Optimization of Multi Species Continuous Cover Forest Management with Stochastic Prices via Determination of the Adaptive Harvest Control Function

Peter Lohmander
Prof. Dr. Optimal Solutions in cooperation with Linnaeus University
Peter@Lohmander.com, www.Lohmander.com

18th Symposium on Systems Analysis in Forest Resources, SSAFR 2019
March 3 - 7, 2019 Puerto Varas, Chile
Our world contains very large areas of mixed species forests with trees of different sizes.
• These forests are important to several industries and to the environment.

• The prices for wood from different species of trees are stochastic.

• This paper presents a new adaptive method for sustainable and economically optimal management of these forests.
Example of a multi species continuous cover forest:

Different species have different colors (blue and green).

Trees have different sizes.
• In principle, this problem could be correctly solved via stochastic dynamic programming, SDP.

• However, SDP can not in reasonable time handle the extremely large number of state dimensions.

• The method used here includes optimization of adaptive control function parameters via repeated full system stochastic simulations, objective function approximation and analytical parameter optimization.
• A computer model was constructed for this purpose.

• The expected present value of a mixed species continuous cover forest is maximized with consideration of correlated stochastic roundwood market prices, dynamically changing tree sizes and local competition.
Method references:
Optimization of adaptive control function parameters via repeated full system stochastic simulations, objective function approximation and analytical parameter optimization

• Lohmander, P., Two Approaches to Optimal Adaptive Control under Large Dimensionality, INTERNATIONAL ROBOTICS AND AUTOMATION JOURNAL, Volume 3, Issue 4, 2017, DOI:10.15406/iratj.2017.03.00062
  http://medcraveonline.com/IRATJ/IRATJ-03-00062.php
  http://www.Lohmander.com/PL_171204aORIG.pdf
  http://www.Lohmander.com/PL_171204aORIG.docx

  https://doi.org/10.1007/978-3-319-66514-6_13
The initial conditions:

A forest with spatially distributed trees of different species, initial diameters and spatial coordinates.
Softwood trees are affected by local competition from other softwood trees.
Hardwood trees are affected by local competition from other hardwood trees.
Trees are affected by local competition from other trees of different species.
A complete numerical forest model has been developed with individual tree growth functions that are sensitive to local competition from neighbour trees.

References to related papers on tree growth functions:

  https://ijms.ut.ac.ir/article_64225_61b32fe374f9df8bca512abbe3b5c379.pdf
  https://ijms.ut.ac.ir/article_64225.html

  HLMM
  https://doi.org/10.1007/s11676-018-0862-8
  Link for reading

• Mohammadi, Z., Mohammadi Limaei, S., Lohmander, P., Olsson, L., Estimation of a basal area growth model for individual trees in uneven-aged Caspian mixed species forests, JOURNAL OF FORESTRY RESEARCH, November 30, 2017
  DOI: https://doi.org/10.1007/s11676-017-0556-7
  https://link.springer.com/article/10.1007%2Fs11676-017-0556-7
References to related papers on tree growth functions (continued):

• Mohammadi Limaei, S., Lohmander, P., Olsson, L.,
  Dynamic growth models for continuous cover multi-species forestry
  in Iranian Caspian forests,
  JOURNAL OF FOREST SCIENCE, 63, 2017 (11): 519-529,
  doi: 10.17221/32/2017-JFS

• Lohmander, P., Optimal Stochastic Dynamic Control of Spatially Distributed Interdependent Production Units. In: Cao BY. (ed) Fuzzy Information and Engineering and Decision. IWDS 2016. Advances in Intelligent Systems and Computing, vol 646. Springer, Cham, 2018
  https://doi.org/10.1007/978-3-319-66514-6_13

• Lohmander, P., Olsson, J.O., Fagerberg, N., Bergh, J., Adamopoulos, S.,
  High resolution adaptive optimization of continuous cover
  spruce forest management in southern Sweden,
  SSAFR 2017, Symposium on Systems Analysis in Forest Resources,
  Clearwater Resort, Suquamish, Washington, (near Seattle), August 27-30, 2017
  SSAFR 2017
• Every five year period, a harvesting team visits the forest.

• Optimal adaptive harvest decisions are taken, based on the prices of the different species, the local competition conditions in the forest, harvest cost and revenue functions and interest rate in the capital market.
Continuous cover forest harvesting in the Swedish mountains, December 2014
• The **optimal adaptive control** of the system works this way:

• **A limit diameter function value, DL**, is calculated.

• The value of the DL is derived **for each tree, in every period**. If the diameter of the tree exceeds the DL, then the tree is instantly harvested.

• Otherwise, the tree is left to continue to grow at least one more period.

• The **parameters of the DL are optimized** via a large number of stochastic full system simulations, expected objective function approximation and analytical control function parameter optimization.
\[ DL1 = \text{Optimal Limit Diameter of Species 1} \]

**Instant Harvest Region**

**Wait Longer Region**

Optimal Control Boundaries as functions of the local competition levels

Deviation of the price of species 1 from the expected price level
Example with particular parameters and particular stochastic price developments
Period = 0
Year = 0

$P_1 = 52.831$
$P_2 = 54.309$

Map after growth and before harvest
Period = 0
Year = 0

P1 = 52.831
P2 = 54.309

Map of harvested trees
Period = 0
Year = 0

P1 = 52.831
P2 = 54.309

Map after harvest
Period = 1
Year = 5

$P_1 = 48.111$
$P_2 = 57.984$

Map before growth and before harvest
Period = 1
Year = 5

P1 = 48.111
P2 = 57.984

Map after growth and before harvest
Period = 1
Year = 5

P1 = 48.111
P2 = 57.984

Map of harvested trees
Then, some periods follow with low prices of both species. 

Trees grow and no harvest takes place. 

Then, we reach period 6.
Period = 6
Year = 30

P1 = 58.572
P2 = 57.650

Map of harvested trees
The stochastic simulation model
Below, the structure of the software is described. (The complete software contains many more details.)

SOFTWARE: AdMultRnd1_EQDIST.bas
Peter Lohmander 190107

"AdmultRndIn.txt" (= INPUT file)

"AdMultRndOut.txt" (= OUTPUT file)

*If needed, maps may be produced.*
SECTION A.
The *initial conditions* of relevance to all calculations are determined.

Generation of 100 market prices series for two species with correlation $p_{corr}$.

Generation of the positions

Generation of the species

Calculation of the distances

Generation of initial diameters
EPVmax = 0

RANDOMIZE seed2

FOR SIMN = 1 TO TOTSIMN

Ds = Dsstart + RND * (Dsstop - Dsstart)
D0 = D0start + RND * (D0stop - D0start)
Dp1 = Dpstart + RND * (Dpstop - Dpstart)
Dp2 = Dpstart + RND * (Dpstop - Dpstart)
Dc1 = Dcstart + RND * (Dcstop - Dcstart)
Dc2 = Dcstart + RND * (Dcstop - Dcstart)

Uniform probability density of control function parameters in six dimensional space
SECTION C. A number of loops of stochastic simulations start here.

FOR series = 1 TO 100

FOR i = 1 TO 100
  d(i) = d0(i)
  NEXT i

FOR T = 0 TO 60

A new series of stochastic prices is selected.

The same initial forest conditions are used for every price series.

A stochastic 300 year simulation starts here.
Comp(i) = 0

FOR j = 1 TO 100
    IF dist(i, j) < 5 THEN Comp(i) = Comp(i) + PI / 4 * d(j) * d(j)
NEXT j

CompBA(i) = 127.32 * Comp(i)

growth occurs

Here, the local competition (In this case expressed as "competing basal area per hectare" within a circle with radius 5 meters) is calculated for every tree.
Harvests of individual trees may or may not occur

The harvest decision of tree \( i \) is set to zero. Then the limit diameter is calculated. In case the tree diameter exceeds the limit diameter, the harvest decision is set to one. In case the tree diameter is below the minimum diameter, harvest is set to zero.

\[
\text{harv}(i) = 0
\]

\[
D_{\text{lim}} = D_0 + D_p \frac{(\text{Price} - \text{meanprice})}{\text{stdev(speci)}} + D_s (\text{speci} - 1) + D_c \text{CompBA}(i)
\]

\[
\text{IF } d(i) > D_{\text{lim}} \text{ THEN } \text{harv}(i) = 1
\]

\[
\text{IF } d(i) < d_{\text{min}}(\text{speci}) \text{ THEN } \text{harv}(i) = 0
\]
The revenues and costs of the harvested trees are calculated.

The discounted net revenue of all harvested trees is added to the present value of the series.

End of SECTION C.
SECTION D.

Results for each control function parameter combination are calculated and printed.

End of SECTION D.

NEXT SIMN
**EXAMPLE**

**Input file**

(CASE 0)

AdMultRndIn.txt

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>INPUT #2, r</td>
<td>0.03</td>
</tr>
<tr>
<td>INPUT #2, pcorr</td>
<td>0.5</td>
</tr>
<tr>
<td>INPUT #2, EP(1)</td>
<td>50</td>
</tr>
<tr>
<td>INPUT #2, EP(2)</td>
<td>50</td>
</tr>
<tr>
<td>INPUT #2, stdev(1)</td>
<td>15</td>
</tr>
<tr>
<td>INPUT #2, stdev(2)</td>
<td>15</td>
</tr>
<tr>
<td>INPUT #2, kheight(1)</td>
<td>40</td>
</tr>
<tr>
<td>INPUT #2, kheight(2)</td>
<td>40</td>
</tr>
<tr>
<td>INPUT #2, dmin(1)</td>
<td>0.2</td>
</tr>
<tr>
<td>INPUT #2, dmin(2)</td>
<td>0.2</td>
</tr>
<tr>
<td>INPUT #2, D0start</td>
<td>0.70</td>
</tr>
<tr>
<td>INPUT #2, D0stop</td>
<td>0.70</td>
</tr>
<tr>
<td>INPUT #2, D0step</td>
<td>0.0</td>
</tr>
<tr>
<td>INPUT #2, Dpstart</td>
<td>-0.30</td>
</tr>
<tr>
<td>INPUT #2, Dpstop</td>
<td>-0.0</td>
</tr>
<tr>
<td>INPUT #2, Dpstep</td>
<td>0.02</td>
</tr>
<tr>
<td>INPUT #2, Dcstart</td>
<td>-0.010</td>
</tr>
<tr>
<td>INPUT #2, Dcstop</td>
<td>0.0</td>
</tr>
<tr>
<td>INPUT #2, Dcstep</td>
<td>0.0</td>
</tr>
<tr>
<td>INPUT #2, Dsstart</td>
<td>0</td>
</tr>
<tr>
<td>INPUT #2, Dsstop</td>
<td>0</td>
</tr>
<tr>
<td>INPUT #2, Dsstep</td>
<td>0.02</td>
</tr>
<tr>
<td>INPUT #2, TOTSIMN</td>
<td>300</td>
</tr>
<tr>
<td>INPUT #2, seed2</td>
<td>1</td>
</tr>
</tbody>
</table>

*Input section in the software*
EXAMPLE AdMultRndOut.txt

Output file
(CASE 0)

Program AdMultRnd_EQDIST by Peter Lohmander 2019:
Parameters from external file:
r = .03  pcorr = .5  EP(1) = 50  EP(2) = 50
stdev(1) = 15  stdev(2) = 15  kheight(1) = 40  kheight(2) = 40
dmin(1) = .2  dmin(2) = .2
D0start = .7  D0stop = .7  D0step = 0
Dpstart = -.3  Dpstop = 0  Dpstep = .02
Dcstart = -.01  Dcstop = 0  Dcstep = 0
Dsstart = 0  Dsstop = 0  Dsstep = .02
TOTSIMN = 300
seed2 = 1
Program AdMultRnd_EQDIST by Peter Lohmander 2019:

Parameters from external file:

- $r = 0.03$  
- $p_{corr} = 0.5$  
- $EP(1) = 50$  
- $EP(2) = 50$
- $stdev(1) = 15$  
- $stdev(2) = 15$  
- $kheight(1) = 40$  
- $kheight(2) = 40$
- $d_{min}(1) = 0.2$  
- $d_{min}(2) = 0.2$
- $D_{0\text{start}} = 0.7$  
- $D_{0\text{stop}} = 0.7$  
- $D_{0\text{step}} = 0$
- $D_{p\text{start}} = -0.3$  
- $D_{p\text{stop}} = 0$  
- $D_{p\text{step}} = 0.02$
- $D_{c\text{start}} = -0.01$  
- $D_{c\text{stop}} = 0$  
- $D_{c\text{step}} = 0$
- $D_{s\text{start}} = 0$  
- $D_{s\text{stop}} = 0$  
- $D_{s\text{step}} = 0.02$
- $TOTSIMN = 300$
- $seed2 = 1$

The output from the simulation model is the input file to the regression software. (Variable transformations will be made within Excel.)

<table>
<thead>
<tr>
<th>D0</th>
<th>Dp1</th>
<th>Dp2</th>
<th>Dc1</th>
<th>Dc2</th>
<th>Ds</th>
<th>r</th>
<th>EPV</th>
</tr>
</thead>
<tbody>
<tr>
<td>700</td>
<td>-149</td>
<td>-113</td>
<td>-90</td>
<td>-40</td>
<td>0</td>
<td>30</td>
<td>5546176</td>
</tr>
<tr>
<td>700</td>
<td>-54</td>
<td>-133</td>
<td>-95</td>
<td>-59</td>
<td>0</td>
<td>30</td>
<td>5419922</td>
</tr>
<tr>
<td>700</td>
<td>-1</td>
<td>-49</td>
<td>-87</td>
<td>-74</td>
<td>0</td>
<td>30</td>
<td>5177052</td>
</tr>
<tr>
<td>700</td>
<td>-74</td>
<td>-139</td>
<td>-49</td>
<td>-45</td>
<td>0</td>
<td>30</td>
<td>5824453</td>
</tr>
<tr>
<td>700</td>
<td>-30</td>
<td>-5</td>
<td>-12</td>
<td>-81</td>
<td>0</td>
<td>30</td>
<td>1807685</td>
</tr>
</tbody>
</table>

Many more rows follow...
Determination of the approximation of the expected present value as a multivariate polynomial of the control function parameters
The expected present value as a quadratic function of the parameters in the DL function (Case 0)

\[ Z = k_0 + k_{p_1} D_{p_1} + k_{c_1} D_{c_1} + k_{p_1p_1} (D_{p_1})^2 + k_{c_1c_1} (D_{c_1})^2 + k_{p_1c_1} D_{p_1} D_{c_1} + k_{p_2} D_{p_2} + k_{c_2} D_{c_2} + k_{p_2p_2} (D_{p_2})^2 + k_{c_2c_2} (D_{c_2})^2 + k_{p_2c_2} D_{p_2} D_{c_2} \]
\[ Z = k_0 + k_{p_1}D_{p_1} + k_{c_1}D_{c_1} + k_{p_1c_1}(D_{p_1})^2 + k_{c_1c_1}(D_{c_1})^2 + k_{p_1c_1}D_{p_1}D_{c_1} + \\
\quad k_{p_2}D_{p_2} + k_{c_2}D_{c_2} + k_{p_2c_2}(D_{p_2})^2 + k_{c_2c_2}(D_{c_2})^2 + k_{p_2c_2}D_{p_2}D_{c_2} \]

\[
\frac{dZ}{dD_{p_1}} = k_{p_1} + 2k_{p_1c_1}D_{p_1} + k_{p_1c_1}D_{c_1} = 0
\]

\[
\frac{dZ}{dD_{c_1}} = k_{c_1} + k_{p_1c_1}D_{p_1} + 2k_{c_1c_1}D_{c_1} = 0
\]

\[
\frac{dZ}{dD_{p_2}} = k_{p_2} + 2k_{p_2c_2}D_{p_2} + k_{p_2c_2}D_{c_2} = 0
\]

\[
\frac{dZ}{dD_{c_2}} = k_{c_2} + k_{p_2c_2}D_{p_2} + 2k_{c_2c_2}D_{c_2} = 0
\]

Objective function

First order optimum conditions
First order optimum conditions of species 1 = "Blue"

Separability

First order optimum conditions of species 2 = "Green"
<table>
<thead>
<tr>
<th>First order optimum conditions</th>
<th>Second order maximum conditions</th>
</tr>
</thead>
</table>
| \[
\frac{dZ}{dD_{p_1}} = k_{p_1} + 2k_{p_1p_1} D_{p_1} + k_{p_1c_1} D_{c_1} = 0
\] | \[
\left| \frac{d^2Z}{dD_{p_1}^2} \right| < 0
\] |
| \[
\frac{dZ}{dD_{c_1}} = k_{c_1} + k_{p_1c_1} D_{p_1} + 2k_{c_1c_1} D_{c_1} = 0
\] | \[
\left| \frac{d^2Z}{dD_{p_1} dD_{c_1}} \right| > 0
\] |
| \[
\frac{dZ}{dD_{p_2}} = k_{p_2} + 2k_{p_2p_2} D_{p_2} + k_{p_2c_2} D_{c_2} = 0
\] | \[
\left| \frac{d^2Z}{dD_{p_2}^2} \right| < 0
\] |
| \[
\frac{dZ}{dD_{c_2}} = k_{c_2} + k_{p_2c_2} D_{p_2} + 2k_{c_2c_2} D_{c_2} = 0
\] | \[
\left| \frac{d^2Z}{dD_{c_2}^2} \right| > 0
\] |

1 = Blue

2 = Green
\[
\frac{dZ}{dD_{p_1}} = k_{p_1} + 2k_{p_1p_1} D_{p_1} + k_{p_1c_1} D_{c_1} = 0
\]
\[
\frac{dZ}{dD_{c_1}} = k_{c_1} + k_{p_1c_1} D_{p_1} + 2k_{c_1c_1} D_{c_1} = 0
\]

\[
\begin{bmatrix}
2k_{p_1p_1} & k_{p_1c_1} \\
k_{p_1c_1} & 2k_{c_1c_1}
\end{bmatrix}
\begin{bmatrix}
D_{p_1} \\
D_{c_1}
\end{bmatrix}
=
\begin{bmatrix}
-k_{p_1} \\
-k_{c_1}
\end{bmatrix}
\]

\[
\begin{bmatrix}
d^2Z \\
d^2Z
dD_{p_1}^2
dD_{c_1}^2
\end{bmatrix}
< 0
\]

\[
\begin{bmatrix}
d^2Z \\
d^2Z
dD_{p_1}dD_{c_1}
dD_{p_1}dD_{c_1}
\end{bmatrix}
> 0
\]

1 = Blue

\[
\begin{bmatrix}
2k_{p_1p_1} \\
k_{p_1c_1}
\end{bmatrix}
< 0
\]

\[
\begin{bmatrix}
2k_{p_1p_1} & k_{p_1c_1} \\
k_{p_1c_1} & 2k_{c_1c_1}
\end{bmatrix}
> 0
\]

1 = Blue
The optimal parameter value via the analytical solution:

\[
D_{p_1} = \begin{bmatrix}
-k_{p_1} & k_{p_1c_1} \\
-k_{c_1} & 2k_{c_1c_1}
\end{bmatrix} \begin{bmatrix}
D_{p_1} \\
D_{c_1}
\end{bmatrix} = \begin{bmatrix}
-k_{p_1} \\
-k_{c_1}
\end{bmatrix}
\]

The second order maximum conditions:

\[
\begin{align*}
2k_{p_1p_1} & < 0 \\
2k_{p_1p_1} & < 0 \\
k_{p_1c_1} & > 0
\end{align*}
\]

\[
\begin{align*}
2k_{p_1p_1} & < 0 \\
4k_{p_1p_1}k_{c_1c_1} - k_{p_1c_1}^2 & > 0
\end{align*}
\]
\[
D_{p_1} = \begin{vmatrix}
-k_{p_1} & k_{p_1c_1} \\
-k_{c_1} & 2k_{c_1c_1} \\
2k_{p_1p_1} & k_{p_1c_1} \\
k_{p_1c_1} & 2k_{c_1c_1}
\end{vmatrix} = \frac{-2k_{p_1}k_{c_1c_1} + k_{c_1}k_{p_1c_1}}{4k_{p_1p_1}k_{c_1c_1} - k_{p_1c_1}^2}
\]

\[2k_{p_1p_1} < 0\]

\[4k_{p_1p_1}k_{c_1c_1} - k_{p_1c_1}^2 > 0\]

With figures from CASE 0, we get:

\[D_{p_1} \approx -217.3312\]

With figures from CASE 0, we get:

\[2(-0.037895) < 0\]

\[4(-0.037895)(-0.54902) - (-0.20816)^2 > 0\]

\[-0.07579 < 0\]

\[0.03988 > 0\]
### Case 0

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Regressionsstatistik</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multipel-R</td>
<td>0,94869828</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R-kvadrat</td>
<td>0,900028426</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Justerad R-kvadrat</td>
<td>0,896569202</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standardfel</td>
<td>357239,6095</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observationer</td>
<td>300</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>ANOVA</strong></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>fg</td>
<td>10</td>
<td>3,32045E+14</td>
<td>3,32045E+13</td>
<td>260,1821758</td>
<td>3,8861E-138</td>
</tr>
<tr>
<td>KvS</td>
<td>Residual</td>
<td>289</td>
<td>3,68822E+13</td>
<td>1,2762E+11</td>
<td></td>
</tr>
<tr>
<td>MKv</td>
<td>Total</td>
<td>299</td>
<td>3,68927E+14</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

|                  |          |       |       |            |           |
| Koefficienter    | Standardfel | t-kvot | p-värde |            |           |
| Konstant         | -2051272,546 | 149401,4285 | -13,72993931 | 2,27661E-33 |
| Dp1              | -27225,34464 | 1108,252192 | -24,56601922 | 9,83949E-73 |
| Dp2              | -16391,15333 | 1043,094784 | -15,71396347 | 1,20624E-40 |
| Dc1              | -101963,8556 | 3172,163703 | -32,14331452 | 1,98109E-97 |
| Dc2              | -42856,37519 | 3055,416205 | -14,02636247 | 1,9043E-34 |
| Dp1Dp1           | -37,89528101 | 3,24952884  | -11,66177771 | 5,16894E-26 |
| Dp2Dp2           | -27,99794181 | 3,055076955 | -9,16439822  | 9,54992E-18 |
| Dc1Dc1           | -549,0229524 | 26,93047653 | -20,38667796 | 7,1657E-58  |
| Dc2Dc2           | -229,0582263 | 27,60918492 | -8,296450147 | 4,13298E-15 |
| Dp1Dc1           | -208,1579104 | 8,435028634 | -24,67779534 | 4,04107E-73 |
| Dp2Dc2           | -105,4858035 | 8,451945763 | -12,48065315 | 6,93176E-29 |

All parameters have the expected signs.

All p-values are very low.

All t-values are very negative.

All estimations have very good precision.
The expected present value as a quadratic function of the parameters in the DL function (Case 0)

\[
EPV = -2051.273 - 27.225 \times D_{p1} - 16.391 \times D_{p2} - 101.964 \times D_{c1} - 42.856 \times D_{c2} - 0.037895 \times D_{p1} \times D_{p1} - 0.027998 \times D_{p2} \times D_{p2} - 0.54902 \times D_{c1} \times D_{c1} - 0.22906 \times D_{c2} \times D_{c2} - 0.20816 \times D_{p1} \times D_{c1} - 0.10549 \times D_{p2} \times D_{c2} ;
\]
Case 1 (r = low)

<table>
<thead>
<tr>
<th>Regressionsstatistik</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Multipel-R</td>
<td>0,89617769</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R-kvadrat</td>
<td>0,803134452</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Justerad R-kvadrat</td>
<td>0,796322495</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standardfel</td>
<td>1602172,583</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observationer</td>
<td>300</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ANOVA</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>fg</td>
<td></td>
<td>KvS</td>
<td>MKv</td>
<td>F</td>
</tr>
<tr>
<td>Regression</td>
<td>10</td>
<td>3,02646E+15</td>
<td>3,02646E+14</td>
<td>117,9006986</td>
</tr>
<tr>
<td>Residual</td>
<td>289</td>
<td>7,41851E+14</td>
<td>2,56696E+12</td>
<td></td>
</tr>
<tr>
<td>Totalt</td>
<td>299</td>
<td>3,76831E+15</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Koefficienter</th>
<th>Standardfel</th>
<th>t-kvot</th>
<th>p-värde</th>
</tr>
</thead>
<tbody>
<tr>
<td>Konstant</td>
<td>-3574468,076</td>
<td>670045,723</td>
<td>-5,334662923</td>
</tr>
<tr>
<td>Dp1</td>
<td>-99493,25346</td>
<td>4970,36509</td>
<td>-20,01729291</td>
</tr>
<tr>
<td>Dp2</td>
<td>-50839,32599</td>
<td>4678,142681</td>
<td>-10,86741672</td>
</tr>
<tr>
<td>Dc1</td>
<td>-337550,3677</td>
<td>14226,73628</td>
<td>-23,72647956</td>
</tr>
<tr>
<td>Dc2</td>
<td>-101690,6792</td>
<td>13703,13913</td>
<td>-7,420976916</td>
</tr>
<tr>
<td>Dp1Dp1</td>
<td>-160,0287581</td>
<td>14,57370873</td>
<td>-10,98064749</td>
</tr>
<tr>
<td>Dp2Dp2</td>
<td>-101,2691704</td>
<td>13,70161764</td>
<td>-7,391037541</td>
</tr>
<tr>
<td>Dc1Dc1</td>
<td>-2031,223255</td>
<td>120,7796392</td>
<td>-16,81759665</td>
</tr>
<tr>
<td>Dc2Dc2</td>
<td>-620,5012689</td>
<td>123,82335569</td>
<td>-5,01117303</td>
</tr>
<tr>
<td>Dp1Dc1</td>
<td>-728,0266741</td>
<td>37,82999212</td>
<td>-19,24469536</td>
</tr>
<tr>
<td>Dp2Dc2</td>
<td>-312,2009082</td>
<td>37,90586322</td>
<td>-8,23621682</td>
</tr>
</tbody>
</table>

All parameters have the expected signs.

All p-values are very low.

All t-values are very negative.

All estimations have very good precision.
### Case 2 (EP1 = high)

**Regressionsstatistik**

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Multipel-R</td>
<td>0,96589442</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R-kvadrat</td>
<td>0,932952031</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Justerad R-kvadrat</td>
<td>0,930632032</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standardfel</td>
<td>485810,3003</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observationer</td>
<td>300</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**ANOVA**

<table>
<thead>
<tr>
<th></th>
<th>fg</th>
<th>KvS</th>
<th>MKv</th>
<th>F</th>
<th>p-värde för F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>10</td>
<td>9,49085E+14</td>
<td>9,49085E+13</td>
<td>402,1346794</td>
<td>3,8197E-163</td>
</tr>
<tr>
<td>Residual</td>
<td>289</td>
<td>6,82074E+13</td>
<td>2,36012E+11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Totalt</td>
<td>299</td>
<td>1,01729E+15</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Koefficienter**

<table>
<thead>
<tr>
<th></th>
<th>Standardfel</th>
<th>t-kvot</th>
<th>p-värde</th>
</tr>
</thead>
<tbody>
<tr>
<td>Konstant</td>
<td>-3642061,571</td>
<td>-17,92608374</td>
<td>7,82048E-49</td>
</tr>
<tr>
<td>Dp1</td>
<td>-42368,95875</td>
<td>-28,11266902</td>
<td>1,11596E-84</td>
</tr>
<tr>
<td>Dp2</td>
<td>-17332,83011</td>
<td>-12,21908241</td>
<td>5,82141E-28</td>
</tr>
<tr>
<td>Dc1</td>
<td>-173652,0372</td>
<td>-40,25475414</td>
<td>1,6411E-120</td>
</tr>
<tr>
<td>Dc2</td>
<td>-46918,99694</td>
<td>-11,29200934</td>
<td>9,74495E-25</td>
</tr>
<tr>
<td>Dp1Dp1</td>
<td>-53,98305371</td>
<td>-12,21602576</td>
<td>5,96751E-28</td>
</tr>
<tr>
<td>Dp2Dp2</td>
<td>-29,16684782</td>
<td>-7,020373672</td>
<td>1,58237E-11</td>
</tr>
<tr>
<td>Dc1Dc1</td>
<td>-828,5692999</td>
<td>-22,62443211</td>
<td>6,35187E-66</td>
</tr>
<tr>
<td>Dc2Dc2</td>
<td>-250,4301089</td>
<td>-6,670000156</td>
<td>1,29957E-10</td>
</tr>
<tr>
<td>Dp1Dc1</td>
<td>-353,1962025</td>
<td>-30,79089775</td>
<td>2,99728E-93</td>
</tr>
<tr>
<td>Dp2Dc2</td>
<td>-111,128594</td>
<td>-9,668564646</td>
<td>2,42745E-19</td>
</tr>
</tbody>
</table>

All parameters have the expected signs.

All p-values are very low.

All t-values are very negative.

All estimations have very good precision.
Case 3 (Stdev1 = high)

<table>
<thead>
<tr>
<th>Regressionsstatistik</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Multipel-R</strong></td>
<td>0,953421533</td>
<td><strong>R-kvadrat</strong></td>
<td>0,909012619</td>
</tr>
<tr>
<td><strong>Justerad R-kvadrat</strong></td>
<td>0,905864266</td>
<td><strong>Standardfel</strong></td>
<td>499842,9392</td>
</tr>
<tr>
<td><strong>Observationer</strong></td>
<td>300</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ANOVA</th>
<th>fg</th>
<th>KvS</th>
<th>MKv</th>
<th>F</th>
<th>p-värde för F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>10</td>
<td>7,2136E+14</td>
<td>7,2136E+13</td>
<td>288,7264618</td>
<td>4,9804E-144</td>
</tr>
<tr>
<td>Residual</td>
<td>289</td>
<td>7,22046E+13</td>
<td>2,49843E+11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Totalt</td>
<td>299</td>
<td>7,93567E+14</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Koefficienter</th>
<th>Standardfel</th>
<th>t-kvot</th>
<th>p-värde</th>
</tr>
</thead>
<tbody>
<tr>
<td>Konstant</td>
<td>-3237805,016</td>
<td>209039,6673</td>
<td>-15,48895029</td>
</tr>
<tr>
<td>Dp1</td>
<td>-43986,50192</td>
<td>1550,645612</td>
<td>-28,36657298</td>
</tr>
<tr>
<td>Dp2</td>
<td>-17701,19141</td>
<td>1459,478593</td>
<td>-12,12843511</td>
</tr>
<tr>
<td>Dc1</td>
<td>-154515,3959</td>
<td>4438,431761</td>
<td>-34,81306106</td>
</tr>
<tr>
<td>Dc2</td>
<td>-43823,70622</td>
<td>4275,080858</td>
<td>-10,25096546</td>
</tr>
<tr>
<td>Dp1Dp1</td>
<td>-61,53616208</td>
<td>4,546679604</td>
<td>-13,53430799</td>
</tr>
<tr>
<td>Dp2Dp2</td>
<td>-30,70382818</td>
<td>4,274606185</td>
<td>-7,182843717</td>
</tr>
<tr>
<td>Dc1Dc1</td>
<td>-803,5153906</td>
<td>37,68061599</td>
<td>-21,32436983</td>
</tr>
<tr>
<td>Dc2Dc2</td>
<td>-224,8491006</td>
<td>38,630252</td>
<td>-5,820544495</td>
</tr>
<tr>
<td>Dp1Dc1</td>
<td>-336,7686865</td>
<td>11,80213334</td>
<td>-28,5345604</td>
</tr>
<tr>
<td>Dp2Dc2</td>
<td>-109,4513287</td>
<td>11,82580346</td>
<td>-9,255297453</td>
</tr>
</tbody>
</table>

All parameters have the expected signs.

All p-values are very low.

All t-values are very negative.

All estimations have very good precision.
Optimization of the control function parameters
Influence of price deviations from expected price levels on instant harvesting decisions

Influence of local competition levels on instant harvesting decisions

\[ 10000 \times \text{coef}(C1) \]

\[ 1000 \times \text{coef}(dP1) \]
Case 0

Expected present value

1000*coef(dP1)  10000*coef(C1)
Influence of deviations from expected price levels on instant harvesting decisions

Influence of local competition levels on instant harvesting decisions
model:

max = EPV;

EPV =
- 2051.273
- 27.225 * Dp1
- 16.391 * Dp2
- 101.964 * Dc1
- 42.856 * Dc2
- 0.037895 * Dp1 * Dp1
- 0.027998 * Dp2 * Dp2
- 0.54902 * Dc1 * Dc1
- 0.22906 * Dc2 * Dc2
- 0.20816 * Dp1 * Dc1
- 0.10549 * Dp2 * Dc2;

@free(Dp1);
@free(Dp2);
@free(Dc1);
@free(Dc2);

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>EPV</td>
<td>6216.348</td>
</tr>
<tr>
<td>DP1</td>
<td>-217.3312</td>
</tr>
<tr>
<td>DP2</td>
<td>-205.7293</td>
</tr>
<tr>
<td>DC1</td>
<td>-51.65963</td>
</tr>
<tr>
<td>DC2</td>
<td>-46.17485</td>
</tr>
</tbody>
</table>
\[
D_{p_1} = \begin{vmatrix}
-k_{p_1} & k_{p_1c_1} \\
-k_{c_1} & 2k_{c_1c_1} \\
2k_{p_1p_1} & k_{p_1c_1} \\
k_{c_1p_1} & 2k_{c_1c_1} \\
\end{vmatrix} = \frac{-k_{p_1} 2k_{c_1c_1} + k_{c_1} k_{p_1c_1}}{4k_{p_1p_1} k_{c_1c_1} - k_{p_1c_1}^2}
\]

\[
2k_{p_1p_1} < 0 \\
4k_{p_1p_1} k_{c_1c_1} - k_{p_1c_1}^2 > 0
\]

The analytical method gave the same answer:

\[
D_{p_1} = \frac{-2(-27.225)(-0.54902) + (-101.964)(-0.20816)}{4(-0.037895)(-0.54902) - (-0.20816)^2}
\]

\[D_{p_1} \approx -217.3312\]

The analytical method also told us that the solution is a unique maximum.

\[
2(-0.037895) < 0 \\
4(-0.037895)(-0.54902) - (-0.20816)^2 > 0 \\
-0.07579 < 0 \\
0.03988 > 0
\]
! Case 1 \( r \) is low_190108_1353;
! Peter Lohmander;

model:

max = EPV;

EPV = 
- 3574.468 
- 99.493 * Dp1 
- 50.839 * Dp2 
- 337.550 * Dc1 
- 101.691 * Dc2 
- 0.160029 * Dp1 * Dp1 
- 0.101269 * Dp2 * Dp2 
- 2.031223 * Dc1 * Dc1 
- 0.620501 * Dc2 * Dc2 
- 0.728027 * Dp1 * Dc1 
- 0.312201 * Dp2 * Dc2 ;

@free(Dp1);
@free(Dp2);
@free(Dc1);
@free(Dc2);

end

(r = low)  

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>EPV</td>
<td>21199.22</td>
</tr>
<tr>
<td>DP1</td>
<td>-205.7134</td>
</tr>
<tr>
<td>DP2</td>
<td>-203.6856</td>
</tr>
<tr>
<td>DC1</td>
<td>-46.22464</td>
</tr>
<tr>
<td>DC2</td>
<td>-30.70111</td>
</tr>
</tbody>
</table>
model:

max = EPV;

EPV =
- 3642.062
- 42.369 * Dp1
- 17.333 * Dp2
- 173.652 * Dc1
- 46.919 * Dc2
- 0.053983 * Dp1 * Dp1
- 0.029167 * Dp2 * Dp2
- 0.82857 * Dc1 * Dc1
- 0.25043 * Dc2 * Dc2
- 0.35320 * Dp1 * Dc1
- 0.11113 * Dp2 * Dc2;

@free(Dp1);
@free(Dp2);
@free(Dc1);
@free(Dc2);

end
Case 3

Stdev1 is high_EQDIST_190108_1424;
Peter Lohmander;

model:

max = EPV;

EPV =
- 3237.805
- 43.987 * Dp1
- 17.701 * Dp2
- 154.515 * Dc1
- 43.824 * Dc2
- 0.061536 * Dp1 * Dp1
- 0.030704 * Dp2 * Dp2
- 0.80352 * Dc1 * Dc1
- 0.22485 * Dc2 * Dc2
- 0.33677 * Dp1 * Dc1
- 0.10945 * Dp2 * Dc2;

@free(Dp1);
@free(Dp2);
@free(Dc1);
@free(Dc2);

end

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>EPV</td>
<td>8320.552</td>
</tr>
<tr>
<td>DP1</td>
<td>-221.0902</td>
</tr>
<tr>
<td>DP2</td>
<td>-202.3298</td>
</tr>
<tr>
<td>DC1</td>
<td>-49.81733</td>
</tr>
<tr>
<td>DC2</td>
<td>-48.20771</td>
</tr>
</tbody>
</table>

(Stdev1 = high)
The optimal control of the forest
OPTIMAL CONTROL:
(of trees of species 1)

Instant Harvest Region

Wait Longer Region

Deviation of the price of species 1 from the expected price level

Local competition level

\[ DL1 = f(dP1, C1) \]
\[ DL1 = \text{Optimal Limit Diameter of Species 1} \]

**Instant Harvest Region**

**Wait Longer Region**

**Optimal Control Boundaries as functions of the local competition levels**

**Case 0**

Deviation of the price of species 1 from the expected price level
Local competition level

Deviation of the price of species 1 from the expected price level

Instant Harvest Region

Wait Longer Region

Optimal Control Boundaries as functions of the diameters of the individual trees

Case 0
Optimal changes of forest control decisions if these parameters change:

- the rate of interest,
- the expected prices,
- the degrees of stochastic price variations
Case 1 (r = low)

• The real rate of interest, r, decreases from 3% to 1%.
The optimal control boundary shifts to North East.
Case 1 (r = low)

• The real rate of interest, r, decreases from 3% to 1%.
• The optimal control boundary shifts to North East.
• To motivate instant harvesting, the level of competition and/or the price has to be higher than before the change.
• The expected size of the trees (when they are harvested) is larger with a low rate of interest.
• The expected present value is 3.41 times higher than if r = 0.03.
Case 2 (EP1 = high)

• The expected price of species 1 increases by 40% (from 50 to 70).
The optimal control boundary of species 1 is rotated to the left.
Consider a small tree (d = 0.3). Assume that the expected price of that species, EP1, increases (from 50 to 70). Consider normal competition C1 = 25. The dP1 needed to motivate instant harvesting increases (from A. to B.). Hence, the probability of instant harvesting of the small tree decreases.
Consider a large tree \((d = 0.5)\).
Assume that the expected price of that species, \(EP1\), increases (from 50 to 70).
Consider normal competition \(C1 = 25\).
The \(dP1\) needed to motivate instant harvesting decreases (from \(C\) to \(D\)).
Hence, the probability of instant harvesting of the large tree increases.
The probability that a small tree (d = 0.3) should be harvested decreases if the expected price of that species increases.

If the expected price of one species increases (and the absolute standard deviation of the price is not changed), it is less likely that it is optimal to harvest trees of that species when they are still small.
If the expected price of one species increases (and the absolute standard deviation of the price is not changed), it is more likely that it is optimal to harvest trees of that species when they have reached some optimal size. Stochastic market price changes influence the optimal harvesting less.
Case 2 (EP1 = high)

- The expected price of species 1 increases by 40% (from 50 to 70).
- The optimal control boundary of species 1 is rotated to the left.
- If the expected price of one species increases (and the absolute standard deviation of the price is not changed), it is less likely that it is optimal to harvest trees of that species when they are still small.
- If the expected price of one species increases (and the absolute standard deviation of the price is not changed), it is more likely that it is optimal to harvest trees of that species when they have reached some optimal size. Stochastic market price changes influence the optimal harvesting less.
- The expected present value is 1.42 times higher than before.
Case 3 (Stdev1 = high)

• The standard deviation of the price of species 1 increases by 100% (from 15 to 30).
The optimal control boundary of species 1 shifts to North East.

Case 0 & Case 3
(d1 = 0.4)
Case 3 (Stdev1 = high)

• The standard deviation of the price of species 1 increases by 100% (from 15 to 30).
• The optimal control boundary of species 1 shifts to North East.
• If harvesting of a tree of species 1 should be optimal, the price has to be higher than before. This is reasonable since the probabilities of high prices are higher than before and we want to harvest when prices are high. Hence, we should request a higher price in order to harvest. Otherwise we can wait longer for a good price.
• The expected present value is 1.34 times higher than before.
CONCLUSIONS
• The Limit Diameter ( = DL) is a function of the tree species.

• Furthermore, if the rate of interest in the capital market increases, the DL decreases.

• The DL is also a decreasing function of the stochastic deviations of the price from the expected values

• and a decreasing function of the local competition from neighbour trees.
A particular tree should be harvested also at a smaller diameter than otherwise

• in case it belongs to a species with lower value of the species parameter in the DL,
• in case the rate of interest increases,
• if the market price of wood from the particular species unexpectedly increases and/or
• if the local competition from neighbour trees increases.
The expected present value of an optimally managed mixed species continuous cover forest is

- a decreasing function of the rate of interest in the capital market,
- an increasing function of the expected price levels of different species,
- an increasing function of the degree of market price variation
APPENDIX:

THE SOFTWARE
REM AdMultRnd1_EQDIST.bas
REM Peter Lohmander 190107_1720
REM

DIM x(100), y(100), d(100), ba(100), dist(100, 100), d0(100), harv(100)
DIM height(100), vol(100), revenue(100), cost(100), species(100), qual(100)
DIM MarketP(100, 2, 100), Comp(100), Volperha(2), PresVal(100)
DIM dmin(2), kheight(2), EP(2), stdev(2), CompBA(100)

PI = 3.141593

OPEN "AdmultRndlIn.txt" FOR INPUT AS #2

OPEN "AdMultRndlout.txt" FOR OUTPUT AS #1

REM SCREEN 12
SCREEN _NEWIMAGE(1800, 800, 12)
COLOR 1, 15
INPUT #2, r
INPUT #2, pcorr
INPUT #2, EP(1)
INPUT #2, EP(2)
INPUT #2, stdev(1)
INPUT #2, stdev(2)
INPUT #2, kheight(1)
INPUT #2, kheight(2)
INPUT #2, dmin(1)
INPUT #2, dmin(2)
INPUT #2, D0start
INPUT #2, D0stop
INPUT #2, D0step
INPUT #2, Dpstart
INPUT #2, Dpstop
INPUT #2, Dpstep
INPUT #2, Dcstart
INPUT #2, Dcstop
INPUT #2, Dcstep
INPUT #2, Dsstart
INPUT #2, Dsstop
INPUT #2, Dsstep
INPUT #2, TOTSIMN
INPUT #2, seed2
PRINT " Program AdMultRnd_EQDIST by Peter Lohmander 2019:"  
PRINT #1, " Program AdMultRnd_EQDIST by Peter Lohmander 2019:"  
PRINT " Parameters from external file: "  
PRINT "  r = "; r; " pcorr = "; pcorr; " EP(1) = "; EP(1); " EP(2) = "; EP(2)  
PRINT "  stdev(1) = "; stdev(1); " stdev(2) = "; stdev(2); " kheight(1) = "; kheight(1); " kheight(2) = "; kheight(2)  
PRINT "  dmin(1) = "; dmin(1); " dmin(2) = "; dmin(2)  
PRINT "  D0start = "; D0start; " D0stop = "; D0stop; " D0step = "; D0step  
PRINT "  Dpstart = "; Dpstart; " Dpstop = "; Dpstop; " Dpstep = "; Dpstep  
PRINT "  Dcstart = "; Dcstart; " Dcstop = "; Dcstop; " Dcstep = "; Dcstep  
PRINT "  Dsstart = "; Dsstart; " Dsstop = "; Dsstop; " Dsstep = "; Dsstep  
PRINT "  TOTSIMN = "; TOTSIMN  
PRINT "    seed2 = "; seed2
PRINT #1, " Parameters from external file: "
PRINT #1, "  r = "; r; pcorr = "; pcorr; EP(1) = "; EP(1); EP(2) = "; EP(2)
PRINT #1, "  stdev(1) = "; stdev(1); stdev(2) = "; stdev(2); kheight(1) = "; kheight(1); kheight(2) = "; kheight(2)
PRINT #1, "  dmin(1) = "; dmin(1); dmin(2) = "; dmin(2)
PRINT #1, "  D0start = "; D0start; D0stop = "; D0stop; D0step = "; D0step
PRINT #1, "  Dpstart = "; Dpstart; Dpstop = "; Dpstop; Dpstep = "; Dpstep
PRINT #1, "  Dcstart = "; Dcstart; Dcstop = "; Dcstop; Dcstep = "; Dcstep
PRINT #1, "  Dsstart = "; Dsstart; Dsstop = "; Dsstop; Dsstep = "; Dsstep
PRINT #1, "  TOTSIMN = "; TOTSIMN
PRINT #1, "  seed2 = "; seed2
drawmaps = 0
INPUT "Draw maps? Then write 1, otherwise 0", drawmaps

IF drawmaps = 0 THEN PRINT " 
IF drawmaps = 0 THEN PRINT " D0   Dp1   Dp2   Dc1   Dc2   Ds   r   EPV"
PRINT #1, " D0   Dp1   Dp2   Dc1   Dc2   Ds   r   EPV"
PRINT #1, " D0   Dp1   Dp2   Dc1   Dc2   Ds   r   EPV"

REM *****************
REM End of SECTION 0.
REM *****************
REM **SECTION A. The initial conditions of relevance to all calculations are determined.**

REM Generation of 100 market prices series for two species with correlation pcorr.

FOR series = 1 TO 100
    pcorr = 0.5
    FOR T = 0 TO 100
        FOR i = 1 TO 2
            epsilon = 0
            FOR ii = 1 TO 12
                epsilon = epsilon + RND
            NEXT ii
            epsilon = (epsilon - 6)
            IF i = 1 THEN randn1 = epsilon
            IF i = 2 THEN randn2 = epsilon
            IF i = 2 THEN randn3 = pcorr * randn1 + (1 - pcorr^2)^.5 * randn2
        NEXT i
        MarketP(T, 1, series) = EP(1) + stdev(1) * randn1
        MarketP(T, 2, series) = EP(2) + stdev(2) * randn3
    NEXT T
NEXT series
PRINT ""
REM Generation of the positions

FOR i = 1 TO 100
    x(i) = 30 * RND
    y(i) = 30 * RND
NEXT i
PRINT ""

REM Generation of the species

FOR i = 1 TO 100
    species(i) = 1
    IF RND > 0.5 THEN species(i) = 2
NEXT i
REM Calculation of the distances

FOR i = 1 TO 100
    FOR j = 1 TO 100
        distx = x(i) - x(j)
        disty = y(i) - y(j)
        dist(i, j) = (distx * distx + disty * disty) ^ 0.5
    NEXT j
NEXT i
REM Generation of initial diameters

FOR i = 1 TO 100
   REM Note that in this version of the code, the initial distributions
   REM of the two species are identical. (Compare next row.)

   IF species(i) > 0.5 THEN GOTO 1010

   1001 REM New diameter suggestion
   dia = 0.1 + RND * 0.4
   freq = 1 - (2 * dia) ^ 2
   test = RND
   IF test > freq THEN GOTO 1001
   d0(i) = dia
   GOTO 1020

1010 REM
1011 REM New diameter suggestion
dia = 0.1 + RND * 0.3
freq = 1 - (2.5 * dia) ^ 2
test = RND
IF test > freq THEN GOTO 1011
d0(i) = dia

1020 REM
NEXT i

REM *****************
REM End of SECTION A.
REM *****************
REM ***********************************************************
REM SECTION B. The control function parameter loops start here.
REM ***********************************************************

EPVmax = 0

RANDOMIZE seed2

FOR SIMN = 1 TO TOTSIMN

    Ds = Dsstart + RND * (Dsstop - Dsstart)
    D0 = D0start + RND * (D0stop - D0start)
    Dp1 = Dpstart + RND * (Dpstop - Dpstart)
    Dp2 = Dpstart + RND * (Dpstop - Dpstart)
    Dc1 = Dcstart + RND * (Dcstop - Dcstart)
    Dc2 = Dcstart + RND * (Dcstop - Dcstart)
REM *******************************************************************
REM SECTION C. A number of loops of stochastic simulations start here.
REM *******************************************************************

FOR series = 1 TO 100
  PresVal(series) = 0
  REM PRINT "Series = "; series
  REM A new simulation is started from year 0
  FOR i = 1 TO 100
    d(i) = d0(i)
  NEXT i
FOR T = 0 TO 60

    MarketP1 = MarketP(T, 1, series)
    MarketP2 = MarketP(T, 2, series)
    year = T * 5
    discf = EXP(-year * r)

    IF drawmaps = 0 THEN GOTO 600
    CLS
    PRINT " Series = "; series; " Period = "; T; " Year = "; year; " P1 = "; MarketP1; " P2 = "; MarketP2
    PRINT " Map before growth and before harvest: "

FOR i = 1 TO 100
    FOR interior = 0.05 TO 1 STEP 0.05
        CIRCLE (INT(20 * x(i) + 100), INT(20 * y(i) + 100)), INT(100 * d(i) * interior / 5), species(i)
    NEXT interior
NEXT i
LINE (100, 100)-(700, 100), 3
LINE (100, 100)-(100, 700), 3
LINE (700, 100)-(700, 700), 3
LINE (100, 700)-(700, 700), 3
SLEEP
600 REM
REM growth occurs
FOR speci = 1 TO 2
    Volperha(speci) = 0
NEXT speci
FOR i = 1 TO 100
    Comp(i) = 0
    FOR j = 1 TO 100
        IF dist(i, j) < 5 THEN Comp(i) = Comp(i) + PI / 4 * d(j) * d(j)
    NEXT j
    CompBA(i) = 127.32 * Comp(i)
    Compadjust = (1 - 0.02 * CompBA(i))
    IF Compadjust < 0 THEN Compadjust = 0
    d(i) = d(i) + 0.1 * (1 - 0.5 * d(i)) * Compadjust
    speci = species(i)
    ba(i) = PI / 4 * d(i) * d(i)
    height(i) = kheight(speci) * d(i)
    vol(i) = 0.5 * ba(i) * height(i)
    Volperha(speci) = Volperha(speci) + vol(i) / 0.09
NEXT i
IF drawmaps = 0 THEN GOTO 601
CLS
PRINT " Series = "; series; " Period = "; T; " Year = "; year; " P1 = "; MarketP1; " P2 = "; MarketP2
PRINT " Map after growth and before harvest: "

FOR i = 1 TO 100
    FOR interior = 0.05 TO 1 STEP 0.05
        CIRCLE (INT(20 * x(i) + 100), INT(20 * y(i) + 100)), INT(100 * d(i) * interior / 5), species(i)
    NEXT interior
NEXT i
LINE (100, 100)-(700, 100), 3
LINE (100, 100)-(100, 700), 3
LINE (700, 100)-(700, 700), 3
LINE (100, 700)-(700, 700), 3
SLEEP
601 REM
REM Harvests of individual trees may or may not occur
Totvolperha = Volperha(1) + Volperha(2)

FOR i = 1 TO 100
    speci = species(i)
    Price = MarketP(T, speci, series)
    meanprice = EP(speci)
    Dp = Dp1
    IF speci > 1.5 THEN Dp = Dp2
    Dc = Dc1
    IF speci > 1.5 THEN Dc = Dc2

REM The harvest decision of tree i is set to zero. Then the limit diameter is calculated.
REM In case the tree diameter exceeds the limit diameter, the harvest decision is set to one.
harv(i) = 0

Dlim = D0 + Dp * (Price - meanprice) / stdev(speci) + Ds * (speci - 1) + Dc * CompBA(i)

IF d(i) > Dlim THEN harv(i) = 1

IF d(i) < dmin(speci) THEN harv(i) = 0
REM The revenues and costs of the harvested trees are calculated.

\[
\text{psizecorr} = -0.25 + 2.5 \times d(i)
\]

IF psizecorr > 1 THEN psizecorr = 1

IF psizecorr < 0 THEN psizecorr = 0

\[
\text{revenue}(i) = \text{Price} \times \text{psizecorr} \times \text{vol}(i) \times \text{harv}(i)
\]

\[
\text{costperm3} = 125 - 250 \times d(i)
\]

IF costperm3 < 25 THEN costperm3 = 25

\[
\text{cost}(i) = \text{costperm3} \times \text{vol}(i) \times \text{harv}(i)
\]

IF revenue(i) < cost(i) THEN harv(i) = 0

NEXT i
REM The discounted net revenue of all harvested trees is added to the present value of the series.
netrev = 0
FOR i = 1 TO 100
    netrev = netrev + (revenue(i) - cost(i)) * harv(i)
NEXT i
PresVal(series) = PresVal(series) + discf * netrev
IF drawmaps = 0 THEN GOTO 602
  CLS
  PRINT " Series = "; series; " Period = "; T; " Year = "; year; " P1 = "; MarketP1; " P2 = "; MarketP2
  PRINT " Map of harvested trees: "

  FOR i = 1 TO 100
    IF harv(i) = 0 THEN GOTO 7070
    FOR interior = 0.05 TO 1 STEP 0.05
      CIRCLE (INT(20 * x(i) + 100), INT(20 * y(i) + 100)), INT(100 * d(i) * interior / 5), species(i)
    NEXT interior
  NEXT i
  7070 REM
  NEXT i
  REM
  LINE (100, 100)-(700, 100), 3
  LINE (100, 100)-(100, 700), 3
  LINE (700, 100)-(700, 700), 3
  LINE (100, 700)-(700, 700), 3
  SLEEP
  602 REM
FOR i = 1 TO 100
    IF harv(i) = 1 THEN d(i) = 0.05
NEXT i

IF drawmaps = 0 THEN GOTO 603
CLS
PRINT " Series = "; series; " Period = "; T; " Year = "; year; " P1 = "; MarketP1; " P2 = "; MarketP2
PRINT " Map after harvest: "

FOR i = 1 TO 100
    FOR interior = 0.05 TO 1 STEP 0.05
        CIRCLE (INT(20 * x(i) + 100), INT(20 * y(i) + 100)), INT(100 * d(i) * interior / 5), species(i)
    NEXT interior
NEXT i
LINE (100, 100)-(700, 100), 3
LINE (100, 100)-(100, 700), 3
LINE (700, 100)-(700, 700), 3
LINE (100, 700)-(700, 700), 3
SLEEP
603 REM
NEXT T
NEXT series

REM *****************
REM End of SECTION C.
REM *****************
SECTION D. Results for each control function parameter combination are calculated and printed.

```
EPV = 0
FOR series = 1 TO 100
   EPV = EPV + PresVal(series) / 100
NEXT series
REM Calculation of EPV per ha
EPV = EPV / 0.09
IF EPV < EPVmax THEN GOTO 900
D0opt = D0
Dp1opt = Dp1
Dp2opt = Dp2
Dc1opt = Dc1
Dc2opt = Dc2
Dsopt = Ds
EPVmax = EPV
900 REM
```
kD0 = D0 * 1000
kDp1 = Dp1 * 1000
kDp2 = Dp2 * 1000
kDc1 = Dc1 * 10000
kDc2 = Dc2 * 10000
kDs = Ds * 1000
kr = r * 1000
kEPV = EPV * 1000

PRINT USING "##################"; kD0; kDp1; kDp2; kDc1; kDc2; kDs; kr; kEPV
PRINT #1, USING "##################"; kD0; kDp1; kDp2; kDc1; kDc2; kDs; kr; kEPV

REM *****************
REM End of SECTION D.
REM *****************

NEXT SIMN
kD0opt = D0opt * 1000
kDp1opt = Dp1opt * 1000
kDp2opt = Dp2opt * 1000
kDc1opt = Dc1opt * 10000
kDc2opt = Dc2opt * 10000
kDsopt = Dsopt * 1000
kEPVmax = EPVmax * 1000

PRINT ""
PRINT " r = "; r
PRINT " The Optimal Solution is (Most values times 1000. Dc1 and Dc2: Values times 10000): "
PRINT " D0   Dp1   Dp2   Dc1   Dc2   Ds   EPV"
PRINT USING "##############"; kD0opt; kDp1opt; kDp2opt; kDc1opt; kDc2opt; kDs; kEPVmax
PRINT "----------------------------------------------------------------------------------------------------------------------------------"
PRINT #1, ""
PRINT #1, " r = "; r
PRINT #1, " The Optimal Solution is (Most values times 1000. Dc1 and Dc2: Values times 10000): "
PRINT #1, "            D0           Dp1           Dp2           Dc1           Dc2            Ds           EPV"
PRINT #1, USING "##############"; kD0opt; kDp1opt; kDp2opt; kDc1opt; kDc2opt; kDsopt; kEPVmax
PRINT #1, "-----------------------------------------------------------------------------"

REM *****************
REM End of SECTION B.
REM *****************

CLOSE #1
CLOSE #1
BEEP
END
EXAMPLE
Input file
(CASE 0)

AdMultRndIn.txt
EXAMPLE AdMultRndOut.txt

Output file

(CASE 0)

Program AdMultRnd_EQDIST by Peter Lohmander 2019:

Parameters from external file:

\[ r = 0.03 \quad \text{pcorr} = 0.5 \quad \text{EP}(1) = 50 \quad \text{EP}(2) = 50 \]
\[ \text{stdev}(1) = 15 \quad \text{stdev}(2) = 15 \quad \text{kheight}(1) = 40 \quad \text{kheight}(2) = 40 \]
\[ \text{dmin}(1) = 0.2 \quad \text{dmin}(2) = 0.2 \]
\[ \text{D0start} = 0.7 \quad \text{D0stop} = 0.7 \quad \text{D0step} = 0 \]
\[ \text{Dpstart} = -0.3 \quad \text{Dpstop} = 0 \quad \text{Dpstep} = 0.02 \]
\[ \text{Dcstart} = -0.01 \quad \text{Dcstop} = 0 \quad \text{Dcstep} = 0 \]
\[ \text{Dsstart} = 0 \quad \text{Dsstop} = 0 \quad \text{Dsstep} = 0.02 \]
\[ \text{TOTSIMN} = 300 \]
\[ \text{seed2} = 1 \]
Program AdMultRnd_EQDIST by Peter Lohmander 2019:

Parameters from external file:

\[ r = 0.03 \quad \text{pcorr} = 0.5 \quad \text{EP}(1) = 50 \quad \text{EP}(2) = 50 \]
\[ \text{stdev}(1) = 15 \quad \text{stdev}(2) = 15 \quad \text{kheight}(1) = 40 \quad \text{kheight}(2) = 40 \]
\[ \text{dmin}(1) = 0.2 \quad \text{dmin}(2) = 0.2 \]
\[ \text{D0start} = 0.7 \quad \text{D0stop} = 0.7 \quad \text{D0step} = 0 \]
\[ \text{Dpstart} = -0.3 \quad \text{Dpstop} = 0 \quad \text{Dpstep} = 0.02 \]
\[ \text{Dcstart} = -0.01 \quad \text{Dcstop} = 0 \quad \text{Dcstep} = 0 \]
\[ \text{Dsstart} = 0 \quad \text{Dsstop} = 0 \quad \text{Dsstep} = 0.02 \]
\[ \text{TOTSIMN} = 300 \]
\[ \text{seed2} = 1 \]

<table>
<thead>
<tr>
<th>D0</th>
<th>Dp1</th>
<th>Dp2</th>
<th>Dc1</th>
<th>Dc2</th>
<th>Ds</th>
<th>r</th>
<th>EPV</th>
</tr>
</thead>
<tbody>
<tr>
<td>700</td>
<td>-149</td>
<td>-113</td>
<td>-90</td>
<td>-40</td>
<td>0</td>
<td>30</td>
<td>5546176</td>
</tr>
<tr>
<td>700</td>
<td>-54</td>
<td>-133</td>
<td>-95</td>
<td>-59</td>
<td>0</td>
<td>30</td>
<td>5419922</td>
</tr>
<tr>
<td>700</td>
<td>-1</td>
<td>-49</td>
<td>-87</td>
<td>-74</td>
<td>0</td>
<td>30</td>
<td>5177052</td>
</tr>
<tr>
<td>700</td>
<td>-74</td>
<td>-139</td>
<td>-49</td>
<td>-45</td>
<td>0</td>
<td>30</td>
<td>5824453</td>
</tr>
<tr>
<td>700</td>
<td>-30</td>
<td>-5</td>
<td>-12</td>
<td>-81</td>
<td>0</td>
<td>30</td>
<td>1807685</td>
</tr>
</tbody>
</table>

Many more rows follow...
Optimization of Multi Species Continuous Cover Forest Management with Stochastic Prices via Determination of the Adaptive Harvest Control Function

Peter Lohmander
Prof.Dr. Optimal Solutions in cooperation with Linnaeus University
Peter@Lohmander.com, www.Lohmander.com

18th Symposium on Systems Analysis in Forest Resources, SSAFR 2019
March 3 - 7, 2019 Puerto Varas, Chile
SSAFR 2019 Song: Methods in Math
Lyrics: Peter Lohmander
Melody: Dire Straits, Brothers in Arms

Inconsistent solutions
We know them by now,
But our background is science,
Where math is key,
Some day you’ll return to
Derivations so pure,
You’ll no longer risk
To be guessing and wrong

Through these fields of confusion,
Incompleteness and lies,
I’ve witnessed your suffering,
As the errors grow wild,
And though they did hurt us so bad
With solutions quite wrong,
You did not desert me
My methods in math

There’s so many different plans
So many different trends
And there is just one solution,
Which is the optimal one

Now our science goes quite well
Our solutions ride high
Now we all know very well
Every lie has to die
But it’s written in the starlight
And every line in your palm
Only fools can make war
On our methods in math

Yoni Schlesinger | Brothers in Arms (Dire Straits)
solo fingerstyle | B&G Little Sister
https://www.youtube.com/watch?v=7QeQhMr33E

SSAFR 2019 in Puerto Varas
March 3-7

Principle of Chile:
\[ Z(x, y) = \frac{x^2 - y^2 + xy}{3} + \sin(xy) - 1 \]
\[ \frac{-\pi}{2} \leq x \leq \frac{\pi}{2}; \frac{-\pi}{2} \leq y \leq \frac{\pi}{2} \]
SSAFR song
by Peter Lohmander 2017-08-29
(Melody: Sancta Lucia)

Systems analysis,
in forest resources,
that's what our planet needs,
let's gather our forces,
to optimize management,
consider logistics,
do not forget the many animals,
climate and fires.
Faustmann song
by Peter Lohmander 1984-12-13
(Melody: Sancta Lucia)

Economic society,
deep in the forest,
we are all gathered here,
in the honor of Faustmann,
the whole world we represent,
and the gold that is green,
which we then transform to money,
in the optimal way.