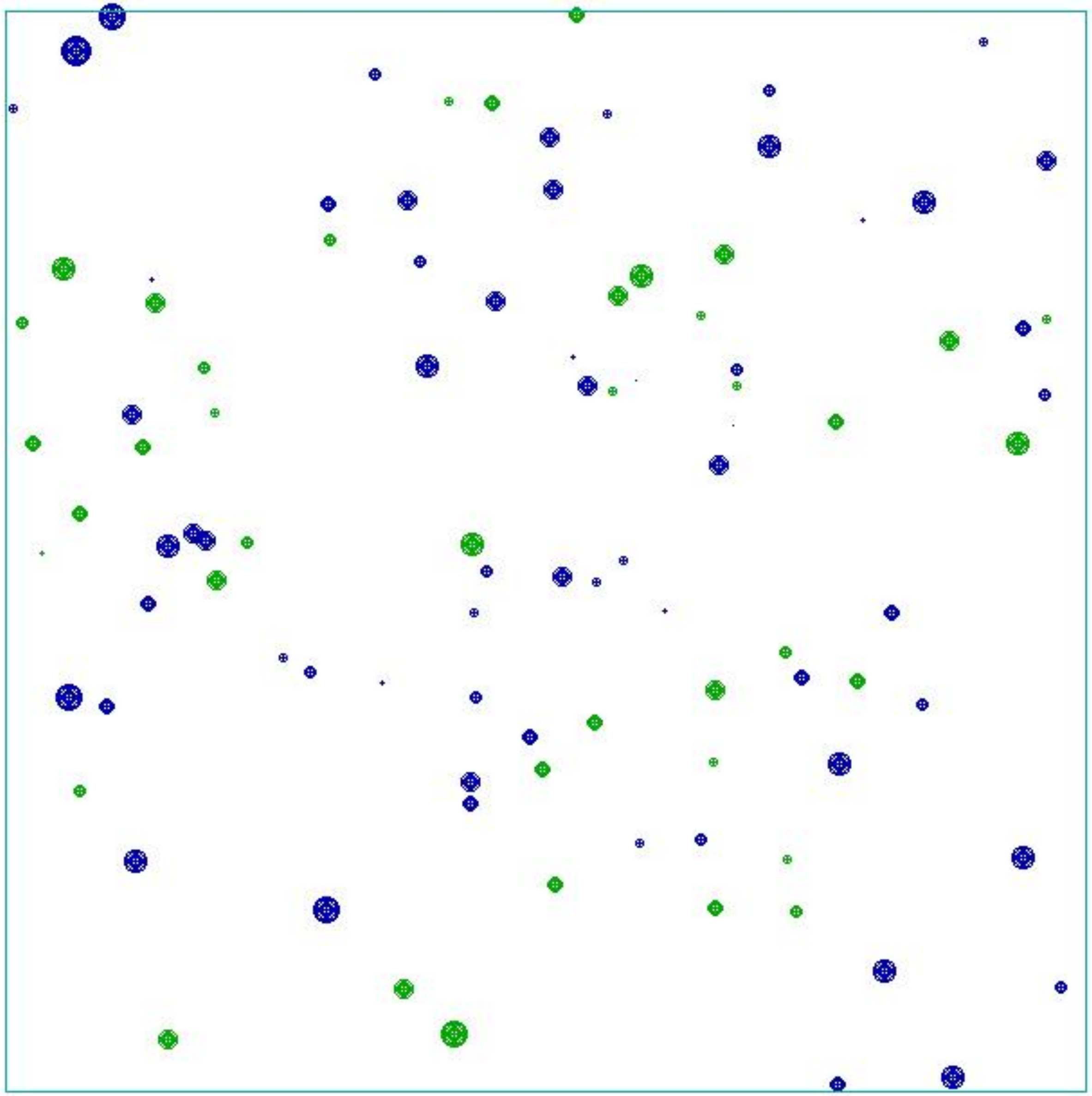
Map before growth and before harvest



Period = 1
Year = 5

P1 = 48.111
P2 = 57.984

Map after growth and before harvest



Period = 1
Year = 5

P1 = 48.111
P2 = 57.984

Map of harvested trees

Then, some periods follow with low prices of both species.

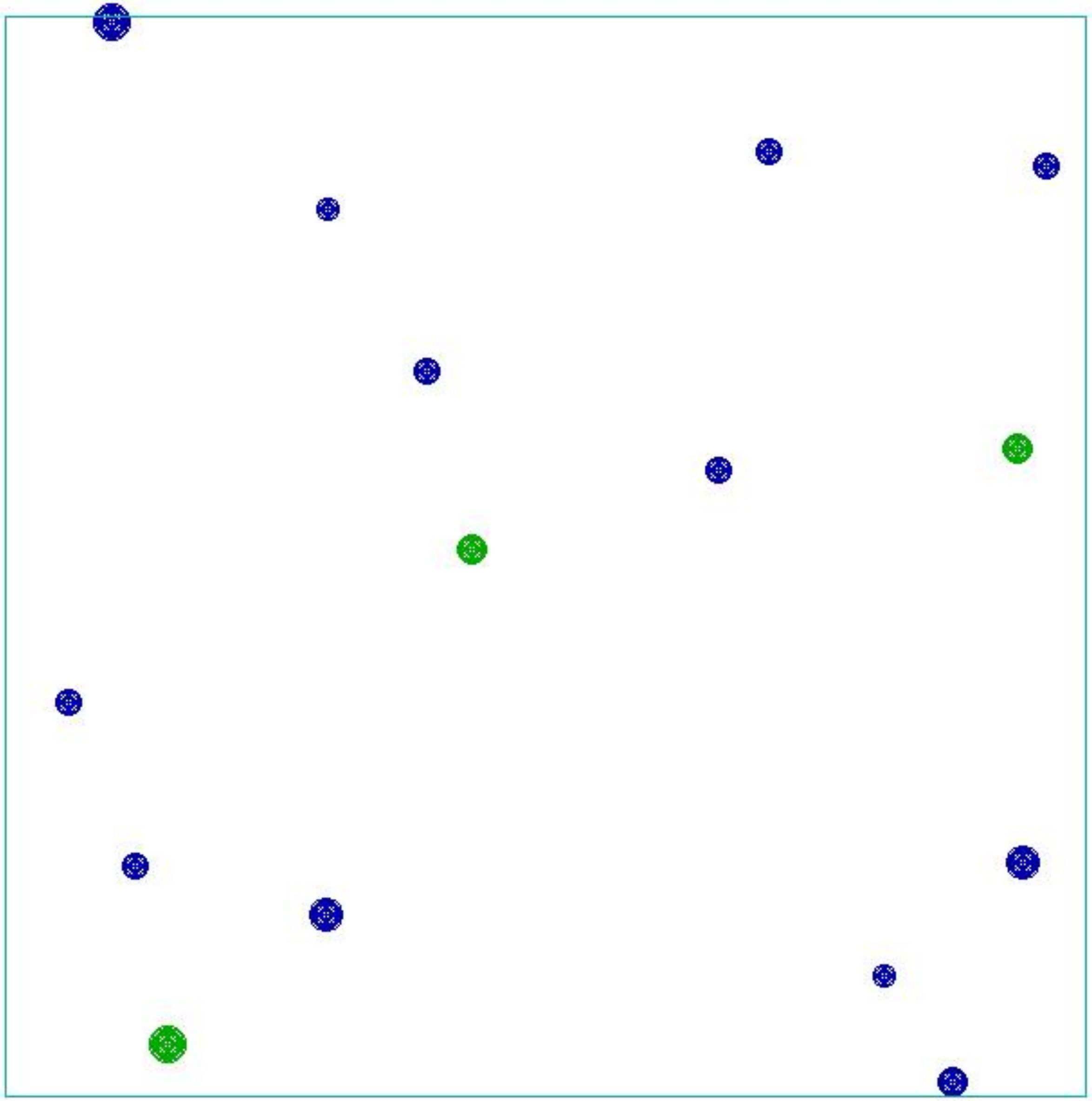
Trees grow and no harvest takes place.

Then, we reach period 6.

Period = 6
Year = 30

P1 = 58.572
P2 = 57.650

Map of harvested trees



The stochastic simulation model

Below, the structure of the software is described.
(The complete software contains many more details.)

SOFTWARE: AdMultRnd1_EQDIST.bas
Peter Lohmander 190107

"AdmultRndIn.txt" (= INPUT file)

"AdMultRndOut.txt" (= OUTPUT file)

If needed, maps may be produced.

SECTION A.
The *initial conditions* of relevance to all calculations are determined.

Generation of 100 market prices series for two species with correlation pcorr.

Generation of the positions

Generation of the species

Calculation of the distances

Generation of initial diameters

SECTION B. The control function parameter loops start here.

```
EPVmax = 0

RANDOMIZE seed2

FOR SIMN = 1 TO TOTSIMN

    Ds = Dsstart + RND * (Dsstop - Dsstart)
    D0 = D0start + RND * (D0stop - D0start)
    Dp1 = Dpstart + RND * (Dpstop - Dpstart)
    Dp2 = Dpstart + RND * (Dpstop - Dpstart)
    Dc1 = Dcstart + RND * (Dcstop - Dcstart)
    Dc2 = Dcstart + RND * (Dcstop - Dcstart)
```

Uniform probability density of control function parameters in six dimensional space

SECTION C. A number of loops of stochastic simulations start here.

FOR series = 1 TO 100

A new series of stochastic prices is selected.

FOR i = 1 TO 100
d(i) = d0(i)
NEXT I

The same initial forest conditions are used for every price series.

FOR T = 0 TO 60

A stochastic 300 year simulation starts here.

Here, the local competition (In this case expressed as ”competing basal area per hectare” within a circle with radius 5 meters) is calculated for every tree.

```
Comp(i) = 0

        FOR j = 1 TO 100
          IF dist(i, j) < 5 THEN Comp(i) = Comp(i) + PI / 4 * d(j) * d(j)
        NEXT j

CompBA(i) = 127.32 * Comp(i)
```

growth occurs

Harvests of individual trees may or may not occur

The harvest decision of tree i is set to zero. Then the limit diameter is calculated. In case the tree diameter exceeds the limit diameter, the harvest decision is set to one. In case the tree diameter is below the minimum diameter, harvest is set to zero.

The limit diameter harvest control function

```
harv(i) = 0

Dlim = D0 + Dp * (Price - meanprice) / stdev(speci) + Ds * (speci - 1) + Dc * CompBA(i)

IF d(i) > Dlim THEN harv(i) = 1

IF d(i) < dmin(speci) THEN harv(i) = 0
```

The revenues and costs of the harvested trees are calculated.

The discounted net revenue of all harvested trees is added to the present value of the series.

NEXT T

NEXT series

End of SECTION C.

SECTION D.

Results for each control function parameter combination are calculated and printed.

End of SECTION D.

NEXT SIMN

EXAMPLE

Input file

(CASE 0)

AdMultRndIn.txt

.03	INPUT #2, r
0.5	INPUT #2, pcorr
50	INPUT #2, EP(1)
50	INPUT #2, EP(2)
15	INPUT #2, stdev(1)
15	INPUT #2, stdev(2)
40	INPUT #2, kheight(1)
40	INPUT #2, kheight(2)
0.2	INPUT #2, dmin(1)
0.2	INPUT #2, dmin(2)
0.70	INPUT #2, D0start
0.70	INPUT #2, D0stop
0.0	INPUT #2, D0step
-0.30	INPUT #2, Dpstart
-0.0	INPUT #2, Dpstop
0.02	INPUT #2, Dpstep
-0.010	INPUT #2, Dcstart
0.0	INPUT #2, Dcstop
0.0	INPUT #2, Dcstep
0	INPUT #2, Dsstart
0	INPUT #2, Dsstop
0.02	INPUT #2, Dsstep
300	INPUT #2, TOTSIMN
1	INPUT #2, seed2

Input section
In the software

EXAMPLE AdMultRndOut.txt
Output file
(CASE 0)

```
Program AdMultRnd_EQDIST by Peter Lohmander 2019:
 Parameters from external file:
  r =  .03  pcorr =  .5  EP(1) =  50  EP(2) =  50
  stdev(1) =  15  stdev(2) =  15  kheight(1) =  40  kheight(2) =  40
  dmin(1) =  .2  dmin(2) =  .2
  D0start =  .7  D0stop =  .7  D0step =  0
  Dpstart = -.3  Dpstop =  0  Dpstep =  .02
  Dcstart = -.01  Dcstop =  0  Dcstep =  0
  Dsstart =  0  Dsstop =  0  Dsstep =  .02
  TOTSIMN =  300
    seed2 =  1
```

```
Program AdMultRnd_EQDIST by Peter Lohmander 2019:
 Parameters from external file:
  r =  .03  pcorr =  .5  EP(1) =  50  EP(2) =  50
  stdev(1) =  15  stdev(2) =  15  kheight(1) =  40  kheight(2) =  40
  dmin(1) =  .2  dmin(2) =  .2
  D0start =  .7  D0stop =  .7  D0step =  0
  Dpstart = -.3  Dpstop =  0  Dpstep =  .02
  Dcstart = -.01  Dcstop =  0  Dcstep =  0
  Dsstart =  0  Dsstop =  0  Dsstep =  .02
  TOTSIMN =  300
    seed2 =  1
```

The output from the simulation model is the input file to the regression software. (Variable transformations will be made within Excel.)

D0	Dp1	Dp2	Dc1	Dc2	Ds	r	EPV
700	-149	-113	-90	-40	0	30	5546176
700	-54	-133	-95	-59	0	30	5419922
700	-1	-49	-87	-74	0	30	5177052
700	-74	-139	-49	-45	0	30	5824453
700	-30	-5	-12	-81	0	30	1807685

Many more rows follow…

Determination of the approximation of the expected present value as a multivariate polynomial of the control function parameters

The expected present value as a quadratic function of the parameters in the DL function (Case 0)

$$Z = k_0 + k_{p_1} D_{p_1} + k_{c_1} D_{c_1} + k_{p_1 p_1} (D_{p_1})^2 + k_{c_1 c_1} (D_{c_1})^2 + k_{p_1 c_1} D_{p_1} D_{c_1} +$$
$$k_{p_2} D_{p_2} + k_{c_2} D_{c_2} + k_{p_2 p_2} (D_{p_2})^2 + k_{c_2 c_2} (D_{c_2})^2 + k_{p_2 c_2} D_{p_2} D_{c_2}$$

$$Z = k_0 + k_{p_1} D_{p_1} + k_{c_1} D_{c_1} + k_{p_1 p_1} (D_{p_1})^2 + k_{c_1 c_1} (D_{c_1})^2 + k_{p_1 c_1} D_{p_1} D_{c_1} +$$
$$k_{p_2} D_{p_2} + k_{c_2} D_{c_2} + k_{p_2 p_2} (D_{p_2})^2 + k_{c_2 c_2} (D_{c_2})^2 + k_{p_2 c_2} D_{p_2} D_{c_2}$$

Objective function

$$\frac{dZ}{dD_{p_1}} = k_{p_1} + 2k_{p_1 p_1} D_{p_1} + k_{p_1 c_1} D_{c_1} = 0$$

$$\frac{dZ}{dD_{c_1}} = k_{c_1} + k_{p_1 c_1} D_{p_1} + 2k_{c_1 c_1} D_{c_1} = 0$$

$$\frac{dZ}{dD_{p_2}} = k_{p_2} + 2k_{p_2 p_2} D_{p_2} + k_{p_2 c_2} D_{c_2} = 0$$

$$\frac{dZ}{dD_{c_2}} = k_{c_2} + k_{p_2 c_2} D_{p_2} + 2k_{c_2 c_2} D_{c_2} = 0$$

First order optimum conditions

$$\frac{dZ}{dD_{p_1}} = k_{p_1} + 2k_{p_1p_1}D_{p_1} + k_{p_1c_1}D_{c_1} = 0$$

$$\frac{dZ}{dD_{c_1}} = k_{c_1} + k_{p_1c_1}D_{p_1} + 2k_{c_1c_1}D_{c_1} = 0$$

First order optimum conditions of species 1 = ”Blue”

Separability

$$\frac{dZ}{dD_{p_2}} = k_{p_2} + 2k_{p_2p_2}D_{p_2} + k_{p_2c_2}D_{c_2} = 0$$

$$\frac{dZ}{dD_{c_2}} = k_{c_2} + k_{p_2c_2}D_{p_2} + 2k_{c_2c_2}D_{c_2} = 0$$

First order optimum conditions of species 2 = ”Green”

First order optimum conditions

$$\frac{dZ}{dD_{p_1}} = k_{p_1} + 2k_{p_1p_1}D_{p_1} + k_{p_1c_1}D_{c_1} = 0$$

$$\frac{dZ}{dD_{c_1}} = k_{c_1} + k_{p_1c_1}D_{p_1} + 2k_{c_1c_1}D_{c_1} = 0$$

$$\frac{dZ}{dD_{p_2}} = k_{p_2} + 2k_{p_2p_2}D_{p_2} + k_{p_2c_2}D_{c_2} = 0$$

$$\frac{dZ}{dD_{c_2}} = k_{c_2} + k_{p_2c_2}D_{p_2} + 2k_{c_2c_2}D_{c_2} = 0$$

Second order maximum conditions

$$\left|\frac{d^2Z}{dD_{p_1}^{\ 2}}\right| < 0$$

$$\begin{vmatrix} \dfrac{d^2Z}{dD_{p_1}^{\ 2}} & \dfrac{d^2Z}{dD_{p_1}dD_{c_1}} \\ \dfrac{d^2Z}{dD_{c_1}dD_{p_1}} & \dfrac{d^2Z}{dD_{c_1}^{\ 2}} \end{vmatrix} > 0$$

$$\left|\frac{d^2Z}{dD_{p_2}^{\ 2}}\right| < 0$$

$$\begin{vmatrix} \dfrac{d^2Z}{dD_{p_2}^{\ 2}} & \dfrac{d^2Z}{dD_{p_2}dD_{c_2}} \\ \dfrac{d^2Z}{dD_{c_2}dD_{p_2}} & \dfrac{d^2Z}{dD_{c_2}^{\ 2}} \end{vmatrix} > 0$$

1 = Blue

2 = Green

$$\frac{dZ}{dD_{p_1}} = k_{p_1} + 2k_{p_1p_1}D_{p_1} + k_{p_1c_1}D_{c_1} = 0$$

$$\frac{dZ}{dD_{c_1}} = k_{c_1} + k_{p_1c_1}D_{p_1} + 2k_{c_1c_1}D_{c_1} = 0$$

$$\left|\frac{d^2Z}{dD_{p_1}{}^2}\right| < 0$$

$$\begin{vmatrix} \frac{d^2Z}{dD_{p_1}{}^2} & \frac{d^2Z}{dD_{p_1}dD_{c_1}} \\ \frac{d^2Z}{dD_{c_1}dD_{p_1}} & \frac{d^2Z}{dD_{c_1}{}^2} \end{vmatrix} > 0$$

1 = Blue

$$\begin{bmatrix} 2k_{p_1p_1} & k_{p_1c_1} \\ k_{p_1c_1} & 2k_{c_1c_1} \end{bmatrix} \begin{bmatrix} D_{p_1} \\ D_{c_1} \end{bmatrix} = \begin{bmatrix} -k_{p_1} \\ -k_{c_1} \end{bmatrix}$$

$$\left|2k_{p_1p_1}\right| < 0$$

$$\begin{vmatrix} 2k_{p_1p_1} & k_{p_1c_1} \\ k_{p_1c_1} & 2k_{c_1c_1} \end{vmatrix} > 0$$

1 = Blue

$$\begin{bmatrix} 2k_{p_1p_1} & k_{p_1c_1} \\ k_{p_1c_1} & 2k_{c_1c_1} \end{bmatrix} \begin{bmatrix} D_{p_1} \\ D_{c_1} \end{bmatrix} = \begin{bmatrix} -k_{p_1} \\ -k_{c_1} \end{bmatrix}$$

$$\left|2k_{p_1p_1}\right| < 0\, f$$

$$\begin{vmatrix} 2k_{p_1p_1} & k_{p_1c_1} \\ k_{p_1c_1} & 2k_{c_1c_1} \end{vmatrix} > 0$$

The optimal parameter value via the analytical solution:

$$D_{p_1} = \frac{\begin{vmatrix} -k_{p_1} & k_{p_1c_1} \\ -k_{c_1} & 2k_{c_1c_1} \end{vmatrix}}{\begin{vmatrix} 2k_{p_1p_1} & k_{p_1c_1} \\ k_{p_1c_1} & 2k_{c_1c_1} \end{vmatrix}} = \frac{-2k_{p_1}k_{c_1c_1} + k_{c_1}k_{p_1c_1}}{4k_{p_1p_1}k_{c_1c_1} - k_{p_1c_1}{}^2}$$

The second order maximum conditions:

$$2k_{p_1p_1} < 0$$

$$4k_{p_1p_1}k_{c_1c_1} - k_{p_1c_1}{}^2 > 0$$

$$D_{p_1} = \frac{\begin{vmatrix} -k_{p_1} & k_{p_1c_1} \\ -k_{c_1} & 2k_{c_1c_1} \end{vmatrix}}{\begin{vmatrix} 2k_{p_1p_1} & k_{p_1c_1} \\ k_{p_1c_1} & 2k_{c_1c_1} \end{vmatrix}} = \frac{-2k_{p_1}k_{c_1c_1} + k_{c_1}k_{p_1c_1}}{4k_{p_1p_1}k_{c_1c_1} - k_{p_1c_1}{}^2}$$

$$2k_{p_1p_1} < 0$$

$$4k_{p_1p_1}k_{c_1c_1} - k_{p_1c_1}{}^2 > 0$$

With figures from CASE 0, we get:

$$D_{p_1} = \frac{-2(-27.225)(-0.54902) + (-101.964)(-0.20816)}{4(-0.037895)(-0.54902) - (-0.20816)^2}$$

$$\boxed{D_{p_1} \approx -217.3312}$$

With figures from CASE 0, we get:

$$2(-0.037895) < 0$$

$$4(-0.037895)(-0.54902) - (-0.20816)^2 > 0$$

$$\boxed{\begin{array}{l} -0.07579 < 0 \\ 0.03988 > 0 \end{array}}$$

Case 0

Regressionsstatistik					
Multipel-R	0,94869828				
R-kvadrat	0,900028426				
Justerad R-kvadrat	0,896569202				
Standardfel	357239,6095				
Observationer	300				
ANOVA					
	fg	KvS	MKv	F	p-värde för F
Regression	10	3,32045E+14	3,32045E+13	260,1821758	3,8861E-138
Residual	289	3,68822E+13	1,2762E+11		
Totalt	299	3,68927E+14			
	Koefficienter	Standardfel	t-kvot	p-värde	
Konstant	-2051272,546	149401,4285	-13,72993931	2,27661E-33	
Dp1	-27225,34464	1108,252192	-24,56601922	9,83949E-73	
Dp2	-16391,15333	1043,094784	-15,71396347	1,20624E-40	
Dc1	-101963,8556	3172,163703	-32,14331452	1,98109E-97	
Dc2	-42856,37519	3055,416205	-14,02636247	1,9043E-34	
Dp1Dp1	-37,89528101	3,24952884	-11,6617771	5,16894E-26	
Dp2Dp2	-27,99794181	3,055076955	-9,16439822	9,54992E-18	
Dc1Dc1	-549,0229524	26,93047653	-20,38667796	7,1657E-58	
Dc2Dc2	-229,0582263	27,60918492	-8,296450147	4,13298E-15	
Dp1Dc1	-208,1579104	8,435028634	-24,67779534	4,04107E-73	
Dp2Dc2	-105,4858035	8,451945763	-12,48065315	6,93176E-29	

All parameters have the expected signs.

All p-values are very low.

All t-values are very negative.

All estimations have very good precision.

The expected present value as a quadratic function of the parameters in the DL function (Case 0)

```
EPV =
- 2051.273
- 27.225 * Dp1
- 16.391 * Dp2
- 101.964 * Dc1
- 42.856 * Dc2
- 0.037895 * Dp1 * Dp1
- 0.027998 * Dp2 * Dp2
- 0.54902 * Dc1 * Dc1
- 0.22906 * Dc2 * Dc2
- 0.20816 * Dp1 * Dc1
- 0.10549 * Dp2 * Dc2 ;
```

Case 1 (r = low)

Regressionsstatistik					
Multipel-R	0,89617769				
R-kvadrat	0,803134452				
Justerad R-kvadrat	0,796322495				
Standardfel	1602172,583				
Observationer	300				
ANOVA					
	fg	KvS	MKv	F	p-värde för F
Regression	10	3,02646E+15	3,02646E+14	117,9006986	8,29119E-96
Residual	289	7,41851E+14	2,56696E+12		
Totalt	299	3,76831E+15			
	Koefficienter	Standardfel	t-kvot	p-värde	
Konstant	-3574468,076	670045,7233	-5,334662923	1,93542E-07	
Dp1	-99493,25346	4970,365069	-20,01729291	1,59232E-56	
Dp2	-50839,32599	4678,142681	-10,86741672	2,71691E-23	
Dc1	-337550,3677	14226,73628	-23,72647956	8,23251E-70	
Dc2	-101690,6792	13703,13913	-7,420976916	1,30491E-12	
Dp1Dp1	-160,0287581	14,57370873	-10,98064749	1,12399E-23	
Dp2Dp2	-101,2691704	13,70161764	-7,391037541	1,57737E-12	
Dc1Dc1	-2031,223255	120,7796392	-16,81759665	9,93045E-45	
Dc2Dc2	-620,5012689	123,8235569	-5,01117303	9,44111E-07	
Dp1Dc1	-728,0266741	37,82999212	-19,24469536	1,0762E-53	
Dp2Dc2	-312,2009082	37,90586322	-8,23621682	6,21763E-15	

All parameters have the expected signs.

All p-values are very low.

All t-values are very negative.

All estimations have very good precision.

Case 2 (EP1 = high)

Regressionsstatistik					
Multipel-R	0,96589442				
R-kvadrat	0,932952031				
Justerad R-kvadrat	0,930632032				
Standardfel	485810,3003				
Observationer	300				
ANOVA					
	fg	KvS	MKv	F	p-värde för F
Regression	10	9,49085E+14	9,49085E+13	402,1346794	3,8197E-163
Residual	289	6,82074E+13	2,36012E+11		
Totalt	299	1,01729E+15			
	Koefficienter	Standardfel	t-kvot	p-värde	
Konstant	-3642061,571	203171,0675	-17,92608374	7,82048E-49	
Dp1	-42368,95875	1507,112637	-28,11266902	1,11596E-84	
Dp2	-17332,83011	1418,505051	-12,21908241	5,82141E-28	
Dc1	-173652,0372	4313,8268	-40,25475414	1,6411E-120	
Dc2	-46918,99694	4155,061826	-11,29200934	9,74495E-25	
Dp1Dp1	-53,98305371	4,419035683	-12,21602576	5,96751E-28	
Dp2Dp2	-29,16684782	4,154600479	-7,020373672	1,58237E-11	
Dc1Dc1	-828,5692999	36,62276674	-22,62443211	6,35187E-66	
Dc2Dc2	-250,4301089	37,54574258	-6,670000156	1,29957E-10	
Dp1Dc1	-353,1962025	11,47079911	-30,79089775	2,99728E-93	
Dp2Dc2	-111,128594	11,49380472	-9,668564646	2,42745E-19	

All parameters have the expected signs.

All p-values are very low.

All t-values are very negative.

All estimations have very good precision.

Case 3 (Stdev1 = high)

Regressionsstatistik					
Multipel-R	0,953421533				
R-kvadrat	0,909012619				
Justerad R-kvadrat	0,905864266				
Standardfel	499842,9392				
Observationer	300				
ANOVA					
	fg	KvS	MKv	F	p-värde för F
Regression	10	7,21363E+14	7,21363E+13	288,7264618	4,9804E-144
Residual	289	7,22046E+13	2,49843E+11		
Totalt	299	7,93567E+14			
	Koefficienter	Standardfel	t-kvot	p-värde	
Konstant	-3237805,016	209039,6673	-15,48895029	8,17263E-40	
Dp1	-43986,50192	1550,645612	-28,36657298	1,65317E-85	
Dp2	-17701,19141	1459,478593	-12,12843511	1,21317E-27	
Dc1	-154515,3959	4438,431761	-34,81306106	2,1481E-105	
Dc2	-43823,70622	4275,080858	-10,25096546	3,10141E-21	
Dp1Dp1	-61,53616208	4,546679604	-13,53430799	1,16395E-32	
Dp2Dp2	-30,70382818	4,274606185	-7,182843717	5,81534E-12	
Dc1Dc1	-803,5153906	37,68061599	-21,32436983	2,8664E-61	
Dc2Dc2	-224,8491006	38,630252	-5,820544495	1,55702E-08	
Dp1Dc1	-336,7686865	11,80213334	-28,5345604	4,69401E-86	
Dp2Dc2	-109,4513287	11,82580346	-9,255297453	4,96251E-18	

All parameters have the expected signs.

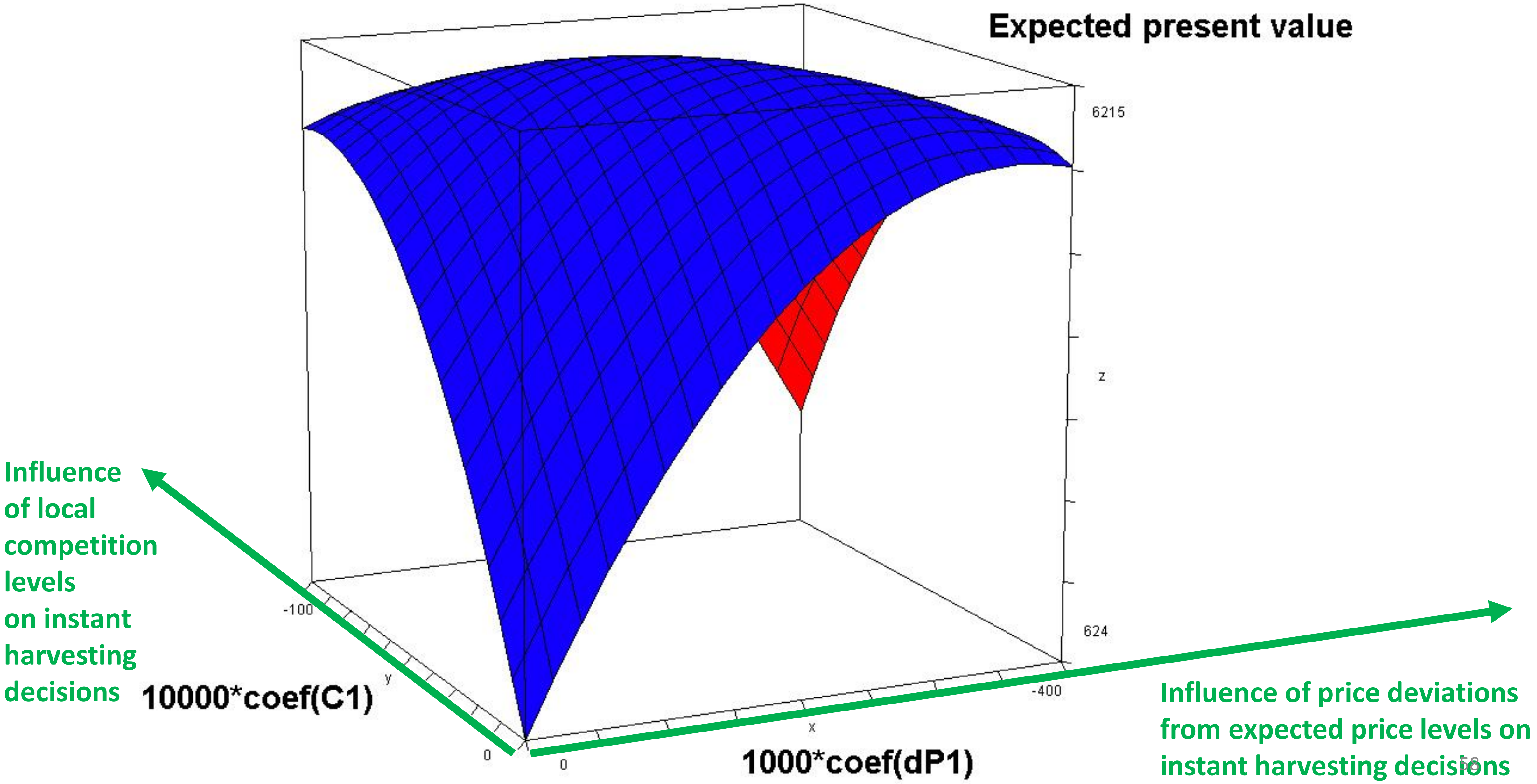
All p-values are very low.

All t-values are very negative.

All estimations have very good precision.

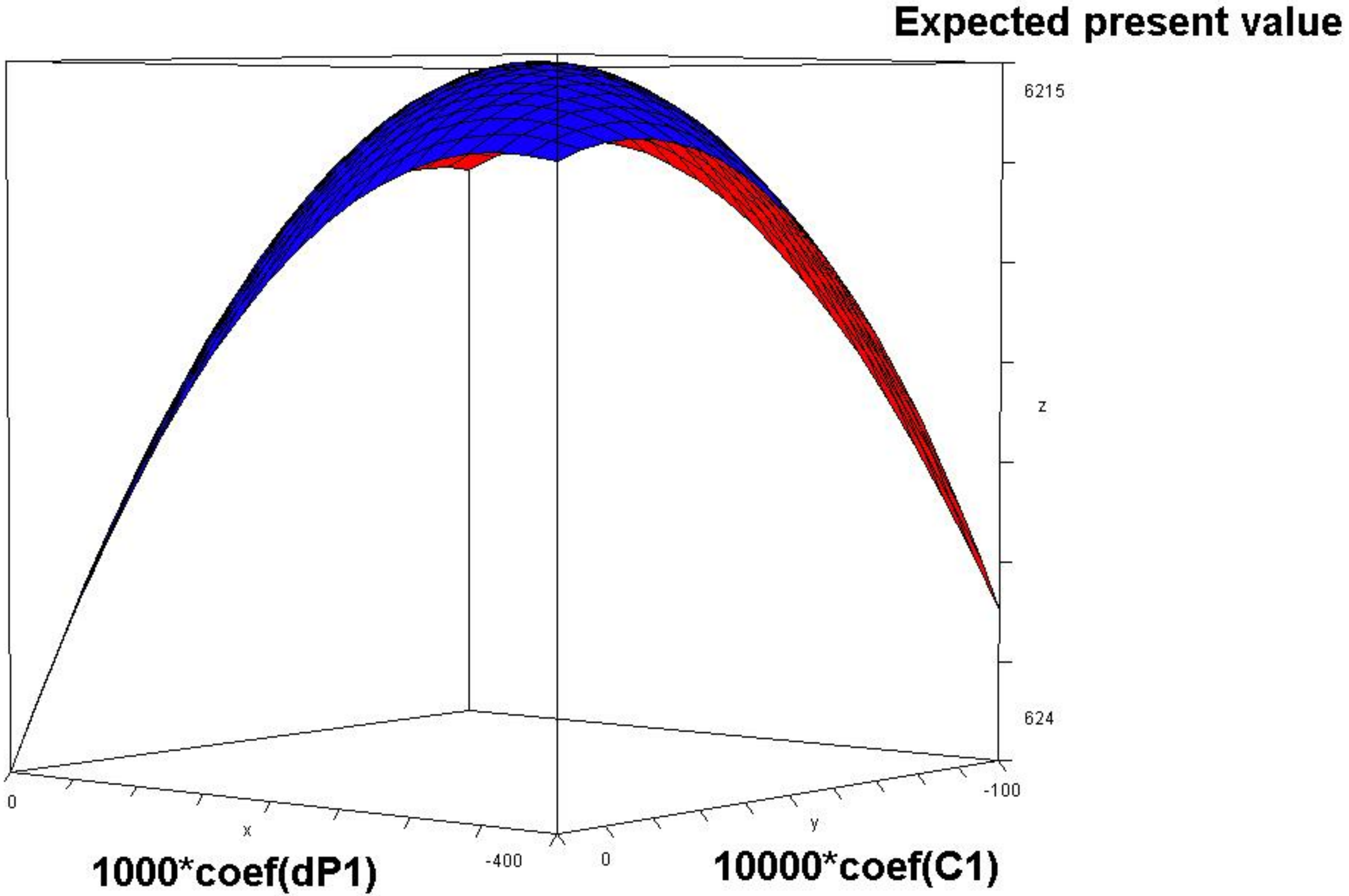
Optimization of the control function parameters

Case 0



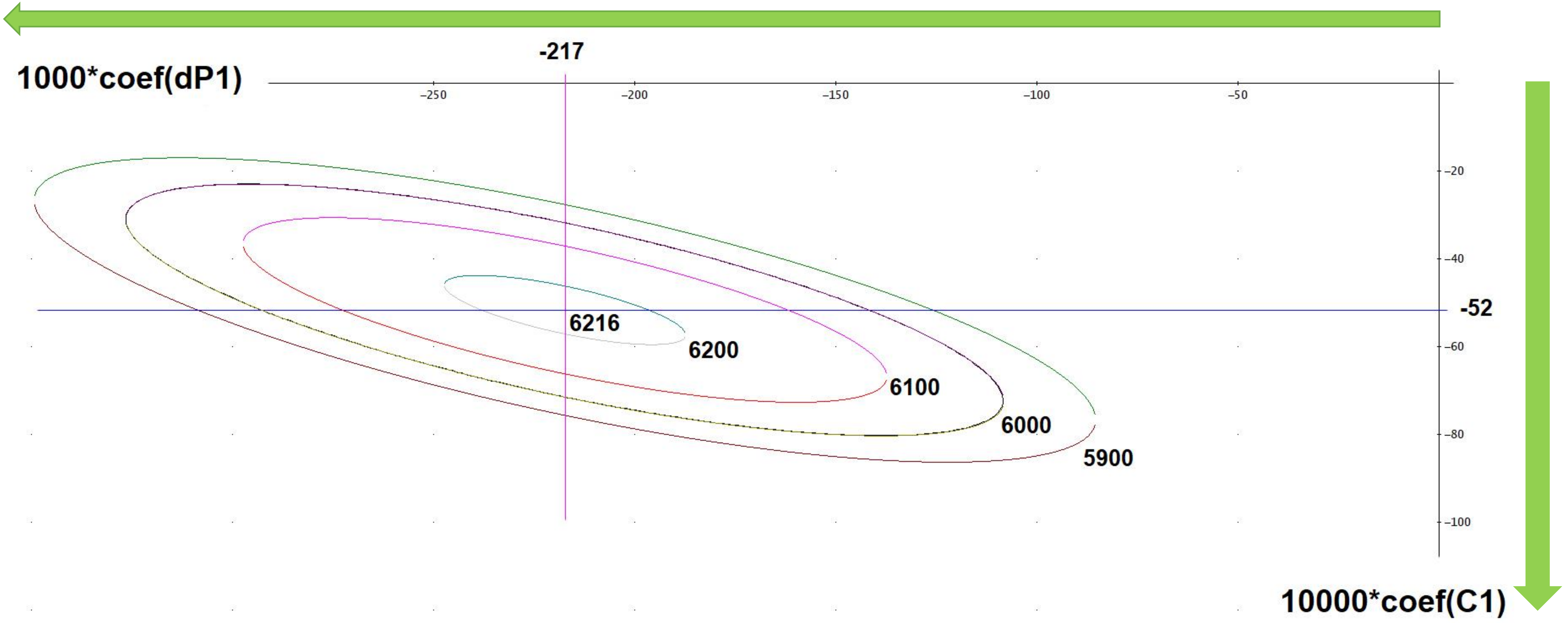
Expected present value
6215
z
624
-100
y
10000*coef(C1)
0
0
x
-400
1000*coef(dP1)
Influence of local competition levels on instant harvesting decisions
Influence of price deviations from expected price levels on instant harvesting decisions

Case 0



Expected present value
6215
z
624
0
x
-400
1000*coef(dP1)
0
y
-100
10000*coef(C1)

Influence of deviations from expected price levels on instant harvesting decisions



Influence of local competition levels on instant harvesting decisions

```
! Case 0_EQDIST_190108_1337;
! Peter Lohmander;

model:

max = EPV;

EPV =
- 2051.273
- 27.225 * Dp1
- 16.391 * Dp2
- 101.964 * Dc1
- 42.856 * Dc2
- 0.037895 * Dp1 * Dp1
- 0.027998 * Dp2 * Dp2
- 0.54902 * Dc1 * Dc1
- 0.22906 * Dc2 * Dc2
- 0.20816 * Dp1 * Dc1
- 0.10549 * Dp2 * Dc2 ;

@free(Dp1);
@free(Dp2);
@free(Dc1);
@free(Dc2);

end
```

Case 0

Variable	Value
EPV	6216.348
DP1	-217.3312
DP2	-205.7293
DC1	-51.65963
DC2	-46.17485

$$D_{p_1} = \frac{\begin{vmatrix} -k_{p_1} & k_{p_1c_1} \\ -k_{c_1} & 2k_{c_1c_1} \end{vmatrix}}{\begin{vmatrix} 2k_{p_1p_1} & k_{p_1c_1} \\ k_{c_1p_1} & 2k_{c_1c_1} \end{vmatrix}} = \frac{-k_{p_1}2k_{c_1c_1} + k_{c_1}k_{p_1c_1}}{4k_{p_1p_1}k_{c_1c_1} - k_{p_1c_1}{}^2}$$

$$2k_{p_1p_1} < 0$$

$$4k_{p_1p_1}k_{c_1c_1} - k_{p_1c_1}{}^2 > 0$$

The analytical method gave the same answer:

$$D_{p_1} = \frac{-2(-27.225)(-0.54902) + (-101.964)(-0.20816)}{4(-0.037895)(-0.54902) - (-0.20816)^2}$$

$$\boxed{D_{p_1} \approx -217.3312}$$

The analytical method also told us that the solution is a unique maximum.

$$2(-0.037895) < 0$$

$$4(-0.037895)(-0.54902) - (-0.20816)^2 > 0$$

$$\boxed{\begin{array}{l} -0.07579 < 0 \\ 0.03988 > 0 \end{array}}$$

```
! Case 1_r is low_190108_1353;
! Peter Lohmander;

model:

max = EPV;

EPV =
- 3574.468
- 99.493 * Dp1
- 50.839 * Dp2
- 337.550 * Dc1
- 101.691 * Dc2
- 0.160029 * Dp1 * Dp1
- 0.101269 * Dp2 * Dp2
- 2.031223 * Dc1 * Dc1
- 0.620501 * Dc2 * Dc2
- 0.728027 * Dp1 * Dc1
- 0.312201 * Dp2 * Dc2 ;

@free(Dp1);
@free(Dp2);
@free(Dc1);
@free(Dc2);

end
```

(r = low) Case 1

Variable	Value
EPV	21199.22
DP1	-205.7134
DP2	-203.6856
DC1	-46.22464
DC2	-30.70111

```
! Case 2 EP1 is high_EQDIST_190108_1408;
! Peter Lohmander;

model:

max = EPV;

EPV =
- 3642.062
- 42.369 * Dp1
- 17.333 * Dp2
- 173.652 * Dc1
- 46.919 * Dc2
- 0.053983 * Dp1 * Dp1
- 0.029167 * Dp2 * Dp2
- 0.82857 * Dc1 * Dc1
- 0.25043 * Dc2 * Dc2
- 0.35320 * Dp1 * Dc1
- 0.11113 * Dp2 * Dc2 ;

@free(Dp1);
@free(Dp2);
@free(Dc1);
@free(Dc2);

end
```

(EP1 = high) Case 2

Variable	Value
EPV	8804.599
DP1	-163.8983
DP2	-205.5634
DC1	-69.85717
DC2	-48.06680

Case 3

(Stdev1 = high)

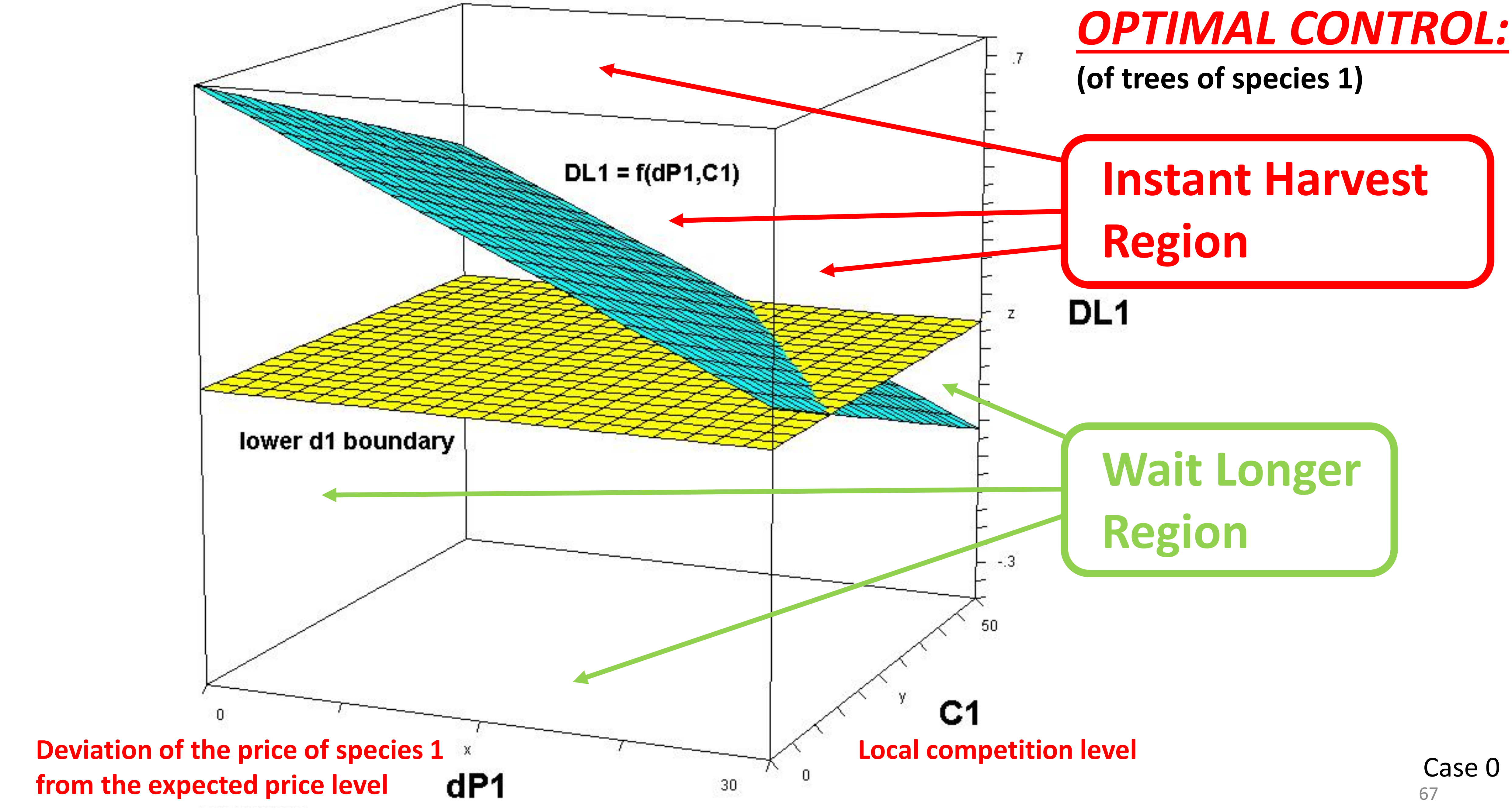
```
! Case 3 Stdev1 is high_EQDIST_190108_1424;
! Peter Lohmander;

model:

max = EPV;

EPV =
- 3237.805
- 43.987 * Dp1
- 17.701 * Dp2
- 154.515 * Dc1
- 43.824 * Dc2
- 0.061536 * Dp1 * Dp1
- 0.030704 * Dp2 * Dp2
- 0.80352 * Dc1 * Dc1
- 0.22485 * Dc2 * Dc2
- 0.33677 * Dp1 * Dc1
- 0.10945 * Dp2 * Dc2 ;

@free(Dp1);
@free(Dp2);
@free(Dc1);
@free(Dc2);

end
```

Variable	Value
EPV	8320.552
DP1	-221.0902
DP2	-202.3298
DC1	-49.81733
DC2	-48.20771

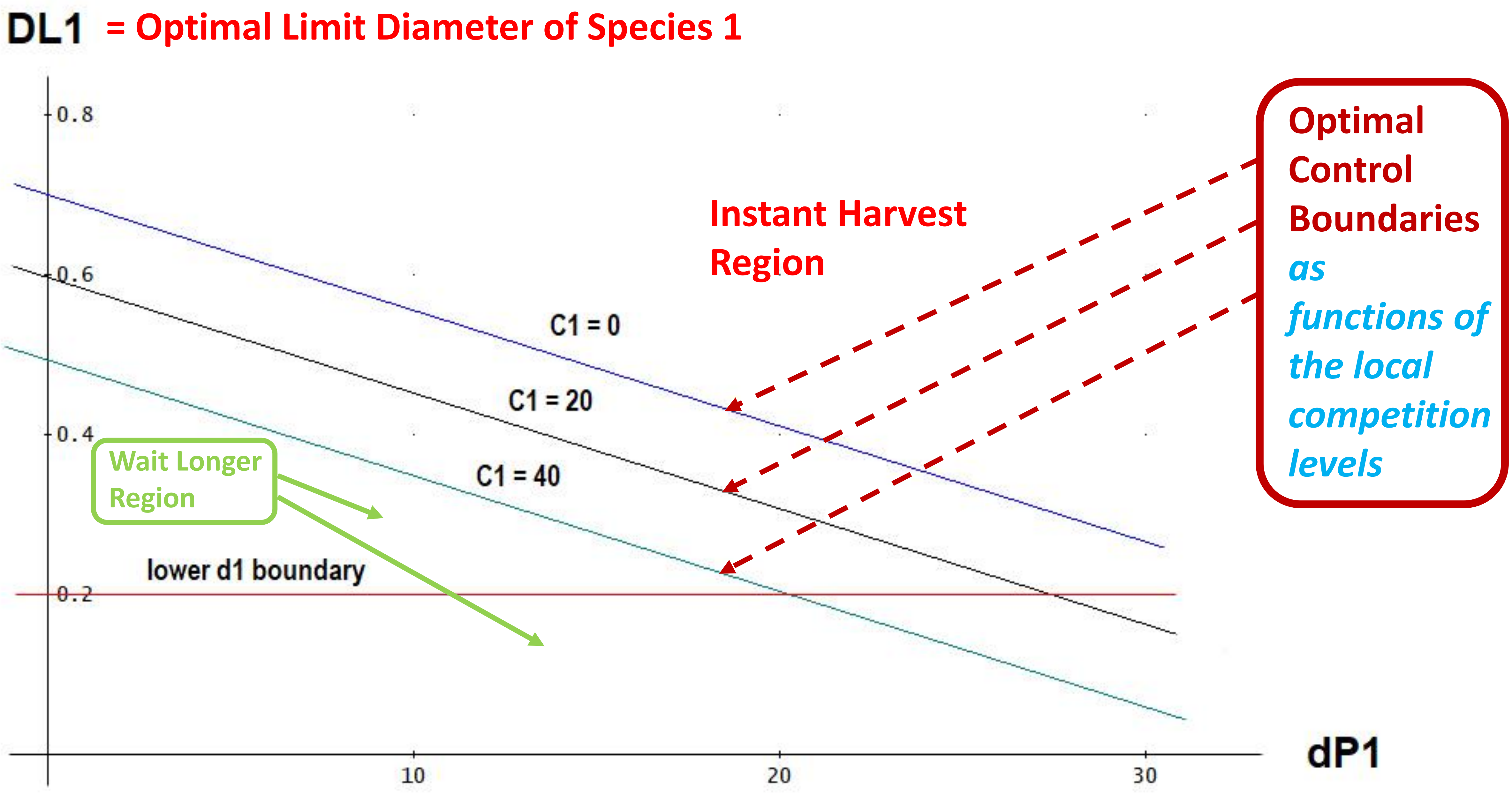
The optimal control of the forest

OPTIMAL CONTROL:

(of trees of species 1)



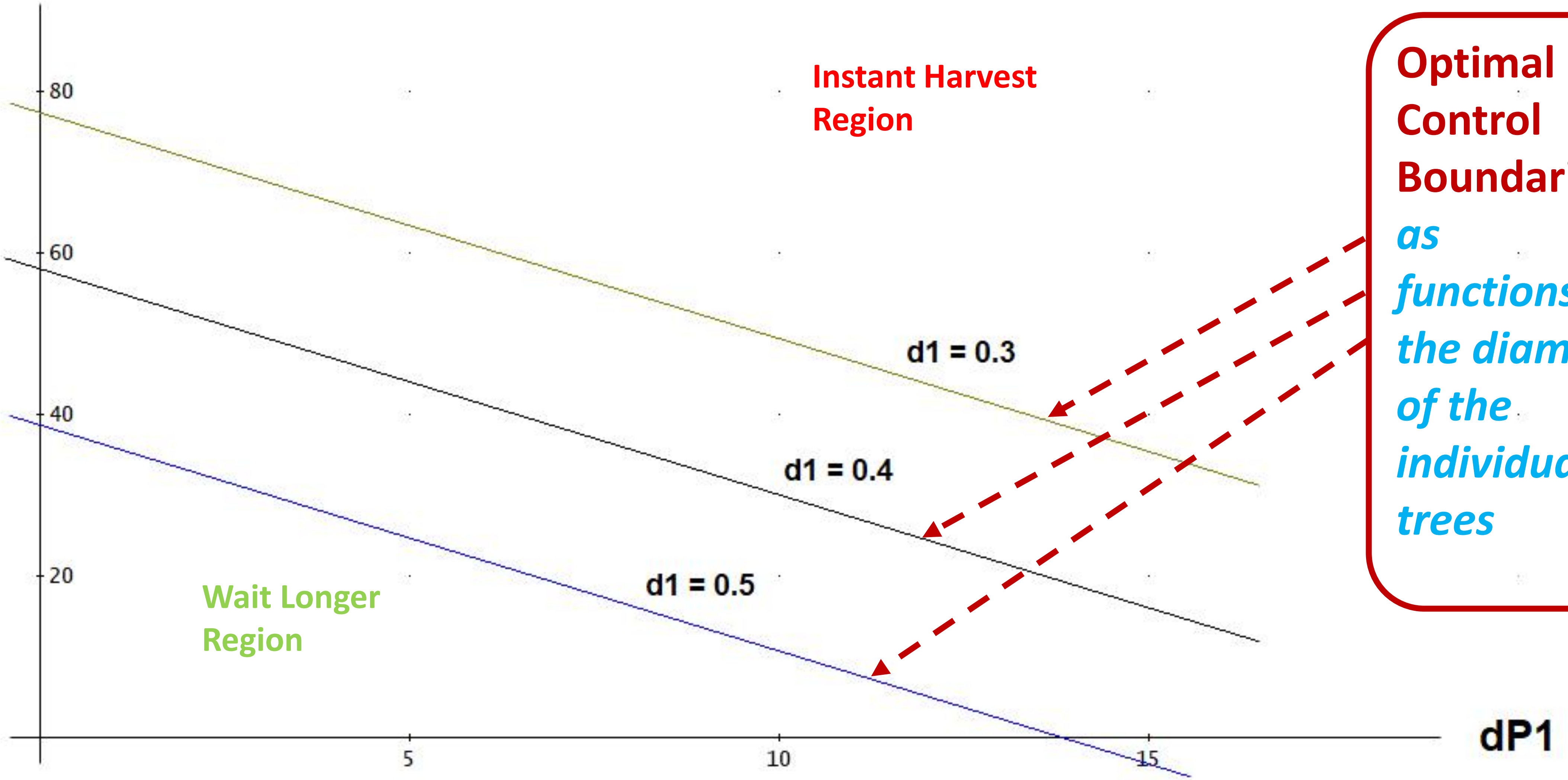
Case 0



Case 0

Deviation of the price of species 1 from the expected price level

C1 = Local competition level



Optimal Control Boundaries ***as functions of the diameters of the individual trees***

Deviation of the price of species 1 from the expected price level

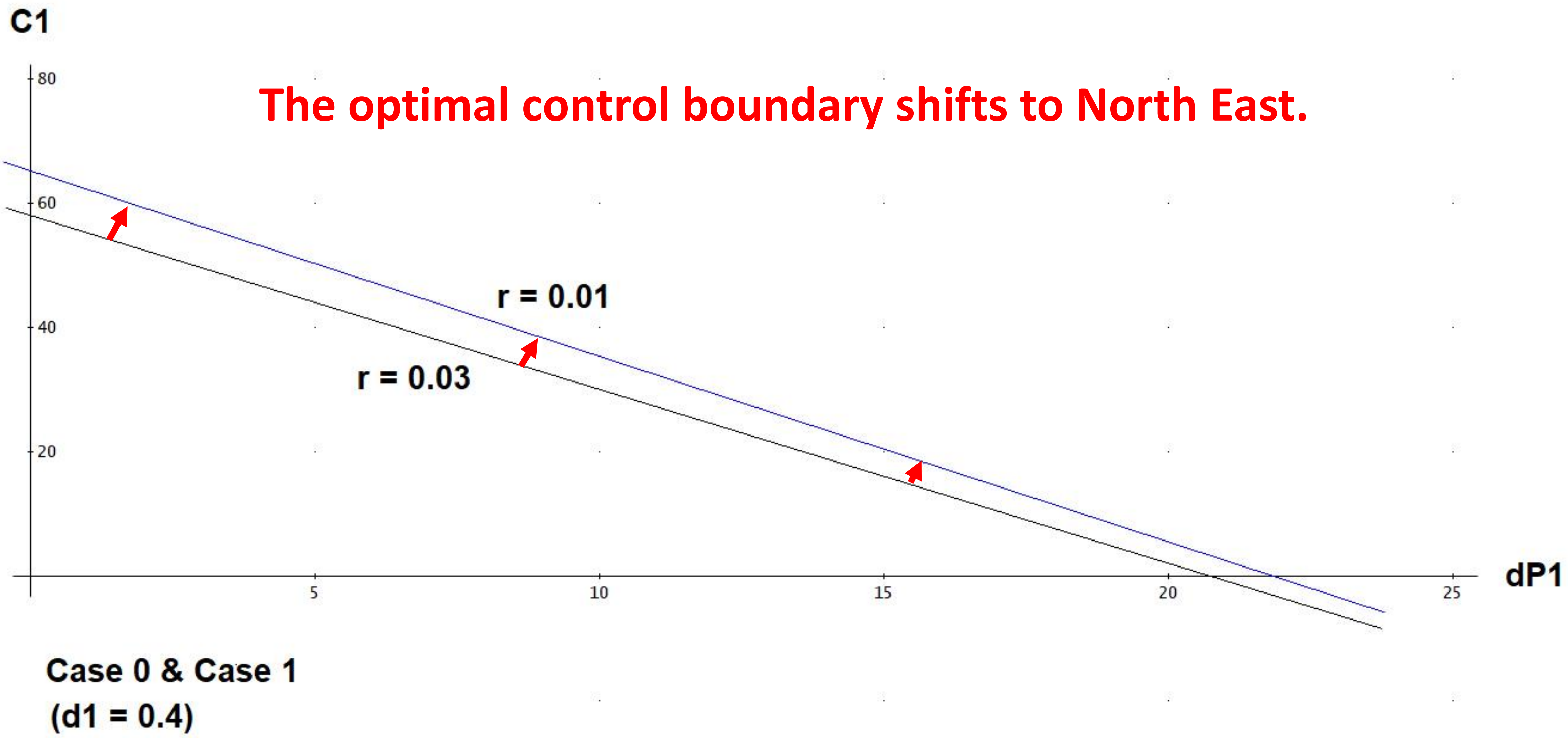
Case 0

Optimal changes of forest control decisions if these parameters change:

- the rate of interest,
- the expected prices,
- the degrees of stochastic price variations

Case 1 (r = low)

- The real rate of interest, r, decreases from 3% to 1%.

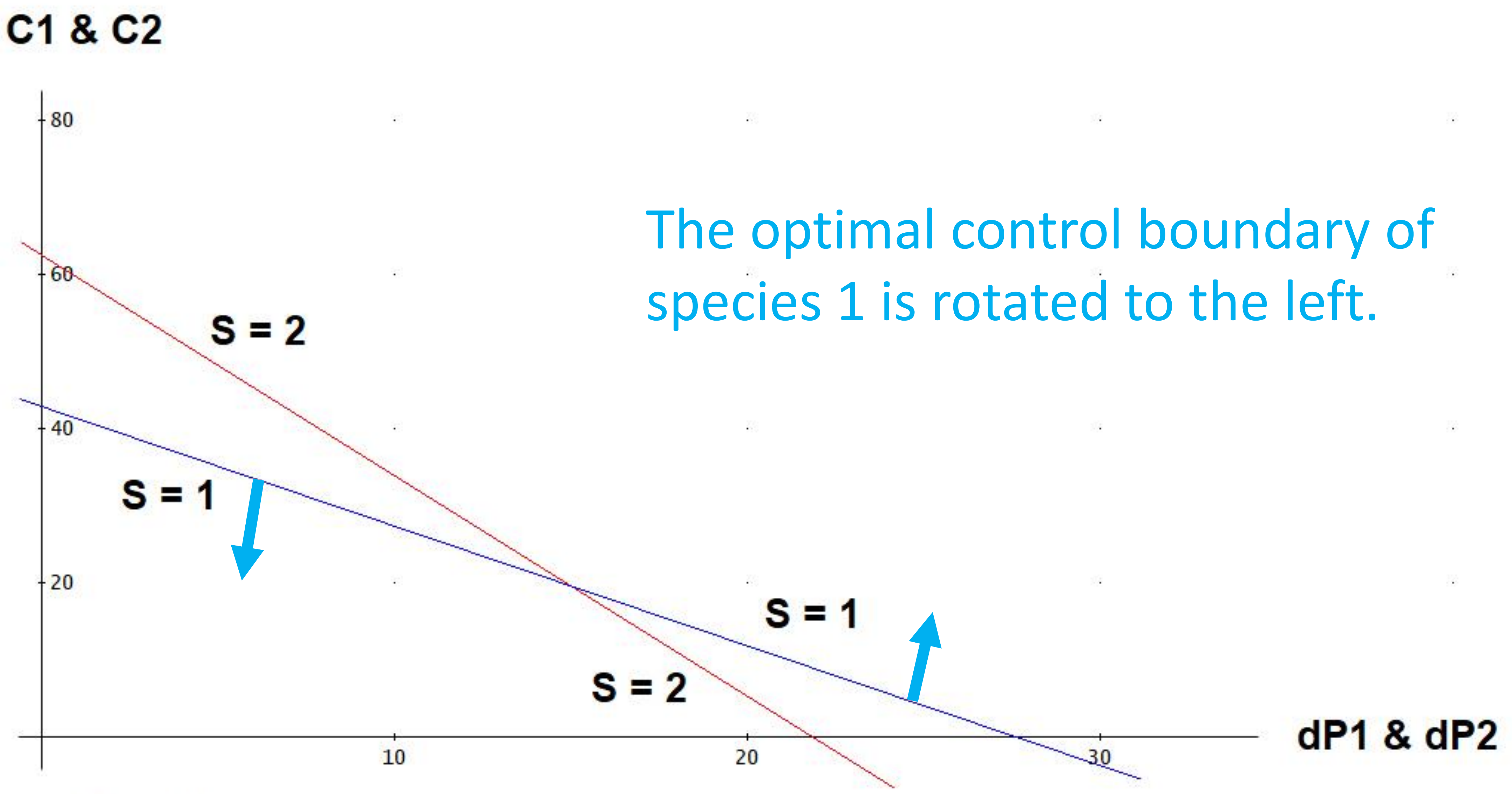

C1
80
60
40
20
The optimal control boundary shifts to North East.
r = 0.01
r = 0.03
5
10
15
20
25
dP1
Case 0 & Case 1
(d1 = 0.4)

Case 1 (r = low)

- The real rate of interest, r, decreases from 3% to 1%.
- The optimal control boundary shifts to North East.
- To motivate instant harvesting, the level of competition and/or the price has to be higher than before the change.
- The expected size of the trees (when they are harvested) is larger with a low rate of interest.
- The expected present value is **3.41 times higher** than if r = 0.03.

Case 2 (EP1 = high)

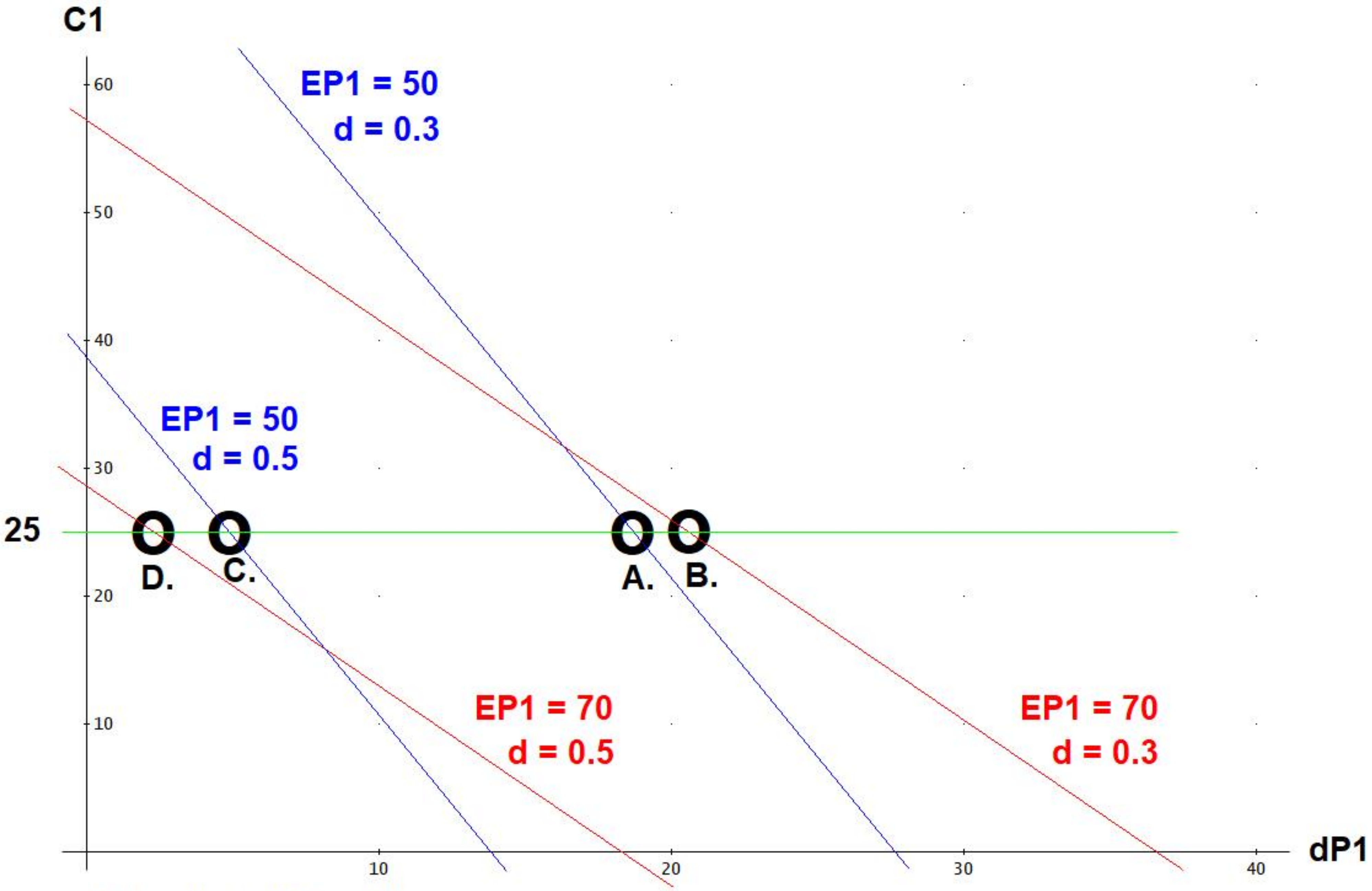
- The expected price of species 1 increases by 40% (from 50 to 70).



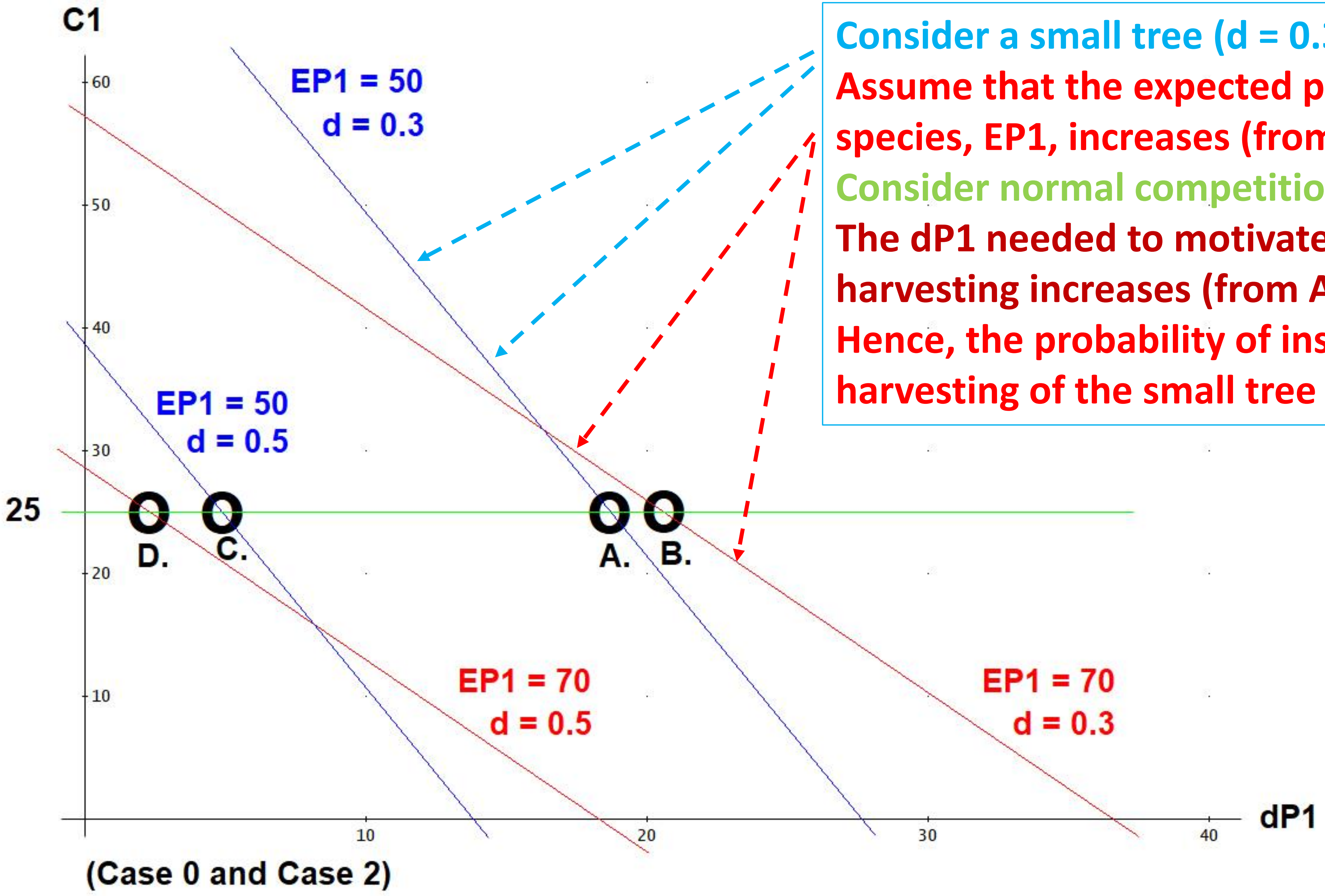
The optimal control boundary of species 1 is rotated to the left.

Case 2

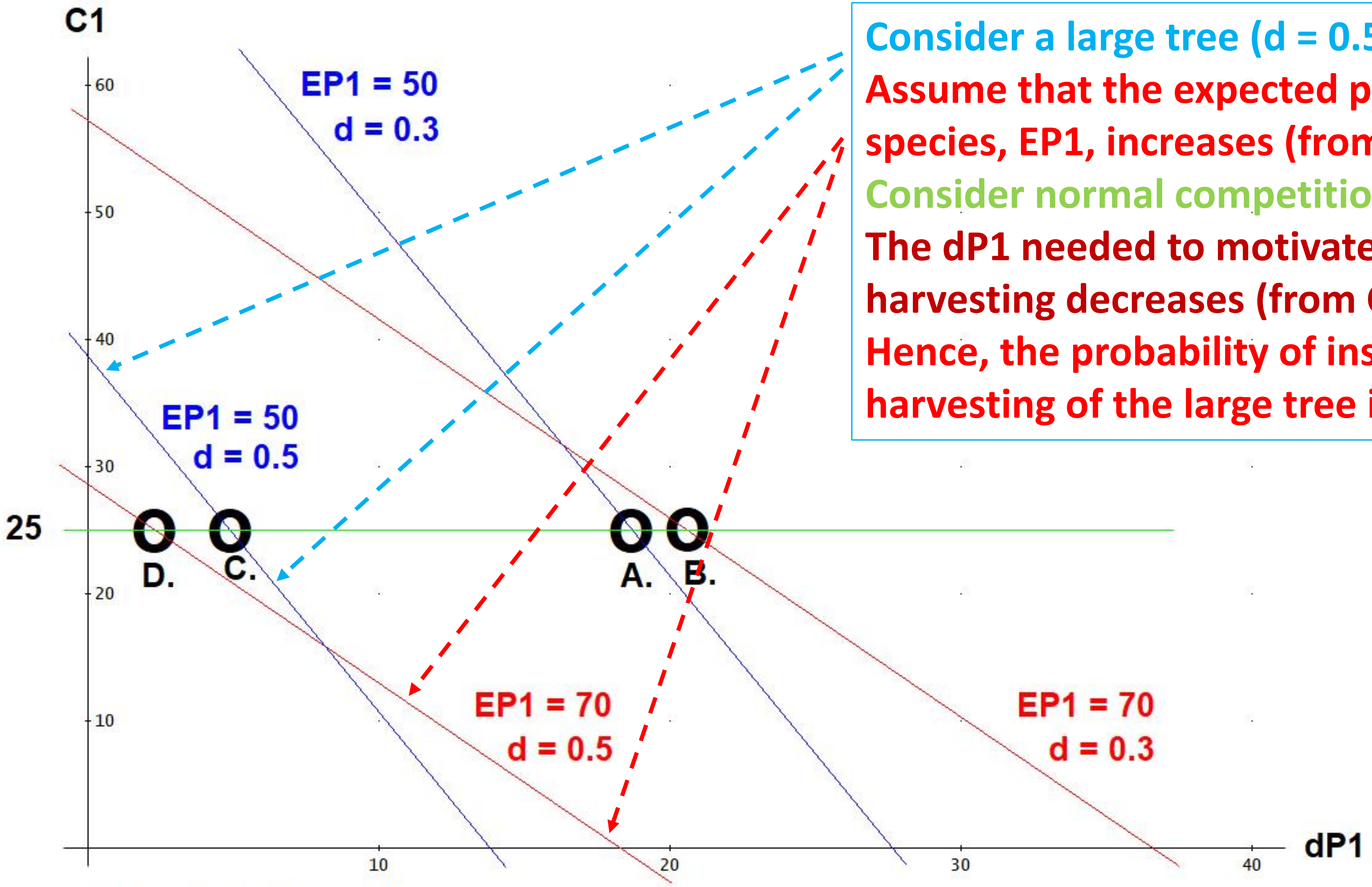
(d1 = d2 = 0.4)



(Case 0 and Case 2)

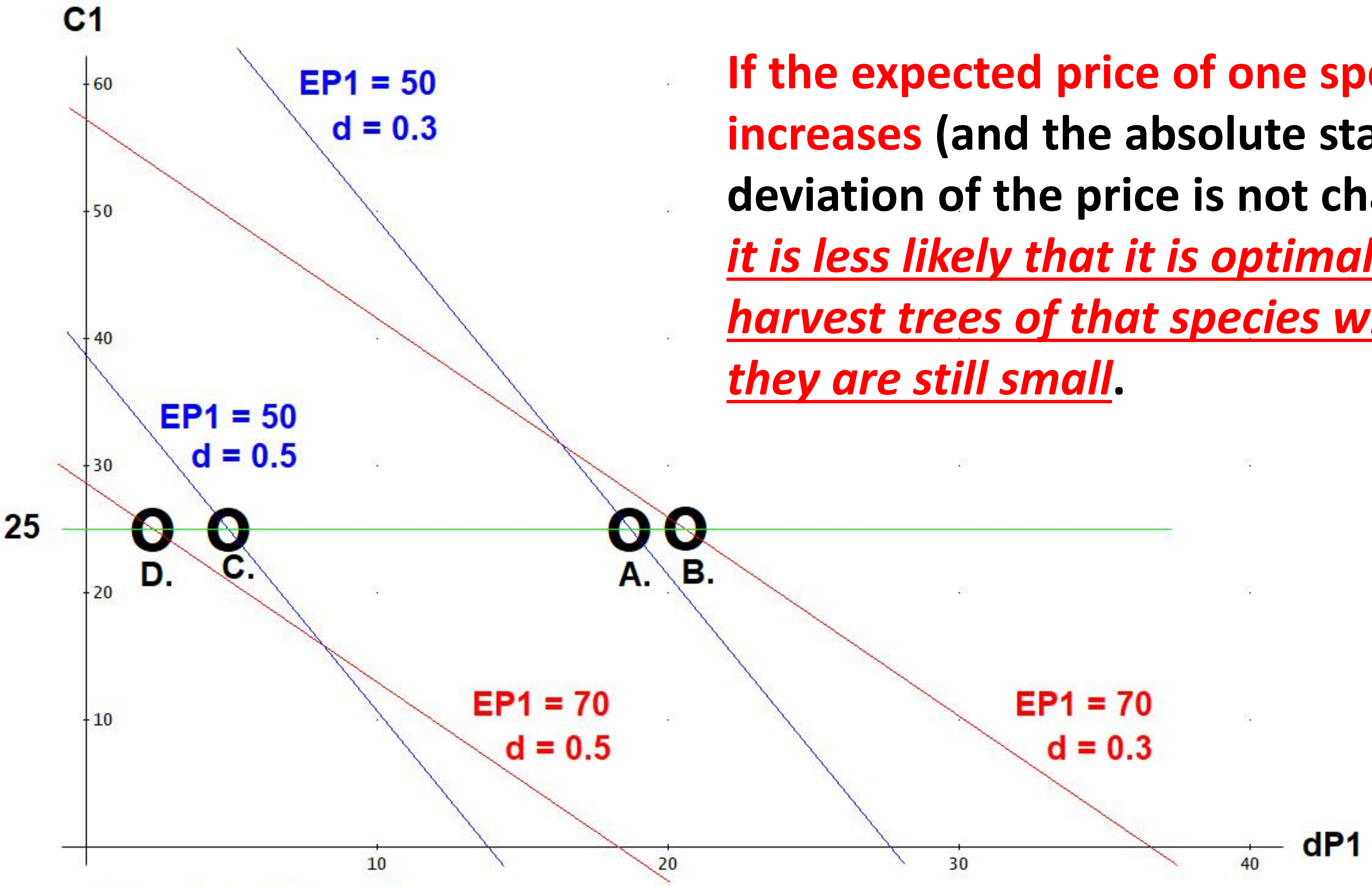


Consider a small tree (d = 0.3).
Assume that the expected price of that species, EP1, increases (from 50 to 70).
Consider normal competition C1 = 25.
The dP1 needed to motivate instant harvesting increases (from A. to B.).
Hence, the probability of instant harvesting of the small tree decreases.



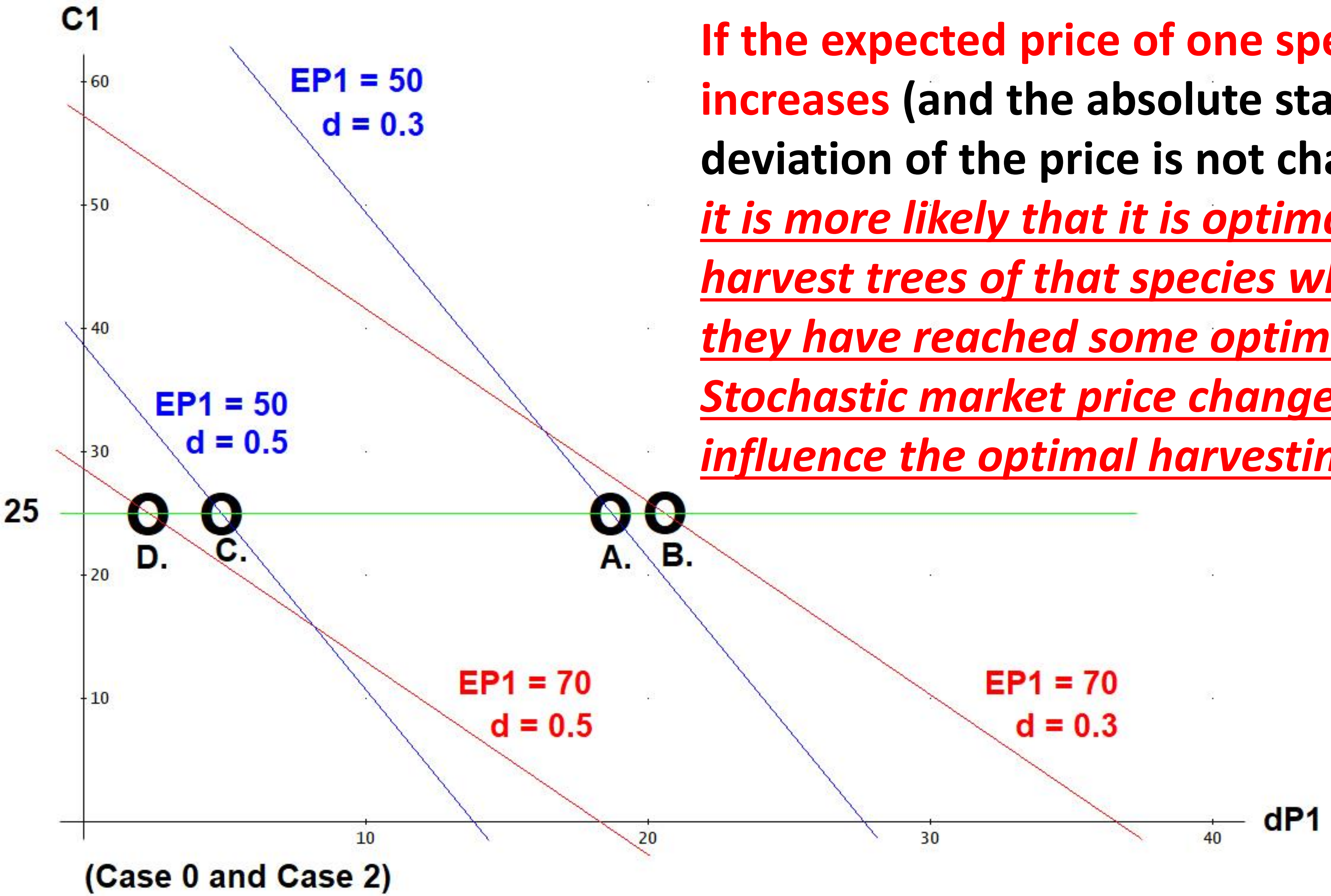
(Case 0 and Case 2)

Consider a large tree (d = 0.5).
Assume that the expected price of that species, EP1, increases (from 50 to 70).
Consider normal competition C1 = 25.
The dP1 needed to motivate instant harvesting decreases (from C. to D.).
Hence, the probability of instant harvesting of the large tree increases.



(Case 0 and Case 2)

If the expected price of one species increases (and the absolute standard deviation of the price is not changed), *it is less likely that it is optimal to harvest trees of that species when they are still small*.



If the expected price of one species increases **(and the absolute standard deviation of the price is not changed),** ***it is more likely that it is optimal to harvest trees of that species when they have reached some optimal size. Stochastic market price changes influence the optimal harvesting less.***

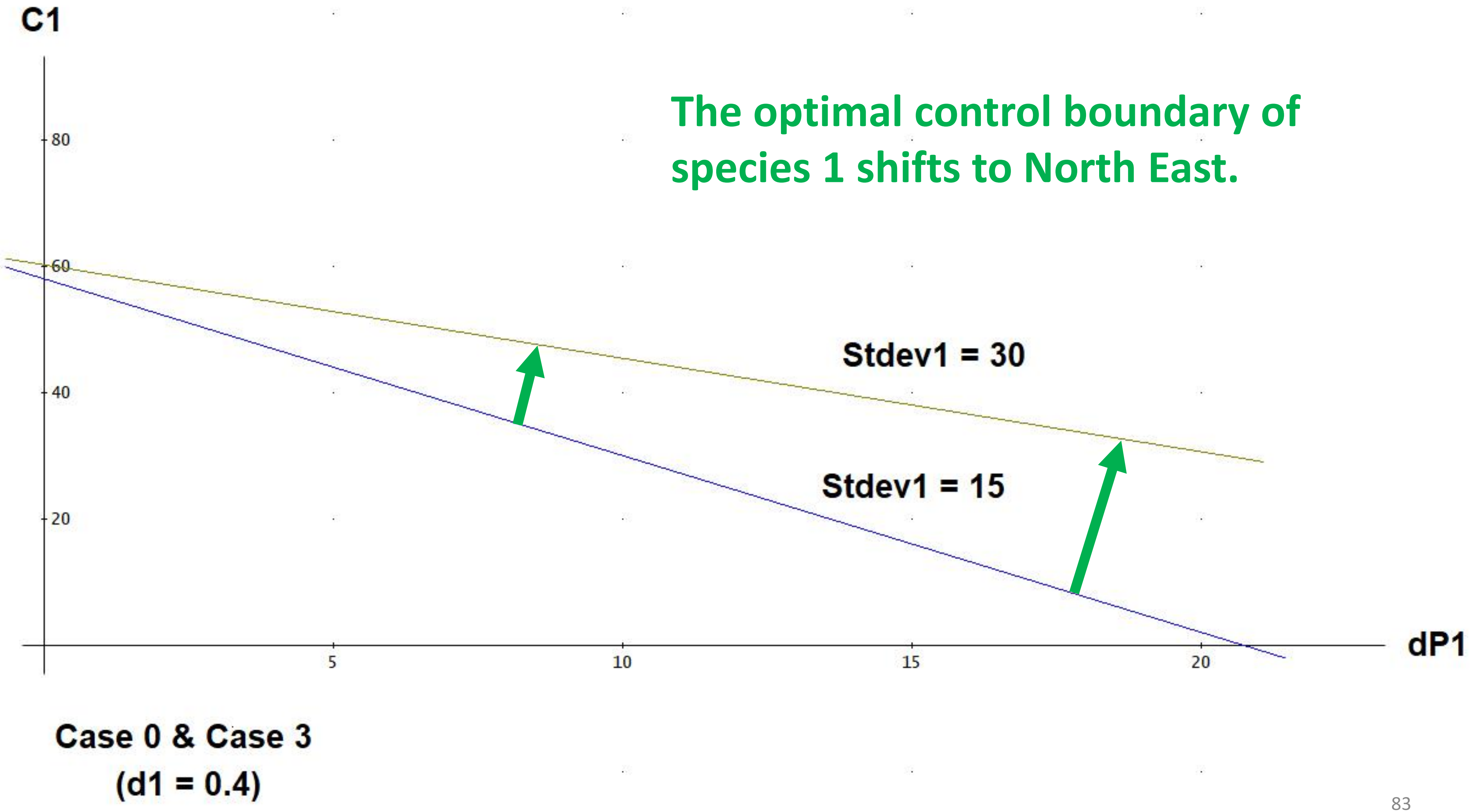
Case 2 (EP1 = high)

- The expected price of species 1 increases by 40% (from 50 to 70).
- The optimal control boundary of species 1 is rotated to the left.
- If the expected price of one species increases (and the absolute standard deviation of the price is not changed), it is less likely that it is optimal to harvest trees of that species when they are still small.
- If the expected price of one species increases (and the absolute standard deviation of the price is not changed), it is more likely that it is optimal to harvest trees of that species when they have reached some optimal size. Stochastic market price changes influence the optimal harvesting less.
- The expected present value is **1.42 times higher** than before.

Case 3 (Stdev1 = high)

- The standard deviation of the price of species 1 increases by 100% (from 15 to 30).

The optimal control boundary of species 1 shifts to North East.



Case 3 (Stdev1 = high)

- The standard deviation of the price of species 1 increases by 100% (from 15 to 30).
- The optimal control boundary of species 1 shifts to North East.
- If harvesting of a tree of species 1 should be optimal, the price has to be higher than before. This is reasonable since the probabilities of high prices are higher than before and we want to harvest when prices are high. Hence, we should request a higher price in order to harvest. Otherwise we can wait longer for a good price.
- The expected present value is **1.34 times** higher than before.

CONCLUSIONS

- The Limit Diameter (= DL) is a function of the tree species.
- Furthermore, if the rate of interest in the capital market increases, the DL decreases.
- The DL is also a decreasing function of the stochastic deviations of the price from the expected values
- and a decreasing function of the local competition from neighbour trees.

A particular tree should be harvested also at a smaller diameter than otherwise

- in case it belongs to a species with lower value of the species parameter in the DL,
- in case the rate of interest increases,
- if the market price of wood from the particular species unexpectedly increases and/or
- if the local competition from neighbour trees increases.

The expected present value of an optimally managed mixed species continuous cover forest is

- a decreasing function of the rate of interest in the capital market,
- an increasing function of the expected price levels of different species,
- an increasing function of the degree of market price variation

APPENDIX:

THE SOFTWARE

```
REM AdMultRnd1_EQDIST.bas
REM Peter Lohmander 190107_1720
REM


DIM x(100), y(100), d(100), ba(100), dist(100, 100), d0(100), harv(100)
DIM height(100), vol(100), revenue(100), cost(100), species(100), qual(100)
DIM MarketP(100, 2, 100), Comp(100), Volperha(2), PresVal(100)
DIM dmin(2), kheight(2), EP(2), stdev(2), CompBA(100)

PI = 3.141593

OPEN "AdmultRndIn.txt" FOR INPUT AS #2

OPEN "AdMultRndOut.txt" FOR OUTPUT AS #1

REM SCREEN 12
SCREEN _NEWIMAGE(1800, 800, 12)
COLOR 1, 15
```

```
REM ************************************************************************************
REM SECTION 0. Parameter inputs from external file and parameter documentation.
REM ************************************************************************************

INPUT #2, r
INPUT #2, pcorr
INPUT #2, EP(1)
INPUT #2, EP(2)
INPUT #2, stdev(1)
INPUT #2, stdev(2)
INPUT #2, kheight(1)
INPUT #2, kheight(2)
INPUT #2, dmin(1)
INPUT #2, dmin(2)
INPUT #2, D0start
INPUT #2, D0stop
INPUT #2, D0step
INPUT #2, Dpstart
INPUT #2, Dpstop
INPUT #2, Dpstep
INPUT #2, Dcstart
INPUT #2, Dcstop
INPUT #2, Dcstep
INPUT #2, Dsstart
INPUT #2, Dsstop
INPUT #2, Dsstep
INPUT #2, TOTSIMN
INPUT #2, seed2
```

```
PRINT " Program AdMultRnd_EQDIST by Peter Lohmander 2019:"
PRINT #1, " Program AdMultRnd_EQDIST by Peter Lohmander 2019:"
PRINT " Parameters from external file: "
PRINT "  r = "; r; " pcorr = "; pcorr; " EP(1) = "; EP(1); " EP(2) = "; EP(2)
PRINT "  stdev(1) = "; stdev(1); " stdev(2) = "; stdev(2); " kheight(1) = "; kheight(1); " kheight(2) = "; kheight(2)
PRINT "  dmin(1) = "; dmin(1); " dmin(2) = "; dmin(2)
PRINT "  D0start = "; D0start; " D0stop = "; D0stop; " D0step = "; D0step
PRINT "  Dpstart = "; Dpstart; " Dpstop = "; Dpstop; " Dpstep = "; Dpstep
PRINT "  Dcstart = "; Dcstart; " Dcstop = "; Dcstop; " Dcstep = "; Dcstep
PRINT "  Dsstart = "; Dsstart; " Dsstop = "; Dsstop; " Dsstep = "; Dsstep
PRINT "  TOTSIMN = "; TOTSIMN
PRINT "    seed2 = "; seed2
```

```
PRINT #1, " Parameters from external file: "
PRINT #1, "  r = "; r; " pcorr = "; pcorr; " EP(1) = "; EP(1); " EP(2) = "; EP(2)
PRINT #1, "  stdev(1) = "; stdev(1); " stdev(2) = "; stdev(2); " kheight(1) = "; kheight(1); " kheight(2) = "; kheight(2)
PRINT #1, "  dmin(1) = "; dmin(1); " dmin(2) = "; dmin(2)
PRINT #1, "  D0start = "; D0start; " D0stop = "; D0stop; " D0step = "; D0step
PRINT #1, "  Dpstart = "; Dpstart; " Dpstop = "; Dpstop; " Dpstep = "; Dpstep
PRINT #1, "  Dcstart = "; Dcstart; " Dcstop = "; Dcstop; " Dcstep = "; Dcstep
PRINT #1, "  Dsstart = "; Dsstart; " Dsstop = "; Dsstop; " Dsstep = "; Dsstep
PRINT #1, "  TOTSIMN = "; TOTSIMN
PRINT #1, "    seed2 = "; seed2
```

```
drawmaps = 0
INPUT "Draw maps? Then write 1, otherwise 0", drawmaps

IF drawmaps = 0 THEN PRINT " "
IF drawmaps = 0 THEN PRINT "          D0          Dp1          Dp2          Dc1          Dc2          Ds          r          EPV"
PRINT #1, " "
PRINT #1, "          D0          Dp1          Dp2          Dc1          Dc2          Ds          r          EPV"

REM *****************
REM End of SECTION 0.
REM *****************
```

```
RANDOMIZE 5

REM ********************************************************************************
REM SECTION A. The initial conditions of relevance to all calculations are determined.
REM ********************************************************************************

REM Generation of 100 market prices series for two species with correlation pcorr.

FOR series = 1 TO 100
   pcorr = 0.5
   FOR T = 0 TO 100
      FOR i = 1 TO 2
         epsilon = 0
         FOR ii = 1 TO 12
            epsilon = epsilon + RND
         NEXT ii
         epsilon = (epsilon - 6)
         IF i = 1 THEN randn1 = epsilon
         IF i = 2 THEN randn2 = epsilon
         IF i = 2 THEN randn3 = pcorr * randn1 + (1 - pcorr ^ 2) ^ .5 * randn2
      NEXT i
      MarketP(T, 1, series) = EP(1) + stdev(1) * randn1
      MarketP(T, 2, series) = EP(2) + stdev(2) * randn3
   NEXT T
NEXT series
PRINT ""
```

```
REM Generation of the positions

FOR i = 1 TO 100
    x(i) = 30 * RND
    y(i) = 30 * RND
NEXT i
PRINT ""


REM Generation of the species

FOR i = 1 TO 100
    species(i) = 1
    IF RND > 0.5 THEN species(i) = 2
NEXT i
```

```
REM Calculation of the distances

FOR i = 1 TO 100
    FOR j = 1 TO 100
        distx = x(i) - x(j)
        disty = y(i) - y(j)
        dist(i, j) = (distx * distx + disty * disty) ^ 0.5
    NEXT j
NEXT i
```

```
REM Generation of initial diameters

FOR i = 1 TO 100
   REM Note that in this version of the code, the initial distributions
   REM of the two species are identical. (Compare next row.)

   IF species(i) > 0.5 THEN GOTO 1010

   1001 REM New diameter suggestion
   dia = 0.1 + RND * 0.4
   freq = 1 - (2 * dia) ^ 2
   test = RND
   IF test > freq THEN GOTO 1001
   d0(i) = dia
   GOTO 1020
   1010 REM
```

```
 1011 REM New diameter suggestion
    dia = 0.1 + RND * 0.3
    freq = 1 - (2.5 * dia) ^ 2
    test = RND
    IF test > freq THEN GOTO 1011
    d0(i) = dia

    1020 REM
NEXT i

REM *****************
REM End of SECTION A.
REM *****************
```

```
REM ********************************************************
REM SECTION B. The control function parameter loops start here.
REM ********************************************************

EPVmax = 0

RANDOMIZE seed2

FOR SIMN = 1 TO TOTSIMN

   Ds = Dsstart + RND * (Dsstop - Dsstart)
   D0 = D0start + RND * (D0stop - D0start)
   Dp1 = Dpstart + RND * (Dpstop - Dpstart)
   Dp2 = Dpstart + RND * (Dpstop - Dpstart)
   Dc1 = Dcstart + RND * (Dcstop - Dcstart)
   Dc2 = Dcstart + RND * (Dcstop - Dcstart)
```

```
REM **************************************************************************
REM SECTION C. A number of loops of stochastic simulations start here.
REM **************************************************************************

FOR series = 1 TO 100
   PresVal(series) = 0
   REM PRINT "Series = "; series

   REM A new simulation is started from year 0

   FOR i = 1 TO 100
      d(i) = d0(i)
   NEXT i
```

```
FOR T = 0 TO 60

    MarketP1 = MarketP(T, 1, series)
    MarketP2 = MarketP(T, 2, series)
    year = T * 5
    discf = EXP(-year * r)

    IF drawmaps = 0 THEN GOTO 600
    CLS
    PRINT " Series = "; series; " Period = "; T; " Year = "; year; " P1 = "; MarketP1; " P2 = "; MarketP2
    PRINT " Map before growth and before harvest: "
```

```
FOR i = 1 TO 100
        FOR interior = 0.05 TO 1 STEP 0.05
          CIRCLE (INT(20 * x(i) + 100), INT(20 * y(i) + 100)), INT(100 * d(i) * interior / 5), species(i)
        NEXT interior
      NEXT i
      LINE (100, 100)-(700, 100), 3
      LINE (100, 100)-(100, 700), 3
      LINE (700, 100)-(700, 700), 3
      LINE (100, 700)-(700, 700), 3
      SLEEP
      600 REM

      REM growth occurs

      FOR speci = 1 TO 2
        Volperha(speci) = 0
      NEXT speci
```

```
FOR i = 1 TO 100
    Comp(i) = 0
    FOR j = 1 TO 100
        IF dist(i, j) < 5 THEN Comp(i) = Comp(i) + PI / 4 * d(j) * d(j)
    NEXT j
    CompBA(i) = 127.32 * Comp(i)
    Compadjust = (1 - 0.02 * CompBA(i))
    IF Compadjust < 0 THEN Compadjust = 0
    d(i) = d(i) + 0.1 * (1 - 0.5 * d(i)) * Compadjust
    speci = species(i)
    ba(i) = PI / 4 * d(i) * d(i)
    height(i) = kheight(speci) * d(i)
    vol(i) = 0.5 * ba(i) * height(i)
    Volperha(speci) = Volperha(speci) + vol(i) / 0.09
NEXT i
```

```
IF drawmaps = 0 THEN GOTO 601
        CLS
        PRINT " Series = "; series; " Period = "; T; " Year = "; year; " P1 = "; MarketP1; " P2 = "; MarketP2
        PRINT " Map after growth and before harvest: "

        FOR i = 1 TO 100
           FOR interior = 0.05 TO 1 STEP 0.05
              CIRCLE (INT(20 * x(i) + 100), INT(20 * y(i) + 100)), INT(100 * d(i) * interior / 5), species(i)
           NEXT interior
        NEXT i
        LINE (100, 100)-(700, 100), 3
        LINE (100, 100)-(100, 700), 3
        LINE (700, 100)-(700, 700), 3
        LINE (100, 700)-(700, 700), 3
        SLEEP
        601 REM
```

```
REM Harvests of individual trees may or may not occur
        Totvolperha = Volperha(1) + Volperha(2)

        FOR i = 1 TO 100
           speci = species(i)
           Price = MarketP(T, speci, series)
           meanprice = EP(speci)
           Dp = Dp1
           IF speci > 1.5 THEN Dp = Dp2
           Dc = Dc1
           IF speci > 1.5 THEN Dc = Dc2

           REM The harvest decision of tree i is set to zero. Then the limit diameter is calculated.
           REM In case the tree diameter exceeds the limit diameter, the harvest decision is set to one.
           harv(i) = 0

           Dlim = D0 + Dp * (Price - meanprice) / stdev(speci) + Ds * (speci - 1) + Dc * CompBA(i)

           IF d(i) > Dlim THEN harv(i) = 1

           IF d(i) < dmin(speci) THEN harv(i) = 0
```

```
REM The revenues and costs of the harvested trees are calculated.
          psizecorr = -0.25 + 2.5 * d(i)
          IF psizecorr > 1 THEN psizecorr = 1
          IF psizecorr < 0 THEN psizecorr = 0
          revenue(i) = Price * psizecorr * vol(i) * harv(i)
          costperm3 = 125 - 250 * d(i)
          IF costperm3 < 25 THEN costperm3 = 25
          cost(i) = costperm3 * vol(i) * harv(i)
          IF revenue(i) < cost(i) THEN harv(i) = 0
       NEXT i
```

```
REM The discounted net revenue of all harvested trees is added to the present value of the series.
        netrev = 0
        FOR i = 1 TO 100
           netrev = netrev + (revenue(i) - cost(i)) * harv(i)
        NEXT i
        PresVal(series) = PresVal(series) + discf * netrev
```

```
IF drawmaps = 0 THEN GOTO 602
    CLS
    PRINT " Series = "; series; " Period = "; T; " Year = "; year; " P1 = "; MarketP1; " P2 = "; MarketP2
    PRINT " Map of harvested trees: "

    FOR i = 1 TO 100
        IF harv(i) = 0 THEN GOTO 7070
        FOR interior = 0.05 TO 1 STEP 0.05
            CIRCLE (INT(20 * x(i) + 100), INT(20 * y(i) + 100)), INT(100 * d(i) * interior / 5), species(i)
        NEXT interior
        7070 REM
    NEXT i
    LINE (100, 100)-(700, 100), 3
    LINE (100, 100)-(100, 700), 3
    LINE (700, 100)-(700, 700), 3
    LINE (100, 700)-(700, 700), 3
    SLEEP
    602 REM
```

```
FOR i = 1 TO 100
        IF harv(i) = 1 THEN d(i) = 0.05
    NEXT i

    IF drawmaps = 0 THEN GOTO 603
    CLS
    PRINT " Series = "; series; " Period = "; T; " Year = "; year; " P1 = "; MarketP1; " P2 = "; MarketP2
    PRINT " Map after harvest: "

    FOR i = 1 TO 100
      FOR interior = 0.05 TO 1 STEP 0.05
        CIRCLE (INT(20 * x(i) + 100), INT(20 * y(i) + 100)), INT(100 * d(i) * interior / 5), species(i)
      NEXT interior
    NEXT i
    LINE (100, 100)-(700, 100), 3
    LINE (100, 100)-(100, 700), 3

    LINE (700, 100)-(700, 700), 3
    LINE (100, 700)-(700, 700), 3
    SLEEP
    603 REM
```

```
   NEXT T
NEXT series

REM *****************
REM End of SECTION C.
REM *****************
```

```
REM ****************************************************************************
REM SECTION D. Results for each control function parameter combination are calculated and printed.
REM ****************************************************************************

EPV = 0
FOR series = 1 TO 100
   EPV = EPV + PresVal(series) / 100
NEXT series
REM Calculation of EPV per ha
EPV = EPV / 0.09
IF EPV < EPVmax THEN GOTO 900
D0opt = D0
Dp1opt = Dp1
Dp2opt = Dp2
Dc1opt = Dc1
Dc2opt = Dc2

Dsopt = Ds
EPVmax = EPV
900 REM
```

```
    kD0 = D0 * 1000
    kDp1 = Dp1 * 1000
    kDp2 = Dp2 * 1000
    kDc1 = Dc1 * 10000
    kDc2 = Dc2 * 10000
    kDs = Ds * 1000
    kr = r * 1000
    kEPV = EPV * 1000

    PRINT USING "##############"; kD0; kDp1; kDp2; kDc1; kDc2; kDs; kr; kEPV
    PRINT #1, USING "##############"; kD0; kDp1; kDp2; kDc1; kDc2; kDs; kr; kEPV

    REM *****************
    REM End of SECTION D.
    REM *****************

NEXT SIMN
```

```
kD0opt = D0opt * 1000
kDp1opt = Dp1opt * 1000
kDp2opt = Dp2opt * 1000
kDc1opt = Dc1opt * 10000
kDc2opt = Dc2opt * 10000
kDsopt = Dsopt * 1000
kEPVmax = EPVmax * 1000

PRINT ""
PRINT " r = "; r
PRINT "  The Optimal Solution is (Most values times 1000. Dc1 and Dc2: Values times 10000): "
PRINT "           D0          Dp1          Dp2          Dc1          Dc2           Ds          EPV"
PRINT USING "##############"; kD0opt; kDp1opt; kDp2opt; kDc1opt; kDc2opt; kDsopt; kEPVmax
PRINT "------------------------------------------------------------------------------------------"
```

```
PRINT #1, ""
PRINT #1, " r = "; r
PRINT #1, "  The Optimal Solution is (Most values times 1000. Dc1 and Dc2: Values times 10000): "
PRINT #1, "            D0          Dp1          Dp2          Dc1          Dc2            Ds          EPV"
PRINT #1, USING "##############"; kD0opt; kDp1opt; kDp2opt; kDc1opt; kDc2opt; kDsopt; kEPVmax
PRINT #1, "-------------------------------------------------------------------------------------------"

REM *****************
REM End of SECTION B.
REM *****************

CLOSE #1
CLOSE #1
BEEP
END
```

EXAMPLE Input file (CASE 0)

AdMultRndIn.txt

```
.03
0.5
50
50
15
15
40
40
0.2
0.2
0.70
0.70
0.0
-0.30
-0.0
0.02
-0.010
0.0
0.0
0
0
0.02
300
1
```

EXAMPLE AdMultRndOut.txt

Output file

(CASE 0)

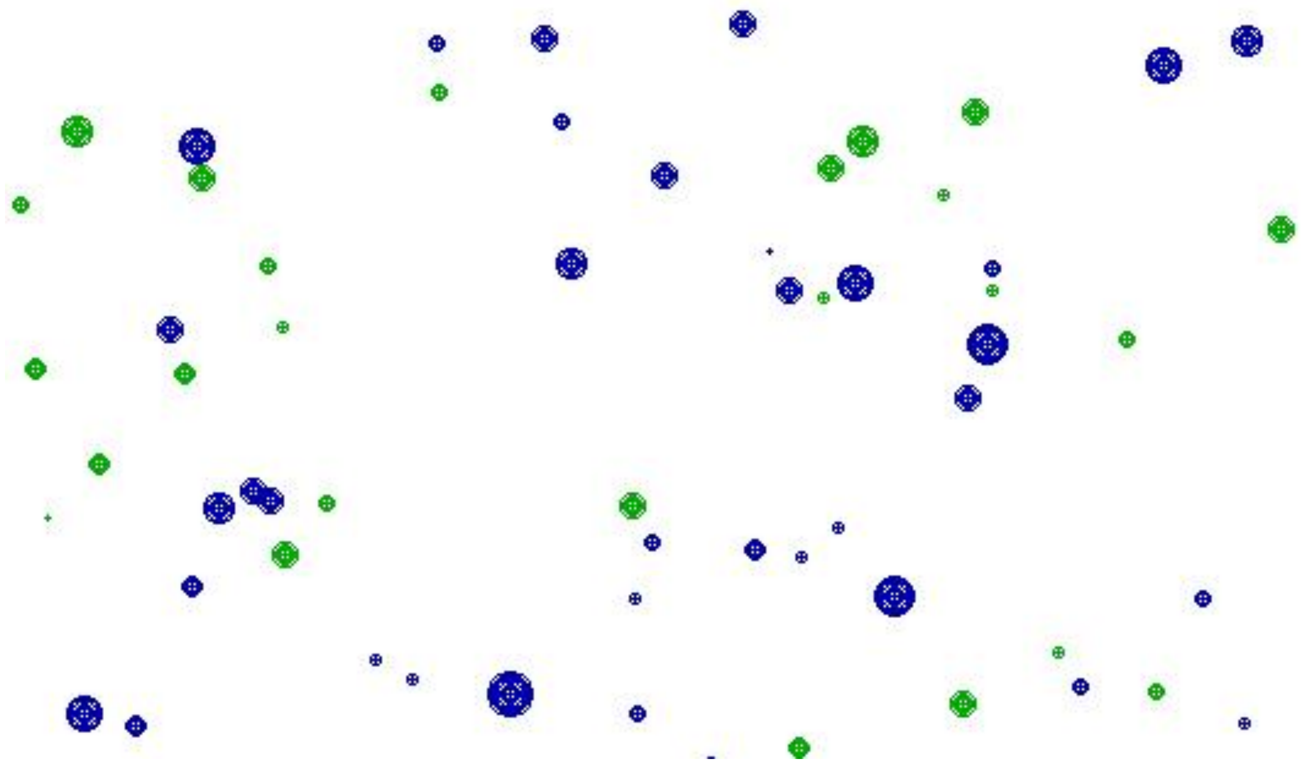
```
Program AdMultRnd_EQDIST by Peter Lohmander 2019:
 Parameters from external file:
  r =  .03  pcorr =  .5  EP(1) =  50  EP(2) =  50
  stdev(1) =  15  stdev(2) =  15  kheight(1) =  40  kheight(2) =  40
  dmin(1) =  .2  dmin(2) =  .2
  D0start =  .7  D0stop =  .7  D0step =  0
  Dpstart = -.3  Dpstop =  0  Dpstep =  .02
  Dcstart = -.01  Dcstop =  0  Dcstep =  0
  Dsstart =  0  Dsstop =  0  Dsstep =  .02
  TOTSIMN =  300
    seed2 =  1
```

```
Program AdMultRnd_EQDIST by Peter Lohmander 2019:
 Parameters from external file:
  r =  .03  pcorr =  .5  EP(1) =  50  EP(2) =  50
  stdev(1) =  15  stdev(2) =  15  kheight(1) =  40  kheight(2) =  40
  dmin(1) =  .2  dmin(2) =  .2
  D0start =  .7  D0stop =  .7  D0step =  0
  Dpstart = -.3  Dpstop =  0  Dpstep =  .02
  Dcstart = -.01  Dcstop =  0  Dcstep =  0
  Dsstart =  0  Dsstop =  0  Dsstep =  .02
  TOTSIMN =  300
    seed2 =  1
```

D0	Dp1	Dp2	Dc1	Dc2	Ds	r	EPV
700	-149	-113	-90	-40	0	30	5546176
700	-54	-133	-95	-59	0	30	5419922
700	-1	-49	-87	-74	0	30	5177052
700	-74	-139	-49	-45	0	30	5824453
700	-30	-5	-12	-81	0	30	1807685

Many more rows follow…

Optimization of Multi Species Continuous Cover Forest Management with Stochastic Prices via Determination of the Adaptive Harvest Control Function



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18th Symposium on Systems Analysis in Forest Resources, SSAFR 2019

March 3 - 7, 2019 Puerto Varas, Chile

SSAFR 2019 Song: Methods in Math

Lyrics: Peter Lohmander
Melody: Dire Straits, Brothers in Arms

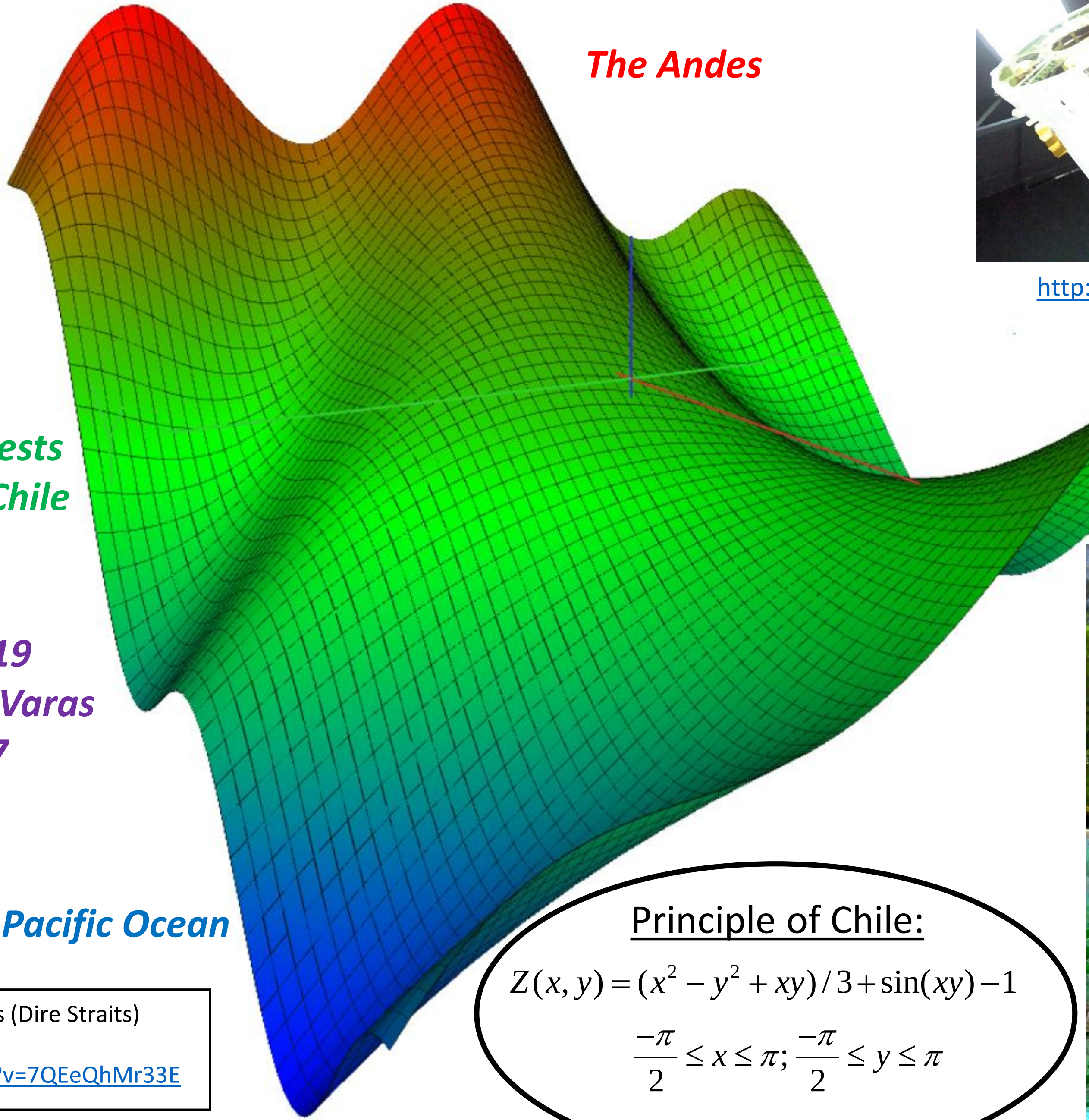
Inconsistent solutions
We know them by now,
But our background is science,
Where math is key,
Some day you'll return to
Derivations so pure,
You'll no longer risk
To be guessing and wrong

Through these fields of confusion,
Incompleteness and lies,
I've witnessed your suffering,
As the errors grow wild,
And though they did hurt us so bad
With solutions quite wrong,
You did not desert me
My methods in math

There's so many different plans
So many different trends
And there is just one solution,
Which is the optimal one

Now our science goes quite well
Our solutions ride high
Now we all know very well
Every lie has to die
But it's written in the starlight
And every line in your palm
Only fools can make war
On our methods in math

Yoni Schlesinger | Brothers in Arms (Dire Straits)
solo fingerstyle | B&G Little Sister
https://www.youtube.com/watch?v=7QEeQhMr33E



SSAFR 2019
in Puerto Varas
March 3-7

http://www.lohmander.com/

Principle of Chile:

$$Z(x,y) = (x^2 - y^2 + xy)/3 + \sin(xy) - 1$$

$$\frac{-\pi}{2} \le x \le \pi; \frac{-\pi}{2} \le y \le \pi$$

SSAFR song

by Peter Lohmander 2017-08-29

(Melody: Sancta Lucia)

Systems analysis,
in forest resources,
thats what our planet needs,
lets gather our forces,
to optimize management,
consider logistics,
do not forget the many animals,
climate and fires.

Faustmann song

by Peter Lohmander 1984-12-13
(Melody: Sancta Lucia)

Economic society,
deep in the forest,
we are all gathered here,
in the honor of Faustmann,
the whole world we represent,
and the gold that is green,
which we then transform to money,
in the optimal way.